A Survey of Vehicle Dynamics Modeling Methods for Autonomous Racing: Theoretical Models, Physical/Virtual Platforms, and Perspectives

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***Abstract*—This paper presents the first survey of vehicle dynamics modeling methods for autonomous racing. Previous surveys have covered dynamics models for standard autonomous vehicles or, alternatively, concentrated on planning and control methods in autonomous racing with vehicle dynamics models briefly mentioned. However, previous surveys overlook the importance of vehicle dynamics under challenging conditions of top speeds and non-steady state driving, which are unique characteristics in autonomous racing. Recognizing the vital role of vehicle dynamics modeling in an autonomous racecar's prediction, planning, and control modules, this survey seeks to ascertain to what degree the nominal full-scale racecar dynamics can be streamlined without sacrificing accuracy for simplicity. Furthermore, this survey provides essential guidance for organizers of virtual autonomous races, helping them choose vehicle dynamics models that meet the required level of precision. This paper begins with a review of previous surveys on vehicle dynamics modeling, highlighting their limitations in the context of autonomous racing. Following this, it investigates the existing dynamics models for autonomous racing vehicles, along with a comprehensive examination of the existing physical/virtual testing platforms. The paper concludes by discussing emerging trends and offering perspectives in the field of vehicle dynamics modeling for autonomous racing, paving the way for groundbreaking research and innovations in autonomous racing.**

***Index Terms*—Autonomous racing, chassis, vehicle dynamics, testing platform, intelligent vehicles for education (IV4E).**

NOMENCLATURE

A	Vehicle front surface.
ADAS	Advanced driver assistance system.
a	Acceleration.
B	Tuning parameter.
C	Coefficient, stiffness.
CM	Customizable models.
D	Longitudinal command.
CG	Vehicle gravity center.
DOF	Degree of freedom.
e	Lateral deviation between a path and CG.
F	Fiala tire model.
F	Force.
FLS	Friction limited by a geometric shape.
FSD	Formula Student Driverless.
g	Gravitational acceleration.
HIL	Hardware-in-the-loop.
h	Height.
I_z	Moment of inertia in vertical direction.
K	Gradient.
LF	Lateral Fiala tire model.
LL	Lateral linear tire model.
LLAB	Lateral and longitudinal acceleration bounded.
LongP	Longitudinal Pacejka tire model.
L	Linear tire model.
l	Wheelbase.
l_{f}	Front axle to CG distance.
l_{r}	Rear axle to CG distance.
LP	Lateral Pacejka tire model.
m	Vehicle mass.
P	Pacejka tire model.
P	Power.
R	Radius.
RC	Radio controlled.
r	Yaw rate.
SBM	Selectable built-in models.
SLP	Simplified lateral Pacejka tire model.
SP	Simplified Pacejka tire model.
s	Coordinate along a path.
T	Torque.
TP	Tunable parameters.
tw	Track width.
u	Longitudinal velocity.
v	Lateral velocity.

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X, Y	CG position in the earth-fixed Cartesian plane.
α	Lateral slip angle.
β	Side slip angle.
$\Delta\psi$	Error between the vehicle heading and the path orientation.
δ	Steering angle.
κ	Driven curvature.
$\bar{\kappa}$	Path curvature.
λ	Longitudinal slip.
μ	Coefficient of friction.
ρ	Air density.
ψ	Vehicle heading angle.
ϵ	Lateral slip.

Subscripts

aero	Aerodynamic.
f	Front.
g	Gear.
L	Linear.
l	Left.
lat	Lateral redistribution.
long	Longitudinal redistribution.
r	Right, rear.
re	Rolling resistance.
tr	Traction.
us	Understeer.
x	Longitudinal direction.
y	Lateral direction.
z	Vertical direction.

I. Introduction

Autonomous racing has been rapidly evolving over the past few years [1]. The absence of a human driver eliminates safety-related issues such as high cockpit temperatures and risks associated with competitive racing. Moreover, the limitations imposed by human reaction times are no longer a factor, which potentially leads to faster speeds than those currently observed in human-driven races such as Formula One and the World Endurance Championship.

Autonomous racing aims for greater speeds and more agile maneuvers, through advancements in software and hardware. Software advancements include better sensor fusion [2], motion planning [3], and tracking control [4] algorithms. Hardware improvements focus on two aspects. The first is upgrading localization/perception sensors like LiDARs [5], inertial measurement units [6], and radars [7]. The second entails evolving chassis design, by using modified passenger cars for full-scale racecars [8] and radio-controlled (RC) cars as the foundation for reduced-scale racecars [9].

The unique challenges of autonomous racing, particularly the notable tire slips during cornering, highlight the inadequacy of traditional kinematic models designed for non-aggressive movements [10], [11]. These kinematic models struggle to predict the directional changes caused by tire slips [12], which results in a gap between theoretical predictions and real-world outcomes [13]. Integrating tire dynamics into a kinematic model adds to the computational complexity [14]. Thus, vehicle dynamics are considered only when the driving conditions become genuinely complicated, such as in an advanced driver assistance system (ADAS) with high-speed lane-keeping assist, adaptive cruise control, and emergency brake assist functions [15]. Another example of the usage of vehicle dynamics model is to guide the optimization design of a racecar chassis, which tightly integrates vehicle mobility with a specific racetrack to achieve extreme speed [16]. In conclusion, accurate modeling of vehicle dynamics is an important aspect in developing a qualified autonomous racecar capable of driving at high speeds.

The importance of dynamics models in autonomous racing cannot be overstated, given that they are crucial for mastering complex racecar behaviors at high speeds. If accurate racecar dynamics models are integrated into the motion planning and tracking control methods for participants in an autonomous race, then they could positively utilize the nonignorable tire slip effects to achieve fastest lap times and implement sophisticated and strategic racing maneuvers on the racetrack [12], [17]. Meanwhile, autonomous race organizers use these dynamics models to guarantee the accuracy of their deployed racing simulation platforms and acquire a good estimation of the baseline performances of the participants. In summary, dynamics modeling is important in autonomous racing.

This study conducts a comprehensive survey of two aspects in autonomous racing: the modeling methods employed for racecar dynamics and the physical/virtual platforms utilized in autonomous racing. First, we review the dynamics models that have emerged due to the rapid advancement in autonomous racing. These models are crucial in assisting participants in navigating the complexities of autonomous racing, particularly when integrated into motion planning and tracking control systems. Second, our survey explores the range of physical and virtual platforms used in this domain. These platforms, which are closely linked with vehicle dynamics, are indispensable for the testing and training of racecars. They provide valuable insights for executing effective racing strategies and assessing the effectiveness and reliability of software. This survey is designed to aid participants in selecting the most appropriate dynamics models and physical/virtual platforms according to their actual needs. However, no such surveys have been previously conducted.

A. Related Works

This subsection reviews previous surveys related to dynamics modeling methods for an autonomous racing car and the physical/virtual platforms used for testing autonomous racing performances. As mentioned, such surveys are lacking. This subsection alternatively investigates existing surveys that are closely aligned with our concerned theme.

1) Vehicle Dynamics Modeling Method: In the field of autonomous racing, dedicated surveys addressing vehicle dynamics models implemented in planning and control algorithms are lacking. To the best of the authors' knowledge, the only existing survey related to autonomous racing is [18], which delves into the perception, planning, and control methods designed for autonomous racing. Although this survey paper recognizes the importance of vehicle dynamics in autonomous racing, it primarily provides a brief overview of model-based and model-free dynamics modeling methods and lacks in-depth elaborations.

Emergency and high-speed scenarios share the features of aggressive movements and non-negligible tire slips inherent in autonomous racing. The planning and control algorithms for on-road autonomous vehicles should be capable of handling these scenarios. Therefore, the review works on planning and control algorithms for autonomous vehicles designed for these scenarios are included to provide a brief understanding of the involved vehicle dynamics models. Schwarting et al. [19] comprehensively reviewed diverse perception, planning, and decision-making strategies, with a focus on the critical importance of vehicle dynamics models during high-speed operations and aggressive maneuvers. However, the review only mentions that vehicle dynamics models should be adopted with consideration of tire forces, which provides no detailed description of the models. Teng et al. [20] reviewed vehicle mobility models under emergency conditions. However, the survey focused solely on data-driven models without elaborating on the specifics of vehicle dynamics models explicitly.

Vehicle dynamics models are important for short-time state estimation of autonomous racing cars in the presence of pronounced tire slips. This importance is well established in the development of ADAS. ADAS-related survey works focusing on model-based vehicle state estimation are encompassed to provide comprehensive insights into vehicle dynamics modeling methods. Given the importance of vehicle dynamics in ADAS research, we also investigate surveys in this domain. Singh et al. [15] systematically summarized methods for estimating vehicle dynamic states and tire road contact parameters. In this context, vehicle dynamics models serve as the link between onboard sensor data and the estimation of states, which forms the basis for the design of chassis control systems. Leung et al. [21] provided an overview of methodologies for vehicle dynamics state estimation by primarily relying on Global Positioning System or inertia navigation system data. Notably, model-based approaches based on vehicle dynamics models have emerged as a cornerstone method. Kanchwala et al. [22] extensively reviewed model-based approaches based on fundamental vehicle dynamics models to control the yaw moment, traction, and side slip in electric vehicles. Similarly, Talvala et al. [23] reviewed electronic stability control systems founded on vehicle dynamics models and confirmed their potential in autonomous racing through rigorous testing at the limits of tire adhesion. Furthermore, Jin et al. [24] and Guo et al. [25] separately explored model- and data-driven approaches to vehicle state estimation. However, the scope of the abovementioned studies primarily centered on selected variables, which resulted in a bias in providing a comprehensive understanding of vehicle dynamics models for researchers in the field of autonomous racing.

Given that previous surveys on vehicle dynamics, particularly for autonomous racing, are absent, we alternatively broaden the scope to generic vehicle dynamics surveys, which still provide some valuable insights. Yang et al. [26] comprehensively summarized advances in vehicle dynamics models across different degrees of freedom (DOFs), by delving into a detailed discussion on the interplay between vehicles and road pavement smoothness. Despite the comprehensiveness of the vehicle dynamics models, their application to autonomous vehicles, particularly in terms of computational efficiency, proved inadequate for the demands of autonomous racing. Kebbati et al. [27] surveyed the prevailing vehicle dynamics models deployed in autonomous driving. However, the review primarily focused on the control of autonomous vehicles, with limited discussion on weight redistribution and tire models. Guiggiani [12] delivered a comprehensive review of vehicle dynamics models related to handling and braking, spanning powertrains, transmissions, braking systems, car bodies, and tires. However, the book centered on manned vehicle handling and vehicle design, rather than on the development of algorithms for motion planning and control. Consequently, many of the models impose computational burdens that might be considered unfeasible in practical applications.

The studies mentioned above either assume that the discussion of vehicle dynamics models is common knowledge for the readers without providing specific details or explore various models that may not be directly applicable to the field of autonomous racing. Consequently, a gap in research remains, which requires the systematic categorization of dynamics models based on different scenarios for their implementation in planning and control algorithms in autonomous racing.

2) Autonomous Racing Testing Platforms: In the domain of established physical and virtual autonomous racing testing platforms, limited attention has been given to comprehensive surveys, with only [18] exploring a few such platforms. Concerning physical platforms, this survey primarily summarizes aspects, such as sensor integration, computational hardware and software, and the associated competition types. With regard to virtual platforms, it provides only brief insights into three platforms. Notably, for the surveyed physical and virtual platforms, the investigation ignores dynamics-related configurations, whether in terms of hardware specifications or the implementation of vehicle dynamics models.

B. Motivations and Contributions

This study aims to provide a comprehensive survey of cutting-edge vehicle platforms in autonomous racing to cater to a diverse audience, including enthusiasts, scholars, race organizers, and participants. Existing surveys often focus on motion planning and control methodologies and utilize vehicle dynamics models that often lack detailed descriptions. Thus, we comprehensively present all theoretical models in the literature especially those applied in the motion planning and control algorithms. Furthermore, we include a detailed list of physical racing cars and virtual platforms used in autonomous racing, along with their associated dynamics configurations, to offer a comprehensive understanding of physical and virtual vehicle testing platforms.

Our work presents the following unique contributions:

1) This survey is the first to systematically investigate the vehicle dynamics modeling methods suitable for motion

planning and tracking control functions in autonomous racing.

2) This survey is the first to systematically review the physical and virtual testing platforms embedded with vehicle dynamics models that are useful for testing autonomous racing cars.

C. Organization

In the rest of the survey paper, Section II reviews the vehicle dynamics models in autonomous racing, including lateral, longitudinal, and tire models. In Section III, an overview of the physical vehicle platforms in autonomous racing is provided. In Section IV, we provide insights into the virtual testing platforms employed for autonomous racing, with a focus on their physics engines and adopted vehicle dynamics models. Section V provides perspectives on the aforementioned vehicle dynamics models and physical/virtual platforms. Section VI finalizes this survey with our conclusions.

II. Vehicle Dynamics Models for Autonomous Racing

In this section, our exploration delves into theoretical vehicle dynamics models, with a focus on those that share a sophisticated balance between accuracy and computational complexity. These models play a critical role in planning and control algorithms designed for autonomous racing events. The diverse nature of autonomous racing requires a range of vehicle dynamics models designed to suit different roles in racing events. Racing participants typically prefer to simplify full-scale vehicle dynamics models before incorporating them into their onboard planning and control algorithms. This preference arises from the necessity for real-time processing throughout the entire race, where the time delay in dynamics model-related calculations can introduce a more significant error compared to the potential inaccuracies of an incomplete theoretical model. Meanwhile, participants also find value in complete versions of these models. These comprehensive models focus on the accuracy of predicted results, serving to validate the consistency of their simplified counterparts. Conversely, organizers of autonomous racing events, especially in virtual realms, prioritize complete vehicle dynamics models. Their objective is to simulate the vehicle's dynamic response in virtual racing simulators as closely as possible to reality. Additionally, they may find utility in simplified models to gain a quick overview of participants' general performance within the virtual racing simulators they create. For readers interested in exploring the complete vehicle dynamics models used to validate simplified models or in the creation of virtual racing simulators, relevant references can be found in [12], [28].

Shifting our focus to the vehicle dynamics models integrated into planning and control algorithms, these models can be briefly classified into three main categories: lateral dynamics models, longitudinal dynamics models, and tire models. Lateral dynamics models are designed to capture the forces and moments that come into play as the vehicle negotiates corners, rendering them a critical aspect of comprehensive vehicle models [29]. Conversely, longitudinal dynamics models focus on the vehicle's forward motion, specifically on longitudinal acceleration and deceleration [30]. On the other hand, tire models are specifically developed to investigate the interaction between the tires and the road surface, allowing for accurate simulation of the tire-generated longitudinal and lateral forces [28].

In addition, the consideration of weight transfer during high-performance driving assumes primary significance, given its substantial impact on the tire models and their integration with the vehicle's lateral dynamics. Weight transfer directly impacts the nominal load distribution on each tire, resulting in variations in its force carrying capability. Consequently, weight transfer serves as a critical linkage between the tire models and the vehicle's lateral or longitudinal models.

While lateral and longitudinal models can be separately applied in specific scenarios [31], [32], they are typically used in conjunction with tire models. Although models within each category fulfill distinct roles, the significance of these models may vary, with some being sophisticated while others being more straightforward, contingent upon the specific application.

Table I presents an overview of the state-of-the-art lateral dynamics and tire models employed in autonomous racing. Initially, the scenarios are categorized based on the scale of the test vehicles, including both real-world applications and simulations. Additionally, four distinct driving scenarios are outlined. The first focuses on time-optimal testing, aimed at solving minimum lap-time problems with a single racecar. The second pertains to driving at the limits, striving to maximize tire lateral grip and enhance vehicle dynamic response under highly dynamic driving conditions. The third involves controlled oversteering with traction loss, commonly referred to as drifting. Last, multi-car competition involves head-to-head racing contests among multiple racecars.

A. Common Assumptions

In the field of vehicle dynamics models, it is common to treat the vehicle body as a single rigid entity [12]. However, to enhance computational efficiency and meet the demands of rapid calculations in a racing context, certain additional assumptions are frequently introduced. These assumptions include the notion that the vehicle body engages in planar motion aligned with the road surface and postulates the road's perfect flatness. It is worth noting that these assumptions disregard any consideration of suspension deflections and tire vertical deformations [12]. Notably, racing-specific chassis typically incorporate stiffer suspensions and tires [85], which serves to mitigate the implications of this simplification. Consequently, the vehicle dynamics model is fundamentally simplified, focusing exclusively on the planar motions of the vehicle.

B. Lateral Dynamics Models

1) Point-Mass Model: The most basic model for racecar dynamics, up to the limits, is the point-mass model. In this model, the vehicle is treated as mass concentrated at its center of gravity (CG). It applies Newton's second law to the point-mass in a 2D plane, including 2 DOFs: longitudinal and lateral. While it considers all the forces acting on the CG as the primary factors

TABLE I
OVERVIEW OF DIFFERENT AUTONOMOUS RACING SCENARIOS AND CORRESPONDING DYNAMICS MODELS IN LITERATURE

Scenario	Lateral Model	Normal Load Variation Type	Tire Model	Related Publications
Reduced-scale car time-optimal testing	Kineto-dynamic single-track	Static	-	[33]
	Dynamic single-track	-	LLAB	[34]
		Static	LL	[35]
			SLP	[36]–[40]
			LP	[41], [42]
		Static & longitudinal	LP	[43]
Reduced-scale multi-car competition	Dynamic single-track	Static	LL	[44]
			SLP	[45]
			LP	[46]
Full-scale car time-optimal testing	Point-mass	-	FLS	[47]–[49]
	Kinematic single-track	-	-	[50]
	Dynamic single-track	Static	LL	[51]
			LF	[52], [53]
			LP	[54], [55]
			SLP	[56]
		Static & aerodynamic	SLP	[57]
		Static & longitudinal	LF	[58]
			SLP	[59]
			SP	[60]
		Static & implicit lateral	F	[61]
		Static, longitudinal & implicit lateral	LP	[62]
	Dynamic double-track	Static, longitudinal & lateral	L	[63]
			LP	[64]
			LF	[65]
		Static, longitudinal, lateral & aerodynamic	LP	[66]
			SP	[67]
			P	[68]
Full-scale car driving at the limit	Point-mass	Static & longitudinal	FLS	[69]
	Dynamic single-track	Static	LL	[70]
			LF	[31]
			SLP	[71]
		Static & aerodynamic	LL	[72]
		Static & longitudinal	LF	[73]
			LongP	[32]
	Dynamic double-track	Static, longitudinal & lateral	P	[74], [75]
Full-scale car drifting	Dynamic single-track	Static	-	[76]
			LF	[77]
			SLP	[78]
Full-scale multi-car competition	Point-mass	-	-	[79]
	Kinematic single-track	-	-	[80], [81]
	Dynamic single-track	Static	SLP	[82]
			LP	[83]
		Static & implicit lateral	F	[84]

influencing the vehicle's motion, it does not explicitly account for yaw dynamics. Consequently, it does not incorporate the lateral forces generated during changes in vehicle orientation, which can impact tire grip and affect vehicle performance. However, empirical studies have indicated that for minimum lap-time problems, the contribution of yaw inertia to vehicle dynamics can be relatively negligible [86]. As a result, in practical terms, the vehicle's yaw dynamics are approximated to remain quasi-steady state [69].

This model relies solely on the vehicle's mass, notwithstanding the consideration of the interaction between the vehicle and the road, as elucidated in Section II-D. In Cartesian coordinates, the state vector is expressed as $[X, Y, \dot{X}, \dot{Y}]^{\mathrm{T}}$ [79]. However, in a broader context, this vector incorporates not only the vehicle states but also its position relative to a desired path, expressed in the form of Frenet coordinates, as illustrated in Fig. 1. In such scenarios, the vehicle's heading angle ψ aligns with the path orientation, leading to a modified state vector $[s, e, u, \Delta\psi]^{\mathrm{T}}$ [47], [69]. The corresponding kinematic and equilibrium equations

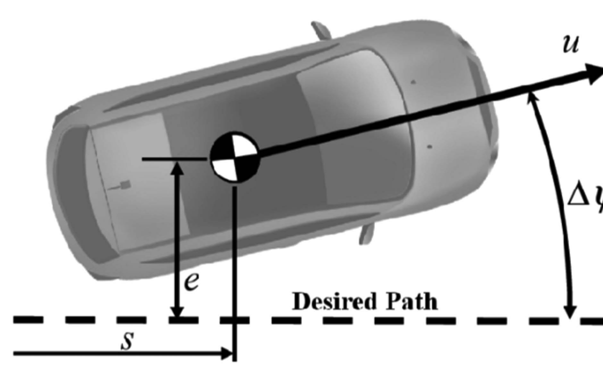

Fig. 1. Vehicle states of point-mass model in the frenet frame.

are expressed as follows:

$$\begin{aligned}\dot{s} &= \frac{u\cos(\Delta\psi)}{1-\bar{\kappa}e},\\ \dot{e} &= u\sin(\Delta\psi),\\ \dot{u} &= \frac{F_\mathrm{x}}{m},\\ \Delta\dot{\psi} &= \frac{F_\mathrm{y}}{mu} - \dot{s}\bar{\kappa}.\end{aligned} \tag{1}$$

The control inputs of the point-mass model in autonomous racing have traditionally been represented either in force form $[F_\mathrm{x}, F_\mathrm{y}]^\mathrm{T}$ [69] or acceleration form $[a_\mathrm{x}, a_\mathrm{y}]^\mathrm{T}$ [48] with adjustments made to the equations to summarize the forces or accelerations along orthogonal directions within the 2D plane. These inputs can be determined arbitrarily or derived through the tire equations, which will be discussed in Section II-D. Furthermore, the control inputs can also be $[F_\mathrm{x}, \kappa]^\mathrm{T}$ expressed as a function of the driven curvature κ, where κ is correlated with the steering wheel angle δ by $\delta = \kappa l$ assuming neutral steer and a small steering angle, with l representing the vehicle wheelbase [47]. Additionally, the point-mass model can be extended to a 3D path by introducing Euler angles into the frame transformation [69].

The point-mass model simplifies vehicle dynamics by not explicitly considering the characteristics of individual axles or tires, including normal load transfers [47], [48], [49], [79]. Thus, its applications are constrained to scenarios involving purely velocity optimization [49] or instances where the controller can perfectly track the lateral and longitudinal accelerations of the racecar [47]. In this approach, the interaction between the vehicle and the road is treated as a unified entity. However, more advanced models for studying vehicle-road interactions are often designed with a focus on individual tires. By distributing the tire forces to each wheel, the model's accuracy in predicting the vehicle's acceleration limits can be improved.

Changes in tire normal loads can lead to variations in cornering stiffness and peak lateral forces for different axles, potentially resulting in tendencies toward oversteering or understeering [28]. To account for these effects, it is possible to incorporate load transfer between the front and rear axles into the point-mass model. Among these considerations, the static load transfer between the front and rear axles can be calculated as follows:

$$F_\mathrm{z,f,static} = \frac{l_\mathrm{r}}{l}\cdot F_\mathrm{z}, F_\mathrm{z,r,static} = \frac{l_\mathrm{f}}{l}\cdot F_\mathrm{z}, \tag{2}$$

and the normal load transfer due to longitudinal acceleration is expressed as [69]

$$F_\mathrm{z,f,long} = -\frac{h_\mathrm{CG}}{l}\cdot F_\mathrm{x}, F_\mathrm{z,r,long} = \frac{h_\mathrm{CG}}{l}\cdot F_\mathrm{x}, \tag{3}$$

where F_z is the vehicle mass multiplied by the gravitational acceleration, and h_CG stands for the height of CG. Consequently, the normal load acting on a single axle is obtained by summing the corresponding static wheel load term F_static with the longitudinal redistribution term F_long.

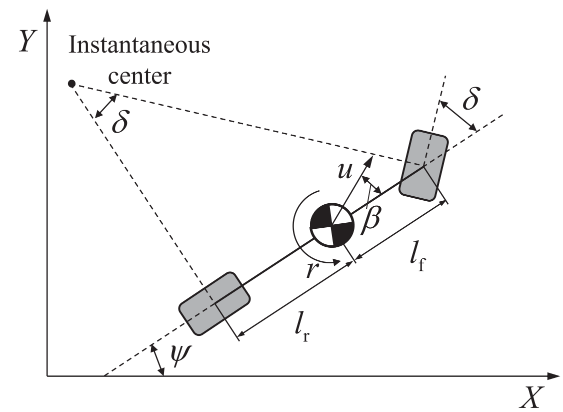

Fig. 2. Kinematic single-track model in Cartesian frame.

2) Single-Track Models: The single-track model, often referred to as bicycle model, represents the merging of the front and rear wheels into a single wheel positioned at the center of their respective axles [87]. This model can be further categorized into kinematic, kineto-dynamic, and dynamic models.

In Fig. 2, the kinematic single-track model is presented, where the vehicle's motion is solely governed by geometric factors, independent of the complexities arising from tire road interactions involving forces and torques. The model features a fixed instantaneous center (Fig. 2), which remains unchanged when the steering angle δ remains constant. For vehicles with front-wheel steering, this model is characterized by only 2 DOFs. The sideslip angle β of this model, expressed within an Earth-fixed Cartesian frame of reference, can be described as follows [50]:

$$\beta = \arctan\left(\frac{l_\mathrm{r}}{l}\cdot\tan\delta\right), \tag{4}$$

and the equations of motion are as follows:

$$\begin{aligned}\dot{X} &= u\cos(\psi+\beta),\\ \dot{Y} &= v\sin(\psi+\beta),\\ \dot{\psi} &= \frac{u}{l_\mathrm{r}}\cdot\sin\beta,\\ \dot{u} &= \frac{F_\mathrm{x}}{m}\cdot\cos\beta.\end{aligned} \tag{5}$$

The input variables of the above system consist of F_x and δ. However, it is important to note that this model disregards the influence of tire slip, which presents a challenge in accurately representing actual dynamics during high-speed cornering [88]. Similar to the point-mass model, the kinematic single-track model was commonly employed when utilizing Frenet coordinates along either the center path [80] or a predefined racing line [50] on the track. In such scenarios, the state vector typically consists of variables s, e, $\Delta\psi$, u, and δ, while the corresponding inputs include F_x and the steering rate ω [50], [80]. Since the kinematic single-track model does not account for the forces exerted on the tires, there is no need to investigate normal load transfers between axles or tires. Consequently, without

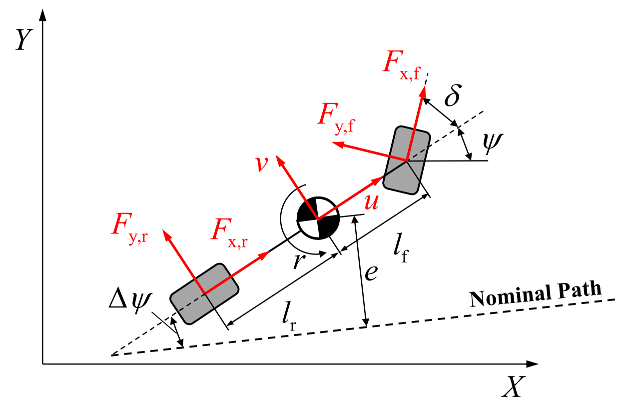

Fig. 3. Dynamic single-track model in Cartesian and Frenet frames.

accounting for tire slip, it can accurately simulate scenarios only when lateral acceleration is negligible.

Assuming that the kinematic model operates under quasi-steady state condition, the concept of an understeer gradient K_{us} is introduced to establish a connection between the yaw rate r and the lateral acceleration a_{y}. This leads to the development of a kineto-dynamic single-track model [33]. In this model, the definition of K_{us} is adjusted to incorporate a correlation between a_{y} and K_{us}. The corresponding equation can be expressed as follows:

$$\delta - \frac{r}{u} \cdot l = K_{\mathrm{us}}(a_{\mathrm{y}}). \tag{6}$$

To accurately depict the vehicle dynamics near the limits of handling while maintaining computational efficiency, the evolution of r is incorporated into the kinematic single-track model as a first-order system. This addition enables a cost-effective modeling approach that captures the vehicle's behavior under challenging conditions. However, it is important to note that the coupling between r and a_{y} is preserved within this model, resulting in the retention of its 2 DOFs. Hence, the applicable scenarios are restricted to situations near quasi-steady state, where the rates of change for yaw rate and lateral acceleration are minimal.

The dynamic single-track model (Fig. 3), widely acknowledged as a prominent model in the field of autonomous racing, assumes nearly equal left and right gear ratios for the steering system [12]. This model accounts for all in-plane rigid body motions, including the addition of yaw motion, thereby encompassing 3 DOFs. When assuming small steering angles rendering $\beta \approx 0$, with respect to the Earth-fixed Cartesian frame, this model can be mathematically expressed as follows [39], [41], [73]:

$$\begin{aligned}
\dot{u} &= \frac{1}{m} \cdot (F_{\mathrm{x,r}} - F_{\mathrm{y,f}} \sin\delta + F_{\mathrm{x,f}} \cos\delta + F_{\mathrm{x,other}}) + vr, \\
\dot{v} &= \frac{1}{m} \cdot (F_{\mathrm{y,r}} + F_{\mathrm{y,f}} \cos\delta + F_{\mathrm{x,f}} \sin\delta) - ur, \\
\dot{r} &= \frac{1}{I_{\mathrm{z}}} \cdot (F_{\mathrm{y,f}} l_{\mathrm{f}} \cos\delta + F_{\mathrm{x,f}} l_{\mathrm{f}} \sin\delta - F_{\mathrm{y,r}} l_{\mathrm{r}}).
\end{aligned} \tag{7}$$

Equation (7) represents the coupling of equilibrium and kinematic formulas. The additional terms beyond the tire forces ($F_{\mathrm{x,f}}$, $F_{\mathrm{y,f}}$, $F_{\mathrm{x,r}}$, and $F_{\mathrm{y,r}}$), which may have impacts on u, are incorporated into the variable $F_{\mathrm{x,other}}$, which will be elaborated in Section II-C. Furthermore, in certain studies investigating reduced-scale platforms driven and braked solely by the rear axle [34], [35], [36], [37], [38], [39], [40] or scenarios such as drifting where front tire longitudinal dynamics are not involved [78], the terms related to $F_{\mathrm{x,f}}$ can be disregarded.

In this dynamic system, the states primarily describe the vehicle's planar motion. Typically, u_{x}, u_{y}, and r were selected as the motion related states. However, several studies have concentrated solely on the lateral response of the system and chose u_{y} and r as the vehicle's motion variables [51]. It is important to emphasize that the use of these specific state variables is not obligatory. Alternative states, such as β and r can also be considered replacements for u_{y} and r [31], [52], [71], [84]. When only two motion-related states are employed, the corresponding dynamic single-track model features only 2 DOFs.

Depending on the specific purpose of the system, additional states are necessary to pinpoint the car's location on the racetrack. For instance, in the Cartesian frame, the model can be expressed as follows [41], [54], [56], [60]:

$$\begin{aligned}
\dot{X} &= u_{\mathrm{x}} \cos\psi - u_{\mathrm{y}} \sin\psi, \\
\dot{Y} &= u_{\mathrm{x}} \sin\psi + u_{\mathrm{y}} \cos\psi, \\
\dot{\psi} &= r,
\end{aligned} \tag{8}$$

or in the Frenet frame with s, e, and $\Delta\psi$ [55], [61], [73] as

$$\begin{aligned}
\dot{s} &= \frac{u \cos\Delta\psi - v \sin\Delta\psi}{1 - \bar{\kappa} e}, \\
\dot{e} &= u \sin(\Delta\psi) + v \cos(\Delta\psi), \\
\Delta\dot{\psi} &= r - \bar{\kappa}\dot{s}.
\end{aligned} \tag{9}$$

The inputs of the system typically consist of two variables, with one related to δ, and the other related to F_{x}. The traction-related input can be selected based on various longitudinal dynamics models, which will be discussed in Section II-C.

For racing cars equipped with mechanically decoupled driving wheels, the distribution of traction force among the wheels can be arbitrarily determined [57]. One potential approach involves introducing an additional input for the torque vectoring moment, which directly influences the variable r in (7) [57]. Likewise, an empirically guided lateral weight transfer ratio can be incorporated to influence the distribution of forces between the driven wheels [84]. Furthermore, for vehicles with rear axle steering, the states of u, v, and r also depend on the rear steering angle coupled with $F_{\mathrm{x,r}}$ and $F_{\mathrm{y,r}}$ [62].

In the dynamic single-track model, it is possible to explicitly account for the normal load transfer between the front and rear axles. Equation (2) is a common consideration in all studies. As indicated in Table I, most studies exclusively examined static weight distribution between the axles for reduced-scale cars [35], [36], [37], [38], [39], [40], [44], [45], [46], primarily due to

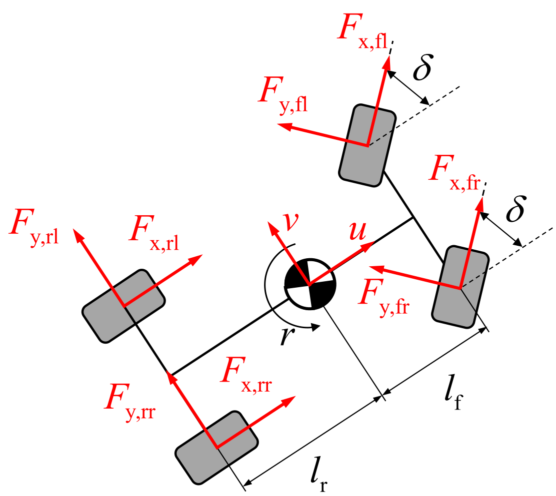

Fig. 4. Schematic of dynamic double-track model.

their exceptionally low CG relative to the ground. Conversely, systems designed for full-scale vehicles often incorporate weight redistribution resulting from longitudinal acceleration, as depicted in (3). However, in such cases, F_x is no longer a single variable but rather a summation of tire forces longitudinal to the vehicle body and $F_{x,other}$. The formula is expressed as

$$F_x = F_{x,r} + F_{x,f} \cos\delta - F_{y,f} \sin\delta + F_{x,other}. \tag{10}$$

Additionally, aerodynamic effects are considered by introducing an additional term that adjusts the vehicle's effective gravity acting on the CG. This term accounts for the partial influence of aerodynamic forces on the vehicle's dynamics. The formula incorporating this aerodynamic effect is expressed as [57]:

$$F_z = mg + C_{lift} u^2, \tag{11}$$

where C_{lift} is the lift coefficient.

Moreover, within the dynamic single-track model, it is not possible to explicitly account for the lateral distribution of the normal force. Nevertheless, an approximation can be applied by adjusting the cornering stiffness C_α on the front and rear axles [62]. It is important to highlight that this modification operates under the assumption of a steady state approximation for weight transfer between the left and right tires [84]. This assumption may not deliver the needed level of accuracy for highly transient dynamics, necessitating more comprehensive vehicle dynamics models. Furthermore, this model fails to deliver accurate simulations when each tire on one axle is independently controlled.

3) Double-Track Model: Fig. 4 illustrates the schematic of the dynamic double-track model, which represents the most complex model in the literature for autonomous racing. This model enables a detailed examination of the influence of forces generated by each tire on the vehicle's dynamic response while largely preserving similar states and inputs as the dynamic single-track model. Therefore, the model accurately captures the dynamic response of the car body when each tire on one axle is independently controlled.

The dynamic double-track model maintains the same 3 DOFs as the dynamic single-track model. Similarly, assuming small and identical steering angles between two front wheels, bearing in mind that $F_{x,f} = F_{x,fl} + F_{x,fr}$, $F_{x,r} = F_{x,rl} + F_{x,rr}$, $F_{y,f} = F_{y,fl} + F_{y,fr}$, and $F_{y,r} = F_{y,rl} + F_{y,rr}$, the resulting equations of motion are [65], [66]

$$\begin{aligned}
\dot{u} &= \frac{1}{m} \cdot (F_{x,r} - F_{y,f} \sin\delta + F_{x,f} \cos\delta + F_{x,other}) + vr, \\
\dot{v} &= \frac{1}{m} \cdot (F_{y,r} + F_{y,f} \cos\delta + F_{x,f} \sin\delta) - ur, \\
\dot{r} &= \frac{1}{I_z} \cdot (l_f (F_{y,f} \cos\delta + F_{x,f} \sin\delta) - l_r F_{y,r} \\
&\quad + \frac{tw_r}{2} \cdot (F_{x,rr} - F_{x,rl}) + \frac{tw_f}{2} \cdot (F_{x,fr} - F_{x,fl}) \cos\delta \\
&\quad + \frac{tw_f}{2} \cdot (F_{y,fl} - F_{y,fr}) \sin\delta).
\end{aligned} \tag{12}$$

It can be observed that the states of u and v maintain a similar structure to (7), but with additional terms that account for the torque generated within the same axle. Furthermore, additional states can be introduced to represent location-dependent variables, as described in (8) and (9).

The double-track model provides a more comprehensive examination of normal load transfer between tires in a quasi-steady state. Employing the same definition of $F_{x,f}$, $F_{x,r}$, $F_{y,f}$, and $F_{y,r}$ as in (12), intermediate variables related to longitudinal and lateral weight redistributions can be introduced as follows [65], [66]:

$$\begin{aligned}
\Delta F_{z,long} &= \frac{h_{CG}}{l} \cdot (F_{x,r} + F_{x,f} \cos\delta - F_{y,f} \sin\delta + F_{x,other}), \\
\Delta F_{z,lat} &= \frac{h_{CG}}{\frac{1}{2}(tw_f + tw_r)} \cdot (F_{y,r} + F_{x,f} \sin\delta + F_{y,f} \cos\delta),
\end{aligned} \tag{13}$$

where tw stands for the track width of different axles.

Occasionally, the aerodynamic lift is consolidated into a single term for load transfer between tires, expressed as

$$F_{z,aero,f} = \frac{1}{4} \cdot C_{lift,f} \rho A u^2, \quad F_{z,aero,r} = \frac{1}{4} \cdot C_{lift,r} \rho A u^2. \tag{14}$$

Finally, as the road surface is horizontal, the normal loads on different tires can be derived as

$$\begin{aligned}
F_{z,fl} &= \frac{\frac{l_r}{l} \cdot mg - \Delta F_{z,long}}{2} - \gamma \Delta F_{z,lat} + F_{z,aero,f}, \\
F_{z,fr} &= \frac{\frac{l_r}{l} \cdot mg - \Delta F_{z,long}}{2} + \gamma \Delta F_{z,lat} + F_{z,aero,f}, \\
F_{z,rl} &= \frac{\frac{l_f}{l} \cdot mg + \Delta F_{z,long}}{2} - (1-\gamma) \Delta F_{z,lat} + F_{z,aero,r}, \\
F_{z,rr} &= \frac{\frac{l_f}{l} \cdot mg + \Delta F_{z,long}}{2} + (1-\gamma) \Delta F_{z,lat} + F_{z,aero,r},
\end{aligned} \tag{15}$$

where γ is a suspension-related roll balance factor, indicating the proportion of lateral transfer supported by the front axle.

TABLE II
STRENGTHS, LIMITATIONS, AND APPLICABLE SCENARIOS OF LATERAL DYNAMICS MODELS

Model Type	Advantages	Disadvantages	Applicable Scenarios
Point-mass	● Enables the simplest analysis of planar motion in vehicle dynamics with only 2 DOFs ● Features a minimal set of optimization variables and constraints ● Demands the least computational resources, thus making it efficient for real-time applications and simulations	● Omits the consideration of yaw dynamics ● Addresses implicitly individual axles or tires ● Disregards kinematic constraints	● Highly limited computational resources ● Purely velocity optimization ● Perfect tracking of lateral and longitudinal accelerations by the controller
Kinematic single-track	● Considers accurately vehicle kinematics with 2 DOFs ● Independent of normal load redistribution ● Requires light computational resources	● Fails to consider tire road interactions involving forces and torques	● Negligible lateral acceleration
Kineto-dynamical single-track	● Considers yaw dynamics correlated with lateral acceleration with 2 DOFs ● Considers roughly tire road interaction under quasi-steady state conditions, while requires computational resources similar to kinematic single-track model	● Relies on empirical models for considering tire road interactions	● Negligible rate of change for yaw rate and lateral acceleration
Dynamic single-track	● Considers all planar rigid body motions with 3 DOFs ● Considers explicitly dynamics of different axles ● Allows integration of longitudinal dynamics models and tire models ● Capable of modeling torque vectoring ● Capable of considering explicitly longitudinal and aerodynamic normal load distribution	● Fails to capture yaw moments resulting from different drive and brake forces across an axle, thus cannot model limited slip differential and brake force distributions systems	● Individually controlled drive and brake forces for each axle
Dynamic double-track	● Investigates explicitly all forces generated by each tire with 3 DOFs	● Requires the highest computational resources for models	● Individually controlled drive and brake forces for each tire on a single axle

It plays a critical role in determining the vehicle's tendency toward understeering or oversteering when operating at the friction limit. Depending on the specific application of the model or considering that the maximum attainable speed may not be exceptionally high, the impact of aerodynamic lift on tire normal load transfer might not be prominently discernible. Consequently, certain studies have chosen to exclude this term from their calculations [63], [64], [65], [74], [75].

The advantages, disadvantages, and applicable scenarios of the aforementioned lateral dynamics models are listed in Table II. It is important to note that the scenarios suitable for simpler vehicle dynamics models are generally applicable to more complex ones, although with the latter requiring increased computational resources.

C. Longitudinal Dynamics Models

Table III presents the longitudinal models employed in the literature concerning autonomous racing. It is notable that some studies concentrated on the lateral dynamics model, assuming a predefined longitudinal velocity, and therefore did not require the development of longitudinal dynamics [31], [51], [79]. Other studies omitted the modeling of how longitudinal forces were generated by the powertrain or braking systems. Consequently, they directly utilized either the longitudinal force denoted as F_x [47], [49], [71] or the longitudinal acceleration denoted as a_x [33], [48], [53] as model inputs. Additionally, a few studies distributed the longitudinal forces to different tires [44], [73], [77], without explicitly modeling their generation.

Apart from traction and braking forces, several supplementary models were employed to determine F_x. These models include factors such as rolling resistance acting on the tires, as well as the aerodynamic drag and gravity force affecting the vehicle's body.

TABLE III
OVERVIEW OF LONGITUDINAL DYNAMICS MODELS IN LITERATURE

Longitudinal Model	Related Publications
Absence of longitudinal forces	[31], [51], [79]
Drive/brake force/acceleration as system input variables	[32], [33], [35], [36], [44], [46]–[50], [53], [55]–[57], [59], [62]–[67], [69], [71], [73], [76]–[78], [80], [84], [89], [90]
Drive/brake force distribution among axles/tires	[38]–[41], [44], [45], [50], [55]–[57], [59], [62]–[68], [71], [73]–[77], [82]–[84], [88]
Electric motor traction modeling	[34], [37]–[42], [45], [56], [67], [68], [70], [82], [83], [88]–[90]
Internal combustion engines (ICE) traction modeling	[72], [76], [91]
Aerodynamic effect modeling	[32]–[34], [38], [40], [49], [54]–[57], [59], [62], [64]–[70], [72], [80], [82], [83], [88], [90], [91]
Rolling resistance modeling	[32]–[35], [38], [40], [42], [54]–[57], [59], [61], [64], [66], [73], [80], [82], [83], [88], [90], [91]
Gravity force modeling	[33], [61], [64], [65], [69]

Furthermore, traction and braking represent the two primary sources of variation in vehicle longitudinal dynamics and serve as the sole controllable force inputs. When operated by a human driver, these forces are influenced by a longitudinal command D, and the forces in relation to D are typically nonlinear. Moreover,

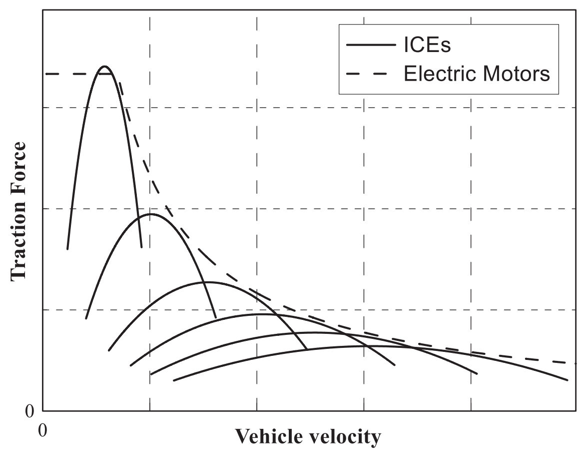

Fig. 5. Schematic traction characteristics of electric motors and ICEs.

in cases where the tire grip is saturated, it becomes crucial to consider the correlation between the generated longitudinal force and D, which must also account for the tire grip [76].

However, some autonomous racing studies omitted the consideration of longitudinal tire slip. This omission may stem from the impracticality of measuring wheel rotational speed [40], [42], or it may result from slippery road conditions combined with nonpneumatic tires [33]. In such cases, the force from the powertrain and braking systems was directly assumed to act on the vehicle body, and the longitudinal force related input was designed as D. Notably, the tire models described in Section II-D solely account for lateral tire slip.

Fig. 5 provides a schematic representation of the maximum traction force that electric motors and ICEs can deliver in relation to vehicle speed [72], [92]. It is evident that these powertrains exhibit different traction force characteristics, necessitating gearshifts in ICE-powered vehicles to align the engine's operating range with varying vehicle speeds.

In the case of electric motors, which are commonly utilized and assume negligible transient influence [93], traction F_{tr} is modeled in the form of a polynomial with respect to both D and u. The formula can be expressed as [38], [40], [83]:

$$F_{\mathrm{tr}} = (C_{\mathrm{tr},1} - C_{\mathrm{tr},2}u)D. \tag{16}$$

In other studies, F_{tr} of the electric motor can be arbitrarily selected within a range limited by $F_{\mathrm{tr,limit}}$, which is formulated based on its power and torque characteristics (Fig. 5). This relationship can be expressed as [67]

$$F_{\mathrm{tr,limit}} = \min\left(\frac{T_{\mathrm{limit}} i_{\mathrm{g}}}{R_{\mathrm{wheel}}}, \frac{P_{\mathrm{limit}}}{i_{\mathrm{g}} u / R_{\mathrm{wheel}}}\right), \tag{17}$$

where T_{limit} and P_{limit} denote the limits of motor torque and power, R_{wheel} is the wheel radius, and i_{g} is the transmission ratio. The first term, F_{tr} is subject to potential limitations by motor torque. It can also be applied in scenarios where F_{tr} is constrained by tire grip. At high velocities, the second term becomes relevant, limiting the traction by power. In some studies, only the power-related term on the right side of (17) is considered exclusively [69].

In cases where the vehicle is powered by an ICE (Fig. 5), it is common to use a lookup table to correlate the current velocity with the appropriate gear transmission ratio [72], [91]. Subsequently, the engine's output torque should be multiplied by this ratio to calculate the final traction force acting on the tires.

The braking force can be incorporated into the model by introducing negative forces solely in the vehicle's heading. The modeling approach for braking forces closely resembles that used for traction force [82].

In specific driving configurations, the force distribution between the front and rear axles can vary, whether it pertains to traction or braking force. The drive force can be adjusted through limited-slip differentials, while the brake force is controlled by a brake ratio setting. Consequently, different distribution ratios can be employed to configure the drive settings and brake ratio [62].

The other factors included by $F_{\mathrm{x,other}}$ that influence the longitudinal dynamics are air drag F_{drag} and resistance F_{re}, and can be expressed as

$$F_{\mathrm{x,other}} = -F_{\mathrm{drag}} - F_{\mathrm{re}}. \tag{18}$$

Regarding air drag, which predominantly affects the vehicle at high speeds, is commonly depicted as a force opposing the vehicle's velocity and acting on the CG. Although various formulations exist, it is typically approximated by the following equation [64]:

$$F_{\mathrm{drag}} = \frac{1}{2} \cdot \rho A C_{\mathrm{drag}} u^2, \tag{19}$$

where ρ is the air density, A is the vehicle front surface, and C_{drag} is the aerodynamic drag coefficient.

The longitudinal force resulting from resistance can be adequately approximated by a single constant [40]. For a more comprehensive representation that implicitly relies on F_{z}, the formula also includes all driveline resistance. This can be expressed as a fourth-order Taylor expansion of the longitudinal velocity [94]:

$$F_{\mathrm{re}} = C_{\mathrm{re},0} + C_{\mathrm{re},1} u + C_{\mathrm{re},2} u^4, \tag{20}$$

where $C_{\mathrm{re},0}$, $C_{\mathrm{re},1}$, and $C_{\mathrm{re},4}$ are predefined constants that may vary depending on the specific characteristics of different test vehicles. Some studies choose to include only the first two terms on the right side, omitting the higher-order refinements.

Furthermore, specialized models have been proposed to specifically investigate the traction source, especially in cases involving electric motors. It is assumed that the traction force provided by the electric motor cannot be sustained at high levels continuously due to the potential for thermal overstressing. This aligns with the primary factor contributing to power loss in electric vehicles [95]. Consequently, the power that the electric powertrain can deliver decreases due to the increase in internal battery resistance for thermal reasons [89]. Additionally, power loss stemming from copper losses, inverter switching, and conduction losses also contributes to a reduction in the power deliverable by an electric powertrain [90]. All of the detailed

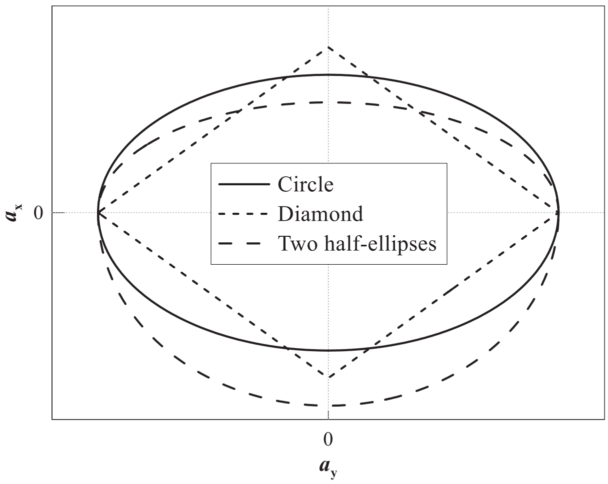

Fig. 6. Shapes used to limit a_x and a_y.

models of these electric systems can be parameterized based on datasheet values and experimental data.

D. Tire Considerations

The tire is the sole component of a vehicle that makes direct contact with the road surface. Consequently, all forces acting on the vehicle in a planar model are eventually transmitted to the tires in various forms.

In the field of autonomous racing, a range of tire models (as shown in Table I) have been employed, spanning from simple to complex. The simplest wheel model constrains the attainable force based on a geometric shape. Subsequently, assuming negligible tire longitudinal slip, certain tire models exclusively analyze lateral forces. Finally, there are tire models that account for slips in both the longitudinal and lateral directions.

Regarding the correlations between tire slip angle and forces, several models have been applied. These include the linear model, the simplified Pacejka model, the Pacejka model, and the Fiala brush model, each with specific underlying assumptions.

1) Lateral and Longitudinal Acceleration Bounded: In studies employing the point-mass model, specific tire characteristics were not individually investigated; rather, the overall performance of the car was considered. In its simplest form, the maximum attainable acceleration must be constrained to prevent entering a phase where tire forces reach saturation [47]. Under these conditions, the maximum forces that the tires can deliver, strongly correlated to their corresponding F_z [69], can be empirically determined.

When integrated with the point-mass model, this leads to bounded resultant forces on the road surface. Various limiting shapes, as depicted in Fig. 6, have been employed in different research studies, such as the diamond shape [47], [49], circle [69], and two half-ellipses [48]. For example, the diamond shape for acceleration bounding can be expressed simply as [47]:

$$\left|\frac{a_x}{a_{x,\max}}\right| + \left|\frac{a_y}{a_{y,\max}}\right| \leq 1, \tag{21}$$

where $a_{x,\max}$ and $a_{y,\max}$ denote the maximum achievable longitudinal and lateral forces provided by the tire. It is noteworthy that the shape and calibration values in this model should be determined while taking into consideration factors such as road surface conditions, tire layout, and current weather conditions [96].

2) Tire Lateral Forces Modeled: Since the longitudinal slip angle is not taken into account, the vehicle's longitudinal behavior is primarily determined by longitudinal models without considering tire saturation. Additionally, the camber angle of a wheel is not considered. This simplification allows for the straightforward derivation of tire slip angles. In the case of a front steering vehicle, based on a single-track model, the lateral slip angles α of its front and rear tires can be expressed as [51]:

$$\begin{aligned} \alpha_f &= \arctan\left(\frac{v + l_f r}{u}\right) - \delta, \\ \alpha_r &= \arctan\left(\frac{v - l_r r}{u}\right). \end{aligned} \tag{22}$$

If tire lateral slip is analyzed using a double-track model, it is important to incorporate the influence of track width into (22) [67]. However, for the sake of brevity, this adjustment is omitted in this context.

Lateral forces F_y exerted by tires within specific lateral slip angles exhibit a nearly linear relationship with the corresponding α. This allows for the use of a linear model to approximate tire behavior in this range, resulting in what is commonly referred to as a linear tire model, expressed as [70]:

$$F_{y,L} = -2C_\alpha \alpha, \tag{23}$$

where C_α, as the product of the surface coefficient of friction μ and F_z, represents the constant cornering stiffness of the respective single tire, regardless of whether it is the front or rear tire. Given that (23) is demonstrated with the single-track model as an illustration, a single tire within this model is employed to represent both tires on a single axle. Consequently, a coefficient of 2 is used.

Various tire models applied in autonomous racing can be broadly categorized into two types, as depicted in Fig. 7. The first category is the semiempirical Fiala brush model [28], while the second category comprises empirical models based on the Pacejka formulation [12]. It is evident from Fig. 7 that within a specific range, F_y linearly decreases with increasing α, corresponding to the effective range of the linear model. As the absolute value of α further increases, F_y gradually approaches saturation and remains approximately constant.

The Pacejka model, often referred to as the Magic Formula, is designed to fit experimental tire response curves [12] and is based on the following equation [54]:

$$\mu_{y,P} = B_1 \sin(B_2 \arctan(B_3\alpha - B_4(B_3\alpha - \arctan(B_3\alpha)))), \tag{24}$$

where B_1 denotes the peak value, B_2 is a shape factor, B_3 represents the stiffness factor, and B_4 stands for a curvature factor. Although variations may exist in different literature sources, such as accounting for aerodynamic effects as an additional coefficient function [66], these variations are mainly focused on

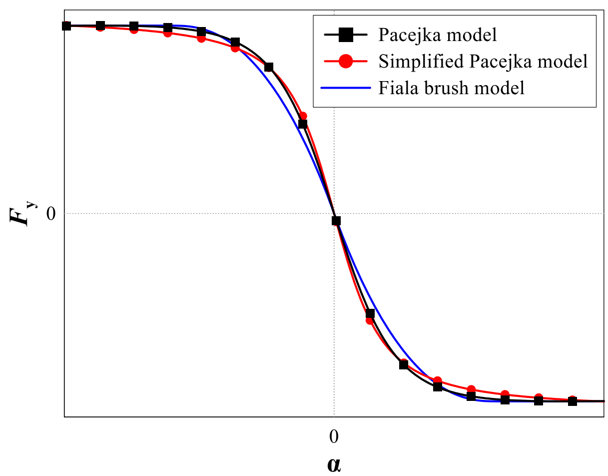

Fig. 7. Model comparison: lateral force F_y vs lateral slip angle α.

the derivation of F_z, which is crucial for lateral tire force [54]. However, the Pacejka model is employed to estimate μ, which is subsequently used to compute lateral force F_y by multiplying it with F_z. In this context, a constant B_1 is adequate for adjusting the maximum value of μ [42].

In pursuit of a more straightforward and computationally efficient alternative to the Pacejka model, a simplified version was proposed, aligning with the demand for rapid computations in racing scenarios. This simplified model is also widely used in autonomous racing and can be expressed as [40]:

$$\mu_{y,SP} = B_5 \sin\left(B_6 \arctan\left(B_7 \alpha\right)\right), \tag{25}$$

where B_5 serves as the shape factor, B_6 represents the stiffness factor, and B_7 signifies the peak lateral force. All these factors collectively define the precise shape of the empirical curve.

In contrast to fitting curves based on the Pacejka model, the brush model strives to depict the complex interplay between the tire and the road, explaining how forces are generated. One example of this is the Fiala model. In the context of autonomous racing, assuming a flexible carcass, a single coefficient of friction μ and a parabolic force distribution across the contact patch [52], the formulation can be expressed as:

$$F_{y,F} = \begin{cases} -C_\alpha \tan\alpha + \frac{C_\alpha^2}{3\mu F_z} \cdot |\tan\alpha| \tan\alpha \\ -\frac{C_\alpha^3}{27\mu^2 F_z^2} \cdot \tan^3\alpha, \ |\alpha| < \arctan\left(\frac{3\mu F_z}{C_\alpha}\right), \\ -\mu F_z \mathrm{sgn}\alpha, \qquad \text{otherwise.} \end{cases} \tag{26}$$

3) Tire Longitudinal and Lateral Forces Modeled: Incorporating longitudinal slip introduces the second direction of tire slip, resulting in the overall tire slip and the induced resultant tire force.

The longitudinal slip, denoted as λ, is defined as [32]

$$\lambda = \frac{\omega R - u}{u}, \tag{27}$$

where ω is the wheel speed and R is the wheel rolling radius. Regarding the simplified Pacejka model, the wheel overall slip, denoted as γ_{SP} and its magnitude $\gamma_{SP} = \sqrt{\lambda^2 + \epsilon^2}$, is the vector sum of λ and the corresponding tire lateral slip ϵ. Equation (25) can then be transformed into

$$\mu_{SP} = B_5 \sin\left(B_6 \arctan\left(B_7 \gamma_{SP}\right)\right). \tag{28}$$

This enables the derivation of the surface friction coefficient μ_{SP}, and the direction of the resultant tire force is simply the reverse of the overall tire slip direction [60].

If the Fiala brush model is applied, an intermediate weighted vector sum γ_F can be defined as [61],

$$\gamma_F = \sqrt{C_\lambda^2 \cdot \left(\frac{\lambda}{1+\lambda}\right)^2 + C_\alpha^2 \cdot \left(\frac{\tan\alpha}{1+\lambda}\right)^2}, \tag{29}$$

and the magnitude of the resultant force is expressed as,

$$F_F = \begin{cases} \gamma_F - \frac{1}{3\mu F_z} \cdot \gamma_F^2 + \frac{1}{27\mu^2 F_z^2} \cdot \gamma_F^3, & \gamma < 3\mu F_z, \\ \mu F_z, & \gamma \geq 3\mu F_z, \end{cases} \tag{30}$$

where C_α and C_λ represent the cornering and longitudinal stiffness, respectively, which can be identified a priori from straight line braking and ramp steer maneuvers [97].

The longitudinal and lateral forces are subsequently calculated as,

$$F_{x,F} = \frac{C_\lambda}{\gamma_F} \cdot \left(\frac{\lambda}{1+\lambda}\right) \cdot F_F, \quad F_{y,F} = \frac{-C_\alpha}{\gamma_F} \cdot \left(\frac{\tan\alpha}{1+\lambda}\right) \cdot F_F. \tag{31}$$

In pure cornering maneuvers, which imply the absence of longitudinal tire slip, the maximum lateral tire slip angle, at which the maximum lateral tire force is achieved, is $\alpha_{max} = \arctan(3\mu F_z / C_\alpha)$, and it is in line with (26).

E. Model-Free Methods

The abovementioned model-based approaches are used to design motion planning and tracking control algorithms relying on a detailed understanding of vehicle dynamics. In contrast, model-free methods, often denoted as end-to-end autonomous driving, leverage data-driven techniques to replace a part of the driving pipeline between perception inputs and the generation of steering and acceleration commands for racecars [18].

In the field of end-to-end driving, with regard to the specific segment of the driving pipeline where the end-to-end pipeline replaces, the approaches can be broadly categorized into two groups. One group involves a partial integration, replacing the segment from perception to motion planning, while maintaining a distinct low-level control controller [98]. The second group directly employs the end-to-end pipeline, establishing a direct connection from perception data to the final actuator outputs [99]. When classified by the type of end-to-end autonomous driving, two fundamental categories are present: imitation learning [100], which involves training by mimicking the behavior of an expert; and reinforcement learning [101], which continuously interacts with the environment, aiming to maximize the specific cumulative reward. Notably, both of these approaches implicitly acquire information about vehicle dynamics during the training process. Additionally, there is a contemporary trend involving physics-informed neural networks, which integrate vehicle dynamics models into their architecture [102]. This approach is employed in autonomous racing to take advantage of both vehicle dynamics models and end-to-end approaches.

TABLE IV
DYNAMICS RELATED HARDWARE OF REDUCED-SCALE AUTONOMOUS RACING CHASSIS

Model Type	**AutoRally** [105][108]	**F1TENTH** [106][109]	**Kyosho Dnano Cars** [40][110]
Dimensions	1:5	1:10	1:43
Powertrain	Brushless motor powered with rear wheel drive	Brushless motor powered with four-wheel drive	DC motor powered with rear wheel drive
Brake	Front disk brake Rear electric brake	Four-wheel electric brake	Rear electric brake
Size	Length: 900 mm; Width: 460 mm	Length: 568 mm; Width: 296 mm	Length: 120 mm; Width: 50 mm
Max speed	18 m/s	22 m/s	3 m/s
Mass	13 kg	3.5 kg	41 g
Wheelbase	570 mm	324 mm	63 mm
Track width	Front: 408 mm; Rear: 418 mm	296 mm	50 mm

III. PHYSICAL AUTONOMOUS RACING PLATFORMS

The rise of autonomous racing has led to the development of diverse racing platforms ranging from reduced-scale to full-scale tailored to the specific needs of researchers. These platforms have served as the foundation for a multitude of studies on perception [9], prediction [44], planning [103], and control [104], conducted by various universities and research institutions.

(a) (b) (c)

Fig. 8. Reduced-scale racing chassis: (a) AutoRally [105]; (b) F1TENTH; (c) kyosho dnano car.

A. Reduced-Scale Racing Platforms

With regard to reduced-scale autonomous racing platforms, emphasis has been placed on crafting compact designs tailored for easily accessible environments to provide a cost-effective means of simulating vehicle dynamics [106].

These reduced-scale racing cars underwent extensive modifications by dedicated students and researchers, who started with commercially available RC cars as their foundation [105]. The final drives of these models are powered by different types of small electrical motors, selected for their compact dimensions and their potential for advanced state estimation and performance optimization [106]. Furthermore, all these vehicles are equipped with Ackermann steering and precisely controlled by a servo motor. The suspensions across reduced-scale cars are uniformly configured in the double wishbone form, which is known for its ease of tuning [107]. These compact structures also lead to a considerably lowered center of gravity, thus mitigating the effect of weight redistribution during acceleration. Additionally, they feature a centered weight distribution and an enhanced power-to-weight ratio, albeit without an incorporated gearbox. Although these platforms share common traits, they have distinctive dynamics-related features. These distinctive features are listed in Table IV.

AutoRally (Table IV and Fig. 8(a)), a 1:5-scale autonomous vehicle testbed, was developed by researchers from the Georgia Institute of Technology [105]. Derived from the original HPI Baja 5SC RC trophy truck, it incorporates a substantial modification, where the initial gasoline engine is replaced by a 10-hp peak output electric motor for exclusively driving the rear axle. This model features a single differential alongside two hydraulic front brakes and two electronic rear brakes. It stands out as the only mainstream reduced-scale platform equipped with physical brakes mirroring the equipment of full-scale vehicles. Moreover, to counteract the negative effects brought by the added mass, the front and rear suspension systems possess an increase in spring rates of up to 2.63 and 3.34 kN/m, respectively, resulting in a spring-to-weight ratio that is comparable to that of a sports car. Additionally, the viscosity of the damper's shock oil is increased to reduce body roll and enhance overall driving dynamics.

The F1TENTH car (Table IV and Fig. 8(b)) is an open platform provided by an international community, and its origins can be traced back to the University of Pennsylvania [111]. This community offers a unique opportunity for researchers and enthusiasts to independently develop identical cars. Constructed on a standardized chassis, this platform facilitates race events that provide a level playing field, and the primary focus is on evaluating the software testing of each team [112]. The F1TENTH car is a transformation of a Traxxas Slash 4×4 Premium Chassis designed at the 1:10 scale [106]. It features a four-wheel drive configuration, with limited-slip differentials mounted on the front and rear axles. Notably, this model lacks a physical braking system; its braking force is skillfully managed by the motor, similar to the drive force delivered to the car. Consequently, the driving and braking forces can be simply described through the motor's performance profile [113].

Several platforms share a 1:10 scale and are grounded on similar stock RC cars. Examples include RoSCAR [114], Berkeley Autonomous Race Car [35], UT Automata robot [115], and MuSHR [9]. They feature similar dynamics-related parameters, and a detailed description is omitted for brevity.

Researchers at ETH Zurich have established 1:43-scale autonomous racing cars (Table IV and Fig. 8(c)) built on the foundation of Kyosho Dnano RC cars [40]. Control signals are transmitted to these cars via Bluetooth, eliminating the need for an onboard computer, which could introduce additional mass. The suspension systems are simplified and realized through elastic aluminum plates [110]. Additionally, the braking system,

TABLE V
DYNAMICS RELATED PARAMETERS OF FULL-SCALE AUTONOMOUS RACING CHASSIS

Model Type	**FSD** [55][125]–[127]	**Autonomous TTS** [52]	**Autonomous Golf GTI** [65]			**Stanford X1** [73][128][129]	**MARTY** [77][119]	**Brilliance® experimental vehicle** [17]
m (kg)	195–275	1500	1776			2000–2009	1450–1700	1528
I_z (kg·m^2)	93–105	2250	3950			2000–3764	2300–2386	2280
l_f (m)	0.77–0.84	1.04	1.19			1.53–1.56	1.39–1.60	1.19
l_r (m)	0.64–0.78	1.42	1.44			1.18–1.23	0.80–1.01	1.60
$C_{\alpha,f}$ (kN/rad)	24–45	160	138 for ice	240 for wet asphalt	176 for dry asphalt	114–150	59–100	58
$C_{\alpha,r}$ (kN/rad)	24–45	180	139 for ice	300 for wet asphalt	226 for dry asphalt	134–280	160–275	68

which exclusively engages the rear axle, relies on the drive motor. This chassis can achieve speeds of up to 3 m/s, which is equivalent to 343 km/h for a full-scale car.

B. Full-Scale Racing Platforms

Although reduced-scale platforms are inexpensive and have a straightforward setup, full-scale cars remain essential for testing software and hardware in autonomous motorsports primarily because they are the only platforms that can replicate the complex and comprehensive dynamic conditions in actual racing environments [18]. Specifically, reduced-scale models struggle to reproduce critical factors, such as power-to-weight ratio, suspension geometry, chassis stiffness, and tire characteristics, inherent to full-scale cars.

Electric motor-powered cars exhibit a linear traction force output (Fig. 5), and those equipped with multiple motors have the ability to distribute traction arbitrarily to any driving wheel [116]. However, electric motors still face limitations in providing sustained high power, which constrain their extensive use in the field of motorsports [89]. Consequently, research on full-scale platforms has expanded beyond electric cars.

One prominent event in this domain is the Indy Autonomous Challenge launched in 2019, which continues to engage university teams in competition. The event employs the Dallara AV-21 (Fig. 9(a)), a chassis derived from the Dallara IL-15 originally designed for Indy Lights [120]. It features a 2.0 L turbocharged four-cylinder engine with a maximum power of 450 HP. It can achieve speeds of up to 240 km/h. Power is transmitted to the rear wheels via a three-plate design clutch, a six-speed semi-automatic gearbox, and a limited-slip differential. Both axles utilize a double wishbone suspension, and the front and rear wing flaps are adjustable [121].

Another notable event is Roborace, which was initially conceived to support Formula E races but discontinued in 2021. The official prototypal chassis provided varied from year to year. Most of the information available pertains to Devbot 2.0 (Fig. 9(b)), which was derived from a Ginetta LMP3 racing chassis [122]. Devbot 2.0 participated in the 2019 Roborace event and remains a valuable research platform for participating universities. In this chassis, each rear wheel is driven by an integrated electrical powertrain, each of which delivers at a maximum 135 kW, thus enabling torque vectoring [123]. The suspension for the front and rear utilizes a double wishbone type.

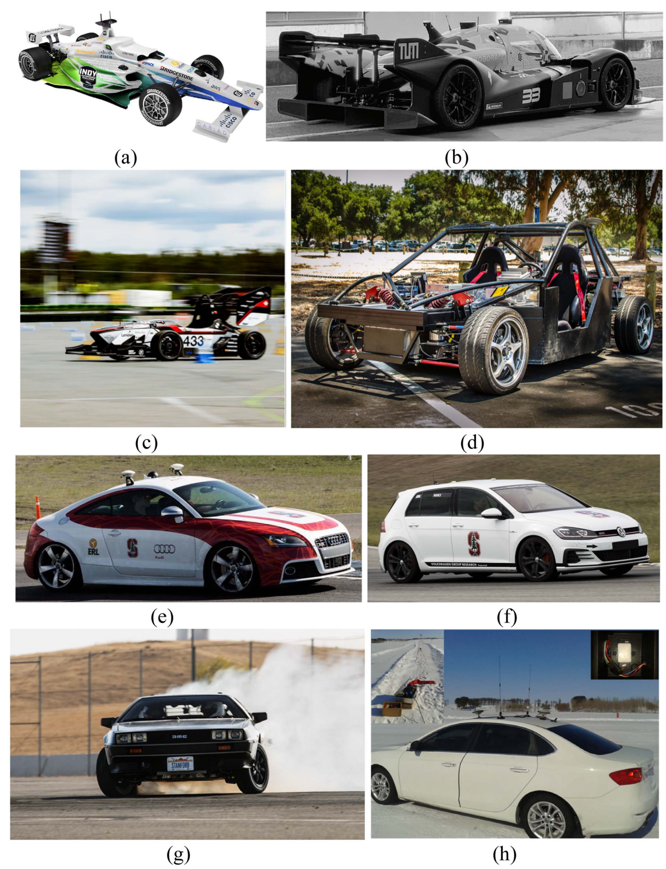

Fig. 9. Full-scale racing chassis: (a) dallara AV-21 [79]; (b) devbot 2.0 [117]; (c) FSD racing car [118]; (d) X1 [73]; (e) autonomous TTS [58]; (f) autonomous Golf GTI [65]; (g) MARTY [119]; (h) brilliance experimental vehicle [17].

The Formula Student Driverless (FSD) Competition offers opportunities for universities worldwide to engage in autonomous racing. While each team is required to fabricate a racing chassis annually, specific rules may vary across branch competitions [124]. As a result, detailed insights into chassis design (Table V and Fig. 9(c)) related to vehicle dynamics have emerged. Some competing groups modified their chassis previously used in the

Formula Student Electric event [130]. With the exception of computers, sensors, servo steering, and braking systems, the other parts of the chassis maintained a framework similar to that of the Formula Student Electric chassis. In single-motor configuration, the electric motor is commonly positioned between the cockpit and rear axis. An intermediate gearbox, a drive shaft, a differential, and two half shafts are utilized to link the motor and drive wheels [131]. With the growing prevalence of distributed power sources, diverse advanced approaches have emerged, featuring motors directly linked to the hubs of individual tires in various configurations [132]. The battery pack is arranged to optimize the car's weight distribution. Additionally, the suspension type commonly selected is the double wishbone, and sophisticated aerodynamic packages may be designed to generate downforce, typically at speeds below 100 km/h [118].

Table V presents typical vehicle dynamics model parameters pertaining to the FSD chassis, and some other full-scale testing platforms with the size of passenger cars. Notably, certain variations in parameters within a model type may be attributed to either distinct chassis designs serving a common purpose or diverse configurations within a single chassis. Furthermore, compared with other test vehicles with similar sizes as passenger cars, the FSD vehicle exhibits notably reduced values across all parameters. This discrepancy arises from its lightweight car body and its specialized design intended for racing purposes.

With regard to passenger car-sized test vehicles with similar dynamic-related parameters, the Center for Automotive Research at Stanford (CARS) has prospectively developed a series of prototypal racing cars initially for advancing by-wire technology and later for autonomous racing. The P1 [133] and X1 (Table V and Fig. 9(d)) [73] platforms were consecutively developed. X1, a platform that is still in use, is a truss-frame car with rear wheels driven by one electric motor through an open differential [134]. All four wheels of the X1 platform can be independently steered by wire and braked by wire, thus opening up possibilities for in-depth explorations of its vehicle dynamic potential.

Furthermore, some production cars have been modified into platforms for testing autonomous racing. The same research group in CARS has transformed three different production cars into autonomous racing cars. Among these cars, Audi TTS (Table V and Fig. 9(e)) [52] and Volkswagen Golf GTI (Table V and Fig. 9(f)) [65] are gasoline-powered, and MARTY (Table V and Fig. 9(g)) [77], a modification of a 1981 DMC Delorean, is equipped with dual electric motors on its rear axle. Notably, the Golf underwent testing under variable tire road conditions. Thus, different values concerning $C_{\alpha,\mathrm{f}}$ and $C_{\alpha,\mathrm{r}}$ are reported in Table V. Similarly, researchers at Tsinghua University have employed a Brilliance experimental vehicle (Table V and Fig. 9(h)) for similar tests [17].

IV. Virtual Autonomous Racing Testing Platforms

The burgeoning field of autonomous racing has elicited substantial interest from research groups and dedicated companies, leading to the development or enhancement of virtual platforms tailored for autonomous racing applications. A comprehensive overview of these virtual platforms, including their physics engine, rendering engine, vehicle dynamics configuration, open-source status, and release year, is presented in Table VI. Additional insights are provided in Section IV-A.

In the vehicle dynamics configuration column, abbreviations, such as customizable models (CM), selectable built-in models (SBM), and tunable parameters (TP), delineate the configurability of vehicle dynamics physics within each platform. SBM denotes a distinct approach for car modeling, and TP involves adjustments to specific parameters without necessitating a change in the modeling method, although it may include modifications to the vehicle type.

Even with the availability of virtual platforms, users still face the arduous task of developing and fine tuning precise vehicle dynamics models tailored to their needs. This task is particularly challenging because users often concentrate on specific model types, whether full- or reduced-scale racecars designed for distinct racing events. In response to these challenges, several teams have attempted to create simulators in specific virtual platforms for addressing this specific need. These simulators, predominantly rooted in Gazebo, are systematically cataloged in Table VII and extensively discussed in Section IV-B. They provide a comprehensive perception, planning, and control algorithm testing ground to researchers or enthusiasts committed to advancing autonomous racing applications.

A. Virtual Testing Platforms

Gazebo (Fig. 10(a)) [135] is a leading robot simulation platform crafted by Open Source Robotics Foundation, the creator of ROS. Fig. 10(a) provides a glimpse of its user interface, and it utilizes third-party physics engines, with ODE serving as the default [158]. Meanwhile, Gazebo can be integrated with physics engines, such as Bullet, DART, and Simbody, each of which can implement vehicle dynamics models based on rigid body dynamics, collision detection, and specific functionalities. However, each physics engine still encounters challenges, such as erratic jumps or positional anomalies [158].

CoppeliaSim (Fig. 10(b)) [136], formerly known as V-REP and developed by Coppelia Robotics, operates on a distributed control architecture. It allows individual control of the object/model through various means, including embedded scripts, plugins, ROS nodes, remote API clients, or custom solutions. CoppeliaSim is designed for rapid algorithm development, prototyping, and verification. It includes a built-in vehicle dynamics engine for basic features, complemented by support for detailed modeling through scripting or external dynamics integration.

dSPACE [137], developed by the dSPACE GmbH, employs a hardware-in-the-loop (HIL) approach to test real electronic control units, thus offering realistic simulated platforms for racing scenarios. While predominantly used to link hardware and virtual platforms, dSPACE allows the integration of various virtual platforms as its physics engine. The 2016 release year in Table VI signifies the incorporation of the HIL concept into ADAS systems and automated driving scenarios.

TABLE VI
VIRTUAL PLATFORMS OF AUTONOMOUS RACING

Virtual Platform	Physics Engine	Rendering Engine	Vehicle Dynamics Configuration	Open-Source	Release Year
Gazebo [135]	ODE, Bullet, DART, and Simbody	ORGE	CM, SBM, and TP	Yes	2004
CoppeliaSim [136]	Bullet, ODE, Vortex, and Newton	Self-developed	CM, SBM, and TP	Yes	2013
dSPACE [137]	Simulink, CarSim, ASM, etc.	Relying on display system	Relying on physics engine	No	2016
Webots [138]	ODE	ORGE	CM, SBM, and TP	No	2004
MORSE [139]	Bullet	Blender	CM, SBM, and TP	Yes	2011
DeepDrive [140]	Unreal	Unreal	TP	No	2018
CARLA [141]	Unreal	Unreal	TP	Yes	2017
AutoDRIVE [142]	PhysX	Unity	-	Yes	2020
LGSVL [143]	Unity	Unity	CM	Yes	2020
Grand Theft Auto V [144]	Self-developed	Self-developed	TP	No	2013
Gran Turismo Sport [145]	Self-developed	Self-developed	TP	No	2017
TORCS [146]	Self-developed	Self-developed	SBM and TP	Yes	2007
CarSim [147]	Self-developed	Relying on display system	CM, SBM, and TP	No	1990
CarMaker [148]	Self-developed	Relying on display system	CM, SBM, and TP	No	1998
rFpro [149]	Self-developed	Self-developed	CM, SBM, and TP	No	2007
Cognata [150]	Self-developed	Not published	TP	No	2016
ANSYS VRXPERIENCE [151]	Self-developed	Self-developed	CM, SBM, and TP	No	2017
NVIDIA Drive Constellation [152]	Self-developed	Self-developed	CM, SBM, and TP	No	2018

TABLE VII
PHYSICAL AUTONOMOUS RACING PLATFORM SIMULATORS

Virtual Platform	Vehicle Dynamics Models or Physics Engine	Simulating Platform	Open-Source	Release Year
RACECAR [153]	Kinematic single-track	Gazebo	Yes	2017
RoboMaker [154]	ODE	Gazebo	Yes	2020
AutoRally [105]	ODE	Gazebo	Yes	2019
F1TENTH [155]	ODE	Gazebo	Yes	2020
Oxford Brookes Formula Student [156]	Dynamic single-track and linear tire model	Gazebo	No	2020
FSSIM [118]	An integrated first principal model	Gazebo	Yes	2019
KIT21d [165]	Self-developed 7 DOFs planar vehicle model and Pacejka tire model	Homemade based on ROS	No	2021

Webots (Fig. 10(c)) [138], provided by Cyberbotics Ltd. in collaboration with the Swiss Federal Institute of Technology in Lausanne, is an open-source mobile robotics simulation platform. It facilitates the simulation of racing cars with configurable dynamics by using the third-party physics engine ODE. Although a free version is available for educational and non-commercial use, the source code remains inaccessible to the public.

MORSE (Fig. 10(d)) [139], jointly developed by LAAS and ONERA, is built on Blender software and exploits the Bullet physics engine. Offering extensive configuration options for vehicle dynamics models and parameters, MORSE enhances the flexibility of simulation platforms.

DeepDrive (Fig. 10(e)) [140], developed by the University of California, Berkeley, is designed specifically for racecars and algorithm testing. Utilizing the Unreal physics engine, it allows users to configure vehicle dynamics models and parameters seamlessly.

CARLA (Fig. 10(f)) [141], a collaboration among Intel Labs, the Toyota Research Institute, and the Computer Vision Center in Barcelona, employs the Unreal engine. CARLA is tailored for the development, training, and validation of autonomous driving systems, but it is also suitable for testing racing scenarios [163].

AutoDRIVE (Fig. 10(g)) [142], a joint development by the SRM Institute of Science and Technology and Nanyang Technological University, utilizes the PhysX physics engine from NVIDIA. This simulator is specifically designed for a reduced-scale racing platform, and preconfigured for educational use, with dynamics models, including aerodynamics, readily set for the specific car.

LGSVL (Fig. 10(h)) [143], developed by LG Electronics America R&D Lab, offers an end-to-end, full-stack simulation compatible with Autoware and Apollo. Featuring a basic vehicle dynamics model for autonomous vehicles, LGSVL also supports the integration of external third-party dynamics models.

Grand Theft Auto V (Fig. 10(i)) [144] from Rockstars, Gran Turismo Sport (Fig. 10(j)) [145] from Polyphony Digital, and TORCS (Fig. 10(k)) [146] developed by individuals are prominent video games utilized in autonomous racing research. Despite being commercial, they serve as valuable virtual platforms because of their robust vehicle dynamics modeling. TORCS, in particular, offers diverse suspension types with corresponding models for user selection.

CarSim (Fig. 10(l)) [147], which is based on VehicleSim and developed by Mechanical Simulation Corporation, is not open-sourced but provides full configurability of vehicle dynamics. Tailored for autonomous racing research, CarSim supports

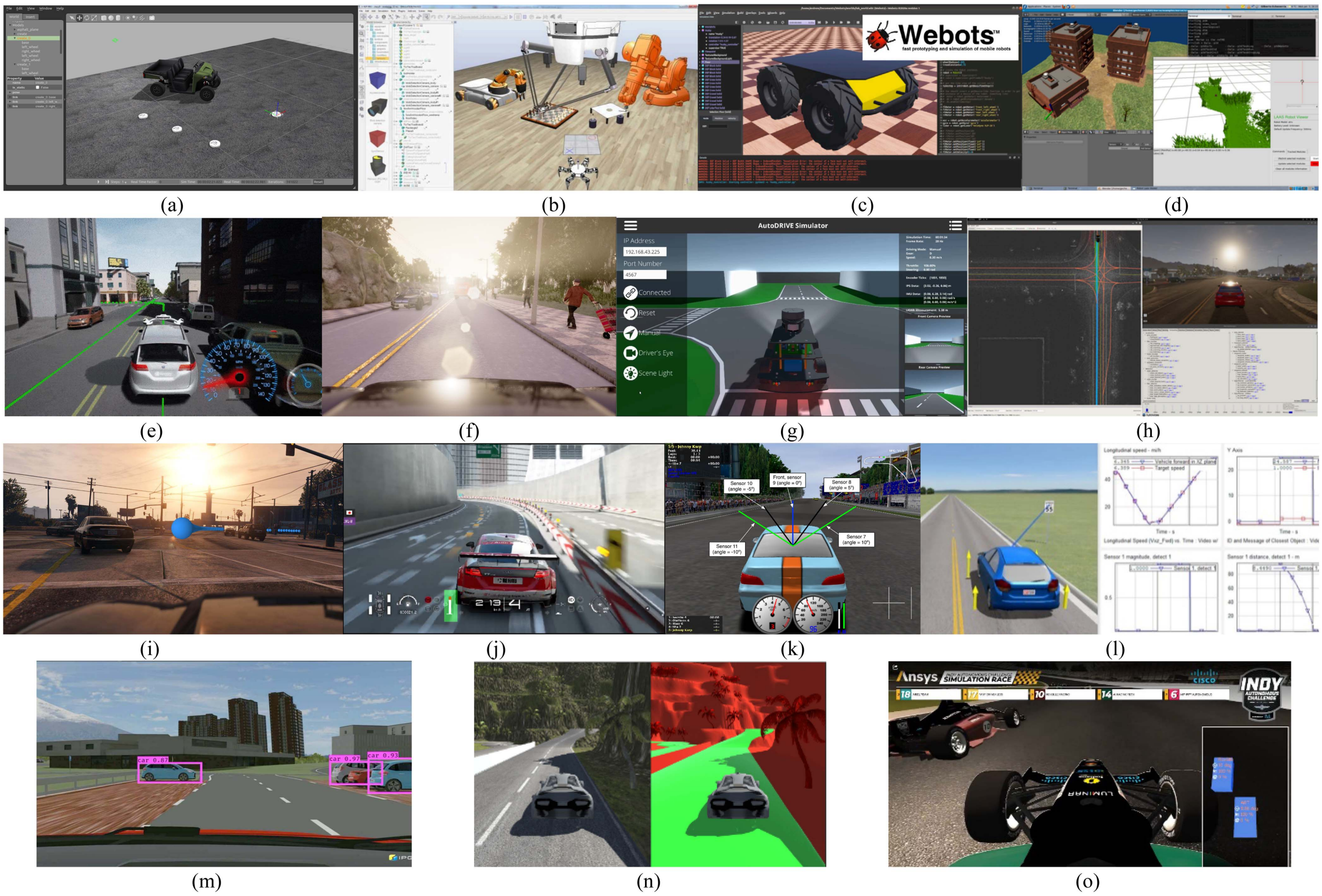

Fig. 10. User interface of the virtual platforms: (a) gazebo [157]; (b) coppeliaSim [136]; (c) webots [158]; (d) MORSE [139]; (e) DeepDrive [159]; (f) CARLA [141]; (g) AutoDRIVE [142]; (h) LGSVL [143]; (i) grand theft auto V [144]; (j) gran turismo sport [145]; (k) TORCS [160]; (l) CarSim [161]; (m) CarMaker [148]; (n) rFpro [149]; (o) VRXPERIENCE [162].

vehicle motion through built-in functions or external models via Simulink, extending its versatility. The provided vehicle dynamics models can be seamlessly integrated with the Unreal rendering engine to ensure high-quality rendering and support HIL.

CarMaker (Fig. 10(m)) [148], developed by IPG Automotive, is designed for recreating real test drives in the virtual world. Similar to CarSim, CarMaker evolved from a tool supporting vehicle dynamics projects and now offers comprehensive simulation platforms and datasets for autonomous vehicle research. Hence, it is a natural choice for autonomous racing virtual testing [164].

rFpro (Fig. 10(n)) [149], owned by AB Dynamics PLC, originated as a project of a Formula One team and is utilized for vehicle dynamics simulation. Focusing on mainstream racetracks, rFpro provides high-fidelity digital models and supports the development, training, testing, and validation of autonomous driving systems.

Cognata [150] from the Cognata Ltd., VRXPERIENCE (Fig. 10(o)) [151] from ANSYS, and DRIVE Constellation [152] from NVIDIA are commercial software that offer physically accurate simulation platforms for autonomous vehicles. VRXPERIENCE serves as the platform for the Indy Autonomous Challenge. While these platforms come with proprietary physics engines, some allow integration of third-party models, such as the dSPACE ASM vehicle dynamics models in the case of the DRIVE Constellation platform.

B. Physical Autonomous Racing Platform Simulators

The RACECAR simulator [153], which was developed by researchers at Massachusetts Institute of Technology (MIT), is a reduced-scale racecar simulator established under the MIT Beaver Works Summer Institute Robotics Program. Although it is primarily a high-school STEM program in robotics, the simulator provides an open-source kinematic single-track model in the Gazebo platform for users to virtually test their vehicles.

RoboMaker [154], developed by Amazon Web Services, incorporates a DeepRacer car model and functions as a virtual racing platform and a reinforcement learning research testbed. Aligned with the simulators for AutoRally [105] and F1TENTH [111], these platforms for reduced-scale racecars opt to use Gazebo as the platform and ODE as the physics engine. Although the dynamics models for this simulation have not been disclosed, the developers of AutoRally acknowledge Gazebo's limitations, especially in high-fidelity simulation when vehicles approach or surpass friction limits. However, these simulators are predominantly geared toward debugging control, perception, and communication software for reduced-scale racecars.

Oxford Brookes Formula Student Team's simulator for their full-scale racecar [156], which based on Gazebo, employs a dynamic single-track model with a linear tire model to showcase its general dynamic behavior.

FSSIM [118], the simulator for the Formula Student racecar of AMZ Driverless in ETH Zurich, is an open-source simulator applicable worldwide, and it is situated in the Gazebo platform. Although the dynamics models applied remain undisclosed, developers confirm that the ODE engine is not used, and an integrated first principal model is adopted.

The KIT21d simulator [165] designed in ROS for the Formula Student team at Karlsruhe Institute of Technology adopts a nonlinear dynamic 7-DOF planar vehicle model with the Pacejka tire model. This model accounts for wheel load transfer, aerodynamic effects, and powertrain characteristics, and strives to accurately reflect real vehicle dynamics.

V. Perspectives of Vehicle Dynamics on Autonomous Racing

The consideration of vehicle dynamics stands out as a critical aspect in the domain of autonomous racing, yet it remains an underdeveloped area when implemented in the autonomous racing car. Meanwhile, artificial intelligence has demonstrated its proficiency in managing a racecar's maximum cornering speed and optimizing lap times on specific racetracks under predefined conditions. Nevertheless, there are still aspects that can be further refined. This section delves into the future developments of enhancing vehicle dynamics models with the goal of seamless integration into the motion planning and control algorithms of autonomous racing cars. Additionally, it provides suggestions for the progress of methodologies in testing autonomous racing cars.

A. Adapting to Model Variation Over Time

The present study primarily emphasizes pushing the lap time limits within determined configurations and environmental conditions, similar to a qualifying race. Under these circumstances, a set of constants in parameter configurations generally suffices to adjust the model [12]. However, when races extend in duration, as in sprint or feature races, several factors influence the outcome. Fuel consumption alters the vehicle's mass, inertia, and the location of CG; tire wear and weather fluctuations change the complexity of tire road interaction. Additionally, rubber accumulates at specific track locations, causing variations in track conditions section by section. All of these factors lead to adjustments in the parameters of vehicle dynamics models. Albeit some can be continuously monitored, modeled, and fine-tuned to approximate real-time conditions, others, particularly tire grip limitations, pose a challenge for accurate estimation [166]. Therefore, maintaining real-time monitoring and fine-tuning of the model may present a notable challenge if autonomous racing is to become a prominent commercial sports activity.

B. Modeling for Thoroughness and Efficiency

Vehicle dynamics models must strike a delicate balance between computational efficiency and accuracy [14]. They should be neither oversimplified to the extent of lacking essential dynamics information, nor become overly complex to be computationally heavy. As onboard computing capabilities advance, there is potential for applying more intricate models. However, a pragmatic approach involves capturing critical physical phenomena through established models while employing machine-learning techniques, as a few researchers did [167], to degrade the description of other mathematical phenomena [58]. Furthermore, the application of physics-informed neural networks may facilitate precise calibration of dynamics models with limited data [102]. This combined approach may offer a practical methodology to accelerate the development of autonomous racing.

C. Balancing Datasets for End-to-End Racing

The training dataset utilized for end-to-end driving can cover a wide range of scenarios pertaining to specified race types when only a single racecar operates on a racetrack under regular conditions. However, the availability of samples related to scenarios involving head-to-head competition and extreme racetrack conditions is limited. Moreover, these samples exhibit substantial diversity. This diversity, coupled with the limited number of samples, results in a substantial imbalance across datasets covering different scenarios [168]. This imbalance poses a notable challenge for end-to-end models, particularly in terms of their generalization to special racing conditions. While the trained end-to-end autonomous racing system may demonstrate satisfactory performance when a single racecar navigates a racetrack under regular conditions, the introduction of competitors and extreme track conditions can lead to a substantial increase in lap times or even safety concerns. Consequently, addressing the challenges associated with data collection and handling imbalanced datasets remains a complex task for the advancement of end-to-end autonomous racing systems.

D. Downscaling Full-Scale Impact

While reduced-scale racing platforms offer a cost-effective means to test various techniques for autonomous racing, there remains a substantial disparity between these models and the full-scale platforms [33]. Aspects, such as tire grip limits, drive and brake characteristics, and weight distribution, differ significantly between these two types of platforms [40]. To bridge this gap, the intentional design of scaled-down platforms tailored for autonomous racing may serve as a solution. This can offer researchers a more direct platform for understanding full-scale vehicles, as they are the ones that participate in races watched by everyone.

E. Testing in Virtual Reality

The limitations of reduced-scale platforms in accurately representing reality, along with the cost constraints associated with full-scale platforms, render them impractical for many researchers. Moreover, deploying perception, planning, and control strategies on real vehicles demands collaboration across diverse teams and can be time-consuming and economically burdensome [169]. To address this challenge, simulative methodologies can be employed [170]. By harnessing the power of social computing and parallel systems [171], a virtual artificial society can be created, enabling a parallel management approach to connect the virtual and real worlds [172]. For instance, the limited physical platforms in use can be linked to their artificial duplicates via wireless sensing networks and wide area networks. This linkage facilitates virtual testing under diverse scenarios, thereby enhancing the performance and effectiveness of algorithms developed by remote users [173]. Consequently, the scarce real-world experiments conducted by small groups of researchers can be transformed into valuable resources, readily accessible for researchers worldwide to conduct precise numerical investigations and perform essential simulations under diverse scenarios.

VI. Conclusion

This survey paper offers a thorough overview of the current state-of-the-art in autonomous racing. Aiming towards elaborating the overlooked aspect of vehicle dynamics embedded in motion planning and tracking control algorithms, the paper provides concise summaries of specific lateral and longitudinal dynamics models, along with key tire considerations. Additionally, it offers a compact review of end-to-end learning-based methods as alternative approaches that implicitly utilize vehicle dynamics information. In order to deliver a comprehensive understanding of dynamics within the hardware context, the paper outlines the vehicle dynamics characteristics of both reduced- and full-scale autonomous racing platforms. These platforms, whether modified from commercial stock or entirely fabricated by dedicated researchers, have been considered. Additionally, this survey provides detailed insights into the virtual testing platforms employed for effective evaluation of motion planning and control algorithms for autonomous racing, highlighting their specific features pertaining to vehicle dynamics.

The field of autonomous racing represents an emerging frontier within intelligent vehicles, gathering increasing interest from researchers in both vehicle dynamics and autonomous driving disciplines. With a growing emphasis on incorporating vehicle dynamics into motion planning and control strategies, coupled with the promising outlook for the development of suitable and widely adopted platforms, we foresee significant advancements in the field of autonomous racing and intelligent vehicles on the horizon.

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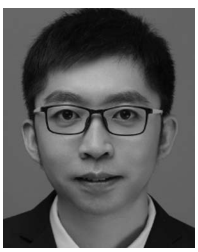

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