Short Papers

Online Competition of Trajectory Planning for Automated Parking: Benchmarks, Achievements, Learned Lessons, and Future Perspectives

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***Abstract*—Automated parking is a typical function in a self-driving car. The trajectory planning module directly reflects the intelligence level of an automated parking system. Although many competitions have been launched for autonomous driving, most of them focused on on-road driving scenarios. However, driving on a structured road greatly differs from parking in an unstructured environment. In addition, previous competitions typically competed on the overall driving performance instead of the trajectory planning performance. A trajectory planning competition of automated parking (TPCAP) has been recently organized. This event competed on parking-oriented planners without involving other modules, such as localization, perception, or tracking control. This study reports the TPCAP benchmarks, achievements, experiences, and future perspectives.**

***Index Terms*—Automated parking, trajectory planning, motion planning, autonomous driving, autonomous racing.**

I. Introduction

Automated parking is about stopping a self-driving car in a specified slot narrowed by surrounding parked cars and/or facilities in a garage or along a roadside [1], [2], [3], [4], [5]. As a critical module in parking a self-driving car, trajectory planning is responsible for getting a spatio-temporal curve that connects the initial and goal poses of the vehicle, which avoids collisions with obstacles observed from the environment, guarantees easy tractability by a low-level controller, and achieves optimality in travel time and/or energy [6], [7]. High-quality parking trajectory planning solutions could save time for the to-be-parked vehicle and even relieve urban traffic congestion [8], [9].

Many autonomous driving competitions were previously launched, part of which involved the parking function. However, few competitions focused on the trajectory planning module for automated parking. Most automated parking competitions require participants to develop a vehicle platform and compete on the overall automated parking performance. Herein, the overall parking performance is determined not only by the trajectory planning module but also by modules such as localization, perception, communication, and tracking control [10]. A side effect of organizing an overall-performance-oriented competition is that the power of trajectory planning is not leveraged alone.

A trajectory planning competition of automated parking (TPCAP) has been launched as a part of the 2022 IEEE International Conference on Intelligent Transportation Systems to show the importance of trajectory planning techniques in automated parking. TPCAP consists of initial and final stages. The initial stage ranges from June 5, 2022, to September 12, 2022, during which 20 benchmark problems should be solved. Each participating team is allowed to upload its solutions to the TPCAP website daily for scoring and ranking. By the end of the initial stage, qualified teams with good scores are sorted for the final stage of the competition, which lasts from October 10, 2022 to October 11, 2022. A team from the Beijing Institute of Technology wins the first prize in TPCAP.

This study reports the organizing procedures of this competition, achievements, learned lessons, and perspectives for future competition chances.

II. Motivations

Trajectory planning competitions related to autonomous driving have been previously launched. Their limitations have motivated us to set up TPCAP.

Previous competitions typically focused on on-road driving scenarios [11]. However, on-road trajectory planners can hardly be applied to parking schemes owing to the following reasons [7], [12]. First, on-road planners do not support driving direction changes. On-road trajectory planners can generate smooth curves by polynomials or splines but cannot generate curves with cusps. Herein, a cusp in a trajectory represents a change in the driving direction, which is commonly necessary during a parking process. Second, on-road driving does not use a vehicle's full mobility. Given that an on-road trajectory is strictly curvature-continuous, a parking trajectory is valid even if the curvature is discontinuous. A curvature-discontinuous trajectory is derived by steering the front wheels with a null longitudinal velocity [13]. In addition, the full steering capability of a vehicle is rarely used

Manuscript received 1 December 2022; revised 7 December 2022; accepted 11 December 2022. Date of publication 13 December 2022; date of current version 23 January 2023. This work was supported in part by the National Natural Science Foundation of China under Grant 62103139, in part by the Natural Science Foundation of Hunan Province under Grant 2021JJ40114, and in part by the 2022 Opening Foundation of State Key Laboratory of Management & Control for Complex Systems under Grant E2S9021119. *(Corresponding author: Xiao Wang.)*

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Digital Object Identifier 10.1109/TIV.2022.3228963

in on-road driving, but is often applied in parking. Third, obstacles in a parking scenario complicate the planning problem. Recall that an on-road trajectory planner is typically developed in the Frenet frame (also known as a curvilinear coordinate system or a road-aligned frame) [14], [15], which makes all the surrounding obstacles located to the left or right side of the ego vehicle. By contrast, parking planners commonly work in the Cartesian frame, wherein obstacles in a cluttered environment render multiple homotopy classes for the parking planner to choose [16], [17]. Herein, an imperfect choice would degrade the parking efficiency or even lead to a solution failure. In conclusion, parking trajectory planning is more challenging than on-road trajectory planning. At present, the solution quality (optimality), computation speed, and algorithm completeness of the existing parking trajectory planners are still imperfect. Therefore, a competition that purely focuses on parking trajectory planning deserves to be organized to arouse researchers' attention.

Some autonomous driving competitions did test the parking function. However, they measured the overall parking behavior, which is influenced by many modules simultaneously, such as localization, perception, planning, and tracking control [18], [19], [20]. If so, the power of a parking trajectory planner cannot be leveraged. Conversely, if there exists a competition that purely focuses on parking trajectory planning, then researchers studying trajectory planning can participate easily without relying on teammates who work on hardware, localization, perception, or tracking control.

Some previous trajectory-planning-oriented competitions requested participants to submit their source codes, which would make underlying participants reluctant. Executing the submitted source codes on a server makes the scoring process of a competition standard and fair, but side effects also exist. First, some source codes might involve intellectual property. Second, those who do not master a specified coding language have no chance to participate in a competition. At this point, scoring on the trajectory planning solutions rather than trajectory planning source codes is a promising alternative. However, the scoring principle should be rigorously defined to maintain competition fairness.

The aforementioned analysis has motivated us to organize a competition to attract research attention in parking-oriented trajectory planning. The competition should focus on trajectory planning alone without involving other onboard modules. The competition should accommodate participants who have concerns about intellectual properties and support different coding language choices.

III. Competition Principle

TPCAP consists of initial and final stages. The initial stage lasts 99 days, whereas the final stage lasts 24 hours.

A total of 20 benchmark problems are released for the initial stage. Each participating team needs to submit the trajectory planning solutions rather than the source codes. Every single participating team has at most one chance to upload its solution batch for scoring daily. Herein, the solution batch consists of the planned trajectories for the aforementioned 20 benchmark cases. The planned trajectories are scored, and the participating team would be informed of the score on the next day. A leaderboard that gets updated daily is deployed to show the 20 leading teams with their scores. For each team, only its best-so-far score is used for ranking and possible showing in the leaderboard. By the end of the initial stage, qualified teams with good enough best-so-far scores are invited to the final stage.

At the final stage, a new set of benchmark problems is released to the qualified teams for possible solutions within 24 hours. Each participating team has only one chance to upload a solution batch. Thereafter, the uploaded solution batches are scored and ranked to define the five leading winners.

IV. Benchmark Design

A total of 20 and eight benchmark problems are defined for the initial and final stages of the competition, respectively. Each benchmark problem is designed with a clear intention. This section introduces those 20 benchmark problems designed for the initial stage of TPCAP (Fig. 1).

Problems 1–3 represent parallel, vertical, and oblique parking cases, respectively. Instead of setting a regular parking slot [21], this work defines the scenario layout through irregularly placed obstacles [22], which would be realistic.

Compared with problems 1–3, problems 4–6 additionally require that the workspace is enriched with many obstacles, most of which do not influence the parking process. Problems 4–6 test whether a trajectory planner can deal with redundant obstacles. If one adopts a numerical-optimization-based planner with full-scale collision-avoidance constraints imposed, then the solution process would be slow. If one adopts a search-/sampling-based planner and checks collisions to those redundant obstacles, then the solution process would be slowed down.

Problems 7 and 8 represent cases with extremely narrow parking slots. Particularly, problem 7 is commonly regarded as the most challenging one among all of the 20 benchmark cases. If a sampling-/search-based planner is adopted, then the sampling/search resolution needs to be high to find zig-zag path primitives precisely. However, setting a high resolution inevitably suffers from the curse of dimensionality, which increases the runtime of a sampling-/search-based planner and even harms algorithm completeness [23]. If a numerical-optimization-based planner is used to deal with problem 7, then the collocation point density needs to be set high, which renders a large-scale mathematical programming problem [21], [24], [25].

Problems 9–12 evaluate the ability of a planner to choose a good homotopy class among multiple ones. Particularly, problems 10–12 contain tiny bottlenecks halfway, which are deployed to test whether a planner can recognize the bottlenecks as passable. If one uses a low-resolution search-/sampling-based planner, then the aforementioned bottlenecks are easily regarded as impassable. If so, the parking car has to detour a longer distance to reach the goal pose, which leads to a poor score. Notably, some well-known planners, such as the hybrid A$*$ (HA) search algorithm [26], only aim to derive a feasible but not optimal homotopy class. Thus, their planning performances may be unstable in dealing with cases with multiple homotopy classes [17].

Problems 13–15 contain sharp obstacles, which are deployed to test whether a planned trajectory is free from subtle collisions. Herein, if collision checks or collision-avoidance constraints are not imposed with accuracy, then the resultant trajectories easily involve collisions [27]. Another interesting phenomenon related to problems 13–15 is that the coordinate values of the vehicles and obstacles are dramatically biased from the origin of the coordinate system. The bias makes classical collision check methods inefficient (e.g., the build-in function *inpolygon* in MATLAB R2021b). The bias also fails a numerical-optimization-based trajectory planner because the embedded optimizer cannot derive derivatives accurately. In this sense, these three cases are used to advocate the importance of eliminating the bias before doing collision checks or imposing collision-avoidance constraints.

Problems 16–18 test the ability of a planner to deal with concave rather than convex obstacles. The reason is that, some previous methods hold an assumption that the polygonal obstacles are all convex [22].

Problem 19 involves a synthetic scenario formed by large- scale obstacles. A parking car is expected to adjust the driving direction halfway

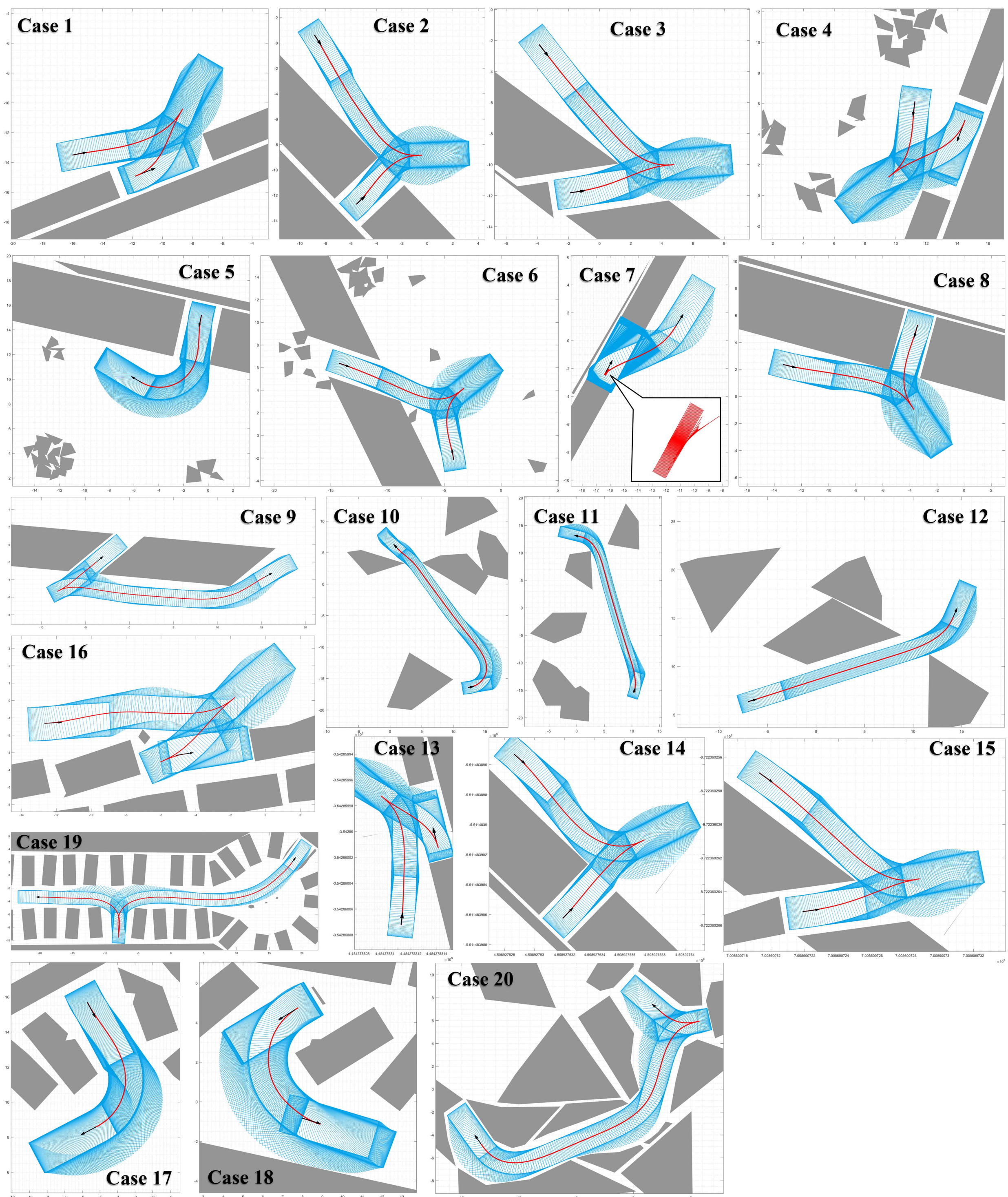

Fig. 1. Twenty benchmark problems defined for initial stage of TPCAP together with best-so-far solutions derived by participating teams.

prior to approaching the goal spot. This case shows the limitation of a search-based planner, such as HA. Concretely, HA prefers to expand nodes close to the goal spot, but the expanded nodes largely differ from the goal pose in their yaw angles. This renders that HA tends to derive a path with many maneuvers close to the goal spot, which consumes much time and thus is not a high-quality solution. Besides that, polygonal obstacles in problem 19 have redundant vertexes, that is, three or more adjacent vertexes are collinear. This design is deployed to test the robustness of a collision checker or a collision-avoidance modeling method.

Problem 20 is about parking in an underground mine scenario narrowed by surrounding obstacles. In this problem, the difference between feasibility and optimality is slim; thus, this problem tests whether those resolution-sensitive planners are efficient.

The aforementioned 20 benchmark problems are provided at https://tpcap.github.io/benchmarks/. We expect that they could be regarded as standard benchmark cases for future research about parking trajectory planning.

V. Scoring Rules

The solution to each benchmark problem is required to store in a .csv file, which contains four columns of time-series data: the time stamp, the x coordinate value of P, the y coordinate value of P, and the orientation angle of the parking car. Herein, P denotes the reference point on the vehicle, which is usually selected as the midpoint along the rear axle. In TPCAP, only four dimensions of data are utilized to score on a candidate trajectory.

A prescreen procedure is conducted before the formal score steps. The legality of a submitted candidate trajectory is examined. If 1) the .csv file contains invalid characters, 2) the data in the .csv file do not form four dimensions, 3) the time stamp column does not form a monotonically increasing sequence, 4) the time stamps are too sparse/dense, or 5) either end of the trajectory is biased from the given initial/goal pose, then the candidate trajectory is regarded as illegal with a sufficiently large penalty. Such an illegal solution is not further evaluated.

As the first step to score a legal candidate trajectory, an infinite-horizon nonlinear model predictive controller is applied to track the submitted candidate trajectory on a virtual chassis with perfect single-track kinematic principle [28]. The derived closed-loop tracking trajectory is further examined.

Ideally, the open-loop candidate trajectory and the closed-loop tracking trajectory should be identical. We penalize the gap between them. If the open-loop trajectory differs too much from the closed-loop one, then the submitted solution is regarded as kinematically infeasible, which yields a rather large penalty in the score. Some prevalent search-based planners that produce curvature-discontinuous trajectories without null-velocity steering actions would be typically recognized as kinematically infeasible. Optimization-based planners with sparse collocation points and an explicit discretization strategy also suffer from this issue.

If the closed-loop tracking trajectory collides with the static obstacles, then a penalty is imposed on the score, which is proportional to the colliding duration. When closed-loop collisions occur, we further investigate whether the submitted candidate trajectory collides with obstacles. If a collision occurs, then a relatively large sunk-cost penalty is imposed for such a mistake.

If a submitted solution does not get penalized by the aforementioned procedures, then it is regarded as feasible. The duration of the parking process and the trajectory smoothness as reflected from the closed-loop tracking trajectory are considered as part of the score after being weighted.

A submitted solution is more advantageous if the scored value is lower. The overall score of a participating team is the sum of its scores on all of the benchmark cases. Recall that an invalid or absent solution is penalized severely. This case renders that a team A that finds feasible solutions to 20 benchmark cases would get a more advantageous score over any team B that completes 19 cases, regardless of how team B's 19 solutions are superior to those of team A. This short-board-effect-embedded policy indicates that a participating team has to finish all of the benchmark cases if it expects to have a good ranking.

VI. Achievements

The initial stage of TPCAP began on June 5, 2022 and ended on September 12, 2022. During that time, as many as 61 participating teams from 24 universities, research agencies, and companies had a total of 466 batches of solutions for scoring. During the last few weeks of the initial stage, the rankings in the leaderboard had changed drastically, which reflects the intensity of the competition. By the end of the initial stage, the scores of the leading teams are close to each other. This result indicates that the pursuit for optimal trajectory has been leveraged by the participating teams. Qualified teams for the final stage are defined as those who find feasible solutions to all of the 20 benchmark problems.

The final stage of TPCAP began on October 10, 2022, and ended on October 11, 2022. During that time, eight benchmark problems were assigned to the 23 qualified teams who had passed the initial stage. Consequently, all of the 23 teams managed to submit their solutions within the required 24 hours. A webinar was launched after the final stage ended, wherein some voluntary teams delivered online technical talks, which are recorded at www.bilibili.com/video/BV1GV4y1L7Yi.

The top five winners of the final stage respectively come from 1) Beijing Institute of Technology, 2) Tianjin Polytechnic University, 3) Massachusetts Institute of Technology and the University of California, Berkeley, 4) Xi'an Jiaotong University, and 5) Tongji University.

VII. Learned Lessons

This section discusses the lessons we have learned from organizing TPCAP.

First, requesting participants to submit solutions rather than source codes is feasible. We noticed that nearly all of the participants who carefully read the competition rules could provide solution files in the correct format. Besides that, the content of a solution file, i.e., the four dimensions of data, is sufficient to evaluate a candidate trajectory. The idea to deploy a controller to track the candidate trajectory is efficient in fairly measuring a submitted solution.

Second, the short-board effect in the scoring rule is efficient in inspiring the participating teams to finish all of the benchmark cases. However, this policy should be held under the prerequisite that the difficulties of the benchmark problems distribute normally; otherwise, the fairness of the competition easily comes into question. Thus, a lesson is that the difficulties of the benchmark problems deserve to be well controlled.

Third, the scoring system should be robust to any underlying input data format that one can ever imagine. In TPCAP, we have tried to carefully set a prescreen rule to avoid scoring on invalid trajectories, but the scoring system still collapsed after processing one solution, which contains complex numbers rather than real ones. Our prescreen system treated letters and special characters as invalid but forgot to monitor a complex number. This mistake makes a trajectory presented in the form of complex numbers evaluated, and the resultant score also happens to be a complex number. The leaderboard of the competition collapsed because ranking on complex numbers is impossible. In this event, the learned lesson is that one cannot be too careful when preparing the prescreen rules for a solution submission system in a competition. That is because every possible type of data, regardless of being valid or invalid, is possible to be uploaded.

Fourth, sampling-, search-, and optimization-based methods are prevalent in our competition, according to the technical talks delivered by the participating teams during the competition webinar. Different types of methods are combined or integrated to deal with some particular cases that are challenging. Learning-based methods which emerged in the past few years [29] do not show advantages in trajectory planning competitions with deterministic obstacles.

Fifth, some participating teams that have passed the initial stage still get penalized during the final stage for submitting kinematically infeasible solutions. A lesson learned here is that low-quality competition performances during the final stage might be avoided by providing straightforward instructions during the initial stage.

Sixth, fairness is always important in running a competition. The scoring rule was slightly modified in the middle of the initial stage to make the collision penalty not too harsh. After the change in the scoring rule, all of the solutions that were submitted prior to the modification had to be re-evaluated immediately so that consistency remained for all of the submitted solutions before and after the modification.

Seventh, setting up a platform to frequently report the best-so-far score values in the middle of the competition is a good idea because it inspires the participating teams to know their positions. However, releasing the dynamic motions in association with the best-so-far score values in the middle of the competition is a rather bad trial. The reason is that releasing the dynamic motions makes the best-so-far homotopy class selections open to those who previously had no ideas, thereby reducing the fairness of the competition. A lesson learned here is that the best-so-far trajectories derived by some leading teams must not be released in the middle of a trajectory planning competition.

VIII. Future Perspectives

TPCAP is the first competition that particularly focuses on trajectory planning of automated parking. Most of the predefined goals in launching this competition have been achieved. The following issues deserve to be addressed in the future.

First, the kinematic principle of a passenger car at a low speed is not as simple as the single-track model we used in TPCAP. A commonly acknowledged dynamic model deserves to be used in evaluating the closed-loop tracking performance of a candidate parking trajectory.

Second, the present rules of TPCAP cannot monitor or forbid the possibility that 1) one participant uses multiple accounts to participate or 2) multiple teams share one batch of solutions. If these things really happen, then the fairness of the entire competition would be fully destroyed because all of the top five prizes might be won by a single person. Although this luckily does not happen, an approach should still be proposed to monitor this kind of action.

Third, the parking scenario deserves to be enriched by considering ground roughness and/or 3D obstacles [30], [31].

Fourth, a future competition might be an event that tests the cooperative trajectory planning capability of multiple vehicles [32], [33] or multi-body vehicles [34] in cluttered workspaces.

Acknowledgment

We appreciate Prof. Li Li and Dr. Shen Li from Tsinghua University for their assistance in organizing this competition. We sincerely thank the Autoware Foundation and Nagoya University for sponsoring this competition by providing prize money to TPCAP winners. The first author would like to thank Yanfeng Ma, Yuxia Sun, Gordon Pan, Yi Liu, Yiqun Dong, and Yuqing Guo for their technical supports.

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