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Suboptimal Longitudinal Reference Trajectory Computation For Time Based Continuous Descent Operations

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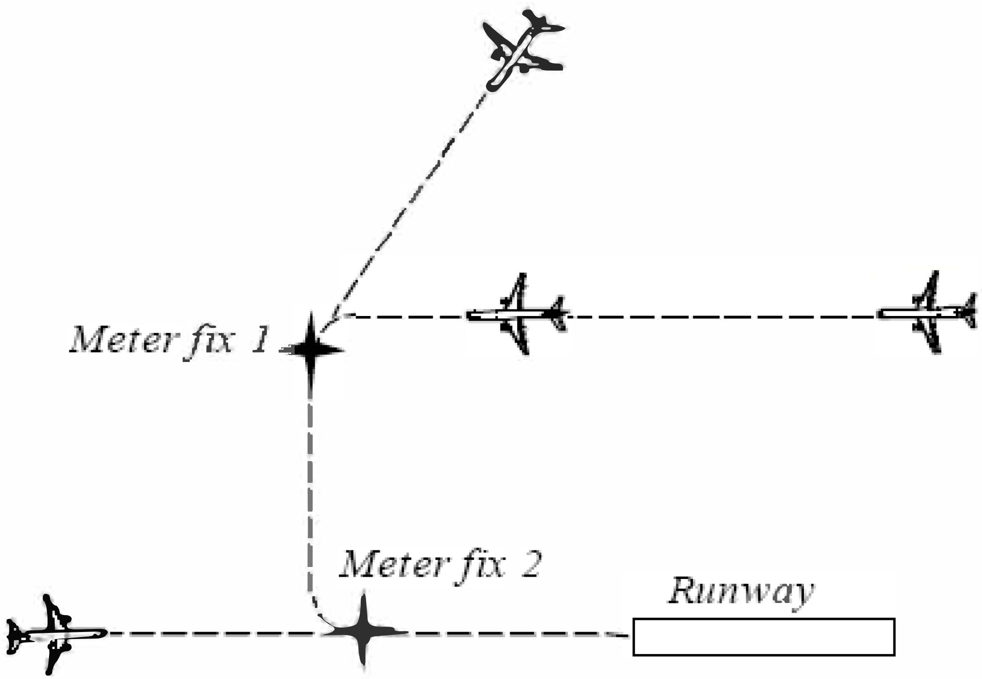
Context

- Continuous Descent Operation (CDO)
  - Continuously descending path, with a minimum of level flight segments
  - Reduction of fuel burn and emissions during descent

- Lot of programs in the world promote CDO
  - NUP2, SESAR (Europe)
  - NextGen (US)
  - AIRE (Europe / FAA)
  - …



Purpose of the presentation

- Futuristic 4D trajectory application
  - Air traffic controller will ask an aircraft to overfly the meter fix at a given altitude and airspeed to be correctly prepared to land
  - Time specified by the air traffic controller to settle properly the arrival sequence (through AMAN for example)
  - This clearance is assumed to be given after the Top Of Descent (TOD) of the aircraft.

- The presentation addresses the issue of computing reference **airspeed and height profiles** for time constrained continuous descent

Overview

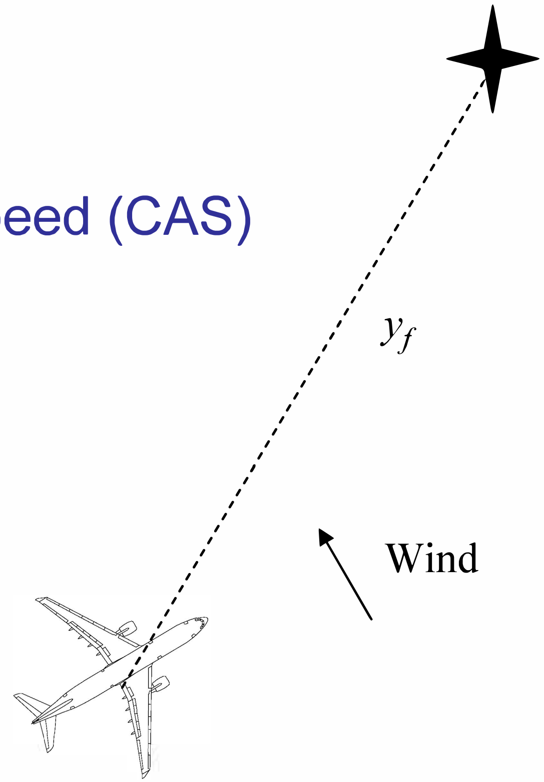
- ***Reference trajectory computation without optimization***
- Reference trajectory computation with optimization
- Simulation results
- Summary and future work

Reference trajectory computation without optimization

- Equations of motion: kinematics equations

$$\begin{cases} \dot{y} = V\cos(\gamma) + w_y(t) \\ \dot{h} = V\sin(\gamma) + w_h(t) \end{cases}$$

- NOT constant wind $w_h(t)$ and $w_y(t)$
- Problem: to compute a reference calibrated airspeed (CAS) and an reference height profile $h(t)$ such that after T sec of flight
  - The aircraft has flown a given distance y_f
  - And has a given final airspeed
  - Given initial height and airspeed
- This is a Two-point Boundary Value Problem (TPBVP)



Reference trajectory computation without optimization

- We will consider the following expression for reference speeds, where parameters a_0, a_1, a_2 and b are free parameters

$$V_r(\tau) = a_0 + \frac{a_1}{b\tau^2+1} + \frac{a_2}{b(\tau-1)^2+1}$$

  - Dimensionless parameter τ is the ratio between actual time t and the duration T to overfly the given meter fix

$$0 \le \tau \stackrel{\text{def}}{=} \frac{t}{T} \le 1$$

  - Integration of the preceding equation (with $l(0) = 0$) leads to the expression of the reference position (either height h or horizontal position y)

$$l(\tau) = T\left(a_0\tau + \frac{a_1}{\sqrt{b}}atan\left(\sqrt{b}\,\tau\right) + \frac{a_2}{\sqrt{b}}\left(atan\left(\sqrt{b}(\tau-1)\right) + atan\left(\sqrt{b}\right)\right)\right)$$

Reference trajectory computation without optimization

- Assuming that parameter *b* is already set, parameters a_0, a_1, a_2 can easily be computed to satisfy the two boundaries constraints

$$
\begin{cases}
V_r(0) \stackrel{\text{def}}{=} V_0 \Leftrightarrow a_0 + a_1 + \dfrac{a_2}{b+1} = V_0 \\
l(1) = d_f \Leftrightarrow a_0 + \dfrac{atan(\sqrt{b})}{\sqrt{b}}(a_1 + a_2) = \dfrac{d_f}{T} \\
V_r(1) = V_f \Leftrightarrow a_0 + \dfrac{a_1}{b+1} + a_2 = V_f
\end{cases}
$$

Reference trajectory computation without optimization

- Two steps process:
  - First compute the reference height *h(t)* such that after *T* sec of flight the aircraft has loss a given difference in altitude Δz_f and has a given vertical peed *Vz(0) and Vz(1)*
  - Then compute the reference calibrated airspeed (CAS) such that

$$\int_0^T TAS\big(h(t), CAS(t)\big)dt = y_f - \int_0^T w_y(t)dt$$

Computed in the first step

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Reference trajectory computation with optimization

- So far parameters b (actually (b_y,b_h)) have been assumed to be known
- Now parameters (b_y,b_h) will be set
  - to minimize fuel consumption
  - while ensuring that the maximum longitudinal and normal accelerations remain lower than

$$(b_h, b_y) = \arg\min \int_0^T f(t)\,dt$$

$$s.t. \begin{cases} \dot{y} = V\cos(\gamma) + w_y \\ \dot{h} = V\sin(\gamma) + w_h \\ h(0) = h_0, \quad \dot{h}(0) = 0 \\ h(T) = h_f, \quad \dot{h}(T) = 0 \\ y(0) = 0, \quad V(0) = V_0 \\ y(T) = y_f, \quad V(T) = V_f \end{cases} \quad and \begin{cases} \dot{V} \leq 2\,ft/s^2 \\ V\dot{\gamma} \leq 5\,ft/s^2 \end{cases}$$

Reference trajectory computation with optimization

- Fuel flow *f* is computed from BADA model

$$f = \max(\eta F, f_{\min}) \quad where \quad \begin{cases} \eta = C_{f1}\left(1 + \dfrac{V}{C_{f2}}\right) \\ f_{\min} = C_{f3}\left(1 - \dfrac{h}{C_{f2}}\right) \end{cases}$$

- Thrust *F* is computed from equation of flight

$$\dot{V} = \frac{F - D}{m} - g \cdot \sin(\gamma)$$

- Scilab routine *fminsearch()* has been used to solve the problem

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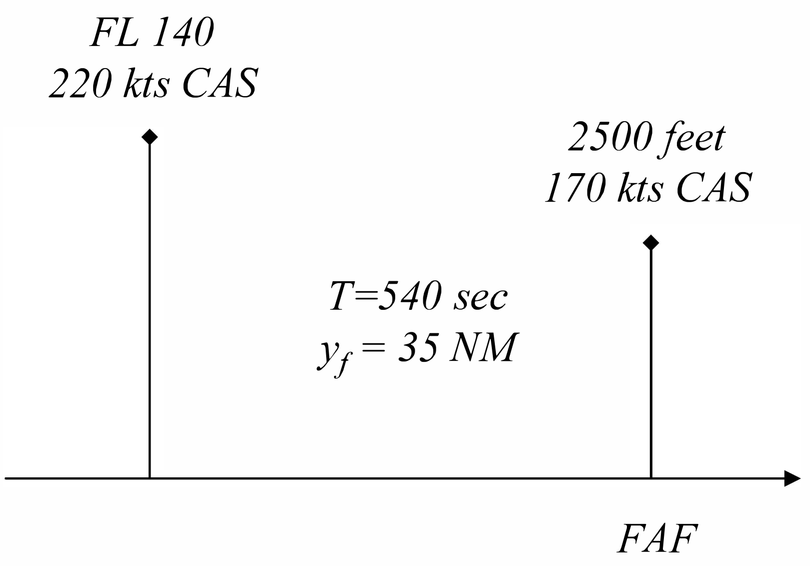
Simulation results

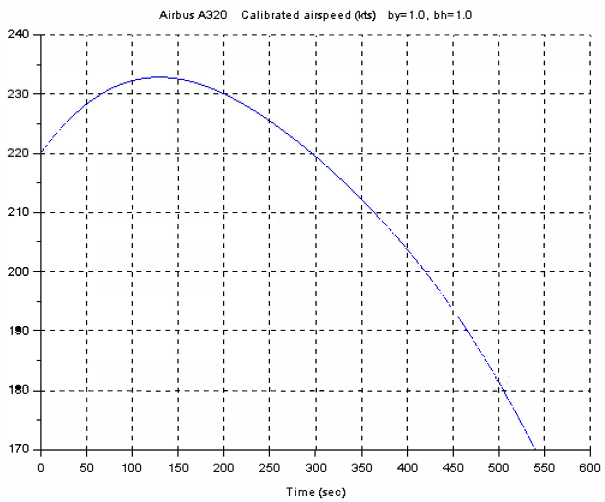
- Scenario:
  - Distance to be flown by the aircraft before reaching the meter fix: 35 NM
  - Aircraft shall descend from *FL 140* to *2500 feet*
  - Initial CAS: 220 kts
  - Final CAS: 170 kts
  - Maneuver duration imposed by ATC: 540 sec (9 min)
  - Vertical wind: constant, + 1 kts
  - Horizontal wind: 20 kts then decreases after 120sec of flight during 30 sec towards -20kts
  - BADA model for an Airbus A320



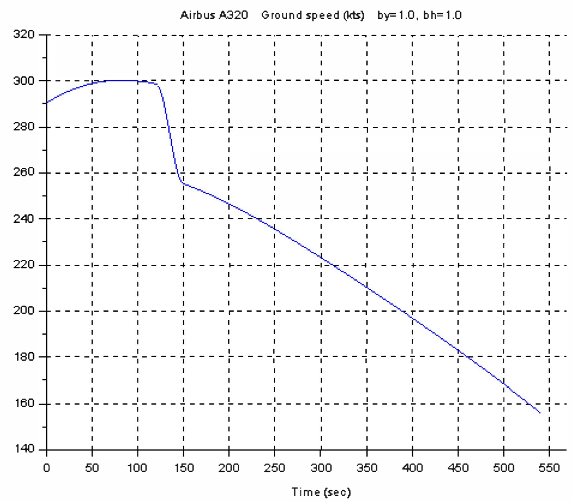
Simulation results

No optimization: $b_y = b_h = 1$

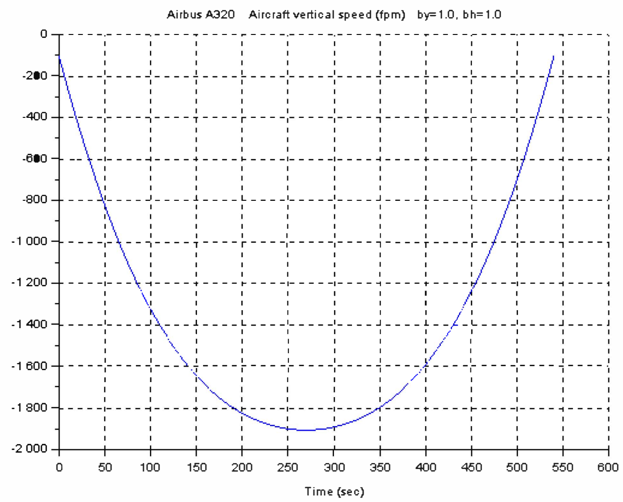
Max. longitudinal acceleration: 0.53 ft/sec²
Max. normal acceleration: 0.28 ft/sec²
Estimated fuel burnt: 96.7 kg.



Calibrated airspeed (CAS in kts)



Ground speed (kts)

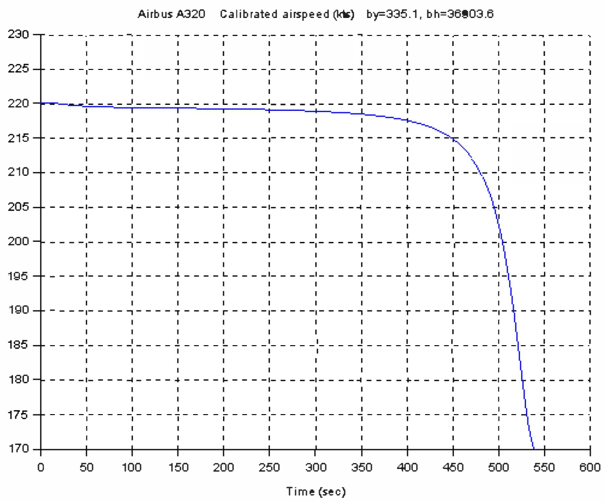


Vertical speed (Min V_z around -1900 fpm)

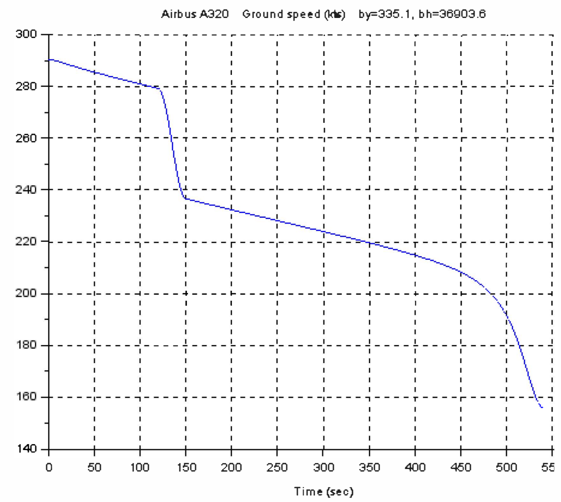
Simulation results

Optimization: b_y = 335.1 and b_h = 36903.6

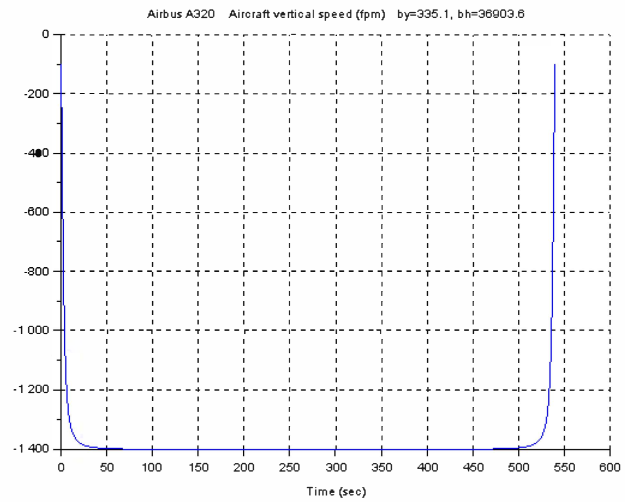
Max. longitudinal acceleration: 2 ft/sec² Max. normal acceleration: 5 ft/sec²
Estimated fuel burnt: 74.8 kg.



Calibrated airspeed (CAS in kts)



Ground speed (kts)



Vertical speed (Vz descent at -1400 fpm)

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Summary and Future work

- Summary
  - Design of airspeed and height profiles dedicated to the achievement of a specified time at a meter fix, with airspeed and altitude constraints
  - Proposed approach based on the shaping of the reference airspeed
- Future work
  - Turns to be managed
  - Extension to successive constraints over fixes

Thank you for your attention