Dynamic Nonlinearity in Piezoelectric Flexural Ultrasonic Transducers

Andrew Feeney, Lei Kang, George Rowlands, Leiqing Zhou, and Steve Dixon

Abstract—**The flexural ultrasonic transducer is a unimorph device which typically comprises a piezoelectric ceramic bonded to a metallic membrane. It is widely applied in industrial applications for metrology and proximity sensing. However, the electromechanical and dynamic characteristics of this class of transducer have only recently been reported, and the influence of different excitation levels on dynamic nonlinearity remains unclear. Dynamic nonlinearity in high-power piezoelectric ultrasonic transducers is familiar, where the performance or dynamic stability of the transducer can significantly reduce under high amplitudes of excitation. Nonlinearity can manifest as measurable phenomena such as resonance frequency drift, influenced by thermomechanical phenomena or structural constraints. There is relatively little reported science of the dynamic nonlinearity in the vibration response of flexural ultrasonic transducers. This study examines the vibration responses of four flexural ultrasonic transducers, showing the existence of dynamic nonlinearity for increases in excitation voltage. An analytical solution of the governing equations of motion for the flexural ultrasonic transducer is presented which complements the experimental investigation, and suggests a close relationship between material properties and nonlinearity. This research demonstrates a detailed dynamic characterization of the flexural ultrasonic transducer, showing the potential for the optimization of dynamic performance in industrial measurement applications.**

Index Terms—**Flexural ultrasonic transducer, dynamic nonlinearity, air-coupled ultrasound, analytical representation.**

I. Introduction

NONLINEARITY describes the response of a system which is disproportionate to the excitation. A system output can exhibit unpredictable or chaotic characteristics, often with high sensitivity to changes in the input condition. Scientifically, the nonlinearity of a system can refer to a range of different phenomena and is applicable to many fields of science and engineering [1]. These phenomena include quantifiable changes in thermal properties, electrical signals, material properties, and mechanical performance, all occurring in response to a variety of triggers. This research focuses on a specific class of nonlinear behavior called dynamic nonlinearity. This can be measured in the vibration response of the flexural ultrasonic transducer at different excitation voltage levels. Dynamic nonlinearity can be generated through geometric-based influences, the type of actuation mechanism, the boundary conditions, and from nonlinear material properties such as damping [2]. The flexural ultrasonic transducer can be operated at different resonant modes of vibration and excitation voltages, but very little is known about their dynamic performance for different drive conditions. There has been only limited study of the dynamic performance of flexural ultrasonic transducers, and continued investigation is essential to ensure they are operated effectively in their wider application. The objectives of this research are to examine the dynamic performance of different piezoelectric flexural ultrasonic transducers, to identify behaviors which can be represented through expressions derived using classical nonlinear dynamic theory, and to investigate the physical characteristics of the transducers, thereby enabling performance optimization.

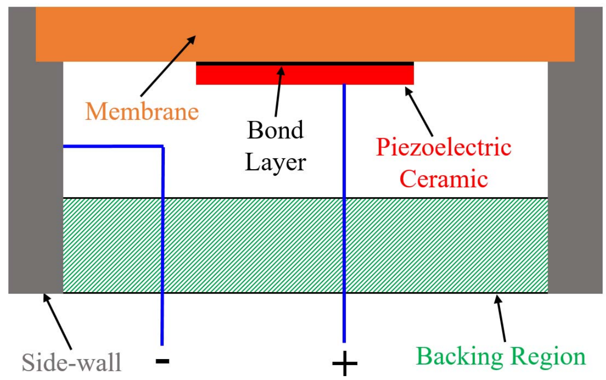

Fig. 1. The assembly of a flexural ultrasonic transducer (cutaway side-view). Transducer dimensions are provided in Table I.

The flexural ultrasonic transducer relies on the resonant vibration of a circular, edge-clamped membrane mechanically coupled to a piezoceramic disc, where the deformation of the membrane generates a desired ultrasonic wave [3]. A typical assembly of a flexural ultrasonic transducer is shown in Fig. 1. The transducer cap comprises a membrane and a side-wall, where the active driving element is attached to the underside of the cap membrane. This cap can either be composed of a single specimen or fabricated in separate parts, as highlighted by the orange membrane component in Fig. 1. The membrane can therefore be composed of a different material to the side-wall. The driving element can be a coil which allows the transducer to be operated via an electromagnetic mechanism. This is an ultrasonic device commonly called an electrodynamic flexural transducer (EDFT) [4]. The driving element utilized in this paper is the piezoelectric ceramic disc.

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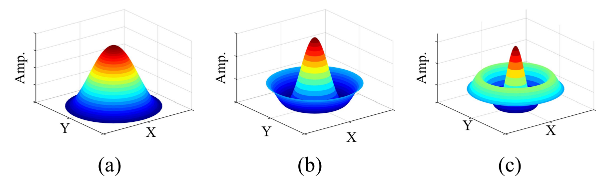

Fig. 2. The first three axisymmetric modes of vibration exhibited by a flexural ultrasonic transducer, showing (a) the (0,0) mode, (b) the (1,0) mode, and (c) the (2,0) mode, all mathematically simulated.

Piezoelectric-based flexural ultrasonic transducers are more widely used than EDFTs due to their superior efficiency and lower cost. A backing region is highlighted in Fig. 1 and can be fabricated from a range of materials including silicone [5]. A silicone backing layer can be used to seal the transducer, although air backing with a rear support composed from acrylonitrile butadiene styrene (ABS) to assist with stability of the transducer during measurement is used in the fabrication of the custom transducers in this study.

There are two key strategies for controlling the resonance of a flexural ultrasonic transducer [3], [5], [6]. The first is through modification of the physical properties of the transducer. For example, modal frequencies can be increased through a reduction in membrane diameter or an increase in membrane thickness. However, there is a practical limit to the permissible dimensional change in a flexural ultrasonic transducer [7]. As an alternative strategy, a higher-order mode can instead be exploited. The axisymmetric modes of a flexural ultrasonic transducer are displayed in Fig. 2 from mathematical simulation and are given the $(n, 0)$ nomenclature in this study [3], [7]. A higher frequency can be generated by exploiting a higher-order mode, without requiring alteration to the geometry of the transducer membrane or its material type. However, operating a flexural ultrasonic transducer at a higher-order mode of vibration can cause a significant reduction in output amplitude and efficiency for a given excitation voltage, and a narrower ultrasound beam profile [7].

The flexural ultrasonic transducer can be considered analytically through a mechanical equivalence, the edge-clamped plate, which can be assumed to represent the transducer membrane [3]. The fundamental axisymmetric vibration modes shown in Fig. 2 are all simulated using this principle, and thus far has been shown to be a reliable method to represent the vibration response of the flexural ultrasonic transducer at resonance. Resonant modes can hence be calculated as a function of the stiffness and geometrical properties of the membrane. There has been significant research dedicated to the mechanics of deformation in plates and the development of plate theory for vibration analysis, particularly for those relating to the study of functionally graded material (FGM) sandwich plates [8]–[12]. For example, Bennoun *et al.* demonstrated a novel plate theory which accounts for shear and stretching deformation without the need for correction [8], and Besseghier *et al.* showed the vibration analysis of functionally graded plates using Hamilton's principle to generate the equations of motion of the plate [9]. Belabed *et al.* considered the vibration response of FGM plates through Hamilton's principle, but defining transverse displacement in the plate as components comprising bending, shear, and stretching, with in-plane displacement divided into components of extension, shear, and bending [10]. A foundation for mathematically representing the dynamics of the flexural ultrasonic transducer can be established by first considering the vibration characteristics of the mobile mass. It has been established that the membrane of the flexural ultrasonic transducer can be approximated as an edge-clamped plate. Based on this equivalence, the theory of plate vibrations can be used to generate an expression to determine the modal frequencies [3], [13], [14]. The differential equation for a thin plate, which is analogous to the transducer membrane, is shown by (1), where D is the plate rigidity, x denotes displacement, r is the position vector and ρ is the volume density.

$$D\nabla^4 x(r,t) + \rho \frac{\partial^2 x(r,t)}{\partial t^2} = 0 \tag{1}$$

The plate rigidity is shown by (2) as a function of Young's modulus E, plate thickness h, and the Poisson's ratio ν of the plate material [14].

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

Given that the plate can be assumed to be circular in the case of the flexural ultrasonic transducer, the expression shown by (3) is applicable to show the relationship between the resonance frequency f and the physical properties of the transducer membrane [3], where z is the radius of the plate. It is assumed that the initial orientation of the plate is flat and that the deflections of the plate are significantly smaller than the plate thickness. The φ parameter is a mode constant possessing nodes relating to the mode of vibration. This can then be used as an approximation for the resonance frequency of the flexural ultrasonic transducer in its entirety, which is useful for the design and optimization process.

$$f = \frac{1}{2\pi}\left(\frac{\varphi}{z}\right)^2 \sqrt{\frac{D}{\rho}} \tag{3}$$

The expressions provided in (1) through to (3) demonstrate the dependence of modal vibration on the stiffness properties of the mobile mass, which can be approximated as the membrane. It is important to clarify that the relationships shown in (1) through to (3) specifically relate to the mechanism by which it can be assumed the flexural ultrasonic transducer operates, via the vibration of an edge-clamped thin plate.

Dynamic nonlinearity in ultrasonic transducers is essential to consider because there are significant consequences on performance efficiency. Documented phenomena include reductions in transducer amplitude, instability, and a drift of resonance frequency [15]–[17]. There are several scientific studies into the nonlinearity in the vibration response of ultrasonic devices, including high-power transducers [15]–[18]. The dynamic nonlinearity associated with the vibration of discs and plates, including piezoelectric elements, has also received significant attention in the scientific literature [19]–[22], but there is little detailed analysis of the nonlinearity associated with flexural ultrasonic transducers, those which generate ultrasonic waves through bending motion.

Dynamic nonlinearity in the vibration response of the flexural ultrasonic transducer operating in a higher-order mode is vital to complement recent scientific advances [7]. This research has significant implications for multiple frequency applications where different modes of vibration can be exploited.

In power ultrasonic transducers operating at sufficiently high amplitude, dynamic nonlinearity can be exhibited as Duffing-type behavior, where characteristics such as the jump phenomenon and amplitude bifurcation are observable [15]–[18]. The thermomechanical influences on the physical properties of piezoelectric ceramics in response to different excitation voltage levels have been attributed to causing such dynamic nonlinearity in power ultrasonic devices [23]. Umeda *et al.* developed a burst excitation method for reducing thermal influences within the piezoelectric ceramic to mitigate dynamic nonlinearity [23]. Duffing-type behavior was not identified in the previous study of dynamic nonlinearity in the vibration response of the flexural ultrasonic transducer [24]. Flexural ultrasonic transducers were operated with burst sinusoidal signals in the (0,0) mode of vibration at increasing excitation voltage levels, where the displacement amplitude was measured as a function of drive frequency. A reduction of resonance frequency was detected as the excitation voltage was increased. The steady-state and natural resonant decay regions of the response were treated separately, shown to both exhibit nonlinear behavior. However, dynamic characteristics attributable to Duffing-type behavior was reported in the study of a device similar to the flexural ultrasonic transducer, the acoustic flexural-mode disc transducer [25]. The construction of this transducer is different to the flexural ultrasonic transducer. The disc transducer comprises a piezoelectric ceramic ring bonded to the top surface of a brass membrane with the reverse side of the membrane attached to a support structure. The disc transducers are not ultrasonic sensors but acoustic, operated around 4 to 5 kHz, and they comprise significantly thinner membranes than those of the flexural ultrasonic transducers used in this study by factors of approximately 10-30. The nonlinear characteristics of these transducers were reported to include resonance frequency drift, hysteresis, amplitude saturation, instability in the amplitude output, non-periodicity of the vibration response resulting from mode coupling, and super-harmonic response [25]. At sub-ultrasonic frequencies, nonlinearity was measured in the response of flexural disc transducers and attributed to structural nonlinearities, although it was reported that additional sources of nonlinearity were possible but undetermined [25].

Previous research has also reported on the analysis of nonlinearity in piezoelectric MEMS devices, for example AlN contour mode resonators (CMRs), which related the nonlinear response of the resonator to its geometrical dimensions and material properties [26]. In the case of CMRs, it was concluded that the nonlinearities were principally thermally-induced. A key development of this research was a robust method of attributing nonlinear behavior of a resonator to specific physical sources. A study by Kaajakari *et al.* demonstrated nonlinear limits for silicon microresonators, showing the existence of capacitive nonlinearity through electrostatic coupling and mechanical nonlinearity arising from geometrical and material phenomena [27]. Specifically with respect to material nonlinearity, it has been demonstrated that Young's modulus can exhibit a nonlinear relationship with applied strain [26], [27]. The nonlinearity arising from the electrostatic condition in devices such as the capacitive micromachined ultrasonic transducer (CMUT) has been shown to be significant compared to that caused through geometric mechanisms [28]. Furthermore, it has been suggested that softening nonlinearities in the dynamic response of a piezoelectric cantilever structure can arise from piezoelectric materials, relating to their associated elastic nonlinear and coupling terms [29].

In this investigation, previous experimental strategies are extended by measuring the dynamic nonlinearity in the vibration response of different designs of flexural ultrasonic transducer, comprising both commercial and custom-fabricated devices. As the excitation voltage is raised, the displacement amplitude of the transducer increases, with the potential to generate dynamic nonlinearity. Classical nonlinear dynamic theory is utilized to show, for the first time, the close correlation between a nonlinear mechanical analog and the experimental data obtained from dynamic characterization measurements of the flexural ultrasonic transducer. An analytical solution to the governing equations of motion of the flexural ultrasonic transducer is presented which captures asymmetry of the response spectra in addition to changes in resonance frequency. Prior research has introduced a third-order beta parameter (β), which is independent of the excitation voltage, used to quantify nonlinearity, and one reported technique used to determine the acoustic nonlinearity parameter is ultrasound spectroscopy [30]. This study introduces a nonlinear equation of motion to describe the phenomena measured through experiment, and the experiments are tailored to investigate potential dynamic nonlinear behavior for different designs of flexural ultrasonic transducer. Although specific designs of flexural ultrasonic transducer are analyzed in this study, the adopted experimental methods, the outcomes, and the mathematical representations of the physical phenomena all exhibit the potential to be applicable to a range of acoustic or ultrasonic transduction systems, for example stack or flextensional transducers. It is anticipated that this investigation will yield optimization strategies for the design and operation of flexural ultrasonic transducers applicable to industrial ultrasonics.

II. Experimental Process

A. Transducer Fabrication and Characterization

Prior research has demonstrated the distinct differences present in nominally identical flexural ultrasonic transducers which are commercially available [24]. This is primarily due to the fabrication process, where consistency is very difficult to achieve with this type of device, as even sub-millimeter differences in membrane geometry can alter the resonance frequencies of the operating mode in the order of kHz. The direct comparison of physical phenomena such as the resonance frequency shift of different transducers, even those which are nominally identical, is therefore not scientifically robust. However, the dynamic characterization process used in this study is a reliable method of demonstrating the vibration response of different transducer configurations for different excitation levels over relatively wide ranges of drive frequency.

In this investigation, two commercial aluminum flexural ultrasonic transducers (Pro-Wave Electronics Corporation) and two custom transducers are analyzed. The membrane diameter and thickness, the membrane material, and the method of transducer assembly are all considered. It should be noted at this stage that it is possible for the dynamic performance of the flexural ultrasonic transducer to be optimized using finite element methods. For example, the peak displacement amplitude of a flexural ultrasonic transducer can be computed for a given membrane material and geometry. The focus of this research is solely on nonlinear dynamics, where the custom flexural ultrasonic transducers have been arbitrarily designed. The commercial devices differ principally in terms of membrane size, defined as the small and large commercial flexural (S/LCF) ultrasonic transducers respectively. The SCF and LCF both possess center operating frequencies of 40.0 ± 1.0 kHz. The vibration responses of the SCF and LCF in their (1,0) modes are measured. Two custom flexural ultrasonic transducers are fabricated, where the brass membrane flexural ultrasonic transducer (FUT_{Br}) is manufactured with a brass membrane and the titanium membrane flexural ultrasonic transducer (FUT_{Ti}) is composed of a single titanium cap. FUT_{Br} is operated in its (0,0) mode of vibration and FUT_{Ti} is designed to operate in its (1,0) mode. The custom transducers are designed using PZFlex® finite element analysis (FEA) software (PZFlex, LLC). Aluminum and titanium are common membrane materials for flexural ultrasonic transducers [3], [5], with specific moduli both in the order of 25 GPa/g.cm^{-3}. Brass possesses a specific modulus around 13 GPa/g.cm^{-3} but is relatively compliant. It is selected as the membrane material of one transducer, being sufficiently different in terms of specific modulus to that of aluminum or titanium.

The dimensions of a flexural ultrasonic transducer membrane are critical to the vibration response. Through fabrication of a transducer cap as a single component by conventional machining, inhomogeneities or warping of the material can be introduced, especially at the interface between the membrane and the side-wall. Furthermore, a filleted profile at the interface between the membrane and the side-wall can be introduced, affecting the mode shapes and resonance frequencies of the transducer. To address this, the cap of FUT_{Br} is composed of two separate parts, one membrane and one side-wall, which are subsequently joined together during the manufacturing process for optimal transducer fabrication quality. The transducer is fabricated with a titanium side-wall. The FEA model of FUT_{Br} is configured as two-dimensional and axisymmetric, with a membrane thickness of 1 mm and a piezoelectric ceramic disc (PIC 151) with a diameter of 6.90 mm and thickness of 0.50 mm. Epoxy resin (Araldite 2014-1) is used to bond the piezoelectric ceramic disc to the membrane and for attachment of the membrane to the side-wall. The geometrical profiles of these bond layers are arbitrarily specified in finite element analysis. FUT_{Ti} is assembled using a cap fabricated from a single specimen of titanium with a membrane thickness of 0.25 mm. Unlike FUT_{Br}, its side-wall and membrane are not fabricated separately, and therefore

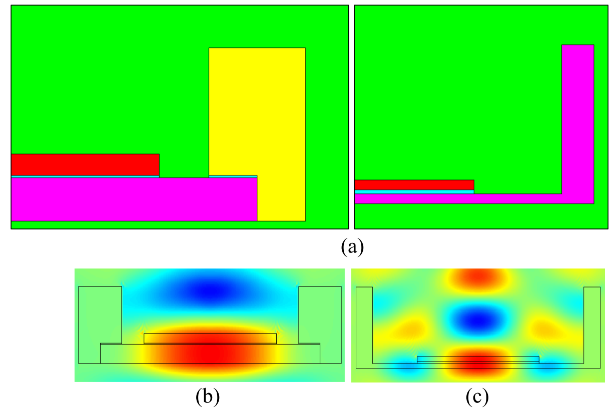

Fig. 3. (a) The PZFlex® 2-D finite element models of FUT_{Br} (left) and FUT_{Ti} (right), with symmetry around the y-axis, (b) the (0,0) mode for FUT_{Br} at 57 kHz, and (c) the (1,0) mode for FUT_{Ti} at 102 kHz. The transducer schematics are not to scale with one another, but are provided to illustrate composition.

the transducer does not require an epoxy resin bond layer deposited around the circumference of the side-wall, as shown by the cyan region in Fig. 3(a). FUT_{Ti} comprises a piezoelectric ceramic (PIC 255) with a diameter of 6 mm and thickness of 0.25 mm, designed to operate in its (1,0) mode in the order of 100 kHz. Key transducer information is shown in Table I, with physical dimensions in millimeters. With reference to the assembly schematic shown in Fig. 1, the total cap diameter includes both the side-wall thickness and membrane diameter. Material properties used in the FEA models are shown in Table II. The FEA models of FUT_{Br} and FUT_{Ti} and the simulation outputs for the (0,0) and (1,0) modes respectively are shown in Fig. 3. The mode shapes shown in Fig. 3(b) and 3(c) can be directly compared with those shown in Fig. 2(a) and 2(b) respectively.

The assembly of FUT_{Br} is undertaken using a bespoke rig. To ensure a perpendicular interface between the membrane and the side-wall of the cap and to centrally align the piezoelectric ceramic on the membrane, the brass membrane is machined from flat sheet. The side-wall is machined from a titanium rod with a notch width of 1.12 mm to accommodate the membrane. The bonding of the piezoelectric ceramic and the side-wall to the membrane is also performed using the bespoke rig. Pressure is applied with a torque wrench through bolts circumferentially positioned on the rig. This ensures the applied pressure to the membrane is as uniform and consistent as possible. The fabrication of FUT_{Ti} is different, where a magnet is used to apply pressure to bond a piezoelectric ceramic disc to the inside of the FUT_{Ti} cap. Once the epoxy resin cures, electrode wires are soldered to each piezoelectric ceramic disc before a protective backing is attached. The flexural ultrasonic transducers are shown in Fig. 4 with a geometric scale.

B. Measurement of Dynamic Nonlinearity

The modes of vibration of each flexural ultrasonic transducer are first measured and verified using laser Doppler vibrometry. A single-point laser Doppler vibrometer (LDV) is used (Polytec OFV-5000), and enables the definition of a drive

TABLE I
DETAILS OF THE FLEXURAL ULTRASONIC TRANSDUCERS

Parameter	SCF	LCF	FUT_{Br}	FUT_{Ti}
Operating Mode	(1,0)	(1,0)	(0,0)	(1,0)
Membrane Material	Aluminium	Aluminium	Brass	Titanium
Side-wall Material	Aluminium	Aluminium	Titanium	Titanium
Cap Height (mm)	12.00	12.00 ± 0.20	4.00	4.00
Cap Diameter (mm)	17.50	25.00	13.70	12.00

TABLE II
MATERIAL PROPERTIES FOR FINITE ELEMENT ANALYSIS

Property	Araldite	Brass	Titanium
Density (kg/m^3)	1146	8292	4480
Longitudinal Wave Velocity (m/s)	2658	4321	6100
Shear Wave Velocity (m/s)	1237	2103	3100
Bulk Damping	4	1.48	0.3
Shear Damping	12.59	8.51	1.2

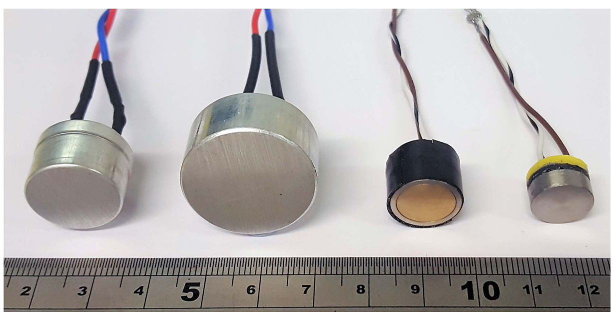

Fig. 4. The flexural ultrasonic transducers used in this study, from left to right: the SCF, LCF, FUT_{Br}, and FUT_{Ti}. The scale is defined in centimeters.

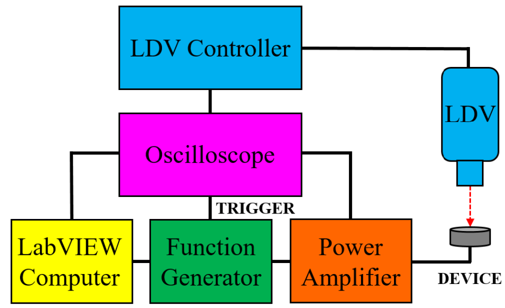

Fig. 5. The experimental setup for dynamic nonlinearity characterization.

frequency range for the nonlinearity characterization of each transducer, which is undertaken by measuring the amplitude as a function of drive frequency around resonance. Once the resonance frequency of the transducer is determined for a specific excitation voltage, a drive frequency range can be defined for which amplitude data can be measured. There are several methods available to characterize the amplitude response of flexural ultrasonic transducers [5], including through an acoustic microphone, a flexural ultrasonic transducer operating in receive mode, and optically through laser Doppler vibrometry. In this research, the LDV is used to measure the mode shapes because it performs with sufficiently high precision over the frequency ranges of interest and because no compensation of air properties or beam characteristics needs to be undertaken, as would be required using an acoustic microphone. The LDV emits a single point laser beam which measures the vibration velocity of a target surface. The laser beam is directed perpendicular to the center of the vibrating surface of the metal membrane of each transducer where the amplitude response for each drive frequency and excitation voltage is collected via a custom-designed LabVIEW virtual instrument. This virtual instrument allows automated data collection which is desirable over relatively wide ranges of drive frequency for small frequency increments, such as those below 100 Hz. A function generator (Tektronix AFG3102C) and an oscilloscope (Agilent DSO-X 3014A, Keysight Technologies) are used in the experimental process for the measurement of dynamic nonlinearity, the schematic for which is shown in Fig. 5.

Each transducer is excited with a continuous-wave sinusoidal signal at increasing levels of excitation voltage. The voltage range and increment both require careful consideration, based on the devices under study. The maximum driving voltage, based on manufacturer information, is 15 V_{RMS} (42.43 $V_{P\text{-}P}$) for the SCF in continuous drive, and 100 $V_{P\text{-}P}$ for a 20-burst drive with 25 ms repetition rate, or 20 V_{RMS} (56.58 $V_{P\text{-}P}$) in continuous drive, for the LCF. The study into disc transducers conducted by Roche *et al.* utilized a maximum driving voltage of 40 V [25]. The excitation voltage range is nominally 20 $V_{P\text{-}P}$ to 40 $V_{P\text{-}P}$, in increments of 5 $V_{P\text{-}P}$. The flexural ultrasonic transducers are operated at higher excitation voltage levels than in prior research [24]. At each voltage level, the transducer is excited around resonance where the drive frequency is increased in 50 Hz steps and the peak-to-peak voltage is recorded for each drive frequency. The continuous-wave sinusoid is applied rather than a burst sinusoid which has been previously reported [5], [6], [31]. The output voltage is then converted to displacement by accounting for the measurement sensitivity of the LDV system. The resonance frequency shift as a function of excitation voltage is then determined for each transducer. An analytical solution of the equations of motion is then presented to complement the experimental results.

III. EXPERIMENTAL RESULTS

The modes of vibration for each transducer around their respective operating frequencies measured using the single-point LDV are shown in Fig. 6, where the amplitude of

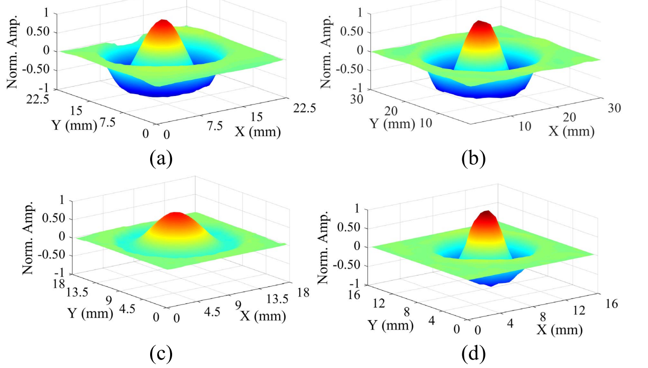

Fig. 6. The axisymmetric modes of vibration measured using laser Doppler vibrometry, showing (a) the (1,0) mode of the SCF, (b) the (1,0) mode of the LCF, (c) the (0,0) mode of FUT_{Br}, and (d) the (1,0) mode of FUT_{Ti}.

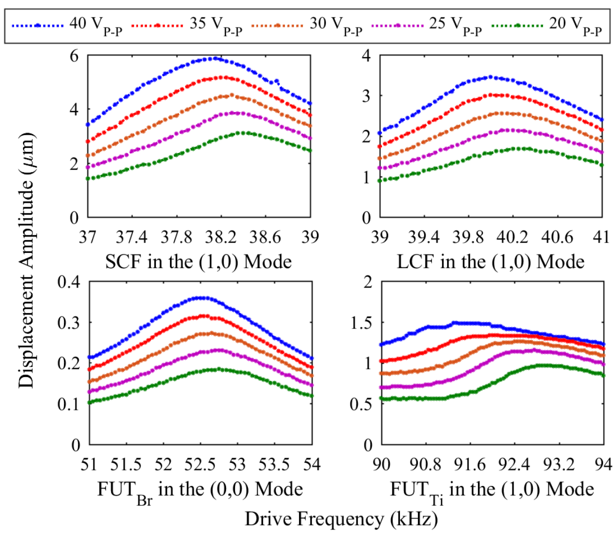

Fig. 7. Amplitude-frequency spectra of the transducers in their operational modes. The axis scales are all different for clarity.

vibration in each case has been normalized for clarity. A close correlation between these mode shapes and those shown in Fig. 2 can clearly be made. It should be noted that the cyan region shown in Fig. 6(c) is representative of the vibratory motion of the FUT_{Br} side-wall. The influence of this transducer assembly incorporating the epoxy resin bond layer between the side-wall and the membrane on the vibration characteristics is evident.

The amplitude-frequency spectra for the transducers around their respective resonance frequencies were then measured, the results of which are shown in Fig. 7. In each case, the vibration response can be attributed with the respective mode of vibration shown in Fig. 6. Peak-to-peak voltage was measured as a function of drive frequency before being converted to displacement. The displacement amplitudes shown in Fig. 7 are absolute and have not been fitted to polynomials. The net shift in resonance frequency for each transducer was calculated from the difference between the frequency at which the peak displacement amplitude was measured at 40 $V_{P\text{-}P}$ with that at 20 $V_{P\text{-}P}$. These results are shown in Table III.

The results shown in Fig. 7 principally illustrate the dynamic performance of each transducer over a 20 $V_{P\text{-}P}$ excitation

TABLE III
RESONANCE FREQUENCY SHIFT BETWEEN 20 $V_{P\text{-}P}$ AND 40 $V_{P\text{-}P}$

Transducer	Operating Mode	Frequency Reduction (Hz)
SCF	(1,0)	200
LCF	(1,0)	200
FUT_{Br}	(0,0)	300
FUT_{Ti}	(1,0)	1400

voltage range and show that even for relatively low-power ultrasonic devices such as the flexural ultrasonic transducer, dynamic nonlinearity can result in distinct resonance frequency reductions, irrespective of the membrane material or transducer assembly configuration. The aluminum SCF and LCF transducers are different in size and also side-wall profile, but both exhibit a resonance frequency reduction of approximately 200 Hz over a 20 $V_{P\text{-}P}$ range for operation in their (1,0) modes of vibration. The resonance frequency reduction for FUT_{Br} is around 300 Hz over a similar voltage range, but the decrease in resonance frequency is relatively large for FUT_{Ti}, in the order of 1400 Hz. All four transducers display evidence of asymmetry in their response profiles around the resonance frequency of the transducer at each excitation voltage level and also nonlinear softening, which is a behavior consistent with prior research [24]. In particular, both asymmetry and the magnitude of resonance frequency reduction are most prominent for FUT_{Ti}. The underlying causes of dynamic nonlinearity are complex. The designs of transducer employed in this research are strategically selected to provide insights into the contributions to dynamic nonlinearity from different design parameters. For example, the membranes of the SCF and LCF are composed of aluminum, where the principal differentiating factor between the two transducers is the membrane size. However, despite the fact that the reduction in resonance frequency for the two transducers has been measured to be around 200 Hz, it cannot be assumed that this response characteristic is directly applicable for two equivalent transducers fabricated from a different material. The influence of the piezoelectric material on the dynamic nonlinearity also cannot be distinguished. Piezoelectric ceramics are known to contribute to nonlinear behavior, and separating the influence of piezoelectric ceramics on dynamic nonlinearity from other physical sources such as material properties, transducer assembly, or structural constraints is difficult even for similar transducers. This is reinforced by the outcomes of previous studies into the dynamic nonlinearity associated with nominally identical transducers [24], [25]. Although it is not possible or the intention of the research presented here to quantify the contribution of each physical mechanism to dynamic nonlinearity, the results presented in this study are supported where relevant with indications towards the dominant factors which appear to influence dynamic nonlinearity.

The membrane shape, boundary condition, and the material type are postulated to both contribute to dynamic nonlinearity, based on the differences in resonance frequency reduction for the transducers and also the supportive literature evidence which shows the nonlinear relationship between applied load and the deflection of edge-clamped circular elastic plates [32]–[34]. Different membrane materials and transducer assembly configurations will directly affect the relationship between applied load and deflection. Therefore there is likely a strong correlation between the structural configuration of the flexural ultrasonic transducer and dynamic nonlinearity. The structural configuration of the transducer is a complex combination of both membrane material type and the edge-clamped boundary condition to which the membrane is subjected. The edge-clamped boundary condition applied to each transducer membrane is not accurately controlled or monitored in the transducer assembly process and therefore cannot be regarded as quantifiably consistent between each transducer. Even minor differences in edge-clamping will produce dissimilarities between nominally identical transducers, for example in terms of efficiency. The amplitude of a transducer is affected by the efficiency, which in turn will directly affect the nonlinearity in the amplitude responses. Additionally, this boundary condition has a close relationship with the properties of the membrane material, principally the Young's modulus, density and mechanical damping, which together dictate the vibration amplitude and resonance frequency for a specific excitation level. It has not been possible in this study to separate the influence of material type from the structural constraint, because the force from the side-wall on the membrane edge cannot be easily quantified. However, it can be stated that there is a clear reduction in resonance frequency for an increase in excitation amplitude for each transducer, suggesting that in addition to the membrane material and boundary condition, the displacement amplitude of the membrane is closely linked to the level of dynamic nonlinearity in the vibration response.

To investigate further, the comparison of dynamic nonlinearity between two different modes of the same transducer was undertaken, by contrasting the amplitude-spectra of FUT_{Ti} operating in the (1,0) mode with those in the (0,0) mode. The experimental mode shape of FUT_{Ti} vibrating in the (0,0) mode at 21.20 kHz is shown in Fig. 8(a). The experimental amplitude-frequency spectra associated with this mode are displayed in Fig. 8(b). Nonlinear softening has been detected in the response of the (0,0) mode, consistent with the observations for the response of the transducer operating in the (1,0) mode. There is also again clear asymmetry in the general response profile. However, the reduction in resonance frequency was calculated to be approximately 1400 Hz for the (1,0) mode, as shown in Table III, but Fig. 8(b) indicates that this resonance frequency reduction is only around 400 Hz for the (0,0) mode. There is a lower level of resonance frequency reduction despite higher levels of displacement amplitude in the (0,0) mode, suggesting that the magnitude of displacement amplitude does not alone influence dynamic nonlinearity. A likely reason for the lower resonance frequency reduction in the (0,0) mode compared to the (1,0) mode is that the piezoelectric ceramic is oscillating with higher frequency in the (1,0) mode and will therefore vibrate with different thermomechanical characteristics. The dynamic behavior of the flexural ultrasonic transducer is dominated by its membrane. Membrane vibration differs depending on mode and frequency, for example via the mode shape. However, through higher frequency operation, the thermal condition of

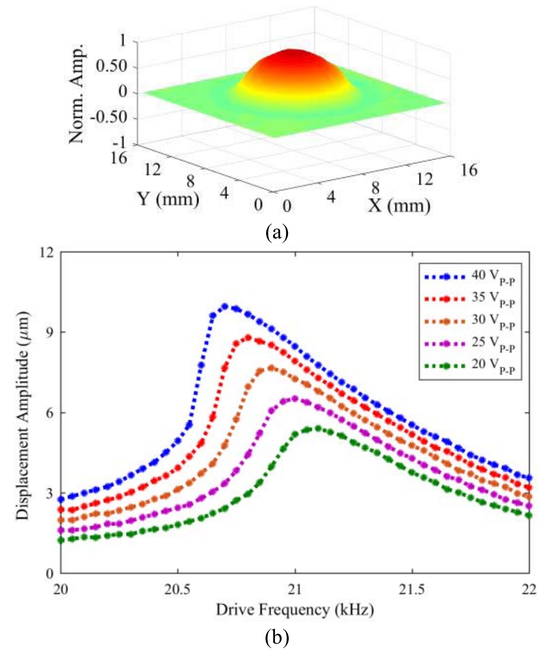

Fig. 8. The dynamics associated with the (0,0) mode of FUT_{Ti}, showing (a) the mode shape and (b) the measured amplitude-frequency spectra.

the piezoelectric ceramic can be influenced. The results in this study were acquired using continuous-wave excitation that can cause heating in piezoelectric ceramics, which can contribute to dynamic nonlinearity [23]. Furthermore, a higher frequency promotes a higher vibration velocity of the membrane providing the amplitude is the same, with a subsequent stress influence. There is hence a complex relationship between multiple physical parameters on nonlinearity, but it is likely that the piezoelectric material is a dominant contributor.

The magnitudes of resonance frequency shift for the SCF, LCF, and FUT_{Br} are broadly similar, despite the transducers all exhibiting key differences, including in terms of size and material type. A key parameter to consider is the compliance of the vibrating membrane, a fundamental physical property of which is Young's modulus. The moduli of aluminum and many classes of brass are lower than that of titanium. It has been found that the magnitudes of dynamic nonlinearity of the transducers fabricated from aluminum and brass materials are generally lower than those measured for FUT_{Ti}, in either the (0,0) or (1,0) mode. However, a wider variety of transducers fabricated from different materials and incorporating different mechanisms of driver element would be necessary to provide more explicit explanations of the contribution from physical properties such as the modulus to the magnitude of dynamic nonlinearity. The compliance of the membrane is also affected by the physical properties and geometrical dimensions of the piezoelectric ceramic, and so also requires consideration.

The underlying causes of nonlinearity will be explored in future research, principally through experiment. In order to achieve this, different driving mechanisms will likely need to be employed, for example through electromagnetism [4]. This will enable the contribution of the transducer driver to dynamic nonlinearity to be quantitatively determined. Furthermore, the investigation of transducer performance for a wider range of excitation voltage can also be useful in identifying practical limits of operation. The dynamic responses associated with all four flexural ultrasonic transducers exhibit important considerations for practical application, particularly the dynamic nonlinearity in response to increases in excitation voltage. There appears to be a strong dependency between dynamic nonlinearity and the transducer membrane configuration. There is measurable resonance frequency reduction for even modest increases in excitation voltage. For applications requiring a single drive frequency, there can be low tolerance for a shift in the resonance frequency of a sensor without a reduction in transducer amplitude. This should be accounted for in measurement systems incorporating flexural ultrasonic transducers. Although distinct physical causes of dynamic nonlinearity cannot be isolated using these experimental results, an analytical representation can be developed which can be useful in the design and optimization of transducers.

IV. Analytical Representation of Nonlinearity

There are different strategies available for representing nonlinearity in the vibration response of ultrasonic transducers. For example, the calculation of the third order elastic parameter, termed the β coefficient, has been shown to be an effective strategy to quantify nonlinearity [26], [35]. The classic nonlinear model of a resonator is the Duffing oscillator [36]–[39]. The associated Duffing equation has been used in a variety of studies to model and quantify nonlinear behavior through the adaption of the classical linear equation of motion for a damped oscillator. In this modified equation of motion, a nonlinear parameter in the form of a third order stiffness constant is present, which defines the softening or hardening nonlinearity of the oscillator response. For example, the nonlinear response of a high-power ultrasonic device was modeled through adaption of the linear piezoelectric constitutive relationships and combined with a simple mass-spring-damper system representation [40]. In the case of the flexural-type disc transducer, nonlinearity was expressed as a one-dimensional oscillator with a cubic term [25], and an equivalent single degree-of-freedom model has also been used in the simulation of the nonlinear dynamics of high-power plate transducers, where both a finite element method and a discrete parameter model were used to demonstrate causes of dynamic nonlinearity in the form of sub-harmonic and super-harmonic vibrations [21]. Sub-harmonic vibration was caused by contact of the vibrating plate and the horn of the transducer, and the source of super-harmonic vibration was determined to be through a distortion of the waveform of the contact force [21]. In a number of these studies, constitutive equations of system components such as the piezoelectric element or the equations used to describe the resonance or motion of the transducer under excitation are adapted in order to model the dynamic nonlinearity [16], [21], [40]. A further example is the linear piecewise model of an impact oscillator which has been used to simulate tapping dynamics related to the cubic nonlinearity in the system [41]. The model was capable of representing nonlinear softening and hardening responses, closely correlating with experimental results. It has also been shown that the nonlinearities associated with the stiffness and coupling in a piezoelectric cantilever can comprise quadratic and cubic terms [29], where the backbone curve, which shows the resonance at different excitations, changes linearly as a function of excitation amplitude for quadratic stiffness nonlinearity and exhibits a quadratic change for cubic stiffness nonlinearity [29]. This backbone curve is produced from the peak amplitude of the vibration response and its associated drive frequency.

There is evidence of asymmetry of the vibration response around resonance in the results shown in Fig. 7 and Fig. 8(b), which all exhibit softening. The asymmetry phenomenon has been identified in amplitude spectra of other studies investigating different transducers, for example those of Aurelle *et al.* [15], Cardoni *et al.* [17], and Leadenham and Erturk [29]. Symmetry in the amplitude response has previously been observed in vibration responses to sufficiently low electrical excitation [15], where the equation of motion for the system can be represented with a single real root. It has been reported that two coefficients of nonlinearity dominate a vibrating system. One coefficient represents the resonant amplitude and the other relates to the hysteresis phenomenon [15]. It has also been established that the latter directly influences the locations of decreasing and increasing frequency, where the larger this nonlinear coefficient, the greater the level of nonlinear softening, and is directly related to the strain in the system. Asymmetry in the amplitude spectra as shown in Fig. 7 and softening nonlinearity are both accounted for through the introduction of these nonlinear coefficients. However, for higher excitation levels, three real roots emerge which are attributable to instability, and indicative of the approach to a Duffing-type response.

In general terms, a mechanical analog model can be used to represent the experimental data in this research. The purpose of this mechanical analog is not to generate a quantifiable simulation, but rather to provide a robust mathematical equivalence of the flexural ultrasonic transducer which can then be used to elucidate key dynamic characteristics. At its fundamental level, the dynamics of a flexural ultrasonic transducer can be demonstrated to conform to the analog which can be given by the standard relationship shown by (4), which possesses a stiffness coefficient K [5], in addition to mass M and damping C.

$$M\ddot{x} + C\dot{x} + Kx = F(t) \tag{4}$$

This expression considers the linear response of the flexural ultrasonic transducer, neglecting influences of nonlinearity. However, the vibration responses of the transducers studied in this research are weakly nonlinear where the amplitudes do not reach the levels required for the transducer to exhibit characteristics such as the jump phenomenon, a classic Duffing-type behavior. The forced vibration with amplitude (A) can be represented by (5) for primary resonance behavior [36], where the force (F) is a function of time (t).

$$F(t) = Acos(\Omega t) \tag{5}$$

This relationship shows that the excitation of the system is in the form of $cos(\Omega t)$, where the resonance frequency (ω_0) can be different to the drive frequency (Ω). The experimental results in this study exhibit general features such as periodicity with that of the drive frequency, softening, and asymmetry in the amplitude response profiles. Using this information, the governing equation of the flexural ultrasonic transducer system is chosen to be (6) [36], which is an expression specifically chosen to represent the nonlinear dynamics in the vibration response of flexural ultrasonic transducers.

$$\ddot{x} + 2\epsilon^2\mu\dot{x} + \epsilon\alpha_2 x^2 + \epsilon^2\alpha_3 x^3 + \omega_0^2 x = \epsilon^2 A cos\Omega t \quad (6)$$

In (3), ϵ is a dimensionless perturbation parameter, μ is a damping term, and the α parameters are coefficients in the series expansion of the stress. A solution of (6) for the case of a primary resonance is given by (7), where the output amplitude is given by a, the phase is denoted by γ, and a detuning exists which is periodic in time with frequency $2\pi\Omega$ and where $\Omega = \omega_0 + \epsilon^2\sigma$ [36].

$$x = acos(\Omega t - \gamma) + \frac{1}{2}\epsilon\alpha_2\omega_0^{-2}a^2 \times [-1 + \frac{1}{3}\cos(2\Omega t - 2\gamma)] + O(\epsilon^2) \quad (7)$$

In (7), the nonlinearity is expressed as part of the stiffness term in the solution to the equation of motion. This correlates with the results shown in this research which suggest that nonlinearity in piezoelectric flexural ultrasonic transducers is dependent, at least in part, on the physical characteristics of the membrane such as its material properties. The a and γ parameters satisfy expressions for the periodic solution which are shown by (8) and (9).

$$\mu a = \left(\frac{A}{2\omega_0}\right) sin\gamma \quad (8)$$

$$a = \frac{1}{\sigma}\left(\frac{9\alpha_3\omega_0^2 - 10\alpha_2^2}{24\omega_0^3}a^3 - \frac{A}{2\omega_0}cos\gamma\right) \quad (9)$$

Softening and hardening nonlinearities can be distinguished through (9), where in the case of this research the condition of $9\alpha_3\omega_0^2 < 10\alpha_2^2$ indicates a softening-type nonlinearity. Regarding the periodic solution, the periodic characteristics of the drive and response signals are displayed in Fig. 9 for reference, using the spectra from the analysis of the SCF operating in the (1,0) mode as illustration.

Using these results, the phase shifts between the drive and response signals are calculated to be 56.6° and 64.5° for excitation voltages of 20 V_{P-P} and 40 V_{P-P} respectively. The phase shift increases with excitation voltage, consistent with the mathematical theory. The asymmetry between the maximum of x and its minimum value is given by (10) and allows the calculation of α_2 from the known variables of a and ω_0. It is only α_2, and not α_3, that contributes to this asymmetry.

$$x_{max} - x_{min} = \frac{\alpha_2 a^2}{\omega_0^2} \quad (10)$$

The mathematical theory also predicts that the relationship between the output and excitation voltages at respective resonance frequencies is linear in each case. Through expansion of the amplitude and phase parameters in (8) and (9), an expression can be obtained which gives the trend in the data between the output amplitude and the excitation voltage in each case. The equation suggests there is a linear relationship which exists between the output amplitude and excitation voltage, and this in explicit terms is given by (11).

$$\frac{a}{A} = \frac{1}{2\omega_0} \quad (11)$$

In order to demonstrate the validity of this theory in practice, the peak output voltage of each transducer at resonance was measured as a function of excitation voltage. This was performed for each voltage level, and the resonance corresponds to the peak voltage measured for the excitation voltage level. The results are shown in Fig. 10, and it should be noted that the measurement sensitivity for each transducer was 100 mm/s/V, with the exception of FUT_{Br} which was 10 mm/s/V. The output

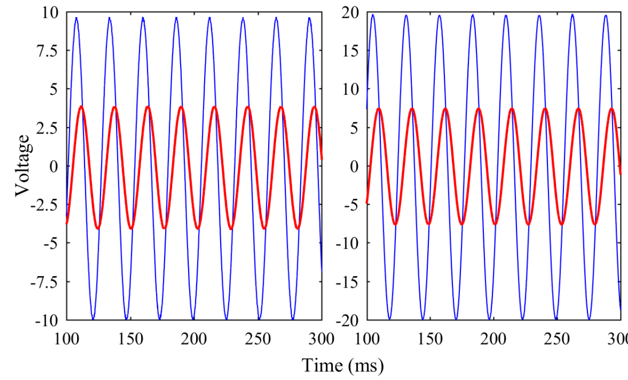

Fig. 9. The periodic drive (blue) and response (red) signals of the SCF operating at resonance in the (1,0) mode through continuous-wave excitation for two different excitation voltages, where the phase difference between each signal can be calculated. Note the difference in ordinate axis scales.

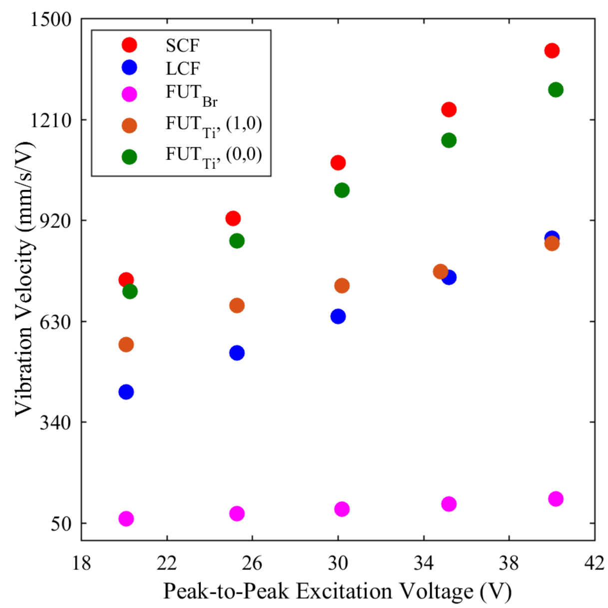

Fig. 10. Experimentally-measured vibration velocity of each transducer as a function of excitation voltage, measured at the respective resonance frequency.

voltages are hence scaled accordingly and directly represent the vibration velocity.

The scaled output voltage, or vibration velocity, of the transducers all exhibit a general linear dependency with excitation, satisfying the mathematical theory, in particular (11). There is a slight deviation of the data spectrum associated with FUT_{Ti} in the (1,0) mode, but in terms of the data range this can be considered as a general linear trend. A significantly wider excitation voltage range would be required to demonstrate that a nonlinear change is exhibited, but based on the available experimental data, this is not evident.

The mathematical theory is consistent with the physical phenomena observed through experiment, principally the existence of softening nonlinearity through resonance frequency reduction, the dominant influence of the membrane material and its physical properties, the asymmetry in the response spectra, the linear relationship between output voltage and excitation voltage, and a rise in the phase shift between the drive and response signals as the excitation voltage is increased. Through this mathematical analysis, a detailed representation of the dynamic performance of flexural ultrasonic transducers has been demonstrated which can be employed in the design, operation, and optimization of measurement systems incorporating flexural ultrasonic transducers.

V. Conclusion

This research has demonstrated the dynamic nonlinearity inherent in the vibration responses of different piezoelectric flexural ultrasonic transducers which comprise both commercial and custom-fabricated, and manufactured from different materials. Laser Doppler vibrometry was used to measure the displacement amplitude of the transducers at different levels of excitation voltage, where relationships between amplitude and drive frequency were established. An analytical solution was developed based on the equations of motion for a flexural ultrasonic transducer and shown to accurately represent the nonlinear dynamics of the transducers. There is strong evidence that the dynamic nonlinearity in piezoelectric flexural ultrasonic transducers is a complex inter-dependency between the transducer membrane material and the nature of the membrane boundary condition. Furthermore, based on the scientific literature, there is likely a strong influence from the thermomechanical behavior of piezoelectric ceramics, and this will be considered in future investigation. It is hoped that this research will be useful in the optimization of transducer performance, for example the selection of an optimum drive frequency for a specific excitation voltage in an industrial metrology application.

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