Correction to "Gaussian Process Online Learning With a Sparse Data Stream"

Jaehyun Lim, Jehyun Park, and Jongeun Choi, *Member, IEEE*

***Abstract*—A letter entitled "Gaussian Process Online Learning With a Sparse Data Stream" suggests an approach for extending the infinite-horizon Gaussian processes (IHGPs, [1]) to deal with a sparse data stream. We point out that there is an error in differentiating the discrete algebraic Riccati equation (DARE), which significantly changes the results of the benchmarking study in a sense that the proposed approach using the solution of the Lyapunov equation does not show outperformance against the original IHGP. In this letter, we provide a correction with details and its consequential implication.**

***Index Terms*—Computer vision for other robotic applications, optimization and optimal control, probability and statistical methods.**

I. Introduction

The aim of this letter is to call the readers' attention to an error we identified in [2], entitled "Gaussian process online learning with a sparse data stream," published in IEEE Robotics and Automation Letters. We show that there is an error in differentiating the discrete algebraic Riccati equation (DARE), and the proposed method in [2] does not improve the performance on average as compared to the original infinite-horizon Gaussian processes (IHGPs, [1]). Therefore, we provide a correction and the discussion of its consequence.

II. Analysis

The IHGP reduces the computational cost and has been successfully implemented on mobile devices for online learning. However, the IHGP fails to yield accurate predictions when applied to a sparse data stream. Therefore, a method for extending the IHGP to deal with a sparse data stream is proposed by [2]. [2] claims to obtain the exact gradient by differentiating the DARE and reformulating it in the form of the discrete Lyapunov equation. Variables in the DARE should be differentiated with respect to hyperparameters. The correct differentiation of the DARE is as follows.

$$
\begin{aligned}
&\left[A^T PA - P - (A^T PC^T)S^{-1}CPA + Q\right]' \\
&= A'^T PA + A^T P'A + A^T PA'^T - P' \\
&\quad - A'^T PC^T S^{-1}CPA - A^T P'C^T S^{-1}CPA \\
&\quad + A^T PC^T S^{-1}S'S^{-1}CPA - A^T PC^T S^{-1}CP'A \\
&\quad - A^T PC^T S^{-1}CPA' + Q' = 0,
\end{aligned} \tag{1}
$$

where the notation $(\cdot)'$ is defined as the partial derivative with respect to the hyperparameter vector θ (i.e., $P' := \partial P/\partial\theta$).

If we substitute $\bar{A}$ and $\bar{Q}$ with $A - C^T S^{-1}CPA$, and $A'^T PA + A^T PA'^T - A'^T PC^T S^{-1}CPA - A^T PC^T S^{-1}CPA' + A^T PC^T S^{-1}R'S^{-1}CPA + Q'$, respectively, (1) becomes the discrete Lyapunov equation $\bar{A}^T P'\bar{A} - P' + \bar{Q} = 0$, which can be solved for P'.

Unfortunately, [2] differentiates the DARE incorrectly assuming that A is constant.

$$
\begin{aligned}
&\left[A^T PA - P - (A^T PC^T)S^{-1}CPA + Q\right]' \\
&= A^T P'A - P' - A^T P'C^T S^{-1}CPA \\
&\quad + A^T PC^T S^{-1}S'S^{-1}CPA \\
&\quad - A^T PC^T S^{-1}CP'A + Q' = 0.
\end{aligned} \tag{2}
$$

Since A is parameterized by the length scale parameter ℓ which determines the smoothness of the GP, assuming that A is constant, online GP learning loses the portion of the update in hyperparameter ℓ from A' via the gradient obtained from (2).

When the data stream is corrupted with a high level of noise and is sparse, the maximum likelihood estimation of the GP hyperparameters and the prediction with GP posterior can be highly sensitive to the noise. However, the results of the Monte-Carlo simulation performed by [2] show the insensitive prediction to the noise, which seems to be obtained due to the fact that the length scale hyperparameter is updated without the contribution of A', i.e., A is considered as a constant value as implied by (2). On the other hand, the results of the Monte-Carlo simulation by [2] could be interpreted in a way to deal with noisy and sparse data streams, i.e., it is better to limit the update of the length scale during GP learning for sparse data streams.

For example, constrained optimization by setting a lower bound on the length scale hyperparameter can overcome the sparse data sampling, which was reported as a revised version of [2] in 2020 IEEE/RSJ International Conference on Intelligent

Manuscript received August 12, 2020; accepted August 12, 2020. Date of current version December 28, 2020. This work was supported in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (2018R1A4A1025986), and in part by the Mid-career Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, and ICT (NRF-2018R1A2B6008063). *(Corresponding author: Jongeun Choi.)*

The authors are with the School of Mechanical Engineering, Yonsei University, 50 Yonsei Ro, Seodaemun Gu, Seoul 03722, South Korea (e-mail: jaehyunlim@yonsei.ac.kr; jhyunpark@yonsei.ac.kr; jongeunchoi@yonsei.ac.kr).

Digital Object Identifier 10.1109/LRA.2020.3043964

Robots and Systems (IROS) October 25-29, 2020, Las Vegas, NV, USA.

III. Conclusion

Although, the extension of the IHGP using the exact gradient via solving the Lyapunov equation is still valid as proposed in [2], due to the error in (2), there is no effect in dealing with a sparse data stream as promised by [2]. A similar level of performance of the original IHGP to the proposed approach by [2] supports the high performance of the original IHGP algorithm as well. The outperformance shown by Monte-Carlo benchmark simulation was a result obtained by limiting the update of the length scale during GP learning (due to the error in (2)), which could be viewed as one of alternative solutions to the proposed problem. When a similar problem needs to be solved, the reader should take this correction letter into account for its solution approach.

References

[1] A. Solin, J. Hensman, and R. E. Turner, “Infinite-horizon gaussian processes,” in *Proc. Adv. Neural Inf. Process. Syst.*, 2018, pp. 3486–3495.

[2] J. Park and J. Choi, “Gaussian process online learning with a sparse data stream,” *IEEE Robot. Autom. Lett.*, vol. 5, no. 4, pp. 5977–5984, Oct. 2020.