

# Vertical Split Ring Resonator Using Vias With Wide Bandwidth and Small Electrical Size

Jongwon Choi, Sangyeol Oh, Sangjin Jo, Won-Sang Yoon, and Jaehoon Lee

**Abstract**—In this letter, we propose a vertically inserted split ring resonator (SRR) in microstrip lines. Since the top and bottom layers of the resonator are electrically connected by using via holes and it has a 3-D shape, it occupies a small electrical size. Through numerical simulations and measurement, it is shown that it can provide a wider range variation of bandwidth than conventional planar SRRs and the resonant frequency and the bandwidth can be adjusted by its length and width, respectively.

**Index Terms**—Electrical size reduction, loaded bandwidth, metamaterials, vertical split ring resonator (VSRR).

## I. INTRODUCTION

RECENTLY the research of metamaterials has been diversely developed. Metamaterials are commonly known as left handed materials, and there are several ways to fabricate them [1], [2]. Among many ways, split ring resonators (SRRs) are commonly used because they are useful in bulk metamaterial media and easy to fabricate, due to its planar shape [3]. However, this 2-D shape occupies the large area and the SRRs show relatively large loss and narrow bandwidth.

To solve these problems, many studies have been conducted [4]–[8]. By achieving the enhanced equivalent capacitance, the broad-side coupled SRR shows more efficient electrical size than the edge-coupled SRR and spiral resonators [3]–[6]. However, this way has a limitation in reducing the electrical size because of the planar shape, and also the increased capacitance acts as a conflicting factor in aspect of broadening bandwidth. To achieve wider bandwidth, it was suggested that enlarging the fractional area occupied by the SRRs in a unit cell [7]. Enhanced fractional area leads to wider bandwidth, but it also causes larger electrical size. To solve the above problem, SRRs with tapered strip width were proposed [8]. The fundamental 2-D design of SRRs also has a limit to reduce the electrical size. To achieve small electrical size, vertical split ring resonator has been proposed in previous paper [9]. This structure acquires small electrical size and good performance. However, it was based on too high dielectric constant material and had relative complicate manufacturing process.

In this letter, we propose another vertical split ring resonator (VSRR) through via holes. It is shown that the

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J. Choi, S. Oh, S. Jo, and J. Lee are with the Department of Radio Engineering, Korea University, Seoul 02841, South Korea (e-mail: ejhoon@korea.ac.kr).

W.-S. Yoon is with the Department of Electronic Engineering, Hoseo University, Asan-si 31499, South Korea.

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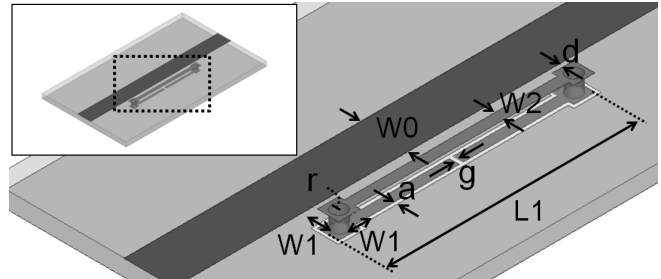


Fig. 1. Microstrip transmission line loaded with VSRR using vias.

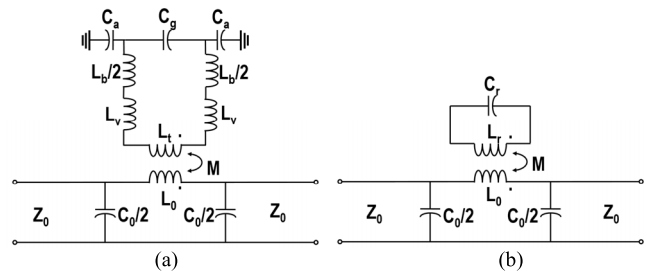


Fig. 2. The equivalent circuit of the transmission line loaded by VSRR using vias. (a) The equivalent circuit model. (b) The simplified circuit model.

bandwidth of VSRR using vias is not only wider than conventional SRRs but also controllable by adjusting the dimensions of the resonator, numerically and experimentally. Therefore, it can support the various range of bandwidth with the dramatic reduction of electrical area.

## II. EQUIVALENT CIRCUIT OF TL LOADED BY VSRR

Fig. 1 shows the TL loaded by VSRR using vias. The VSRR is composed of the TMs on the top and bottom layers and their electrical connections using via holes. The TL on the bottom layers are split by a small gap. The electromagnetic fields from the main TL are coupled to the VSRR. To predict the frequency response of the TL loaded by VSRR, we propose an equivalent circuit model as illustrated in Fig. 2(a), where  $L_0$  and  $C_0$  are reactant components of the TL with  $Z_0$ ,  $L_r$  is an inductance of the resonator,  $C_g$  is a gap capacitance, and  $C_a$  is a capacitance between VSRR and the ground. The effect of via-pads is neglected for the simplification.  $L_r$  can be represented as

$$L_r = L_t + L_b + 2 \cdot L_v \quad (1)$$

where,  $L_t$ ,  $L_b$  and  $L_v$  are inductances of VSRR components in top and bottom layers, and a via. The capacitance of VSRR,  $C_r$ , can be obtained from Y- $\Delta$  transformation as

$$C_r = C_g + C_a/2. \quad (2)$$

From the above results, we can obtain a simplified model as shown in Fig. 2(b). The reactive components in (1) and (2),



Fig. 3. Top views of four different structures for comparison. (a) Fundamental VSRR structure. (b) VSRR with the short length ( $L_1 = 10$  mm). (c) VSRR with the small width ( $W_2 = 0.15$  mm). (d) Dual VSRRs ( $n = 2$ ).

TABLE I  
EXTRACTED CIRCUIT PARAMETERS OF EQUIVALENT CIRCUIT

	$L_1$ [nH]	$L_2$ [nH]	$L_v$ [nH]	$C_s$ [fF]	$C_a$ [pF]	$M$ [nH]
Fig. 3(a)	6.73	5.15	0.12	16.4	1.06	1.64
Fig. 3(b)	2.99	2.25		16.4	0.46	0.73
Fig. 3(c)	13.4	8.75		3.43	0.64	1.53

TABLE II  
RESONANT FREQUENCY AND LOADED BANDWIDTH FROM EQUIVALENT CIRCUITS AND HFSS SIMULATION

	$f_r$ [GHz]			Fractional $BW_L$ [%]		
	Eq. (3)	Sim.	Exp.	Eq. (4)	Sim.	Exp.
Fig. 3(a)	2.74	2.89	3.07	3.87	3.79	3.74
Fig. 3(b)	5.91	5.99	6.02	3.60	3.02	3.39
Fig. 3(c)	2.63	2.54	2.57	1.74	1.23	1.43

a mutual inductance,  $M$ , can be calculated by the aid of [10]. The resonant frequency is given by

$$\omega_r = 1/\sqrt{L_r C_r}, \quad (3)$$

and a loaded bandwidth can be modified from [11] as follows:

$$BW_L = \frac{1}{L_r \sqrt{L_r C_r}} \cdot \frac{M^2}{2 \cdot Z_0} \quad (4)$$

From above equation, we can infer that the bandwidth can be controlled by the mutual inductance  $M$ , the inductance and capacitance of resonator,  $L_r$  and  $C_r$ , respectively.

### III. THE DESIGN OF VSRR

To investigate these electromagnetic properties of the proposed VSRR, we made several numerical simulations with the various dimensions as shown in Fig. 3. The width of TL for matching to  $50 \Omega$ ,  $W_0 = 2.347$  mm and the dielectric Duroid substrate with a thickness  $h = 0.78$  mm, a relative dielectric constant  $\epsilon_r = 2.2$ , and a loss tangent  $\tan \delta = 0.0009$  were used. The dimensions of fundamental VSRR as shown in Fig. 3(a) are  $W_1 = W_2 = 1$  mm,  $L_1 = 20$  mm,  $d = 0.2$  mm,  $a = 0.15$  mm,  $g = 0.2$  mm and  $r = 0.25$  mm. To adjust the center frequency and loaded bandwidth, the length  $L_1$ , and the width  $W_2$ , were changed as shown in Fig. 3(b)  $L_1 = 10$  mm, in Fig. 3(c)  $W_2 = 0.15$  mm. For a larger bandwidth, the number of resonators,  $n$  was changed as shown in Fig. 3(d).

S-parameters of TLs loaded by VSRR using vias were simulated by ANSYS HFSS, as shown in Fig. 4 and their resonant frequencies and loaded bandwidths were calculated by Eq. (3) and (4). In Table I, the circuit parameters of TLs loaded by single VSRR as shown in Fig. 3(a), (b) and (c) were calculated by [10]. Their resonant frequencies and loaded bandwidths were compared with simulation results in Table II. Fig. 4(a) shows the simulated S-parameters of the TL with the

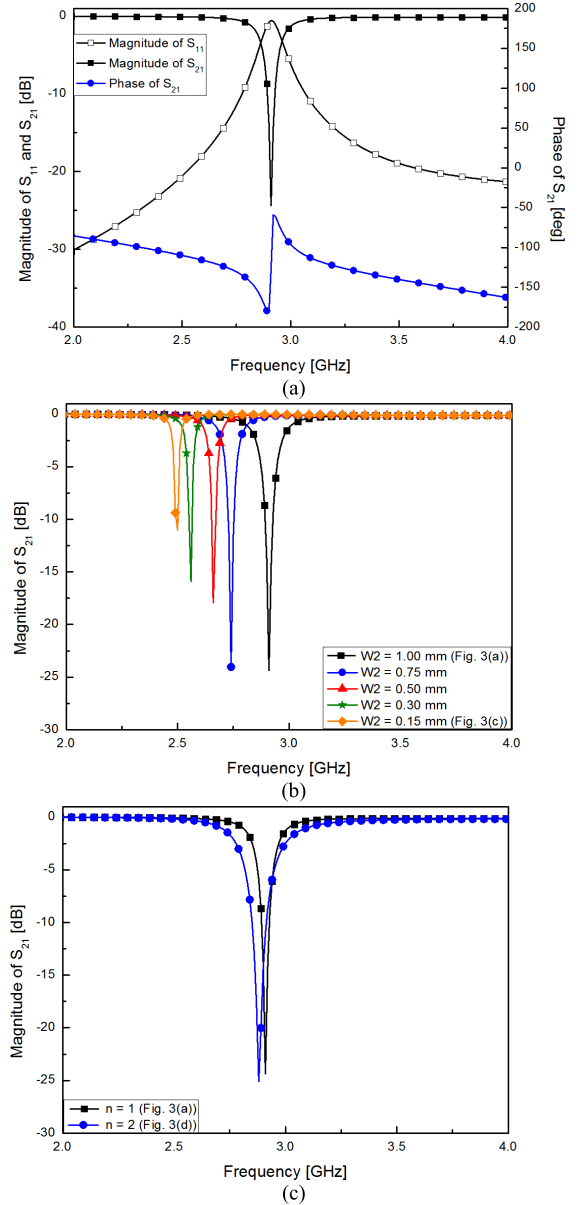


Fig. 4. Simulated S-parameters of TLs loaded by VSRR. (a) S-parameters of Fig. 3(a). (b) Magnitude of  $S_{21}$  with respect to  $W_2$ . (c) Magnitude of  $S_{21}$  with respect to  $n$ .

fundamental VSRR in Fig. 3(a). At the resonance frequency, there is a strong resonance and the slope of  $S_{21}$  phase becomes positive. The positive slope of  $S_{21}$  phase leads to a negative group delay ( $= -\partial\phi/\partial\omega$ ) which can be observed in left-handed media [12].

From the comparison of Fig. 3(a) and (b), the decrease of  $L_1$  leads to smaller  $L_r$  and  $C_r$ , and higher resonant frequency as shown in Table I and II. The lengths of VSRRs,  $L_1$  are approximately estimated at a quarter-wavelength,  $\lambda/4$ . From the comparison of Fig. 3(a) and (c), the width of VSRR,  $W_2$ , makes a relatively small influence on the resonant frequency. Narrower  $W_2$  changes the reactance of the resonator and it results in slightly lower  $f_r$  as shown in Table II and Fig. 4(b).

As illustrated in (4), the bandwidth of VSRR can be controlled by the reactance of the resonator and the mutual inductance,  $M$ . Therefore, the bandwidth is highly dependent

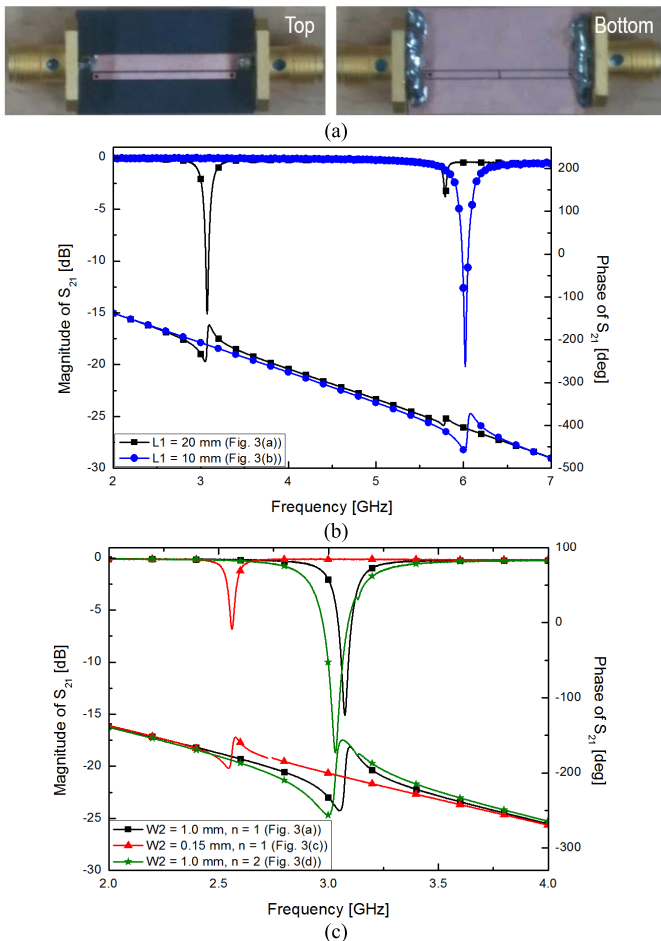


Fig. 5. Measure results. (a) Photograph of the fabricated structure in Fig. 3(a). (b)  $S_{21}$  with respect to  $L_1$ . (c)  $S_{21}$  with respect to  $W_2$  and  $n$ .

on the width of resonator,  $W_2$ . The thinner width makes larger  $L_r$ , smaller  $C_r$ , and slightly smaller  $M$  as shown in Table I. However, from Eq. (4), the loaded bandwidth is highly dependent on  $L_r$  and  $M$ , rather than  $C_r$ . Therefore, the decreasing of  $W_2$  brings on narrowed bandwidth as shown in Fig. 4(b). It is possible to control the bandwidth by the width of VSRR. In addition, the number of resonators,  $n$  is also able to control the bandwidth as shown in Fig. 4(c). A pair of VSRRs has a fractional loaded bandwidth of 7.13 %.

To verify the theory and simulation, some TLs with VSRRs were implemented and their  $S_{21}$  were measured. Fig. 5 shows the measurement results with various sizes of VSRRs. The measured resonant frequencies of four different structures in Fig. 3(a), (b), (c) and (d) are 3.07, 6.02, 2.57, and 2.99 GHz, respectively. The experimental results of loaded bandwidths are 3.74, 3.39, 1.43, and 7.30 %, respectively. These results reveal good agreements with simulations in Table II. In Fig. 5(b), the resonant frequency is changed from 3.07 GHz (black line) to 6.02 GHz (blue line) by  $L_1$ , and in Fig. 5(c), the fractional loaded bandwidth is changed 3.74 % (black line) to 1.43 % (red line) by  $W_2$  and to 7.30 % (green line) by  $n$ . These show that 62 % narrower and 95 % wider than the fundamental VSRR structure. Therefore, it is possible to adjust the bandwidth with wide varying range. These VSRRs occupy small electrical area because of the 3-D shape.

TABLE III  
COMPARISON OF RESONANT FREQUENCY AND LOADED BANDWIDTH

	$f_r$ [GHz]	BW <sub>L</sub> [%]	RL [dB]	Size of a resonator [ $\lambda \times \lambda$ ]
Uniform SRRs in [8]	2.89	1.85	12.5	$0.13 \times 0.13$
Tapered SRRs in [8]	2.89	2.99	15.2	$0.13 \times 0.13$
Fig. 3(d) in this letter	3.03	7.30	18.7	$0.27 \times 0.01$

Finally, Table III shows a comparison between dual VSRRs in Fig. 3(d) and planar SRRs in [8]. The measured values of dual VSRRs in Fig. 3(d) indicate a higher loaded bandwidth, lower rejection level (RL), and smaller electrical size at the similar resonant frequency. The circumstances of each ones are not exactly same, so the above comparison cannot be objective one. But we can just estimate the general tendency of difference.

#### IV. CONCLUSION

We proposed the novel resonator, vertically inserted SRR. This resonator consists of top and bottom layers and via holes and has 3-D shape. Due to its structure, the VSRR occupies smaller electrical area than planar SRRs and it can provide a wide variation of loaded bandwidth than planar ones. The bandwidth and resonant frequency of VSRR can be controlled by adjusting its width and length, respectively. The measured results show that the fractional bandwidth of VSRR varies from 1.43 % to 7.30 % with a small electrical size.

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