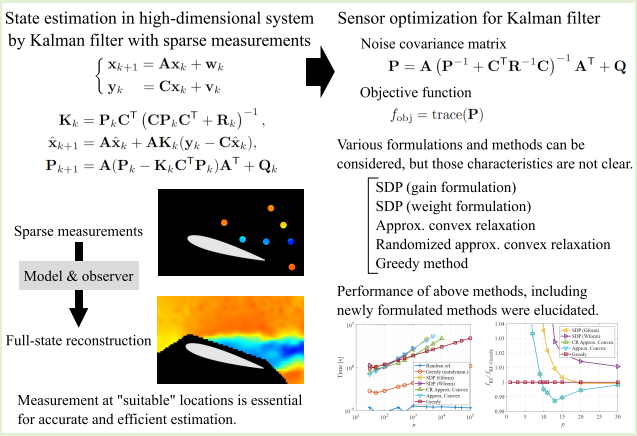
Assessment of Sensor Optimization Methods Toward State Estimation in a High-Dimensional System Using Kalman Filter

Takayuki Nagata, Yasuo Sasaki, Keigo Yamada, Masahito Watanabe, Daisuke Tsubakino, and Taku Nonomura

***Abstract*—The characteristics of the sensor selection method based on various algorithms for state estimation in a high-dimensional system using the Kalman filter were investigated. Sensors were selected based on the error covariance matrix of the Kalman filter. The performance of the sensor selection methods based on different algorithms, semidefinite programming (SDP), approximate convex relaxation, and greedy algorithm, including newly formulated methods, were compared by varying the number of potential sensor locations and the number of sensors to be selected under several noise ratio conditions. Two sensor selection methods for state estimation in a high-dimensional system using a Kalman filter were newly proposed based on the SDP with gain formulation and approximate convex relaxation, and the characteristics of the method including the previously proposed method were compared in a high-dimensional system. Although the condition was limited at $n < \mathcal{O}(10^3)$, the approximate convex relaxation method and its randomized method are effective in terms of computational time and objective value for a small-scale problem. The objective value obtained by the greedy method shows the best performance compared to the other methods in almost all investigated conditions. Particularly, the greedy method outperforms other methods when the number of selected sensors is small. In addition, only the greedy method can handle the large-scale problem of $n > \mathcal{O}(10^4)$. Overall, the greedy-based method is found to be favorable in large-scale problems in terms of computation time and the performance of the obtained sensor set.**



***Index Terms*— Kalman filter, optimal design of experiment, sensor selection.**

I. Introduction

MEASUREMENTS of physical phenomena are an important topic in various fields. This may involve surface or volume measurements, and in most cases, such measurements are performed by discretely installed point sensors. This kind of situation can be seen in various types of measurements, such as global positioning system [1], [2], acoustic measurements [3], [4], infrastructure health monitoring [5], [6], [7], [8], environment monitoring [9], [10], brain source localization [11], and seismic wave reconstruction [12], [13]. Although each sensor can only measure quantities at a particular location, full-state recovery can be achieved from sparse observations by solving an inverse problem. It is necessary to carefully determine the position of the sensor and maximize the information obtained by sparse observations to perform the required measurement with the minimum number of sensors. This is referred to as the sensor placement/selection problem. The objective of this problem is to determine the p sensor locations from the n potential sensor locations.

Sensor placement/selection problems are formulated as combinatorial optimization problems known as NP-hard problems. The exact solution can be obtained by exhaustive

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search or global optimization techniques, such as branch and bound [14], [15], but these techniques can only be used for the problem of choosing a small number of sensor locations from a small number of potential sensor locations because of the expensive computational cost. Therefore, an approximate method that can find a suboptimal solution with a reasonable computational cost is an interesting topic.

There are several approximated ways to solve sensor selection problems. Joshi and Boyd [16] formulated an approximate convex relaxation method. Nonomura et al. [17] accelerated the method for large-scale problems by applying the randomized subspace Newton method. In addition, the sensor selection methods based on semidefinite programming (SDP) are also formulated [18], [19]. Another sensor selection method based on proximal optimization [20], [21], [22], [23] has been formulated and applied to the sensor selection in dynamical and nondynamical systems. Although the local optimal solution could be obtained, the greedy method is often applied to large-scale sensor selection problems [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37] because of less computational cost. The improved method of the pure greedy algorithm in terms of the objective value [38], [39] and computational time [40], [41] have also been addressed.

In terms of the sensor selection method for the dynamical system, methods for the Kalman filter have also been developed. Oshman [42] applies V-lambda filtering to choose between a priori selection matrices, determined to adequately represent the sensor space. Liu et al. [43] and Ertin et al. [44] conducted sensor selection as a greedy entropy minimization problem using Bayesian filtering for object tracking. Feng et al. [45] applied a branch-and-bound method to determine the best single sensor selection at each time step. Although these studies achieved some success, scalability is one of the critical issues for solving large-scale problems such as sensor selection/scheduling in large-scale networks and large-scale state estimation problems.

Weimer et al. [46] attempted this problem with a relaxation approach. They considered a relaxation of a nonconvex combinatorial optimization problem and demonstrate its applicability to large-scale sensor networks. Fardad et al. [47] formulated sensor and actuator selection based on SDP and solved the problem using the sparsity-promoting framework. Their framework selects actuators and sensors in dynamical systems based on the obtained block-sparse feedback and observer gains. Lin et al. [20] attempted a similar optimization problem using the proximal optimization algorithm, and Dhingra et al. [48] and Zare and Jovanović [21] extended those methods. Masazade et al. [49] also formulated a method based on the sparsity-promoting framework for selecting sparse sensors so that the estimation error using the extended Kalman filter is minimized.

The greedy algorithm is another choice as same as an optimization in a nondynamical system. Although the convex relaxation-based approach can achieve the global optimal solution of the relaxed problem, there are no guarantees on the performance of the approximated solution with respect to the exact solution. On the other hand, a greedy algorithm results in an approximate solution with performance within a $(1 - (1/e))$ of the optimal when the objective function has appropriate characteristics [50].

Shamaiah et al. [51] considered the sensor selection problem in the context of state estimation in linear dynamical systems via Kalman filtering. In each step of the Kalman filter algorithm, the problem was formalized as an optimization of a submodular function over uniform matroids. Jawaid and Smith [52] showed that the most commonly used estimation error metrics in the Kalman filter are not, in general, submodular functions and provide sufficient conditions on the system for which the estimation error is a submodular function. Tzoumas et al. [53] showed that the log determinant of the error covariance matrix is a supermodular and nonincreasing set function with respect to the choice of the sensor set. Based on their results, an efficient approximation algorithm was provided for the solution to their sensor selection problem, along with its worst case performance guarantees. Zhang et al. [54] established performance bounds for sensor selection algorithms based on system dynamics and proved the optimality of greedy algorithms for a specific class of systems. However, as a drawback, they discovered that certain common objective functions are generally not submodular or supermodular. This poses challenges in evaluating the performance of greedy algorithms for sensor selection, except in specific scenarios. Nonetheless, through simulations, they demonstrated that these greedy algorithms exhibit strong practical performance. Hashemi et al. [40] proposed a randomized greedy method that selects sensors for state estimation in large-scale linear time-varying dynamical systems based on the study by Mirzasoleiman et al. [55]. They provided a performance guarantee for the proposed algorithm and demonstrated that the randomized method is superior to the common greedy and semidefinite problem relaxation methods in terms of computational time while providing the same or better utility.

In addition to the performance guarantees, the greedy method has desirable properties in large-scale problems. The greedy method tends to outperform the convex relaxation-based approach in terms of the optimization results when the problem size is increased [38], [51], [56], and the greedy method has an advantage in terms of the computational cost. Based on these theoretical studies, the sensor selection methods based on the greedy algorithm for the Kalman filter are expected to be further investigated in terms of their applicability to a more significant large-scale problem. Particularly, a problem can have more than $\mathcal{O}(10^4)$ potential sensor locations in some applications, such as data-driven sensor selection [24]. In this case, the issue of the computational cost is severe. In addition, the relative performance between various optimization algorithms may change depending on the problem size.

In the present study, we assess the characteristics of sensor optimization methods based on various algorithms for state estimation in a high-dimensional system using the Kalman filter. The sensor selection methods based on different algorithms, SDP, approximate convex relaxation, and greedy algorithm, including newly formulated methods are

implemented, and those performance and characteristics are assessed by varying the number of potential sensor locations, the number of sensors to be selected, and the ratio of the observation noise and system noise. Sensors are selected based on the error covariance matrix of the Kalman filter, and p sensors are selected from the n potential sensor locations. The sensor subset gives an observation vector of a linear function of latent variables superimposed with independent identically distributed zero-mean Gaussian random noise. The main contributions of the present article are summarized as follows.

1) Two new sensor selection methods for state estimation in a high-dimensional system using a Kalman filter were proposed based on the SDP with gain formulation and approximate convex relaxation, and the characteristics of the method including the previously proposed method were compared in a high-dimensional system.
2) It was confirmed that although the condition was limited when the ratio of the system noise and observation noise was similar, the approximate convex relaxation method and its randomized method are effective in terms of computational time and objective value at $p \gtrapprox r$.
3) The computational time of the approximate convex relaxation method is the shortest for the small-scale problem, but only the greedy method can handle the large-scale problem of $n > \mathcal{O}(10^4)$.
4) Overall, it is found that the greedy method is better in large-scale problems in terms of computation time and the performance of the obtained sensor set.

The MATLAB code used in the present study can be found at https://github.com/Fluid-Dynamics-Lab-Nagoya-University/Sensor-Selection-for-Kalman-Filtering [57].

II. Sensor Selection Problems

A. Problem Formulation

We aim to find a set of sensors that is suitable for the estimation of the state in a linear-time-invariant (LTI) discrete-time system. In this study, we focus on the dynamical systems with $r(\in \mathbb{N})$ state variables and $p(\in \mathbb{N})$ measurements as

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k & \text{(1a)} \\ \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k & \text{(1b)} \end{cases}$$

where $\mathbf{x}_k \in \mathbb{R}^r$, $\mathbf{y}_k \in \mathbb{R}^p$, $\mathbf{A} \in \mathbb{R}^{r\times r}$, and $\mathbf{C} \in \mathbb{R}^{p\times r}$ are the state vector, the observation vector at the instance $k \in \mathbb{Z}$, the system matrix, and the observation matrix, respectively. In addition, $\mathbf{w}_k \in \mathbb{R}^r$ and $\mathbf{v}_k \in \mathbb{R}^p$ are process noise and observation noise, respectively. The observation vector $\mathbf{y}_k$ is composed of measurement values of selected sensors, and therefore, the observation matrix $\mathbf{C}$ varies in dependence on selected sensors. If we denote the observation vector for all $n(\in \mathbb{N})$ sensors, the corresponding observation noise, and the corresponding observation matrix by $\mathbf{y}_{\text{all},k} \in \mathbb{R}^n$, $\mathbf{v}_{\text{all},k} \in \mathbb{R}^n$, and $\mathbf{U} \in \mathbb{R}^{n\times r}$, respectively, we have the following full observation equation:

$$\mathbf{y}_{\text{all},k} = \mathbf{U}\mathbf{x}_k + \mathbf{v}_{\text{all},k} \tag{2}$$

where $\mathbf{v}_{\text{all},k}$ is the white Gaussian noise that follows the normal distribution $\mathcal{N}(0, \sigma_v^2\mathbf{I}_n)$. Selecting S_1th, S_2th, ..., S_pth entries from the all-measurement vector $\mathbf{y}_{\text{all},k}$ is formulated by multiplication by the following sensor location matrix $\mathbf{H} \in \mathbb{R}^{p\times n}$:

$$\mathrm{H}_{i,j} = \begin{cases} 1, & \text{if } j = S_i \quad \forall i \in \{1, 2, \ldots, p\} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The observation equation (1b) by selected sensors is obtained by multiplying $\mathbf{H}$ by (2) and setting as follows:

$$\mathbf{y}_k = \mathbf{H}\,\mathbf{y}_{\text{all},k}, \quad \mathbf{v}_k = \mathbf{H}\,\mathbf{v}_{\text{all},k}, \quad \mathbf{C} = \mathbf{H}\,\mathbf{U}. \tag{4}$$

The estimation error of the state depends on the observation system (1b) and naturally on selected sensors in state estimation based on the model (1).

B. State Estimation Based on Kalman Filter

The sequential state estimation based on the same linear dynamical system as (1) is considered

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k & \text{(5a)} \\ \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k. & \text{(5b)} \end{cases}$$

Here, the pair $(\mathbf{A}, \mathbf{C})$ is assumed to be detectable. The state is estimated sequentially in the Kalman filter by using the following equations [58]:

$$\mathbf{K}_k = \mathbf{P}_k\mathbf{C}^\mathsf{T}\left(\mathbf{C}\mathbf{P}_k\mathbf{C}^\mathsf{T} + \mathbf{R}_k\right)^{-1} \tag{6}$$

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{A}\mathbf{K}_k(\mathbf{y}_k - \mathbf{C}\hat{\mathbf{x}}_k) \tag{7}$$

$$\mathbf{P}_{k+1} = \mathbf{A}(\mathbf{P}_k - \mathbf{K}_k\mathbf{C}^\mathsf{T}\mathbf{P}_k)\mathbf{A}^\mathsf{T} + \mathbf{Q}_k \tag{8}$$

where $\mathbf{K}_k \in \mathbb{R}^{r\times p}$ is the Kalman gain matrix, $\hat{\mathbf{x}}_k \in \mathbb{R}^r$ is the estimated state vector, and $\mathbf{P}_k \in \mathbb{R}^{r\times r}$ is the error covariance matrix with respect to the state vector at time step k. Here, $\mathbf{R} = \mathrm{E}[\mathbf{v}_k\mathbf{v}_k^\mathsf{T}]$ and $\mathbf{Q} = \mathrm{E}[\mathbf{w}_k\mathbf{w}_k^\mathsf{T}]$ are observation and system noise covariance matrix, whereas $\mathrm{E}[\circ]$ represents the expected value.

The state vector and error covariance matrix are given as $\hat{\mathbf{x}}_{t_0}$ and $\mathbf{P}_{t_0}$ at the initial time step $t_0 \in \mathbb{Z}$. Then, the Kalman gain $\mathbf{K}_k$ at time $k = t_0$ is computed with (6). The estimated state $\hat{\mathbf{x}}_k$ is updated based on the observation $\mathbf{y}_k$ in accordance with $\mathbf{K}_k$ and (7), and the covariance matrix $\mathbf{P}_k$ is updated via (8). The sequential state estimation is conducted by executing this update procedure at each timestep $k \in \{t_0, t_0 + 1, \ldots\}$. The variables $\hat{\mathbf{x}}_k$ and $\mathbf{P}_k$ are equivalent to the mean and the covariance matrix with respect to $\mathbf{x}_k$ given $\{\mathbf{y}_{k'}\}_{k'=t_0}^{k-1}$, respectively.

The error covariance matrix converges to a unique matrix due to $(\mathbf{A}, \mathbf{C})$ detectable [59], when the procedure is repeated infinite times. Hence, in the limit of $t_0 \to \infty$, the error covariance matrix $\mathbf{P}_k$ converges to a constant matrix $\mathbf{P}$ for each $k \in \mathbb{N}$. The limit matrix is a unique and positive-definite solution of the following algebraic Riccati equation:

$$\mathbf{P} = \mathbf{A}\left(\mathbf{P}^{-1} + \mathbf{C}^\mathsf{T}\mathbf{R}^{-1}\mathbf{C}\right)^{-1}\mathbf{A}^\mathsf{T} + \mathbf{Q}. \tag{9}$$

In the present study, the process noise $\mathbf{w}_k \in \mathbb{R}^r$ and the observation noise $\mathbf{v}_k \in \mathbb{R}^p$ are assumed to be random white noise that follows the normal distributions $\mathcal{N}(0, \sigma_v^2\mathbf{I}_r)$ and $\mathcal{N}(0, \sigma_w^2\mathbf{I}_p)$, respectively, for simplicity. Then, the noise covariance matrices are simplified as follows:

$$\mathbf{R} = \sigma_v^2\mathbf{I}_p, \quad \mathbf{Q} = \sigma_w^2\mathbf{I}_r. \tag{10}$$

In summary, the error covariance matrix after convergence is given as follows:

$$\mathsf{E}\left[\left(\mathbf{x}_k-\hat{\mathbf{x}}_k\right)\left(\mathbf{x}_k-\hat{\mathbf{x}}_k\right)^{\mathsf{T}}\right]=\mathbf{P}. \tag{11}$$

III. Sensor Selection Methods

The sensor selection problem is the combinatorial optimization problem that selects the observations for reducing the error in the estimated state $\hat{\mathbf{x}}_k$. This corresponds to determining the appropriate observation matrix $\mathbf{C}$. As described in Section II-B, the error covariance matrix of the true state and estimated state can be calculated. Therefore, the estimation error can be minimized by constructing $\mathbf{C}$ so as to minimize the characteristic value of the error covariance matrix $\mathbf{P}$. Several representative criteria, such as the determinant, trace norm, largest or smallest eigenvalue, are described in the optimal design of experiment [60]. In the present study, we employed the A-optimality criterion, which corresponds to evaluation of the trace norm of the error covariance matrix $\mathbf{P}$. The objective function f_{obj} is accordingly defined by

$$f_{\text{obj}}=\text{trace}(\mathbf{P}) \tag{12}$$

and the sensing locations are determined so that f_{obj} is minimized. The minimization of this objective function corresponds to the minimization of the average estimation error.

Sensor selection by optimizing the objective function (12) can be achieved via several different ways. We compare several methods based on different approaches, the SDP-based approach, the convex relaxation approach, and the greedy-based approach. We prepare two methods with different formulations for the SDP-based approach. One is the SDP-based approach in weight formulation [(SDP Wform)] [19], and the other is the SDP-based approach in gain formulation [(SDP Gform)] which is newly formulated in the present study. The weight formulation intends to optimize the weight vector that indicates the sensor location to be used. On the other hand, the gain formulation intends to optimize the gain matrix that recovers the state. The convex relaxation approach (Approx. Convex.) is also newly implemented in the present study. This method is based on the sensor selection method for a nondynamical system [16], and it was extended to the sensor selection for the state estimation using the Kalman filter. The greedy-based approach (Greedy) is straightforwardly implemented for sensor selection for estimation via the Kalman filter. This method approximates the solution of the optimization problem by solving small subproblems and combining the optimal solutions obtained in each subproblem. This method may reach the local optimal but the computational cost can be reduced.

It should be noted that there is another approach that is based on proximal optimization [21], but this method is not treated in the present article because of the slight difference in the problem formulation and difficulty of the determination of the hyperparameter.

A. SDP (Weight Formulation)

This formulation was described by Yang et al. [19]. They assumed that the pair $(\mathbf{C}_i, \mathbf{A})$ is detectable and $(\mathbf{A}, \sqrt{\mathbf{Q}})$ is controllable. The original paper considered the vector sensors in the node candidate but the internode noise correlation was not considered. In the present study, one sensor per one node is considered, and therefore, the formulation is simplified and then employed. Hence, $\mathbf{R}$ should be assumed to be $\sigma_v^2\mathbf{I}$, and the Riccati equation below is considered

TABLE I
Optimization Problem and Objective Function

Method	Problem	Objective function
Greedy	Single-sensor subprob.	Original
SDP	Relaxed convex opt. prob.	Convex func. w/ constraint
Approx. Convex	Relaxed convex opt. prob.	Convex func. w/ constraint

$$\mathbf{P}=\mathbf{Q}+\mathbf{A}\left(\mathbf{P}^{-1}+\sigma_v^2\mathbf{U}^{\mathsf{T}}\mathbf{H}^{\mathsf{T}}\mathbf{H}\mathbf{U}\right)^{-1}\mathbf{A}^{\mathsf{T}}. \tag{13}$$

Here, $\mathbf{H}^{\mathsf{T}}\mathbf{H}$ is a diagonal matrix of which the entry is one at the index corresponding to sensor positions and zero for other positions. This $\mathbf{H}^{\mathsf{T}}\mathbf{H}$ is relaxed, and the following equation is obtained:

$$\mathbf{P}=\mathbf{Q}+\mathbf{A}\left(\mathbf{P}^{-1}+\sigma_v^2\mathbf{U}^{\mathsf{T}}\mathbf{Z}\mathbf{U}\right)^{-1}\mathbf{A}^{\mathsf{T}}. \tag{14}$$

Then, the problem seeks optimal selections minimizing the trace of the steady-state estimation error covariance can be considered under the constraint of limited available sensors as follows:

$$\begin{aligned}&\min_{\mathbf{z}}\ \text{trace}(\mathbf{P}(\mathbf{z}))\\&\text{s.t. }\mathbf{P}=\mathbf{A}\left(\mathbf{P}^{-1}+\sigma_v^{-2}\mathbf{C}^{\mathsf{T}}\mathbf{Z}\mathbf{C}\right)^{-1}\mathbf{A}^{\mathsf{T}}+\mathbf{Q}\\&\quad\sum_{i=1}^{N}z_i\le p\\&\quad z_i\in\{0,1\}\end{aligned} \tag{15}$$

where $\mathbf{z}$ and z_i are the weighting vector and its ith elements that indicate whether the corresponding sensor should be selected or not, and $\mathbf{Z}$ is a diagonal matrix of which the diagonal entries are corresponding to the weight vector $\mathbf{z}$. Based on [19], the problem above can be transformed into SDP which can be solved by CVX. See [19] for the details of the derivation of the formulation

$$\begin{aligned}&\min_{\mathbf{z},\mathbf{X},\mathbf{M},\mathbf{G},\mathbf{Z}}\ \text{trace}(\mathbf{X})\\&\text{s.t.}\begin{bmatrix}\mathbf{X}&\mathbf{I}\\\mathbf{I}&\mathbf{M}\end{bmatrix}\succeq\mathbf{0}\\&\begin{bmatrix}\mathbf{M}&\mathbf{M}\mathbf{A}-\mathbf{G}\tilde{\mathbf{C}}\mathbf{A}&\mathbf{M}-\mathbf{G}\tilde{\mathbf{C}}&\mathbf{G}\\\mathbf{A}^*\mathbf{M}-\mathbf{A}^*\tilde{\mathbf{C}}^*\mathbf{G}^*&\mathbf{M}&\mathbf{0}&\mathbf{0}\\\mathbf{M}-\tilde{\mathbf{C}}^*\mathbf{G}^*&\mathbf{0}&\mathbf{Q}^{-1}&\mathbf{0}\\\mathbf{G}^*&\mathbf{0}&\mathbf{0}&\mathbf{Z}\end{bmatrix}\succeq\mathbf{0}\\&\sum_{i=1}^{N}z_i\le p,\quad z_i\in\{0,1\}.\end{aligned} \tag{16}$$

The matrix $\mathbf{X}$ corresponds to the covariance matrix $\mathbf{X}=\mathbf{P}$. Here, the signal-to-noise-ratio-based normalized full observation matrix $\tilde{\mathbf{C}}=\sigma_v^{-1}\mathbf{C}$ is introduced. The matrix $\mathbf{Q}$ is replaced by $\sigma_w^2\mathbf{I}_r$ before solving this problem in the present study, although this formulation allows a nondiagonal matrix of $\mathbf{Q}$.

This algorithm gives us the nonzero entries of z of the larger number than what users specified by the variable p. Therefore, the indices of the p largest entries of z are selected and used as the sensor location.

B. SDP (Gain Formulation)

This formulation is proposed in the present study. This was inspired by the sensor selection formulation for the continuous-time system implemented by Zare et al. [61], and extended to the discrete-time system in this article. Unfortunately, proximal gradient with the weight formulation adopted by Zare et al. [61] is not introduced because it cannot be applied to the discrete-time system and also it does not work even if we translate the problems from discrete-time system to continuous-time system, compared with other algorithms shown here.

The following observer can be constructed for the linear system in (1):

$$\begin{aligned}\hat{\mathbf{x}}_{k+1} &= \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{L}(\mathbf{y}_k - \hat{\mathbf{y}}_k)\\ &= \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{L}\mathbf{U}(\mathbf{x}_k - \hat{\mathbf{x}}_k) + \mathbf{L}\mathbf{v}_{\mathrm{all},k}.\end{aligned} \tag{17}$$

Here, $\hat{\mathbf{x}}$ gives zero-mean estimate of $\mathbf{x}$, and the estimation error $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ is written as follows:

$$\tilde{\mathbf{x}}_{k+1} = (\mathbf{A} - \mathbf{L}\mathbf{U})\tilde{\mathbf{x}}_k + \mathbf{w}_k + \mathbf{L}\mathbf{v}_{\mathrm{all},k}. \tag{18}$$

The observer minimizes H_2 norm of the system (18) is a Kalman filter and its gain $\mathbf{L}$ becomes the Kalman gain $\mathbf{K}$ introduced above. Therefore, it can be obtained by solving the following problem (as noted in the Appendix):

$$\begin{aligned}&\min_{\mathbf{L},\mathbf{X}} \ \mathrm{trace}(\mathbf{X}\mathbf{Q} + \mathbf{X}\mathbf{L}\mathbf{R}_{\mathrm{all}}\mathbf{L}^*)\\ &\text{s.t. } \mathbf{X} - (\mathbf{A} - \mathbf{L}\mathbf{U})^*\mathbf{X}(\mathbf{A} - \mathbf{L}\mathbf{U}) - \mathbf{I} \succ \mathbf{0}\\ &\qquad \mathbf{X} \succ \mathbf{0}\end{aligned} \tag{19}$$

where $\mathbf{R}_{\mathrm{all}} = \mathrm{E}[\mathbf{v}_{\mathrm{all}}\mathbf{v}_{\mathrm{all}}^{\mathsf{T}}]$ represents the full observation noise covariance. This formulation can be transformed into this problem by changing the variables

$$\begin{aligned}&\min_{\mathbf{X},\mathbf{Y}} \mathrm{trace}(\mathbf{X}\mathbf{Q} + \mathbf{X}^{-1}\mathbf{Y}\mathbf{R}_{\mathrm{all}}\mathbf{Y}^*)\\ &\text{s.t. } \mathbf{X} - (\mathbf{X}\mathbf{A} - \mathbf{Y}\mathbf{U})^*\mathbf{X}^{-1}(\mathbf{X}\mathbf{A} - \mathbf{Y}\mathbf{U}) - \mathbf{I} \succ \mathbf{0}\\ &\qquad \mathbf{X} \succ \mathbf{0}\end{aligned} \tag{20}$$

where $\mathbf{L} = \mathbf{X}^{-1}\mathbf{Y}$. Then, this problem can be transformed into the following SDP using the Schur complement, which can be solved by CVX:

$$\begin{aligned}&\min_{\mathbf{X},\mathbf{Y},\mathbf{V},\gamma} \gamma\\ &\text{s.t. } -\mathrm{trace}(\mathbf{Q}\mathbf{X}) - \mathrm{trace}(\mathbf{V}) + \gamma > \mathbf{0}\\ &\qquad \begin{bmatrix} \mathbf{X} - \mathbf{I} & (\mathbf{X}\mathbf{A} - \mathbf{Y}\mathbf{U})^*\\ (\mathbf{X}\mathbf{A} - \mathbf{Y}\mathbf{U}) & \mathbf{X}\end{bmatrix} \succ \mathbf{0}\\ &\qquad \begin{bmatrix} \mathbf{V} & \mathbf{R}_{\mathrm{all}}^{1/2}\mathbf{Y}^*\\ \mathbf{Y}\mathbf{R}_{\mathrm{all}}^{1/2} & \mathbf{X}\end{bmatrix} \succ \mathbf{0}.\end{aligned} \tag{21}$$

We confirmed that the SDP formulation above gives us the Kalman filter gain $\mathbf{L} = \mathbf{X}^{-1}\mathbf{Y}$ in the steady state and the covariance matrix $\mathbf{X} = \mathbf{P}$. The sparsity-promoting term is added to the problem to obtain the sparse sensor with optimal gain value

$$\begin{aligned}&\min_{\mathbf{X},\mathbf{Y},\mathbf{V},\gamma} \gamma + \lambda\sum_i \|\mathbf{Y}_i\|_2^2\\ &\text{s.t. } -\mathrm{trace}(\mathbf{Q}\mathbf{X}) - \mathrm{trace}(\mathbf{V}) + \gamma > \mathbf{0}\\ &\qquad \begin{bmatrix} \mathbf{X} - \mathbf{I} & (\mathbf{X}\mathbf{A} - \mathbf{Y}\mathbf{U})^*\\ (\mathbf{X}\mathbf{A} - \mathbf{Y}\mathbf{U}) & \mathbf{X}\end{bmatrix} \succ \mathbf{0}\\ &\qquad \begin{bmatrix} \mathbf{V} & \mathbf{R}_{\mathrm{all}}^{1/2}\mathbf{Y}^*\\ \mathbf{Y}\mathbf{R}_{\mathrm{all}}^{1/2} & \mathbf{X}\end{bmatrix} \succ \mathbf{0}\end{aligned} \tag{22}$$

where λ is the weight for the regularization term. This parameter was set so that the group norm becomes close to but larger than the number of sensors to be selected. Here, we consider the column-norm-based group ℓ_1 norm [62]. This procedure does not give us the sensors of the exact number we expect, even if we adjusted the regularization term by try-and-error processes. In addition, our experience to the gain formulation of linear inverse problem, in which the proximal gradient method was adopted, showed that the formulation with the group ℓ_1 norm cannot obtain a small number of sensors, while the formulation with the specified group ℓ_0 norm gives us the better sensors of a specified number than the greedy method in almost all the cases [22], [23]. However, the gain formulation above adopts SDP and the penalty term should be the group ℓ_1 norm owing to its convexity, which does not seem to give us the sensors of a small number. Actually, the sensors of a small number that we expect cannot be obtained and usually more of sensors remain after the optimization in the numerical experiments shown later. Therefore, the sensor nodes with larger gain which is corresponding to the larger norm of the column of $\mathbf{L}$ are selected after the optimization. The same strategy was adopted in the study of Zare et al. [61]. The proximal gradient method with the penalty term of the group ℓ_0 norm seems to work better when a small number of sensors to be selected, such a formulation for a discrete-time system has not been found and cannot be formulated in the present study. Such an attempt is also left for future study.

Finally, it should be noted that this formulation in (22) can handle the nondiagonal $\mathbf{R}$ and nondiagonal $\mathbf{Q}$ that can be handled by other formulations. This is a clear advantage of this gain formulation, whereas the importance of sensor selection in correlated noises has been addressed in the previous studies [18], [23], [32], [33]. Although this point should be studied in detail, the focus of the present study is on the comparison of the sensor selection for the Kalman filter with independent noises, and therefore, this point is also left for future study.

C. Approximate Convex Relaxation (Weight Formulation)

Here, we try the (empirically) convex weight formulation for the discrete-time KF, similar to the SDP formulation by Yang et al. [19]. Substitute sensor candidate and sensor location matrices in the Riccati equation of (9).

We straightforwardly extended the method based on the Newton method proposed by Joshi and Boyd [16] and its application to the observability Gramian in linear dynamical systems of Yamada et al. [34], as a comparison with an existing algorithm. In the present article, this function is

optimized by the standard Newton method of Algorithm 1, and its customized randomization approach [17] of Algorithm 2 to alleviate its expensive computation, as will be explained hereafter. Similar to the algorithm of the weight formulation of SDP, this algorithm returns the indices of the p largest entries of $\mathbf{z}$ as the sensor locations.

The objective function of the original method is the maximization of the logarithm of the determinant of the FIM. The relaxation here also requires the independent noise assumption, and therefore, the same problem as in (15) is considered as a basic problem. Then, the constraint of $z_i \in (0, 1)$ is replaced by the penalty term of logarithms, and the form that can be solved by the Newton method is derived. Even if the algorithm is the same, the deviation for the Kalman filter problem is conducted for the first time and the performance of the algorithm which depends on the objective function, the size of the problem, etc., should be carefully investigated. The approximate relaxed sensor selection problem with minimization of the trace of the inverse of the FIM can be written as follows:

$$\begin{aligned} \min \phi(z) &= \operatorname{trace}(\mathbf{P}(\mathbf{z})) \\ &\quad -\kappa \sum_{i=1}^{n} \left[\log(z_i) + \log(1 - z_i)\right] \\ \text{s.t. } & \mathbf{1}^{\mathsf{T}}\mathbf{z} = p. \end{aligned} \tag{23}$$

A positive parameter κ controls the quality of the approximation. A set of n potential measurements is characterized by $\mathbf{u}_1, \ldots, \mathbf{u}_n \in \mathbb{R}^m$. The vector $\mathbf{1}$ is the vector with all entries one. Finally, the obtained objective function ϕ is convex and smooth in the range we investigated, although the rigorous proof cannot be given in the present study.

A Newton step $\delta\mathbf{z}$ is determined by minimizing the second-order approximation of the objective function and by satisfying the constraint $\delta\mathbf{z} = 0$ as found in [16]

$$\delta\mathbf{z} = \left(\nabla^2\phi\right)^{-1}\left(-\nabla\phi + \frac{\mathbf{1}^{\top}\left(\nabla^2\phi\right)^{-1}\nabla\phi}{\mathbf{1}^{\top}\left(\nabla^2\phi\right)^{-1}\mathbf{1}}\mathbf{1}\right). \tag{24}$$

Each entry of the gradient of ϕ is defined as

$$\{\nabla\phi\}_i = \operatorname{trace}\left(\frac{\partial\mathbf{P}}{\partial z_i}\right) + \kappa\left(\frac{1}{z_i} - \frac{1}{1 - z_i}\right) \tag{25}$$

and that of the Hessian of ϕ is defined as

$$\left\{\nabla^2\phi\right\}_{ij} = \operatorname{trace}\left(\frac{\partial^2\mathbf{P}}{\partial z_i \partial z_j}\right) - \kappa\delta_{ij}\left(\frac{1}{z_i^2} + \frac{1}{(1 - z_i)^2}\right). \tag{26}$$

Unfortunately, it should be noted again that we could not give the proof for the semidefinite positivity of the Hessian due to cumbersome calculus, whereas it is numerically confirmed to be positive semidefinite in the range we investigated. The proof of these characteristics will be left for future study.

The Newton step is derived for our method by substituting derivatives of $\mathbf{P}$ into (24)–(26). Differentiating both sides of (14) with respect to z_i gives the following Lyapunov equation:

$$\mathbf{P}_i = -\mathbf{A}\mathbf{W}^{-1}\left(-\mathbf{P}^{-1}\mathbf{P}_i\mathbf{P}^{-1} + \sigma_v^2\mathbf{u}_i\mathbf{u}_i^T\right)\mathbf{W}^{-1}\mathbf{A}^{\mathsf{T}} \tag{27}$$

where

$$\mathbf{P}_i = \frac{\partial\mathbf{P}}{\partial z_i} \tag{28}$$

$$\mathbf{W} = \left(\mathbf{P}^{-1} + \sigma_v^2\mathbf{U}^{\mathsf{T}}\mathbf{Z}\mathbf{U}\right). \tag{29}$$

Matrix calculus based on the vector identity or the use of other numerical algorithms, which is implemented as "dlyap" function in MATLAB, for example, can be used for this solution. The "vec" form could reduce the numerical operations by reusing the common inverse matrix of large size. (The "dlyap" function is used for the solutions of the discrete-time Lyapunov equations, which adopts the subroutine libraries, Subroutine Library in Systems and Control Theory (SLICOT) [63], [64], [65], [66]. It should also be noted that there is another approach for solving the equation like [67], for instance.)

Then, the second derivative using (27) gives another Lyapunov equation for $\mathbf{P}_{ij} = (\partial^2\mathbf{P}/\partial z_i \partial z_j)$

$$\mathbf{P}_{ij} - \mathbf{A}\mathbf{W}^{-1}\mathbf{P}^{-1}\mathbf{P}_{ij}\mathbf{P}^{-1}\mathbf{W}^{-1}\mathbf{A}^{\mathsf{T}} = \sum_{i=1,2,4,5} \mathbf{L}_i \tag{30}$$

where

$$\mathbf{L}_1 = \mathbf{A}\mathbf{W}^{-1}\mathbf{W}_j\mathbf{W}^{-1}\mathbf{W}_i\mathbf{W}^{-1}\mathbf{A}^{\mathsf{T}}, \tag{31}$$

$$\mathbf{L}_2 = \mathbf{A}\mathbf{W}^{-1}\mathbf{W}_i\mathbf{W}^{-1}\mathbf{W}_j\mathbf{W}^{-1}\mathbf{A}^{\mathsf{T}}, \tag{32}$$

$$\mathbf{L}_4 = -\mathbf{A}\mathbf{W}^{-1}\mathbf{P}^{-1}\mathbf{P}_j\mathbf{P}^{-1}\mathbf{P}_i\mathbf{P}^{-1}\mathbf{W}^{-1}\mathbf{A}^{\mathsf{T}}, \tag{33}$$

$$\mathbf{L}_5 = -\mathbf{A}\mathbf{W}^{-1}\mathbf{P}^{-1}\mathbf{P}_i\mathbf{P}^{-1}\mathbf{P}_j\mathbf{P}^{-1}\mathbf{W}^{-1}\mathbf{A}^{\mathsf{T}}, \tag{34}$$

$$\mathbf{W}_i = \frac{\partial\mathbf{W}}{\partial z_i} \tag{35}$$

$$= \left(-\mathbf{P}^{-1}\mathbf{P}_i\mathbf{P}^{-1} + \sigma_v^2\mathbf{u}_i\mathbf{u}_i^T\right). \tag{36}$$

The Riccati equation (14) is solved for $\mathbf{P}(\mathbf{z})$ in (23) by "idare" command in MATLAB and the objective function is updated inside Newton iterations with the step size adjusted. Although the Riccati equation is solved only $\mathcal{O}(1)$ times per time increment, a plenty number of evaluations of the Lyapunov equations is required to fill in the gradient vector and the Hessian for the Newton system construction. The number of the equations is up to the square of the size of the solution vector $\mathbf{z}$, which is overwhelming.

From the computational point of view, another Newton algorithm leveraging randomization is proposed here as summarized in Algorithm 2. The sketch matrix compresses the solution space of the sensor selection problem as a projection onto a small subspace

$$\mathcal{I}_{\tilde{n}} = \left\{i'_1, \ldots, i'_\rho, \ldots, i'_{\tilde{n}}\right\}. \tag{37}$$

The permutation operation defines the sketch matrix $\mathbf{S}_{\tilde{n}}$ in this study as previously performed in [17] and [34], which should be referred to for more description of the randomization. Despite a heuristic procedure, sampling is not drawn uniformly from the candidate subspace. A fraction of the sampled subspace (half, in this study) is designated by the indices of the largest components of $\mathbf{z}$ in the preceding iteration, while a uniform sampling from the remaining candidates fills the subspace selection to enhance exploration. The first and second derivatives in (25) and (26) are obtained only over these selected indices. The total amount of the evaluation is

reduced by linear and quadratic to the compression rate. The iteration is repeated until global convergence is ensured by thresholding of the step sizes for several consecutive updates.

Algorithm 1 Newton Algorithm

Input: $\mathbf{C} \in \mathbb{R}^{n\times r}$, $\mathbf{A} \in \mathbb{R}^{r\times r}$, $p \in \mathbb{N}$
Output: Indices of chosen p sensor positions $\mathcal{I}_p$
  Set an initial weight $\mathbf{z} \leftarrow \mathbf{1}p/n$
  while convergence condition not satisfied **do**
    Calculate ∇f by (25) and $\nabla^2 f$ by (26)
    Calculate $\delta\mathbf{s}$ by (24)
    Obtain step size t by backtracking line search
    Set $\mathbf{z} \leftarrow \mathbf{z} + t\delta\mathbf{z}$
  end while
  Return the indices of p-largest components of $\mathbf{z}$ as $\mathcal{I}_p$

Algorithm 2 Customized Algorithm for CR Approximated Convex Relaxation

Input: $\mathbf{C} \in \mathbb{R}^{n\times r}$, $\mathbf{A} \in \mathbb{R}^{r\times r}$, $p > 0$, $\tilde{n} > 0$
Output: Indices of chosen p sensor positions $\mathcal{I}_p$
  Set $\mathbf{z} \leftarrow \mathbf{1}p/n$
  while convergence condition not satisfied **do**
    Select $\mathcal{I}_{\tilde{n}}$ [(37)] and set $\mathbf{S}_{\tilde{n}}$
    Calculate subsampled derivatives $\nabla\tilde{f}$ and $\nabla^2\tilde{f}$
    Calculate $\delta\tilde{\mathbf{z}}$
    Obtain step size t by backtracking line search
    Set $\mathbf{z} \leftarrow \mathbf{z} + t\left(\mathbf{S}_{\tilde{n}}^{\top}\delta\tilde{\mathbf{z}}\right)$
  end while
  Return the indices of p-largest components of $\mathbf{z}$ as $\mathcal{I}_p$

D. Greedy Algorithm

The standard greedy algorithm was straightforwardly applied for minimizing the objective function (12). Greedy sensor selection for a Kalman filter was considered with bounds on the worst case performance by Zhang et al. [54]. The empirical approximation ratio of greedy is typically better than other sensor selection algorithms. However, some algorithms can obtain better objective value depending on the objective function or the problem size [22], [23], [28]. From the viewpoint of the computational time, the greedy algorithm generally has an advantage compared with other algorithms such as SDP, approximated convex relaxation method, and proximal optimization. However, computational time of the greed method tends to be long for the sensor selection with the objective function that the computational cost is high in the high-dimensional system because the number of evaluation of the objective function is large [34].

Algorithm 3 shows the standard greedy algorithm. The objective function $f(\mathcal{S}\cup\{s\})$ is computed for all $s \in \{1, 2, \ldots, n\}\backslash\mathcal{S}$ at each greedy iteration of a number $k \in \{1, 2, \ldots, p\}$. The optimal s_k of the single-sensor subproblem, which maximizes the temporary objective value, is found. The optimal s_k is added to the temporary solution $\mathcal{S}$, and the baseline observation matrix $\tilde{\mathbf{C}}$ is updated as $\tilde{\mathbf{C}} \leftarrow [\tilde{\mathbf{C}}^{\top}, \mathbf{U}_{s_k}^{\top}]^{\top}$, where $\mathbf{U}_{s_k}$ is s_kth row of $\mathbf{U}$. The greedy solution $\mathcal{S}$ with $|\mathcal{S}| = p$ is obtained after p iterations.

Algorithm 3 Greedy Algorithm for Sensor Selection

1: Set $\mathcal{S} \leftarrow \emptyset$ and $\tilde{\mathbf{C}} \in \mathbb{R}^{0\times r}$
2: **for** $k = 1, \ldots, p$ **do**
3: **for** $s \in \{1, 2, \ldots n\}\backslash\mathcal{S}$ **do**
4: $\mathbf{C} = \left[\tilde{\mathbf{C}}^{\top}, \mathbf{U}_{:s}^{\top}\right]^{\top}$
5: $g_s = f_{\text{obj}}(\mathcal{S}\cup\{s\})$
6: **end for**
7: $s_k \in \text{argmax}_{s\in\{1,2,\ldots n\}\backslash\mathcal{S}}\, g_s$
8: $\mathcal{S} \leftarrow \mathcal{S}\cup\{s_k\}$, $\tilde{\mathbf{C}} \leftarrow \left[\tilde{\mathbf{C}}^{\top}, \mathbf{U}_{s_k}^{\top}\right]^{\top}$
9: **end for**
10: **return** $\mathcal{S}$

Zhang et al. [54] gave sufficient conditions that the greedy sensor selection with objective function for the Kalman filter gives optimal solution. This is a special condition and does not apply in most cases. Therefore, it is generally not possible to determine whether the objective function has submodularity. For large-scale problems, it is extremely difficult to evaluate whether the submodularity inequality holds for a given objective function for all combinations. Although submodularity is not known in general conditions, the study by Zhang et al. [54] showed that these greedy algorithms perform well in practice via simulations.

The problem we treated in the present study has a large number of potential sensing locations but the number of latent variables is small. The number of sensors to be selected is the same level as that of the latent variables. Hence, the scale relation of each parameter in the problem intended in the present study is $p \approx r \ll n$. In the case of the convex relaxation method, the Newton method was used. The calculation of Hessian P_{ij} is the most computationally expensive, and it takes $\mathcal{O}(n^2r^3 + n^3)$. The problem we treated in the present study is $r \ll n$, and thus, the number of latent variables does not have a large impact on the computational cost but the dominant parameter might be changed depending on the problem. When the CR approximated convex relaxation method is used, the set of the sensor candidate is compressed from the size of n to n_c, and the complexity per iteration is reduced to $\mathcal{O}(n_c^2r^3 + n_c^3)$ instead of the increase in the number of iterations to achieve convergence. Here, the compression ratio n_c/n is the hyperparameter, and it is typically set around $n_c/n = 1/10$.

The greedy algorithm needs the solution of the algebraic Riccati equation (9) for all potential sensing locations. The MATLAB "idare" function that solves (9) was used in the present study, and it takes the complexity of $\mathcal{O}(r^3)$ for solving the Riccati equation [68]. Hence, the complexity of selecting p sensors from n potential sensing locations is $\mathcal{O}(nr^3p)$ as stated in [54].

The problem we treated in the present study has a large number of potential sensing locations but the number of latent variables is small. The number of sensors to be selected is the same level as that of the latent variables. Hence, the scale relation of each parameter in the problem intended in the

present study is $p \approx r \ll n$. The SDP (Gform), SDP (Wform), and convex relaxation methods require iterative calculation, and the number of iterations which is needed for convergence is depending on the conditions.

IV. Numerical Experiments

The performance of the proposed methods was evaluated by applying those methods to the data generated by the random system. Linear systems with random stable eigenvalues are generated for this purpose using the MATLAB "drss" function. This problem has an averaged absolute eigenvalue of 0.5 for the system matrix $\mathbf{A}$. The number of latent variables was set to be $r = 10$. The conditions of $p < r$ and $p > r$ correspond to the underdetermined and overdetermined conditions for the linear inverse problem on the nondynamical system, respectively. However, it is not simply applicable for the dynamical system, influences of the number of sensor candidates, number of selected sensors, and ratio of system noise and observation noise were investigated. It should be noted that the number of latent variables is also an important parameter in this problem, but we omit investigation of the influence on the performance of the sensor set because it seems characterized by the ratio of the number of the latent variables and selected sensors, as reported by Yamada et al. [34] in the study of sensor selection based on observability Gramians.

1) Influence of Number of Potential Sensor Locations on Computational Time: Influences of the number of potential sensor locations on the computational time are investigated. The range of the number of sensor candidates was set to be between $30 \leq n \leq 10^5$, and the number of sensors to be selected and noise ratio were fixed at $p = 20$ and $\sigma_w/\sigma_v = 10^{-1}$, respectively. The computations for each condition were conducted 20 times with different data, and the average value was calculated.

Fig. 1 shows the computational time with respect to the number of potential sensing locations. The computational time of the methods based on the trace of the error covariance matrix is much longer than that of the random selection and the A-optimality-based greedy method for a nondynamical system [30]. Although the computational time of the greedy method is much shorter than that of the convex relaxation and SPD methods in the case of the objective function for a nondynamical system, the computational time of all methods for the Kalman filter is antagonizing in the small-scale problems. The approximate convex relaxation method and its randomized method are the fastest when the number of potential sensing locations is $n \leq 10^2$. At $n > 10^2$, the computational time of the convex relaxation method is the same level as the SDP methods, and the increment of the computational time with respect to the number of potential sensing locations is similar. The increment of the computational time with respect to the number of potential sensing locations is the same for the randomized approximate convex relaxation method, but the absolute computational time is reduced with the benefit of the randomized algorithm. The randomized convex relaxation is competitive in terms of the computational time with the greedy method at $n \leq 10^3$. An increase in the computational time of the greedy method by increasing the number of potential sensing locations is smaller than that of the approximate convex relaxation and SDP methods, and thus, the advantage in terms of the computational time is pronounced in the large-scale problem. Only the greedy method is available due to computational cost in a large-scale problem of $n > \mathcal{O}(10^4)$.

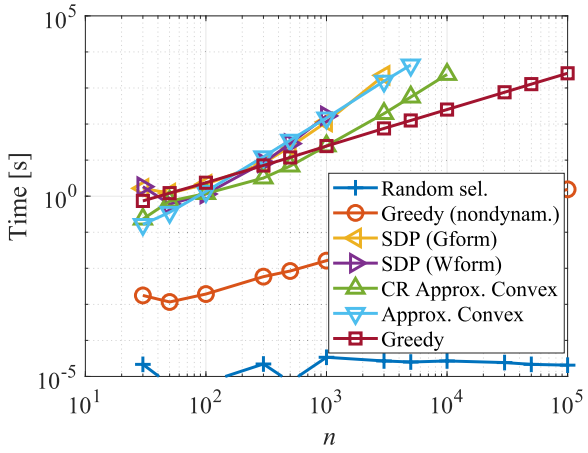

Fig. 1. Computational time with respect to the number of potential sensing locations ($p = 20$).

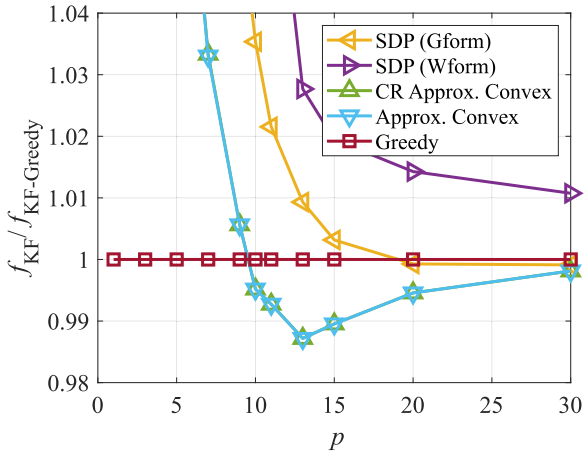

Fig. 2. Comparison of the normalized objective value for a different number of selected sensors at $\sigma_w/\sigma_v = 10^0$.

2) Influence of Number of Selected Sensors on Objective Value: Influences of the number of selected sensors on the objective value are investigated in several noise ratio conditions. The range of the number of selected sensors was $1 \leq p \leq 30$. The number of potential sensor locations was fixed at $n = 1000$. The computations for each condition were conducted 50 times with different data, and the average value was calculated.

Fig. 2 shows the objective value with respect to the number of selected sensors. The objective values obtained by each method were normalized by those obtained by the greedy method, and the variances of the system noise and observation noise are equivalent. The smaller objective value indicates a better optimization result.

The objective values differ from each other, particularly at smaller p. When the number of selected sensors is small, the objective value obtained by the greedy method is the best, and the objective values obtained by other methods are

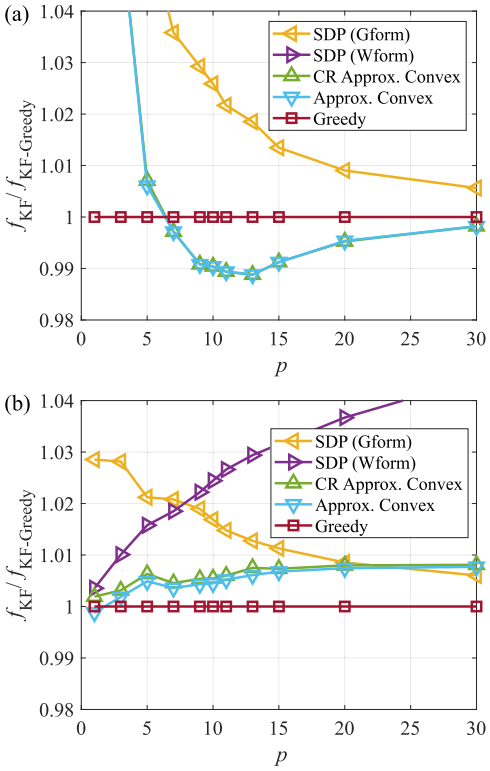

Fig. 3. Normalized objective value with respect to the number of selected sensors when the observation noise is stronger than the system noise: (a) $\sigma_w/\sigma_v = 10^{-1}$ and (b) $\sigma_w/\sigma_v = 10^{-3}$.

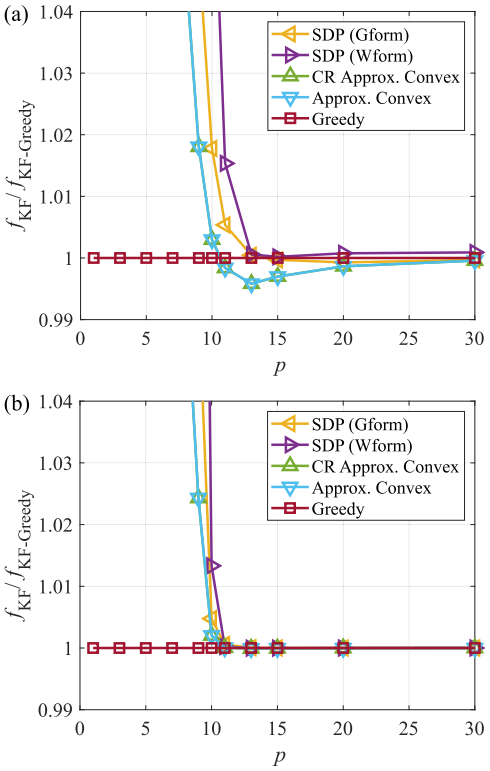

Fig. 4. Normalized objective value with respect to the number of selected sensors for different noise ratios when the system noise is stronger than the observation noise: (a) $\sigma_w/\sigma_v = 10^{1}$ and (b) $\sigma_w/\sigma_v = 10^{3}$.

much higher than the greedy method. On the other hand, the objective value obtained by the approximate convex relaxation method is the best at $p \geq 10$, and the advantage of these methods remains at $p \leq 30$. The difference between each method becomes small as the number of selected sensors increases. The SPD with the weight formulation does not work well in the present condition, even though the SDP with the gain formulation can outperform the greedy method at $p > 20$. Overall, the greedy method shows good performance in the investigated range of the number of sensors to be selected.

The trend in the objective value changes because of the noise ratio. Figs. 3 and 4 show the normalized objective value with respect to the number of sensors selected for different noise ratios. The general trend at $\sigma_w/\sigma_v = 10^{-1}$ is the same as the case at $\sigma_w/\sigma_v = 10^{0}$. The behavior of the approximate convex relaxation method is almost the same as the case of $\sigma_w/\sigma_v = 10^{0}$. However, the difference between the SDP method with gain formulation and the greedy method becomes large when the number of selected sensors is large, and the performances of the SDP methods get worse. At $\sigma_w/\sigma_v = 10^{-3}$, the performance of the greedy method is the best in all investigated ranges of the number of selected sensors.

The result in the case that the strength of the system noise is stronger than the observation noise is shown in Fig. 4. When the system noise is stronger, the greedy method outperforms other methods at $p < 10$ the same as $\sigma_w/\sigma_v = 10^{0}$. The approximate convex relaxation methods can outperform other methods in $p < 10$, but the influence of the optimization method on the obtained objective value becomes small when the number of selected sensors is large. This trend is further pronounced at $\sigma_w/\sigma_v = 10^{3}$, and there is no difference in the obtained objective value in the most range of p.

3) Influence of Noise Ratio on Objective Value: Influences of the noise ratio on the obtained objective value are investigated at $p < r$ ($p = 5$) and $p > r$ ($p = 15$) conditions. The range of the noise ratio was set to be between $10^{-5} \leq \sigma_w/\sigma_v \leq 10^{5}$, and the number of potential sensor locations was fixed at $n = 1000$. The computations for each condition were conducted 50 times, and the average value was calculated.

Fig. 5 shows the influence of the noise ratio on the obtained objective values at $p = 5$ and $p = 15$. The greedy method provides the best or better result for all noise ratios investigated for both the conditions of $p < r$ and $p > r$. Particularly, the greedy method outperforms other methods for $p = 5$.

For $p < r$, the approximate convex relaxation method and the SDP with gain formulation are competitive with the greedy method only at $\sigma_w/\sigma_v = 10^{-5}$, but the greedy method should be used for all ranges of σ_w/σ_v. The behavior of the randomized approximate convex relaxation method is almost the same as that of the convex relaxation method in the most range of the noise ratio, but the negative influence of the randomization becomes apparent as the observation noise becomes strong.

For $p > r$, the advantage of the greedy method is decreased. Particularly, the obtained objective values are almost the same as each other when the system noise is stronger. On the other hand, there is a difference between each method when the

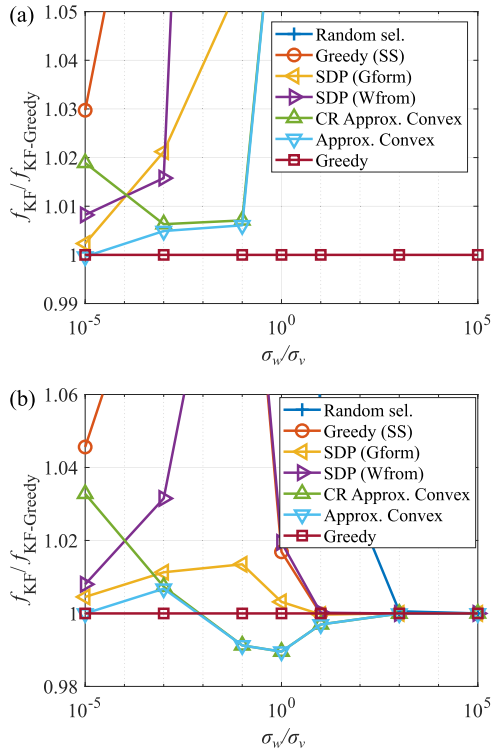

Fig. 5. Influence of the noise ratio on the obtained objective values at (a) $p = 5$ and (b) $p = 15$.

observation noise is stronger. The convex relaxation method and SDP with gain formulation also provide a good optimization result, as well as the greedy method. Furthermore, although the range is limited around $\sigma_w/\sigma_v = 10^0$, the approximate convex relaxation provides the best results. The influence of the noise ratio on the performance difference between the approximate convex relaxation method and the randomized approximate convex relaxation method is similar to the condition of $p < r$. The negative influence of the randomization becomes apparent as the observation noise becomes strong. Overall, the greedy method provides better performance constantly for the wide range of the noise ratio and the number of selected sensors.

V. Conclusion

Two new sensor selection methods for state estimation in a high-dimensional system using a Kalman filter were proposed based on the SDP with gain formulation and approximate convex relaxation, and the characteristics of the method were compared including the previously proposed. Even if the algorithm is the same, the performance of the algorithm depends on the objective function, the size of the problem, etc. In this research, we provided a new formulation for the objective function with the error covariance matrix of the Kalman filter, and the characteristics of the sensor selection method based on various algorithms for state estimation in a high-dimensional system using the Kalman filter were investigated.

The sensor selection methods based on different algorithms, SDP, approximate convex relaxation, and greedy algorithm, including newly formulated methods were implemented, and those performance and characteristics were assessed by varying the number of potential sensor locations, noise ratio, and the number of sensors to be selected. Sensors are selected based on the error covariance matrix of the Kalman filter, and p sensors are selected from the n potential sensor locations. The sensor subset gives an observation vector of a linear function of latent variables superimposed with independent identically distributed zero-mean Gaussian random noise.

Although the condition was limited when the ratio of system noise and observation noise was similar, the approximate convex relaxation method and its randomized method are effective in terms of computational time and objective value at $p \gtrapprox r$. The objective value obtained by the greedy method shows the best results compared with those of the other methods in almost all investigated conditions. Particularly, the greedy method outperforms other methods when the number of sensors is small. In addition, only the greedy method can handle the large-scale problem of $n > \mathcal{O}(10^4)$. Overall, it is found that the greedy method is better in large-scale problems in terms of computation time and the performance of the sensor set obtained.

Appendix

Minimization of the stochastic H_2 norm of (19) corresponds to the following equation with given impulse response after transformation:

$$\min\ J = \mathrm{E}\left(\sum_{k=0}^{\infty} \|\tilde{\mathbf{x}}_k\|_2^2\right). \tag{38}$$

Here,

$$\begin{aligned} J &= \mathrm{E}\left(\sum_{k=0}^{\infty} \|\tilde{\mathbf{x}}_k\|_2^2\right) \\ &= \mathrm{trace}\left((\mathbf{R} + \mathbf{L}\mathbf{Q}\mathbf{L}^*)\sum_{k=0}^{\infty}\left((\mathbf{A}'^*)^k(\mathbf{A}')^k\right)\right) \end{aligned} \tag{39}$$

whereas

$$\mathbf{A}' = \mathbf{A} - \mathbf{L}\mathbf{C}. \tag{40}$$

Therefore, the problem above can be rewritten as follows:

$$\min\ J = \mathrm{trace}\left((\mathbf{R} + \mathbf{L}\mathbf{Q}\mathbf{L}^*)\mathbf{X}\right) \tag{41}$$

$$\mathbf{X} \succ \sum_{k=0}^{\infty}\left((\mathbf{A}'^*)^k(\mathbf{A}')^k\right). \tag{42}$$

Here, (42) leads to the following Lyapunov-type inequality:

$$\mathbf{X} - \mathbf{A}'^*\mathbf{X}\mathbf{A}' \succ \mathbf{I} \tag{43}$$

which corresponds to

$$\mathbf{X} - (\mathbf{A} - \mathbf{L}\mathbf{C})^*\mathbf{X}(\mathbf{A} - \mathbf{L}\mathbf{C}) - \mathbf{I} \succ \mathbf{0}. \tag{44}$$

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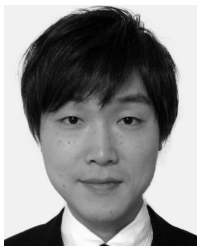

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