Weak Feature Extraction and Strong Noise Suppression for SSVEP-EEG Based on Chaotic Detection Technology

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***Abstract*—Brain computer interface (BCI) is a novel communication method that does not rely on the normal neural pathway between the brain and muscle of human. It can transform mental activities into relevant commands to control external equipment and establish direct communication pathway. Among different paradigms, steady-state visual evoked potential (SSVEP) is widely used due to its certain periodicity and stability of control. However, electroencephalogram (EEG) of SSVEP is extremely weak and companied with multi-scale and strong noise. Existing algorithms for classification are based on the principle of template matching and spatial filtering, which cannot obtain satisfied performance of feature extraction under the multi-scale noise. Especially for the subjects produce weak response for external stimuli in EEG representation, i.e., BCI-Illiteracy subject, traditional algorithms are difficult to recognize the internal patterns of brain. To address this issue, a novel method based on Chaos theory is proposed to extract feature of SSVEP. The rule of this method is applying the peculiarity of nonlinear dynamics system to detect feature of SSVEP by judging the state changes of chaotic systems after adding weak EEG. To evaluate the validity of proposed method, this research recruit 32 subjects to participate the experiment. All subjects are divided into two groups according to the preliminary classification accuracy (mean acc >70% or <70%) by canonical correlation analysis and we define the accuracy above 70% as group A (normal subjects), below 70% as group B (BCI-Illiteracy). Then, the classification accuracy and information transmission rate of two groups are verified using Chaotic theory. Experimental results show that all classification methods using in our study achieve good performance for normal subjects while chaos obtain excellent performance and significant improvements than traditional methods for BCI-Illiteracy.**

Manuscript received December 10, 2020; revised February 14, 2021; accepted April 11, 2021. Date of publication April 19, 2021; date of current version May 12, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 51775415, in part by the Xi'an Key Laboratory of Brain Computer Interaction and Neuroregulation Digital Medicine, and in part by the Xi'an Science and Technology Plan Project under Grant 20KYPT0001-10. *(Corresponding author: Guanghua Xu.)*

This work involved human subjects or animals in its research. Approval of all ethical and experimental procedures and protocols was granted by the Ethics Committee of Xi'an Jiaotong University.

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Digital Object Identifier 10.1109/TNSRE.2021.3073918

***Index Terms*—Brain computer interface (BCI), electroencephalogram (EEG), steady-state visual evoked potentials (SSVEP), Chaotic detection (Chaos), BCI-Illiteracy.**

I. Introduction

WITH the development of information technology, weak signal detection has been widely used, involving many fields such as signal detection for early fault, seismic wave and bioelectrical information [1], [2].

Brain-computer interface (BCI) is an artificially constructed communication pathway between human brain and external environment, i.e., the potential activity of brain can be acquired and decoded into commands to control external devices using BCIs [3]. According to different signal acquisition methods, BCI can be divided into invasive and non- invasive. Among them, non- invasive BCIs are widely used to control the external devices by recording scalp EEG due to its convenient operation [4]. VEP is the response of the brain to external visual stimuli. Studies [5] have shown that when human subject to periodic visual stimuli, an electrical activity component corresponding to the stimulation frequency will be generated in the visual cortex region of the brain, that is, steady-state visual evoked potential (SSVEP). Compared with other paradigms, SSVEPs have more stable and significant feature representations and higher information transmission rate (ITR), as well as simpler system and experimental operation [6]. Commonly, the system framework of non- invasive BCI based on EEG consists of three parts: data acquisition, signal processing and external device control. Among them, signal processing is the key of the whole system. However, the evoked EEG amplitude is generally in the microvolt level and often submerged in multi-scale and strong noise, which puts forward higher requirements for the feature extraction for SSVEP [7]. At present, the mainstreams of SSVEP feature extraction algorithms are mostly based on spatial filtering, canonical correlation analysis (CCA) [8], minimum energy combination (MEC) [9] and common spatial pattern (CSP) [10]. In these methods, CCA has been widely used due to its high efficiency and simplicity [11]. To improve the performance of this benchmark method, there are various improved CCA algorithms were proposed. In 2014, Wang et al [12] proposed extended

CCA (ECCA) using pre-trained strategy, which collected data across subjects/target frequencies as templates and performed feature fusion to extract features. In 2015, Chen [13] et al proposed the filter bank CCA (FBCCA), which further improved the recognition accuracy and information transmission rate by conducting band-pass filtering of different sub-bands and combining with ECCA. In 2017, Naskanishi [14] et al proposed the task-related component analysis (TRCA), which decomposed the raw EEG signals into useful components and noise parts and carried out ECCA analysis for the useful components. Most of these algorithms are based on the principle of spatial filtering and template matching. They can achieve satisfied results in the certain range, but there still are several challenges for SSVEP feature recognition.

Firstly, EEG signal has weak amplitude and retains strong nonlinear and non-stationary. Additionally, useful component of signal is often coupled with multi-scale noise signal. Excessive noise suppression will cause attenuation of useful signal, while ignoring the influence of noise makes it difficult to recognize patterns. How to effectively suppress noise to achieve optimal identification is still to be explored [15].

Secondly, due to the differences in physiological structure and state, there are significant differences for the feature representations of EEG signals across subjects/sessions [16]. Traditional methods use pre-training data to train the model, which increases the calibration time and reduces the efficiency of BCI. However, training-free method would lead to low accuracy.

Thirdly, numerous studies have shown that even after a long time of training, some users still cannot control a specific BCI system. One of the possible reasons is that features are extremely weak and overwhelmed by irrelevant components. Therefore, improving the performance of the classification model is conducive to solving the problem of BCI-Illiteracy [17].

In recent years, weak signal detection technology has been widely used in bioelectrical signal processing fields. Chaotic detection (Chaos) is a signal detection method based on nonlinear-chaos theory, which mainly uses the feature of the nonlinear dynamics system to detect weak signals [18]. During the calculation, a chaotic system is firstly constructed by nonlinear equation, then judge the state changes after adding the collected signal into the chaotic system. The existence of weak feature is determined by whether this system changes from chaotic state to large-scale periodic state. The principle of chaos is to establish the mapping relationship between frequency feature of EEG and dynamics state of chaotic system, while this correspondence is unique [19]. Therefore, Chaos has huge potential in suppressing noise during feature extraction for EEG. To solve the limitations of feature extraction for SSVEP in current studies, we proposed a novel framework to recognize the target patterns, which modeled the signal as the external driving force and put it into nonlinear oscillation system, then performed EEG pattern recognition by judging the state change of the system. By experimental verification and conclusion, the main contributions of this paper are as follows:

Firstly, to address the issues of feature extraction for SSVEP under the multi-scale and strong noise, a feature detection method based on Chaos theory is proposed. By associating the state transformation of chaotic system with the specific features existence of EEG signals, the target frequency of SSVEP signals can be accurately recognized under different levels of noise.

Secondly, in view of the limitations of the traditional methods for state discrimination of chaotic system, this study proposes a fast state detection method for chaotic system based on spectrum difference, which is named spectrum symmetry of chaotic system (SSCS). We apply it to the frequency detection of SSVEP and obtain high classification accuracy and ITR, which creates a novel methodology to feature extraction for SSVEP.

Thirdly, we apply the proposed method for BCI-Illiteracy subjects in SSVEP task. Experimental results proved that it can significantly improve the identification accuracy of BCI-Illiteracy and realize the effective detection of these subjects.

II. Methodology

A. Principle of SSVEP Detection Based on Chaotic Theory

Chaos theory focus on the research for the complex and unpredictable trajectory of chaotic motion in definite nonlinear system, which is always confined to a finite area but along with complex and unrepeated behaviors. Meanwhile, noise immunity is one of the main characteristics of chaotic motion, which is also the theoretical basis of weak signal detection. In recent years, the Van der Pol equation and Duffing equation have been used to construct chaotic systems for weak signal detection and achieve good results [20], [21]. In this study, the chaotic system based on Duffing equation was used to establish feature extraction model in SSVEP-BCI task.

The Duffing equation describes a typical nonlinear vibration system and can be used to simulate many nonlinear vibration phenomena, such as ship lateral shaking, structural vibration or chemical bond damage. The differential equation is generally expressed as:

$$\ddot{x} = \varphi(x)\dot{x} + f(x) = E(t) \tag{1}$$

where $\ddot{x}$ is accelerated velocity, $\dot{x}$ is velocity and x is displacement. $\varphi(x)$ represents damping force, $f(x)$ represents restoring force and $E(t)$ is driving force.

Based on this mathematical model, many engineering problems can be further reduced to the Lennar equation as shown below:

$$\ddot{x} + g(x)\dot{x} + f(x) = E(t) \tag{2}$$

Duffing presents the standard Duffing equation based on equation 2 to describe the spring effect, the mechanical problem is shown below:

$$\ddot{x} + k\dot{x} + ax + bx^3 = \gamma\cos(\omega t) \tag{3}$$

where k is damping coefficient, $ax+bx^3$ is nonlinear restoring force, a and b are linear stiffness of the spring and nonlinear

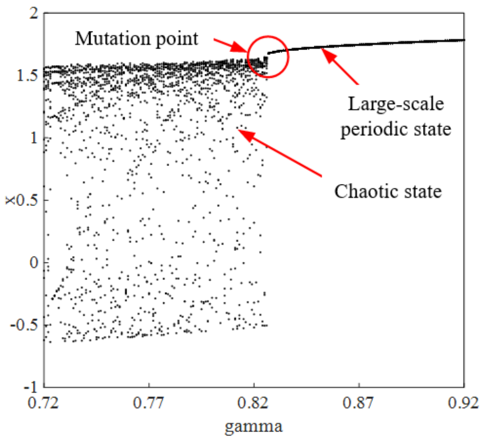


Fig. 1. A bifurcation diagram of dynamics system from chaotic state to large-scale periodic state. It is solved by differential equations for chaotic system and describe the relationship between the time series and displacement state of system in different states.

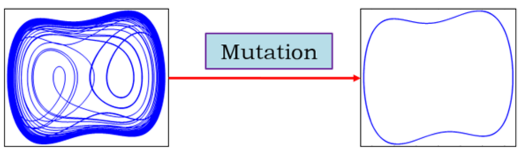


Fig. 2. A phase transition diagram of dynamics system from chaotic state to large-scale periodic state. It represents the dynamic relationship between velocity variable and displacement variable.

stiffness of the spring respectively. $\gamma \cos(\omega t)$ is driving force and γ is amplitude.

As a kind of nonlinear dynamics system, chaotic system has various dynamics characteristics and multiple parameter combination. Among them, various driving force will cause different dynamics characteristics of chaotic system, which can be reflected by the bifurcation diagram. Among all the state of a chaotic system, the change from chaotic state to large-scale periodic state has mutability and uniqueness (Fig.1), which forms the basis of weak signal chaos detection.

In practical application, when weak signals with the same frequency as driving force are introduced into the critical chaotic system, the state of system will change due to the disturbance to internal periodic driving force of the system (Fig 2).

EEG based on SSVEP has stable feature of periodic frequency, so it can be introduced into the chaotic system as an external disturbance. Based on this principle, the target frequency detection of SSVEP can be realized by discriminating the change of chaotic system state after adding EEG to chaotic system.

The expression of Chaos model of weak periodic signal can be written as follows:

$$\ddot{x} + k\dot{x} - ax + bx^3 = \gamma \cos(\omega t) + S(\omega t) \tag{4}$$

where $S(t)$ represents the EEG with specific frequency. With the increase of γ value, the output state of the system will show the alternation of chaotic state, critical state and large-scale periodic state. To address this issue, scale transformation method was adopted to detect signals of random frequency. If we define $t = R\tau$, the displacement, accelerated velocity and velocity are redefined as:

$$x(t) = x(R\tau) = x_\tau(\tau) \tag{5}$$

$$\dot{x}(t) = \frac{dx(t)}{dt} = \frac{1}{R}\frac{dx_\tau(\tau)}{d\tau} = \frac{1}{R}\dot{x}_\tau(\tau) \tag{6}$$

$$\ddot{x}(t) = \frac{1}{R^2}\frac{d^2x_\tau(\tau)}{d\tau^2} = \frac{1}{R^2}\ddot{x}_\tau(\tau) \tag{7}$$

Put the above formula into Formula 3, and the Duffing oscillator equation can be expressed by the following equation:

$$\frac{1}{R^2}\ddot{x}(R\tau) + k\dot{x}(R\tau) - ax(R\tau) + bx^3(R\tau) = \gamma \cos(\omega\tau) \tag{8}$$

The dynamics equation of duffing oscillator can be expressed as:

$$\begin{aligned} Ry &= \dot{x} \\ \dot{y} &= R(-ky + ax - bx^3 + \gamma \cos(\omega\tau)) \end{aligned} \tag{9}$$

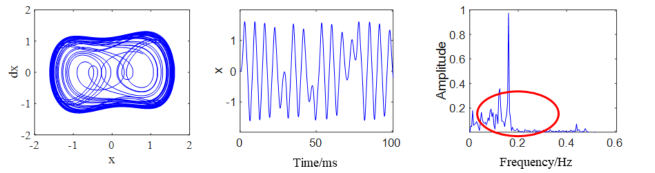
As a feature extraction model for weak signal, there are two necessary conditions must be met: a definite critical point for phase transition, the retentivity of chaotic state interval. The key factor to satisfy these conditions is the parameters selection of oscillator equation. In the previous study [23], we discussed the influence of these parameters on the performance of the detection model. Therefore, we use the conclusions in the previous study and set the Duffing equation in the following form:

$$\ddot{x} + 0.5\dot{x} - 0.6x + 0.1x^3 = \gamma \cos(\omega t) + S(\omega t) \tag{10}$$

B. A Fast-Quantitative Identification Method for Chaotic System State Based on Spectral Difference

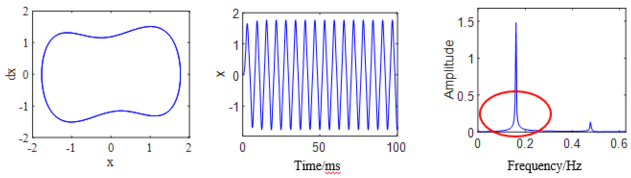
During Chaos for weak signal, the state discrimination of chaotic system is a very important step. At present, the typical discriminant methods of chaotic systems include intuitive method and quantitative method. Among them, the intuitive method determines whether the state changes by analyzing the rule of phase plane diagram for chaotic system [24]. As shown in the previous section, this method is simple and do not require complex calculations, but it is a subjective criterion and cannot be applied to automated detection processes. Another method is quantitative calculation, which calculates the characteristic variables of the chaotic system and determines the change of chaotic state. The typical quantitative index contains Lyapunov exponents, Fractal dimension and Kolmogorov entropy [25]–[27]. Quantitative calculation can guarantee the accuracy of discrimination, but it takes a long-calculation time and enough data length of detected sample. To solve these limitations, this study proposes a fast-quantitative identification method for chaotic system state based on symmetry of spectrum of system variables. The specific research ideas are as follows:

During the calculation for oscillator equitation, one iteration of the equation yields a set of analytic solutions. By recording the whole process of iteration, we can draw phase trajectory using velocity and displacement from each step (Figure 3a).



(a) Phase trajectory (b) Time domain wave (c) Frequency Spectrum

Fig. 3. (a) Phase trajectory in chaotic state (b) Displacement-iteration time waveform in chaotic state (c) Frequency Spectrum for displacement-iteration.



(a) Phase trajectory (b) Time domain wave (c) Frequency Spectrum

Fig. 4. (a) Phase trajectory in periodic state (b) Displacement-iteration time waveform in periodic state (c) Frequency Spectrum for displacement-iteration time waveform.

Moreover, by taking the displacement obtained from each step as the dependent variable of iteration time, we can plot one-dimensional time series waveform (Figure 3b). While the difference of time waveform inevitably leads to the difference of frequency spectrum (Figure 3c), which contain the discriminant information between different system state. Therefore, the problem of state discrimination can be transformed into the problem of identifying the frequency feature for displacement-iteration time waveform. To visually display the spectral differences between time series between chaotic states and large-scale periodic states, the phase trajectories, time waveforms and corresponding spectrum of two states are shown in Figure 3 and Figure 4.

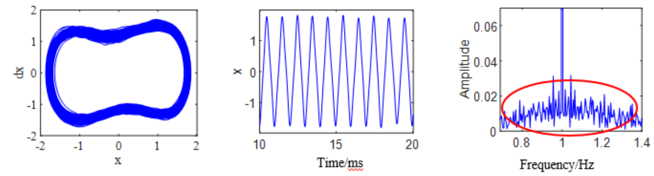
The displacement-time (x-t) waveform of dynamics system in chaotic state is irregular, which essentially is complex aperiodic signals (Fig 3b).

While waveform in large-scale periodic state is a stable periodic signal (Fig 4b). Therefore, it is easy to conclude that that the frequency spectrums of the two signals show asymmetry and symmetry respectively (Fig 3c, Fig 4c).

Under the influence of noise, the phase trajectories of large-scale periodic state become rough, which results in the fluctuation of spectrum and thus brings calculation error (Fig 5). However, we find that the data on both sides of main peak of spectrum in chaotic state show asymmetry, while it shows symmetry in periodic state whether suffering from noise. Therefore, the quantitative discrimination method can be designed based on the phenomenon of symmetry difference.

In this study, we propose a novel quantitative state discrimination method based on the symmetry information of spectrum, which is named as spectrum symmetry of chaotic system (SSCS). The calculation is shown as follows:

Firstly, calculate equation (10) using 4-order Runge-Kutta method. Then, transform the time-series wave of $x-t$ and $\dot{x}-t$



(a) Phase trajectory (b) Time domain wave (c) Frequency Spectrum

Fig. 5. (a) Phase trajectory in periodic state with Gaussian noise (b) (c) Same as above waveform in chaotic state (c) Frequency Spectrum for displacement-iteration time waveform.

into frequency domain using fast Fourier transform (FFT).

$$Y = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt \tag{11}$$

Next, calculate the value of SSCS

$$SSCS = \frac{\sum_{i=1}^{M-1} Y_i}{\sum_{j=M+1}^{2M-1} Y_j} \tag{12}$$

where Y is FFT value of spectrum and M is the abscissa value corresponding to the main peak.

Target-external signal can cause mutation for state in the chaotic system. However, if the phase of driving force is opposite to the external signal, the effect of mutation will become weaker or disappear. To avoid the error of identification, we search suitable phase interval using fixed step after calculation by SSCS.

In the experimental part, we will analyze the parameters selection of chaotic system and discuss the verification results of feature extraction for EEG-SSVEP.

C. Feature Extraction for SSVEP Based on Chaotic Detection Technology

When human receive a visual stimulus such as flicker or flashing at a fixed frequency, the potential activity of the cerebral cortex is modulated to produce a continuous response related to the stimulus frequency (same or multiples). This response has a periodic rhythm similar to the external visual stimuli, that is, SSVEP [6]. The feature representation of SSVEP is that the power spectrum of EEG can produce the same spectrum peak or harmonic as the stimulus frequency. By detecting the frequency corresponding to the spectral peak, the intention of the subject can be recognized. However, this raw EEG is generally weak, which is easily submerged by multi-scale noise and difficult to extract. According to the above research, Chaos method has the advantages of high sensitivity for weak signal and noise immunity, so it is suitable for the identification of target frequency for SSVEP. In our study, the framework of Chaos for SSVEP is as Figure 6.

There is a prerequisite for the detection, that is, input signal should transform into one-dimensional vector before input into system. However, EEG are often collected in the form of multi-channels. To make full use of the advantages of multi-channel EEG signals and meet the requirements of the chaotic calculation, we adopt the common average reference [28] to conduct dimension reduction processing for multi-channel EEG signals

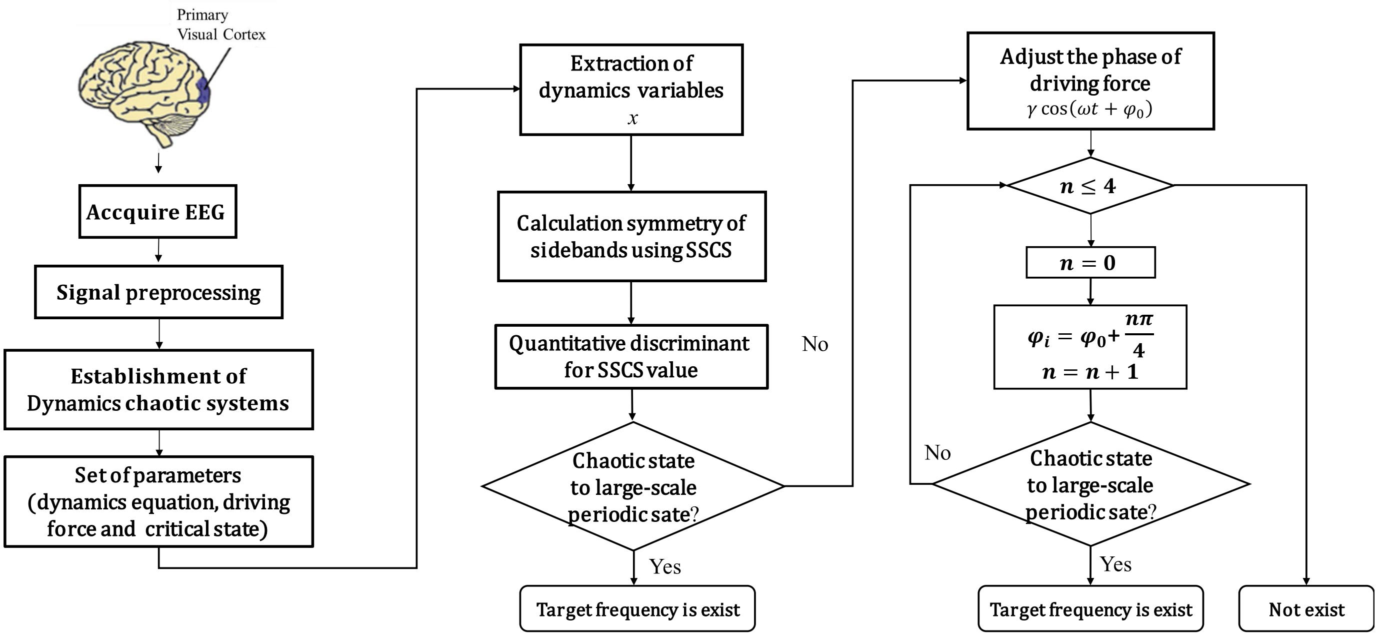


Fig. 6. Detection Framework for SSVEP based on Chaos.

during preprocessing. The definition is as following:

$$V_i^{CAR} = V_i - \frac{i}{n}\sum_{j=1}^{n} V_j \tag{13}$$

where V_i^{CAR} is potential difference between electrode i and reference electrode, n is the number of channels. In our experiment, we select Oz channel as target channel.

In the application of EEG-BCIs, research show that 20% subjects cannot expertly control BCI systems with an accuracy less than 70% even they have received long time training [29], [30]. The phenomenon of BCI-Illiteracy may result from two main reasons. One reason is that due to the differences of physiological structure across individuals, nerves to be detected are located in the sulci of the brain or close to another large-group of neurons [31]. Therefore, the specific potential activity cannot be detected on the scalp for this kind of users. Another reason of BCI-Illiteracy is that subjects produces too much muscle artifact or irrelated potential activity in BCI tasks, while weak target features are submerged by these multi-scale noise. Researchers have explored extensive research to address this challenge, such as optimizing signal processing algorithms [32], [33] or using hybrid brain-machine interface paradigm [34] to enhance the brain response of subjects.

In our study, we define BCI-Illiteracy into two types:

- BCI-Illiteracy-I: The brain is unable to respond to the specific external stimuli and produce correspond feature of EEG
- BCI-Illiteracy-II: The brain can produce a response for specific external, but the representation of EEG is too weak to detect.

From above study, the chaotic system is highly sensitive to the signal with specific frequency, so we adopt Chaos technology to extract feature in SSVEP task for the BCI-Illiteracy-II.

D. Performance Evaluation

To evaluate the performance of SSVEP-BCIs based on Chaos, classification accuracy and ITR are used testify the performance. The mathematical definition of ITR is shown as follows:

$$ITR = \left\{log_2 N + Plog_2 P + (1-P)\times log_2\frac{1-P}{N-1}\right\} \times (60/T) \tag{14}$$

where N represents the number of instructions sets, P represents the classification accuracy, and T represents the data length for analysis.

Meanwhile, the contrast of chaos and typical algorithms—standard CCA, multivariate synchronization index (MSI) [35] and FBCCA were adopted.

III. Experiment and Results

A. Participants

32 subjects (males: females = 1:1, mean age± SD, 23.7 ± 3.2 years) participated in this experiment. All the subjects had normal or corrected normal visual acuity. This study was approved by the ethics committee of the Xi'an Jiaotong University. All subjects were informed of all procedures and signed an informed consent agreement.

B. Data Acquisition

EEG were recorded using Neuracle NeuSen W system with 8 electrodes placed at POz, PO3, PO4, PO5, PO6, Oz, O1 and O2 based on the rule of international 10/20 system. And the reference electrode was placed at CPz and the ground at AFz. EEG signal were sampled at 1000Hz and filtered by a 50Hz notch filter and filtered by a 0.1-100Hz bandpass filter. Each subject participated in 3 sessions experiment and each session contain 25 trials. The experimental process is shown in Figure 7:

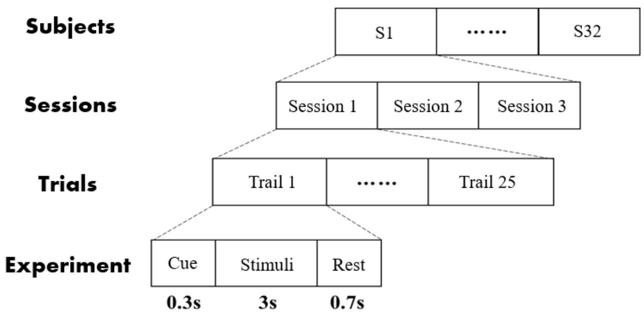


Fig. 7. Experimental scheme.

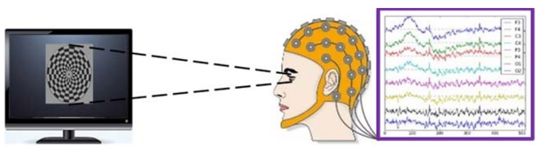


Fig. 8. Motion stimuli in SSVEP experiment.

C. SSVEP Experiment

The frequency range of visual stimuli is 3Hz$\sim$20Hz and the interval is 0.5Hz. To ensure the consistency of frequency, the system will automatically adjust the frequency of driving force by indexing the label of stimulus frequency before detection. As shown in Figure 8, we adopted the stimulation pattern based on the motion of ring-shaped checkerboard proposed in previous studies [36]. Compared with traditional SSVEP patterns (light-flashing pattern), few harmonic responses were elicited by ring-shaped checkerboard pattern and the frequency energy of it was concentrative [37]. One stimulus appears on the display screen at a time and is randomly selected within frequency range.

D. Time-Frequency Analysis for EEG

Prior to the formal experiment, CCA was used to classify the operation level for BCI of all subject(Group A:acc>70%, Group B: acc<70%). After testing, 13 subjects were assigned to group A and B respectively. To show the difference of feature distribution in normal subjects and BCI-Illiteracy, Short-time Fourier transform (STFT) was used to transform EEG signal into time-frequency domain to visually display the difference.

As shown in group A (Fig 9), the spectrums corresponding to target frequency (6Hz) obtain high resolution and the irrelated component is relatively weak. However, the spectrums from group B show that the target feature is extremely weak and submerged by multi-scale and strong noise even after pre-processing of EEG. Therefore, it is not difficult to infer that the traditional decoding algorithm tends to show worse performance of feature extraction for BCI-Illiteracy.

E. Parameter Selection of Chaotic System

As mentioned above, the transformation of the system from chaotic state to large-scale periodic state is unique. By setting the system to the critical state and introducing an external

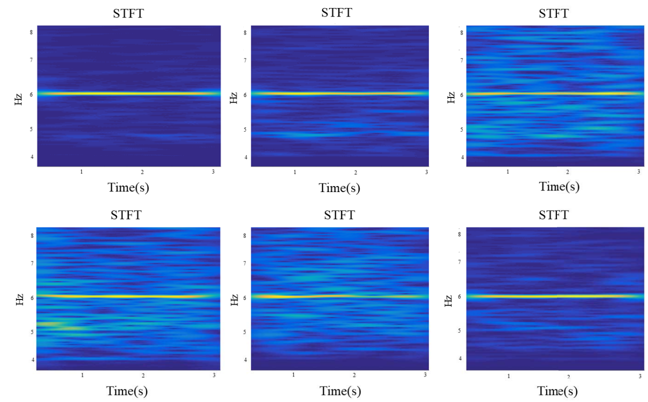


Fig. 9. Power spectrum of EEG from Group A.

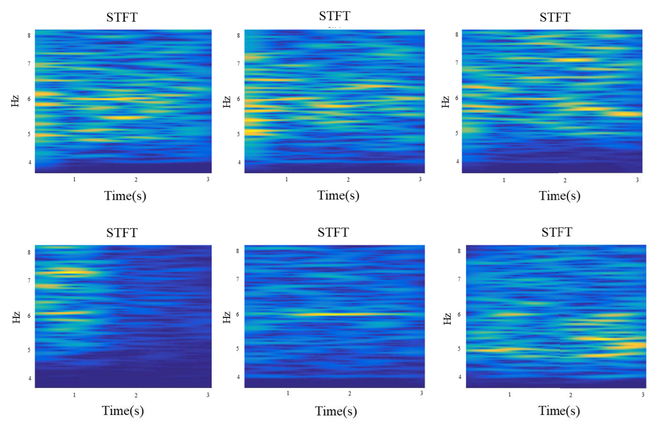


Fig. 10. Power spectrum of EEG from Group B.

signal into system, the existence of target frequency can be detected by determining whether the system has changed the state. To explore the specific value of the abrupt transition under the equation 10, we applied the bifurcation graph method of chaotic system to search this point (Figure 11).

As shown in Figure 11, when value of γ is between 1.980$\sim$2.294, system is in chaotic state. While γ belong to 2.295 to 3.000, system show in periodic state. It can be concluded that phase transition critical point is 2.295. Therefore, the equation for feature detection in our research is defined as:

$$\ddot{x} + 0.5\dot{x} - 0.6x + 0.1x^3 = 2.295\cos(\omega t) + EEG(\omega t) \quad (15)$$

Meanwhile, we set four variables $x, \dot{x}, x - \dot{x}, x + \dot{x}$ to calculate the value of SCCS in the chaotic state and periodic state respectively.

From Table I, we can find that the value of SSCS for four variables are greater than 2.0. Among them, x can obtain the largest mean value and standard deviation, while $\dot{x}$ is the smallest.

From Table II, it is not difficult to find that all variables are close to 1, which reveal the symmetry of the spectrum in large-periodic states. Compare the results of Table I and II, the rang of distribution of SSCS between different variables are shown in Figure 12.

TABLE I
THE VALUE OF SSCS IN CHAOTIC STATE

γ	SSCS			
	x	$\dot{x}$	$x-\dot{x}$	$x+\dot{x}$
2.20	5.728	1.839	3.755	3.445
2.21	6.137	2.112	4.000	3.919
2.22	5.595	2.014	3.725	3.599
2.23	5.887	2.309	4.049	3.958
2.24	8.030	2.675	4.796	5.319
2.25	5.326	2.119	4.032	3.434
2.26	6.452	2.395	4.256	4.199
2.27	6.697	2.516	4.623	4.220
2.28	5.374	2.091	3.732	3.582
2.29	4.525	2.313	3.748	3.160
Mean± SD	5.975± 0.948	2.238± 0.250	4.071± 0.382	3.878± 0.615

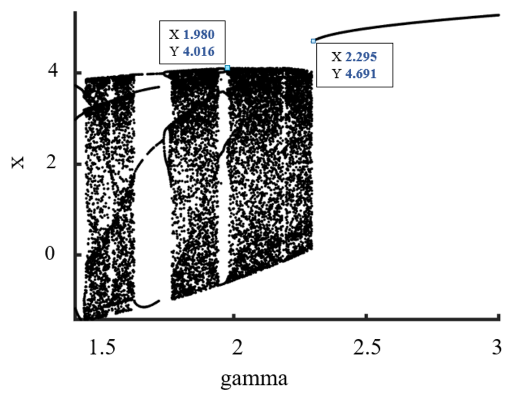


Fig. 11. Bifurcation diagram of equation 10.

TABLE II
THE VALUE OF SSCS IN LARGE-PERIODIC STATE

γ	SSCS			
	x	$\dot{x}$	$x-\dot{x}$	$x+\dot{x}$
2.30	0.832	1.169	1.322	0.724
2.31	0.768	1.171	1.239	0.748
2.32	0.759	1.169	1.187	0.766
2.33	0.740	1.167	1.146	0.782
2.34	0.725	1.165	1.114	0.796
2.35	0.714	1.163	1.087	0.809
2.36	0.704	1.161	1.063	0.821
2.37	0.696	1.160	1.042	0.933
2.38	0.690	1.159	1.024	0.844
2.39	0.685	1.159	1.008	0.855
Mean± SD	0.733± 0.047	1.164± 0.004	1.123± 0.101	0.798± 0.043

We define the difference value of SSCS between chaotic state and large-periodic state for all variables as inter-class. Among all variables, the displacement x can get the maximum inter-class. Therefore, x is adopted to be the discriminant index of SSCS and the threshold is set as 2.0.

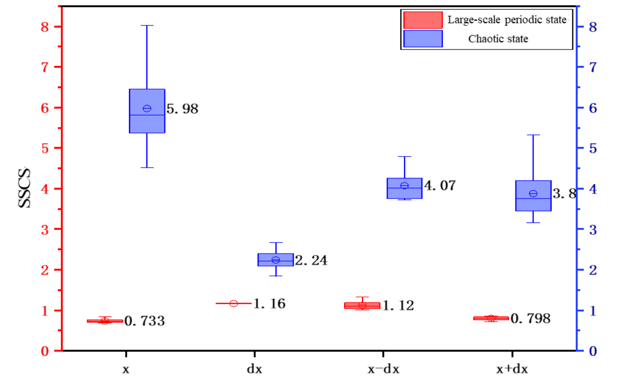


Fig. 12. The value distribution of different variables for large-scale periodic state and chaotic state.

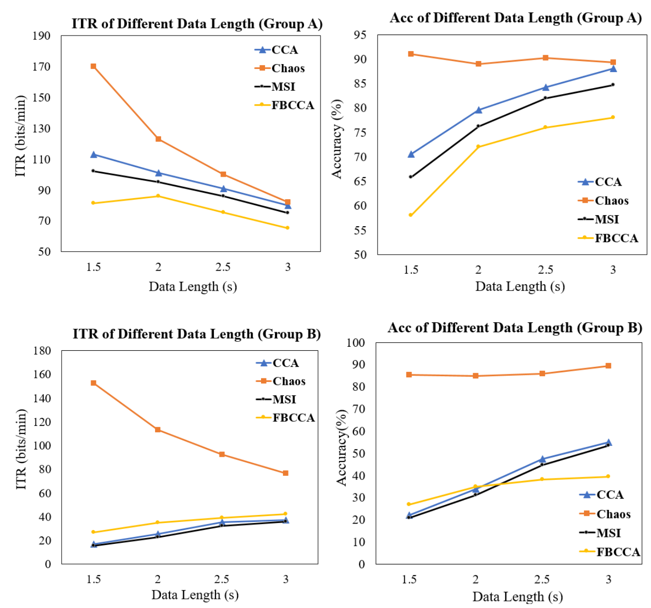


Fig. 13. ITR and accuracy in different data length of two groups.

F. BCI Performance

During the detection, the data length affects the reliability of the state change in chaotic system. For example, too short length may not induce a phase change in the system even if it met the requirement of threshold condition. To obtain the best ITR, we compared classification results during different data length (3s, 2.5s, 2s, and 1.5s) for two group.

Firstly, we analyzed the mean accuracy and ITR of 16 subjects in group A (Fig 13 and Table IV). As shown in above, almost all algorithms can achieve best ITR at the data length of 1.5s. For normal subjects, ITR can be effectively improved by shortening the length of data and Chaos tends to grow faster compare with three other methods. Among all subjects, S15 can obtain the highest ITR = 198.37bits/min.

Same analysis were carried out on group B. The experimental results are shown in Figure 13 and Table V. Unlike group A, ITR will decreases as the data gets shorter for all

TABLE III
THE COMPUTING TIME AMONG DIFFERENT DATA LENGTH

	Data Length			
	3s	2.5s	2s	1.5s
CCA	0.0147s	0.0135s	0.0083s	0.0071s
MSI	0.0084s	0.0078s	0.0074s	0.0070s
FBCCA	0.9225s	0.8133s	0.6883s	0.5598s
Chaos	0.0524s	0.0435s	0.0430s	0.0425s

methods except Chaos, and they obtain worse classification performance in group B. However, Chaos remain the similar results as group A and obtain excellent classification performance than other methods. The ITR of Chaos achieved the improvement of 114.68±10.88 compare with CCA under the 'best' data length. Paired t-tests were performed to compare the accuracy between Chaos and other methods. The results showed the accuracy rate of the Chaos was significantly higher than that of CCA ($p<0.001$), MSI ($p<0.001$) and FBCCA ($p<0.001$) under the data length from 1.5s to 3s. Among all subjects, S11 can achieve the score of 187.34 that even better than best subjects from group A. Moreover, we found that Chaos has good generalization ability across subjects/data lengths, which is suitable for the complex and variable environment in various SSVEP-BCI system.

From the experimental results in two group, we found that all methods show reliable performance for normal subjects, while only Chaos obtain expected results for BCI-Illiteracy. Note that Chaos can achieve best classification performance under different data length and CCA obtained similar results with MSI. However, we found that FBCCA haven't obtained expected results as described in previous studies [13]. One possible explanation is that the principle of FBCCA is to design spatial filter by weighting the main component and harmonic component of signal. However, EEG response lacks of harmonic components due to the spectral characteristics of ring-shaped checkerboard pattern. Therefore, it is difficult to obtain expected performance using FBCCA without harmonic component.

To ensure the timeliness of the application for Chaos, we compared the calculation time of all methods. (Table III).

From Table III, MSI can obtain fastest calculation time while FBCCA takes longest time. Though Chaos consume a longer calculate-time (six times higher than MSI), but the time-consuming can meet the requirements of timeliness in control of SSVEP-BCI.

G. Ability of Noise Suppression

One of the advantages of chaotic detection is its excellent performance of noise suppression. To verify this conclusion, we evaluate the accuracy of classification methods after adding different intensity levels of Gaussian noise to the raw EEG signal. In the experiment, we use the signal-noise rate (SNR) as equation 14 to quantify the noise intensity:

$$SNR = \frac{P_s}{P_n} \tag{16}$$

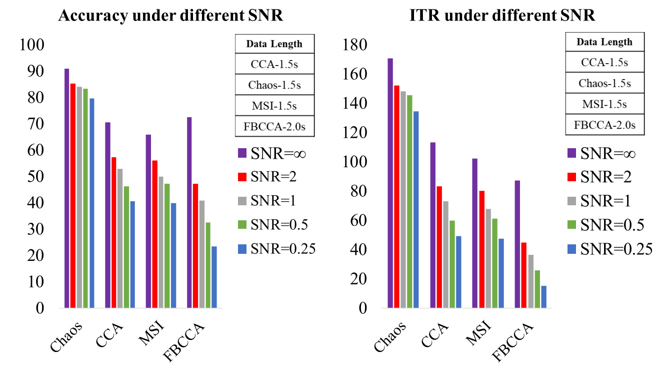


Fig. 14. ITR and accuracy under different noise levels.

where P_s and P_n is power of raw EEG signal and noise. To reduce the influence by other factors, we selected EEG data with best performance (ITR) group A to quantitatively evaluate the accuracy in different noise levels. We evaluate the accuracy and ITR results of EEG after adding four levels Gaussian noise.

As shown in Fig 14, with the decrease of SNR, the classification performance of all methods are reduced in various degree. Among four methods, the classification results based on chaos obtain best robustness and ability of noise suppression. Note that the performance of CCA, MSI and FBCCA are easily disturbed by external noise and they only achieve average accuracy less than 40% under the SNR = 0.25 even for normal subjects with best data length. While detection results using Chaos can obtain the accuracy of 79.50±4.64. Therefore, even strong noise may bring negative influence for classification performance, Chaos can provide reliable results of pattern recognition in SSVEP task.

IV. DISCUSSION

Previous research indicates [15] that noise suppression may attenuate or damage useful information of EEG. And when EEG cannot meet the requirement of stability, the suppression of useful signal may cause more negative effects than ignore of noise. While most of BCI-Illiteracy-II may not produce stable EEG, that is why most algorithms fail to decode EEG from these subjects with high accuracy and sensitivity. It has been demonstrated in the above studies that frequency feature detection based on Chaos theory achieve good performance no matter in normal subjects or BCI-Illiteracy. The reason why it differs from other algorithms is that Chaos apply the mechanism of specific dynamical response to external stimuli in chaotic system to weak signal detection, while this dynamics response is unique. Therefore, the multi-scale and strong noise cannot interfere with this model for pattern recognition, i.e., noise immunity. As mentioned above, BCI-Illiteracy-II can produce specific response to external stimuli, which can be detected by Chaos though it is extremely weak. Meanwhile, it's worth noting that Chaos is a training -free algorithms, which do not require any training data and allow user immediately start operation for BCI system.

Quantitative detection for phase transition in chaotic systems is a key step to realize feature recognition for weak

TABLE IV
CLASSIFICATION ACCURACY OF GROUP A (D = 1.5s)

Subject	Accuracy % (CCA) D=1.5s	ITR bits/min (CCA)	Accuracy % (Chaos) D=1.5s	ITR bits/min (Chaos)	Accuracy % (MSI) D=1.5s	ITR bits/min (MSI)	Accuracy % (FBCCA) D=1.5s	ITR bits/min (FBCCA)
S1	84.00	147.23	77.33	128.15	85.33	151.27	60.00	83.99
S2	53.33	70.33	92.00	172.80	50.67	64.78	48.33	58.41
S3	93.33	177.47	92.00	172.80	93.33	177.47	66.67	98.59
S4	69.33	107.19	96.12	187.33	70.67	110.56	63.78	91.65
S5	61.33	87.9	88.00	159.57	62.67	91.07	63.33	90.68
S6	78.67	131.84	96.33	177.47	41.33	46.67	65.02	97.58
S7	77.33	128.15	97.33	192.64	74.67	120.96	58.72	81.19
S8	57.33	78.96	94.67	182.33	50.67	64.78	67.33	100.61
S9	82.67	143.28	96.00	187.33	81.33	139.41	38.67	39.81
S10	69.33	107.19	92.00	172.80	66.67	100.61	50.01	63.42
S11	46.67	56.73	90.67	168.27	38.67	41.85	54.33	71.22
S12	56.00	76.04	73.33	117.43	48.00	59.40	49.33	61.73
S13	64.00	94.20	97.33	192.64	58.67	81.93	56.67	76.59
S14	60.00	84.93	82.67	143.28	60.00	84.93	53.33	69.46
S15	90.67	168.27	98.67	198.37	59.33	163.87	71.01	110.98
S16	85.33	151.26	93.33	177.47	80.00	135.59	69.00	106.78
Ave	70.58	113.17	91.11	170.67	65.75	102.20	58.47	81.42

TABLE V
CLASSIFICATION ACCURACY OF GROUP B (D = BEST DATA LENGTH)

Subject	Accuracy % (CCA) D=3s	ITR bits/min (CCA)	Accuracy % (Chaos) D=1.5s	ITR bits/min (Chaos)	Accuracy % (MSI) D=3s	ITR bits/min (MSI)	Accuracy % (FBCCA) D=3s	ITR bits/min (FBCCA)
S1	42.67	24.56	89.33	163.87	40.00	22.12	45.42	27.17
S2	61.33	53.99	76.00	124.53	53.33	35.17	62.50	45.34
S3	40.00	22.11	84.00	147.23	46.67	28.38	55.00	36.94
S4	53.33	35.17	78.67	131.84	48.00	29.70	70.83	55.49
S5	54.67	36.59	93.33	177.47	48.00	29.70	61.67	44.37
S6	54.67	36.59	90.67	168.28	52.00	33.77	52.08	33.86
S7	52.00	33.76	89.33	163.87	56.00	38.02	66.67	50.30
S8	68.00	51.84	82.67	143.28	61.33	43.99	39.58	21.74
S9	44.00	25.81	77.33	128.15	53.33	35.17	67.08	50.81
S10	54.67	36.59	85.33	151.26	58.67	40.96	58.33	40.59
S11	62.67	45.53	96.00	187.34	65.33	48.69	71.67	56.56
S12	69.33	53.59	89.33	163.87	66.67	50.30	53.75	35.61
S13	50.67	32.39	82.67	143.28	26.67	11.24	62.50	45.34
S14	58.67	40.96	90.67	168.27	60.00	42.47	38.33	20.63
S15	54.67	36.58	76.00	124.53	56.00	38.02	33.33	16.39
S16	58.67	40.94	85.33	151.26	64.00	47.10	65.83	49.29
Ave	55.01	37.72	85.41	152.40	53.50	35.93	55.92	39.40

signal. In previous studies, Lyapunov exponents is the most commonly used for quantitative detection, which is calculated by ratio of convergence and divergence of phase trajectory [38]. However, it takes a large amount of computing resources and time-consuming that is not suitable for the real-time detection in SSVEP-BCI task. In our study, we propose a novel method-SCCS to realize quantitative detection for system state. The essence of it is the dynamic observation of the solution of Duffing equations and dynamics feature extraction for this nonlinear system, which makes full use of the analytical results in the process of equation solution and realize discrimination without other complicated operation.

Compared to the calibration-based method (TRCA or multiset CCA), Chaos is a training-free method but the setting of parameters for dynamics system requires pre-search with repeat experiments which takes a long time. Therefore, it is valuable to develop a calibration-based chaos detection method. Based on the framework of Chaos detection, we consider it is a promising approach to extract the reproducibility and generality components of dynamics response after adding different EEG template signals and explore the best parameters combination of variables during these response.

There is an important prior of Chaos for SSVEP detection, that is, computer obtain the stimulus frequency in advance. The stimulated pattern with different frequency in our experiment is displayed in turns, whose label is easy to be indexed to adjust internally frequency of system. However, the application of Chaos for SSVEP-BCIs exists some limitations. For example, the 'freedom' control of mobile robot and wheelchair cannot be realized by Chaos-SSVEP because it depends on the improvisational intentions of users that cannot be detected by computer in advance. Therefore, we will explore the

SSVEP-BCIs based on Chaotic detection in ‘freedom’ control for various machines in the future research.

V. Conclusion

In this study, we proposed a novel feature detection algorithm for SSVEP based on chaos theory. Compared with tradition methods, this model transforms the problem of target frequency detection into state discrimination of nonlinear dynamics system and has the advantages of noise immunity and training-free. An experiment contains thirty-two subjects and thirty-five stimuli frequency were used to testify the performance of Chaos model. Results indicated that proposed model obtain significantly improvement of ITR and classification accuracy than traditional method, especially for BCI-Illiteracy subjects. Therefore, we can conclude that feature detection based on Chaos is more suitable for the scene with multi-scale and strong noise and used by BCI-Illiteracy.

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