

# Transmission Cost Allocation by Power Tracing Based Equivalent Bilateral Exchanges

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**Abstract**—The allocation of transmission cost provides important references and signals for system expansions and investments. This paper proposes a power tracing based equivalent bilateral exchange (PTEBX) method in which network users are responsible for not only their induced power flows, but also power flows induced by whom they have equivalent bilateral exchanges with. The equivalent bilateral exchanges are recognized based on the power tracing. To evaluate the performance of different methods of allocating transmission cost, seven criteria are put forward that take into consideration characteristics of power systems. Theoretical analysis is then conducted to certify whether the methods satisfy the criteria. The results indicate that only the PTEBX method is able to satisfy all the seven criteria. Numerical examples based on the IEEE-30 system are presented to further demonstrate the applicability of the proposed method.

**Index Terms**—Bilateral contract trading, equivalent bilateral exchanges, power tracing, transmission cost allocation, wind power trading.

## NOMENCLATURE

### A. Indices

$g (g \in G)$	Index of generators.
$l (l \in L)$	Index of loads.
$i, j, k$	Index of nodes.
$\omega$	Index of scenarios.

### B. Constants, Variables and Functions

$f_{g,ij}$	Allocation proportion of generator $g$ in branch $ij$ .
$f_{l,ij}$	Allocation proportion of load $l$ in branch $ij$ .
$F_{g,ij}$	Contribution proportion of generator $g$ in branch $ij$ from generators' perspective.
$F_{l,ij}$	Contribution proportion of load $l$ in branch $ij$ from generators' perspective.
$P_g$	Power output of generator $g$ (MW).
$P_l$	Power input of load $l$ (MW).
$P_{ij}$	Power flow in branch $ij$ (MW).
$\Gamma_+(\cdot)$	Set of outgoing line of node $\cdot$ .
$\Gamma_-(\cdot)$	Set of incoming line of node $\cdot$ .

$r$	Proportion of contribution to an EBE by the generator.
$C_{ij}$	Transmission cost of branch $ij$ (\$/h).
$EBE_{gl}$	Equivalent bilateral exchange between generator $g$ and load $l$ (MW).
$\Delta P_{ij}^g$	Variance of power flow in branch $ij$ induced by unit incremental power of generator $g$ (MW).
$\Delta P_{ij}^l$	Variance of power flow in branch $ij$ induced by unit incremental power of load $l$ (MW).
$\Delta P_{g,ij}$	Variance of power flow in branch $ij$ induced by total power of generator $g$ (MW).
$\Delta P_{l,ij}$	Variance of power flow in branch $ij$ induced by total power of load $l$ (MW).
$P_{g,ij}$	Generator $g$ 's induced power flow in branch $ij$ (MW).
$P_{l,ij}$	Load $l$ 's induced power flow in branch $ij$ (MW).
$I_{g,ij}$	Generator $g$ 's induced current in branch $ij$ (MW).
$I_{l,ij}$	Load $l$ 's induced current in branch $ij$ .
$I_{ij}$	Current in branch $ij$ .
$I_g$	Current output of generator $g$ .
$I_l$	Current input of load $l$ .
$\gamma_{gl,ij}$	Generation shift distribution factor of transaction between generator $g$ and load $l$ to branch $ij$ .
$z_{ij}$	Series impedance of the $\pi$ equivalent circuit of branch $ij$ .
$y_{ij}^{sh}$	Shunt admittance of the $\pi$ equivalent circuit of branch $ij$ .
$UP_{g,ij}$	Unitary participation of generator $g$ in the power flow of branch $ij$ .
$UP_{l,ij}$	Unitary participation of load $l$ in the power flow of branch $ij$ .
$Var(\cdot)$	The variance of $\cdot$ .

## I. INTRODUCTION

THE allocation of transmission cost plays an indispensable role in fairly reflecting the usage proportions on power networks among generators and loads, and providing efficient signals for expansions and investments [1]–[4]. The ongoing reformation of the power industry in China is gradually restructuring the power system from vertically integrated towards competitive and decentralized [5]. As direct electricity purchase practices emerge and stochastic renewable energy takes on a larger generation part, the electricity market will require an equitable and reasonable approach for allocating transmission cost.

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Substantial research has been conducted on this issue. Typical methods can be classified as follows [6]–[19]:

- 1) short/long-run marginal cost (SRMC/LRMC);
- 2) pro-rata;
- 3) MW-mile;
- 4) contract path (CP);
- 5) marginal participation (MP);
- 6) “with and without” (WW);
- 7) power tracing (PT);
- 8) equivalent bilateral exchanges (EBX);
- 9)  $Z_{\text{bus}}$ ;
- 10) co-operative game theory, i.e., the Aumann-Shapley value (AS) method.

The SRMC/LRMC method calculates the short/long-run marginal cost of the network under the assumption that the existing network is at its capacity limit [6]. The pro-rata method deems that the utilization of the network is only dependent on the amount of power purchased/sold in the system [7]. The MW-mile method calculates the MW-mile usage of the network and allocates the usage proportionately [8]. In the CP method, contract paths are artificially predetermined; however, the real power flows rarely follow the predetermined contract paths [9]. The MP method considers a unit increment in generation and load as the evaluation of network usage [10]. The WW method defines the use of a network user by differences in power flow between ‘with’ and ‘without’ it [11]. The PT method attributes the power flow to generators and loads according to the Kirchhoff Current Law (KCL) [1]. The EBX method decomposes the generations and loads to multiple fictitious bilateral exchanges according to their proportion of the total system generation and load [12]. The  $Z_{\text{bus}}$  method adopts  $Z_{\text{bus}}$  matrix as the reference to determine the usage proportion of network users [13]. In the co-operative game theory method, all network users are regarded as collaborators and transmission cost is fairly allocated based on their contributions [14]–[16]. Other approaches involve graph theory [17] and artificial neural network [18]; however, these methods are not appropriate since they tend to neglect the physical laws of power systems or they induce heavy calculations [19].

The aforementioned 10 methods have their individual advantages and drawbacks [13], [16], [20]. This paper proposes seven criteria to evaluate the transmission cost allocation methods. The power tracing based equivalent bilateral exchanges (PTEBX) method is put forward and certified to be the unique method that meets all the criteria. Numerical examples are then provided to compare the different methods and validate the efficiency of the PTEBX method.

## II. CRITERIA ON THE ALLOCATION METHOD

In order to fairly and reasonably evaluate the methods for allocating transmission cost, the following criteria are established based on economic theories [25] as well as actual situations and developmental trends of power system [3], [4], [10].

### A. Efficiency

An efficiency method is needed to assure that the total

transmission costs can be fully recovered, which is expressed as

$$\sum_g f_{g,ij} + \sum_l f_{l,ij} = 1. \quad (1)$$

### B. Non-negativity

The allocation of transmission cost should be non-negative as long as a generator/load utilizes the network, i.e.,

$$\forall g, f_{g,ij} \geq 0; \quad \forall l, f_{l,ij} \geq 0. \quad (2)$$

This is because the generator/load benefits from the network as the network realizes the power transaction.

### C. Consideration of Network Position

The network user’s position in the network should be taken into account in allocating the transmission cost. For example, a network user may take less responsibility for power flow of a transmission line that is electrically far from it.

### D. Monotone Non-decreasing to Amount of Power

The proportion of transmission cost should be non-decreasing/non-increasing if the user trades more/less power when other users do not increase/decrease their transactions, represented as

$$\forall g, f'_{g,ij} \geq f_{g,ij}; \quad \forall l, f'_{l,ij} \geq f_{l,ij} \quad (3)$$

when

$$P'_g > P_g; \quad P'_l > P_l \quad (4)$$

and

$$\forall g', P'_{g'} \leq P_{g'}, g' \in G \setminus g; \quad \forall l', P'_{l'} \leq P_{l'}, l' \in L \setminus l. \quad (5)$$

Here,  $P'_g/P_l$  is increased power output/input of generator  $g$ /load  $l$ ,  $f'_{g,ij}/f_{l,ij}$  is allocation proportion of generator  $g$ /load  $l$  by  $P'_g/P_l$ . This criterion assures that more uses bring more charges. The equal signs in (3) hold only if the increased power does not affect the power flow.

### E. Signal to Trade

The allocation results should provide signals to trade. In the view of supply-demand interaction, which is attracting increasingly growing attention these days [4], both supply and demand sides are responsible for the safe and economic operation of the power system. The transmission cost allocation method, therefore, should provide cost reduction for the network users to encourage them to be responsible for mitigating congestion. Given a condition that a generator/load wants to sell/purchase more power, it will choose a load/generator in which the bilateral trade induces less power flow through lower transmission cost, which is mathematically defined as,

$$\forall g, f'_{g,ij} > f''_{g,ij}; \quad \forall l, f'_{l,ij} > f''_{l,ij} \quad (6)$$

when

$$P'_{g_1,ij} > P''_{g_2,ij}; \quad P'_{g_1,ij} > P''_{g_2,ij} \quad (7)$$

and

$$\forall g', g' \in G \setminus g, P'_{g'} = P''_{g'}; \quad \forall l', l' \in L \setminus l, P'_{l'} = P''_{l'} \quad (8)$$

where  $P'_{gl_1,ij}$ ,  $P''_{gl_2,ij}$ ,  $P'_{g_1l,ij}$ ,  $P''_{g_2l,ij}$  are induced power flow by transaction  $g-l_1$ ,  $g-l_2$ ,  $g_1-l$ ,  $g_2-l$ , respectively.  $f'_{g,ij}$ ,  $f''_{g,ij}/f'_{l,ij}$ ,  $f''_{l,ij}$  are the corresponding allocation proportions.

### F. Less Fluctuant to Different Scenarios

As renewable energy takes on a larger part of power generation, the transmission cost allocation is calculated considering different scenarios for different system operation modes. Therefore, the allocation results should be less fluctuant to different scenarios for easy realization, which is described by  $Var(f'_{g,ij})$  and  $Var(f'_{l,ij})$ . This criterion provides no strict value for deciding whether it is satisfied, but can be adopted for comparison among several methods. In this paper, we deem that this criterion is not satisfied if counter flows are not charged. This is because power flow direction of a specific line could be altered under different system operation modes. In this case, a slight change in output/input power may induce considerable change in transmission cost allocation.

### G. Equal Treatment on Different Transaction Types

Both pool market trade and bilateral exchanges should be treated equally. Furthermore, the allocation should consider the entirety of the bilateral exchanges. To be specific, the differences in charges executed on transaction parties in a bilateral exchange should not be too large.

Assume a bilateral exchange between generator  $g$  and load  $l$ , and let  $f_{g,ij}$ ,  $f_{l,ij}/f'_{g,ij}$ ,  $f'_{l,ij}$  denote the allocation proportion without/with the bilateral exchange, the difference in charges on the transaction parties is defined as

$$D_{gl,ij} = |(f'_{g,ij} - f_{g,ij}) - (f'_{l,ij} - f_{l,ij})|, \quad (9)$$

and the value of which should be as small as possible to satisfy this criterion.

## III. PRINCIPLE OF POWER TRACING BASED EQUIVALENT BILATERAL EXCHANGES

The key to the PTEBX method is to determine what proportion of each generation is attributed to each load. In [12], generations/loads are uniformly decomposed to form the equivalent bilateral exchange (EBE). However, this settlement ignores the network position without consideration of KCL. As a result, a generator/load may take the same responsibility for power flows of different lines, neglecting the different electric distances. This is unreasonable and against the criterion ‘‘consideration of network position.’’ As PT is widely utilized to recognize the proportion of contribution of a specific generator/load to different loads/generators, as well as taking into account network positions, it is adopted in the PTEBX method to recognize the EBEs.

### A. Power Tracing

The PT provides the allocation proportion of generator  $g$  and load  $l$  to branch  $ij$ , as  $f_{g,ij}$  and  $f_{l,ij}$ .

$$f_{g,ij} = \frac{F_{g,ij}}{\sum_g F_{g,ij} + \sum_l F_{l,ij}} \quad (10)$$

$$f_{l,ij} = \frac{F_{l,ij}}{\sum_g F_{g,ij} + \sum_l F_{l,ij}} \quad (11)$$

$$\text{s.t. } F_{g,ij} = \frac{b_{g,ij} \cdot P_g}{P_{ij}} \quad (12)$$

$$F_{l,ij} = \frac{b_{l,ij} \cdot P_l}{P_{ij}} \quad (13)$$

$$B = CA^{-1} \quad (14)$$

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ \frac{P_{ij}}{P_j} & \text{if } ij \in \Gamma_-(j) \\ 0 & \text{else} \end{cases} \quad (15)$$

$$c_{k,ij} = \begin{cases} \frac{P_{ij}}{P_k} & \text{if } ij \in \Gamma_+(k) \\ 0 & \text{else} \end{cases} \quad (16)$$

$a_{ij}$ ,  $b_{g,ij}/b_{l,ij}$ ,  $c_{k,ij}$  are elements of matrix  $A$ ,  $B$ ,  $C$ , respectively. Equation (15) represents contributions of generations to node input. Equation (16) represents proportions of outgoing line to the node.

With the help of PT, we can arrive at the proportion of responsibility a generator or a load takes on for the power flow of a specific branch.

### B. Equivalent Bilateral Exchange Recognition

Assume load  $l$  is at node  $k$ ; the EBE between generator  $g$  and load  $l$  is the difference in the uses of generator  $g$  to all incoming lines and outgoing lines of node  $k$ , represented as

$$EBE_{gl} = \sum_{ij \in \Gamma_-(k)} P_{ij} \cdot F_{g,ij} - \sum_{ij \in \Gamma_+(k)} P_{ij} \cdot F_{g,ij} \quad (17)$$

$$\text{s.t. } \sum_l EBE_{gl} = P_g \quad (18)$$

$$\sum_g EBE_{gl} = P_l. \quad (19)$$

It can be viewed that the generators are decomposed to the loads according to their uses of branches, which are recognized by the PT method. Similarly, the EBE can also be calculated from the perspective of load. Assume generator  $g$  is at node  $k$ ; the EBE is illustrated as

$$EBE_{gl} = \sum_{ij \in \Gamma_+(k)} P_{ij} \cdot F_{l,ij} - \sum_{ij \in \Gamma_-(k)} P_{ij} \cdot F_{l,ij}. \quad (20)$$

### C. Transmission Cost Allocation

In view of the PTEBX method, it is each EBE that induces the power flow instead of the individual generator and load. Therefore, a generator should be responsible for part of the load use with which it has EBEs. On the other hand, a load should be responsible for part of the generator use with which it has EBEs.

As a result, generator  $g$ 's allocation proportion of transmission cost on branch  $ij$  using PTEBX method is defined as

$$f_{g,ij}^H = \sum_l \left( \left( f_{g,ij} \cdot \frac{EBE_{gl}}{P_g} + f_{l,ij} \cdot \frac{EBE_{gl}}{P_l} \right) \cdot r \right). \quad (21)$$

The EBE's utilization to the branch  $ij$  depends on the ratio it takes in the total output of the generator and input of the load. Thus both the transaction parties are responsible for the EBE's utilization, while the generator takes the  $r$  part.

Similarly, load  $l$ 's allocation proportion of branch  $ij$  using PTEBX method is defined as

$$f_{l,ij}^H = \sum_g \left( \left( f_{l,ij} \cdot \frac{EBE_{gl}}{P_l} + f_{g,ij} \cdot \frac{EBE_{gl}}{P_g} \right) \cdot (1-r) \right). \quad (22)$$

If the cost of branch  $ij$  is  $C_{ij}$ , the total transmission costs of generator  $g$  and load  $l$  can be represented as

$$C_g^H = \sum_{ij} f_{g,ij}^H \cdot C_{ij} \quad (23)$$

$$C_l^H = \sum_{ij} f_{l,ij}^H \cdot C_{ij}. \quad (24)$$

#### D. A Simple Test

To better illustrate the calculation process and superiority of the proposed PTEBX method, a simple test is proposed, shown in Fig. 1.

Case 3.1: Fundamental case, as Fig. 1(a).

Case 3.2: 20 MW more power traded from generator 1 and load 2, as in Fig. 1(b).

Case 3.3: 20 MW more power traded from generator 3 and load 2, as in Fig. 1(c).

Case 3.4: 20 MW more power traded from generator 1 and load 4, as in Fig. 1(d).

Case 3.5: 20 MW more power traded from generator 3 and load 4, as in Fig. 1(e).

Since the PTEBX method is a combination of PT and EBX methods, we make a simple comparison among the three methods. Case 3.1 is first utilized to show the calculation process of the PTEBX method, and the remaining cases are calculated for comparison purposes.

The results of power tracing for Case 3.1 can be easily generated as both generator 1 and load 4 take 50% of the total transmission cost. The EBEs are recognized by PTEBX as illustrated in Table I with comparisons to EBX method.

TABLE I  
EQUIVALENT BILATERAL EXCHANGES (MW)

Generator	PTEBX		EBX	
	Load 2	Load 4	Load 2	Load 4
Generator 1	20	30	6.67	43.33
Generator 3	0	100	13.33	86.67

The final transmission cost allocation results are presented in Table II, together with results of the other cases under different methods. This comparison shows the superiority and improvements offered by the PTEBX method over PT and the EBX method.

For the PT method, the results are dependent on the direction of the power flow, but are not sensitive enough to distinguish different transaction amounts with same power flow direction. For the EBX method, there are other drawbacks:

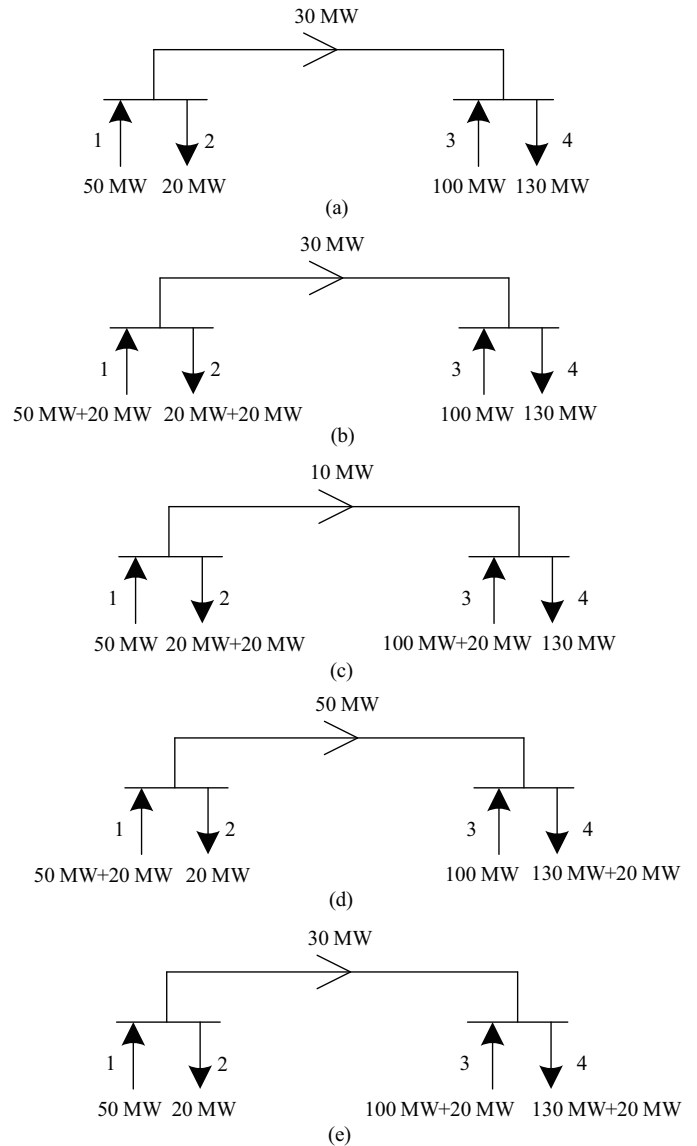


Fig. 1. Simple test to describe the PTEBX method.

TABLE II  
TRANSMISSION COST ALLOCATION RESULTS

Method	Case	Generator 1	Load 2	Generator 3	Load 4
PT	3.1	0.5	0	0	0.5
	3.2	0.5	0	0	0.5
	3.3	0.5	0	0	0.5
	3.4	0.5	0	0	0.5
	3.5	0.5	0	0	0.5
EBX	3.1	0.3823	0.1177	0.1177	0.3823
	3.2	0.3473	0.1527	0.1527	0.3473
	3.3	0.2880	0.2120	0.2120	0.2880
	3.4	0.4200	0.0800	0.0800	0.4200
	3.5	0.3787	0.1213	0.1213	0.3787
PTEBX	3.1	0.3077	0.1000	0.1923	0.4000
	3.2	0.3077	0.1430	0.1923	0.3570
	3.3	0.3077	0.1000	0.1923	0.4000
	3.4	0.3334	0.0714	0.1666	0.4286
	3.5	0.3000	0.1000	0.2000	0.4000

- 1) It is unreasonable that the allocation proportion of generator 1 is same to load 4 (and that of generator 2 is same to load 3) since their traded power is not equal.

- 2) It is unreasonable that the proportion of generator 1 decreases while its traded power increases in Case 3.2 when compared to Case 3.1. There are similar results for proportions of load 4 in Case 3.3 and load 2 in Case 3.5 when compared to Case 3.1.
- 3) Power flow is mitigated in Case 3.3 compared to Case 3.2, while load 2 takes larger proportion in Case 3.3. So the transmission cost allocation method fails to provide price signals beneficial for system operation.

Thus, in the PTEBX method, the above drawbacks of PT and EBX methods are overcome.

#### IV. CRITERIA BASED COMPARATIVE ANALYSIS OF METHODS

In this section, we conduct theoretical analysis to determine that transmission cost allocation methods meet the criteria raised in Section II. We focus on the following methods: pro-rata, MP, WW, PT, EBX,  $Z_{bus}$ , AS, and PTEBX. Other methods referred to in Section I are reasonable approaches, but are not selected for analysis here for the sake of targeted comparison and length of paper.

To be specific,

- 1) The SRMC method has been determined as unable to recover the total transmission costs. Moreover, it does not satisfy the criterion “efficiency” in the first place.
- 2) The LRMC method is settled from a long-term perspective, while all other methods that are based on instant power flow, are considered as acceptable from short-term perspective. It is not necessary to compare methods based on different time scales.
- 3) The MW-mile method calculates the use of a specific transaction by removing all other generators and loads. This calculation is similar to the WW method [21].
- 4) The CP method is highly dependent on artificially pre-determined power flow, which could be totally different under different market operation conditions. Moreover, extra handlings are needed to improve this method [9].

For the sake of simplicity, we only present tariffs for the generators of the available methods. Tariffs for the loads can be acquired similarly.

##### A. Pro-rata

The pro-rata method deems that the allocated proportion is only related to the amount of power, defined as

$$f_{g,ij}^A = \frac{P_g}{\sum_g P_g + \sum_l P_l}. \quad (25)$$

The criteria “efficiency,” “non-negativity,” and “monotone non-decreasing to amount of power” are satisfied apparently, while criterion of “consideration of network position” is not satisfied.

Whoever the network user trades with, the allocated proportion stays the same if the trade amount is unchanged. Therefore, the criterion “signal to trade” is not satisfied. Since counter flows are not treated differently and are charged, the criterion “less fluctuant to different scenarios” is satisfied.

The criterion “equal treatment on different transaction types” is satisfied since each network user is charged only based on the amount of power, without discrimination in the pool market trade or in bilateral contract. Furthermore, for a bilateral contract, since both transaction parties meet the same amount of power, they meet the same allocation proportion of transmission cost as well.

##### B. Marginal Participation

The MP method deems that the network users should be charged in proportion to the variance of power flow induced by their unit incremental power, which is defined as

$$f_{g,ij}^B = \frac{\Delta P_{ij}^g \cdot P_g}{\sum_g \Delta P_{ij}^g \cdot P_g + \sum_l \Delta P_{ij}^l \cdot P_l}. \quad (26)$$

If the *variance* is calculated by the absolute value, impact of counter flow is treated equally to positive flow, defined as

$$f_{g,ij}^{B'} = \frac{|\Delta P_{ij}^g| \cdot P_g}{\sum_g |\Delta P_{ij}^g| \cdot P_g + \sum_l |\Delta P_{ij}^l| \cdot P_l}. \quad (27)$$

The criterion “efficiency” is apparently satisfied while the criterion “non-negativity” is satisfied only if the allocation proportion is defined as  $f_{g,ij}^{B'}$  and  $f_{l,ij}^{B'}$ . The criterion “consideration of network position” is satisfied because the allocation proportion is dependent on the power flow.

Assume a  $\Delta P_g$  increment in  $P_g$ , since DC power flow is often used in calculating transmission cost allocation problem in real-world practice. We then have

$$\Delta P_{ij}^g = \Delta P_{ij}^{g+\Delta g}. \quad (28)$$

If  $\Delta P_{ij}^g$  is considered as negative, then we can only make sure that

$$f_{g+\Delta g,ij}^{B'} > f_{l,ij}^{B'} \quad (29)$$

since only the use of the absolute value of  $\Delta P_{ij}^g$  assures that the  $\Delta P_g$  increment will lead to positive influence on power flow. Therefore, only if the power flow variance is considered by its absolute value, does the method satisfy the criterion “monotone non-decreasing to amount of power.”

Assume load  $l$  wants to purchase  $\Delta P_l$  power, either from generator  $g_1$  or  $g_2$ . Transaction  $g_1 - l$  induces less power flow in branch  $ij$  than transaction  $g_2 - l$ , represented as

$$(\Delta P_{ij}^{g_1} + \Delta P_{ij}^l) \cdot \Delta P_l < (\Delta P_{ij}^{g_2} + \Delta P_{ij}^l) \cdot \Delta P_l \quad (30)$$

For transaction  $g_1 - l$  and  $g_2 - l$ , the allocation proportions of the load are calculated by

$$f_{l+\Delta l, g_1-l, ij}^B = \frac{\Delta P_{ij}^l \cdot (P_l + \Delta P_l)}{\sum_{g \in G \setminus g_1} \Delta P_{ij}^g \cdot P_g + \Delta P_{ij}^{g_1} \cdot (P_{g_1} + \Delta P_l) + \sum_l \Delta P_{ij}^l \cdot (P_l + \Delta P_l)} \quad (31)$$

$$f_{l+\Delta l, g_2-l, ij}^B = \frac{\Delta P_{ij}^l \cdot (P_l + \Delta P_l)}{\sum_{g \in G \setminus g_2} \Delta P_{ij}^g \cdot P_g + \Delta P_{ij}^{g_2} \cdot (P_{g_2} + \Delta P_l) + \sum_l \Delta P_{ij}^l \cdot (P_l + \Delta P_l)} \quad (32)$$

Because both  $\Delta P_{ij}^{g_1}$  and  $\Delta P_{ij}^{g_2}$  can be negative, we cannot determine that  $f_{l+\Delta l, g_1-l, ij}^B < f_{l+\Delta l, g_2-l, ij}^B$ . Similarly, the relative value of  $f_{l+\Delta l, g_1-l, ij}^B$  and  $f_{l+\Delta l, g_2-l, ij}^B$  also cannot be determined as that of  $|\Delta P_{ij}^{g_1}|$ , and thus  $|\Delta P_{ij}^{g_2}|$  cannot be assured. Therefore, the criterion “signal to trade” is not satisfied.

As counter flows are only considered when the power flow variance is calculated by the absolute value, the criterion “less fluctuant to different scenarios” is satisfied when applying the absolute value for calculating the power flow variance.

Different transaction types are treated without discrimination, but generators and loads are treated separately. Therefore, the criterion “equal treatment on different transaction types” is not satisfied.

### C. With and Without

The WW method is similar to the MP method. The only difference is that the WW method considers the network users’ entire power as the inducement of power flow variance, instead of unit incremental power. The tariff of this method is defined as

$$f_{g, ij}^C = \frac{\Delta P_{g, ij} \cdot P_g}{\sum_g \Delta P_{g, ij} \cdot P_g + \sum_l \Delta P_{l, ij} \cdot P_l}. \quad (33)$$

If the *variance* is calculated by the absolute value, impact of counter flow is treated equally to positive flow, defined as

$$f_{g, ij}^{C'} = \frac{|\Delta P_{g, ij}| \cdot P_g}{\sum_g |\Delta P_{g, ij}| \cdot P_g + \sum_l |\Delta P_{l, ij}| \cdot P_l}. \quad (34)$$

Because both  $\Delta P_{g, ij}$  and  $\Delta P_{l, ij}$  in (33) can be negative, an analysis as to whether the WW method satisfies the criteria is similar to the analysis of the MP method. Therefore, their conclusions are also the same.

### D. Power Tracing

The tariff of the PT method is demonstrated in Section III A. According to (10) and (11), criteria “efficiency,” “non-negativity” and “consideration of network position” are satisfied.

The key idea of the PT method is that a network user’s usage to the power flow of a branch equals to the proportion that its induced current takes in the entire current, represented as

$$f_{g, ij}^D = \frac{P_{g, ij}}{P_{ij}} = \frac{I_{g, ij}}{I_{ij}}. \quad (35)$$

According to (15) and (16),  $I_{g, ij}$  or  $I_{l, ij}$  may either increase or decrease when  $P_g$  or  $P_l$  increases. Therefore, there is no guarantee of the criterion “monotone non-decreasing to amount of power” and “signal to trade.”

Furthermore, counter flows are not charged so the criterion “less fluctuant to different scenarios” is not satisfied.

Different transaction types are treated without discrimination, while the entirety of bilateral exchanges is not taken into account. Therefore, the criterion “equal treatment on different transaction types” is not satisfied.

### E. Equivalent Bilateral Exchange

In the EBX method, the generations and loads are decomposed based on their proportions in the system’s total generation and load. The tariff of this method is defined as

$$f_{g, ij}^E = \frac{P_{g, ij}}{\sum_g P_{g, ij} + \sum_l P_{l, ij}} \quad (36)$$

$$\text{s.t. } P_{g, ij} = \frac{1}{2} \cdot P_{gl, ij} = \sum_l \frac{1}{2} \cdot |\gamma_{gl, ij}| \cdot P_g \cdot \frac{P_l}{\sum_l P_l}. \quad (37)$$

According to (36)–(37), criteria “efficiency” and “non-negativity” are satisfied, while the criterion “consideration of network position” is not satisfied.

If we assume a  $\Delta P_g$  increment in  $P_g$ , then the corresponding allocation proportion for generator  $g$  is changed under two conditions, i.e.,

$$f_{g+\Delta g, ij}^E = \frac{P_{g, ij} + \Delta P_{g, ij}}{(\sum_g P_{g, ij} + \sum_l P_{l, ij}) + \Delta P_{g, ij} + \Delta P_{g, ij}} \quad (38)$$

$$f_{g+\Delta g, ij}^E = \frac{P_{g, ij} + \Delta P_{g, ij}}{(\sum_g P_{g, ij} + \sum_l P_{l, ij}) - \Delta P_{g', ij} + \Delta P_{g, ij}}. \quad (39)$$

Equation (38) represents the condition that the  $\Delta P_g$  increment in  $P_g$  corresponds to a  $\Delta P_{l, ij}$  increment in load. In this case, as the relative value of  $\Delta P_{g, ij}$  and  $\Delta P_{l, ij}$  cannot be assured, so there is no guarantee that

$$f_{g+\Delta g, ij}^E > f_{g, ij}^E. \quad (40)$$

Equation (39) represents the condition that the  $\Delta P_g$  increment in  $P_g$  corresponds to changes in the other generator’s induced power flow variance in branch  $ij$ , expressed as  $P_{g', ij}$ . In this case, the relative value of  $\Delta P_{g, ij}$  and  $P_{g', ij}$  cannot be assured; therefore (40) cannot be guaranteed. The criterion “monotone non-decreasing to amount of power” is not satisfied.

Assume a transaction  $g_1 - l$  between load  $l$  and generator  $g_1$ , and a transaction  $g_2 - l$  between load  $l$  and generator  $g_2$ . The incremental power flows induced by the two transactions meet

$$\gamma_{g_1 l, ij} < \gamma_{g_2 l, ij} \quad (41)$$

But we cannot assure that

$$|\gamma_{g_1 l, ij}| < |\gamma_{g_2 l, ij}|. \quad (42)$$

Therefore, the criterion “signal to trade” is not satisfied.

Since the generation shift distribution factor  $\gamma_{gl, ij}$  is used as its absolute value, counter flows are taken into consideration. As a result, the criterion “less fluctuant to different scenarios” is satisfied.

The EBX method treats all transactions as bilateral exchanges; it satisfies the criterion “equal treatment on different transaction types.”

### F. The $Z_{bus}$ Method Using DC Power Flow

We simplify the  $Z_{bus}$  method in [12] by using DC power flow, and then the tariff is defined as,

$$f_{g,ij}^F = \frac{|P_{g,ij}|}{\sum_g |P_{g,ij}| + \sum_l |P_{l,ij}|} \quad (43)$$

$$\text{s.t. } P_{g,ij} = V_i a_{ij}^g I_g \quad (44)$$

$$a_{ij}^g = (z_{ig} - z_{jg}) \cdot y_{ij} + z_{ig} \cdot y_{ij}^{sh}. \quad (45)$$

According to (43)–(45), the criteria “efficiency,” “non-negativity” and “consideration of network position” are satisfied.

If we assume an increment  $\Delta P_g$  in the generation of generator  $g$ , then  $I_g$  increases. Although the value of  $a_{ij}^g$  could be either positive or negative, power flow induced by generator  $g$  increases in absolute value, as

$$|P_{g+\Delta g,ij}| > |P_{g,ij}|. \quad (46)$$

Therefore, the criterion “monotone non-decreasing to amount of power” is satisfied.

Assume a transaction  $g_1-l$  between load  $l$  and generator  $g_1$ , and a transaction  $g_2-l$  between load  $l$  and generator  $g_2$ . The incremental uses to branch  $ij$  induced by the two transactions meet

$$P_{g_1+\Delta l,ij} < P_{g_2+\Delta l,ij}. \quad (47)$$

However, both  $P_{g_1+\Delta l,ij}$  and  $P_{g_2+\Delta l,ij}$  could be either positive or negative, so the relative value of  $|P_{g_1+\Delta l,ij}|$  and  $|P_{g_2+\Delta l,ij}|$  cannot be ascertained. The criterion “signal to trade” is not satisfied.

Counter flows are charged, as the absolute value is used in (43), so the criterion “less fluctuant to different scenarios” is satisfied.

Different transaction types are treated without discrimination, while the entirety of bilateral exchange is not taken into account. Therefore, the criterion “equal treatment on different transaction types” is not satisfied.

### G. Aumann-Shapley

We simplify the AS method in [15] by using DC power flow, and then the tariff is defined as,

$$f_{g,ij}^G = \frac{|P_{g,ij}|}{\sum_g |P_{g,ij}| + \sum_l |P_{l,ij}|} \quad (48)$$

$$\text{s.t. } P_{g,ij} = I_g \cdot UP_{g,ij} \quad (49)$$

$$UP_{g,ij} = z_{ig} \cdot \left( \sum_g \left( \frac{z_{ig} - z_{jg}}{z_{ij}} + z_{ig} \cdot y_{ij}^{sh} \right) \cdot I_g \right) + \left( \sum_g z_{ig} \cdot I_g \right) \cdot \left( \frac{z_{ig} - z_{jg}}{z_{ij}} + z_{ig} \cdot y_{ij}^{sh} \right) \quad (50)$$

According to (48)–(50), network users’ uses of branch  $ij$  are calculated similar to those in the  $Z_{bus}$  method. Values of  $UP_{g,ij}$  and  $UP_{l,ij}$  also could be either positive or negative as  $a_{ij}^g$  and  $a_{ij}^l$  in (45). Therefore, the conclusions are same to those in the  $Z_{bus}$  method.

### H. Power Tracing Based Equivalent Bilateral Exchange

In Section III, principles of the PTEBX method have been comprehensively demonstrated. As the PTEBX method can be viewed as a combination of the PT method and the EBX method, the mutual characteristics remain. Therefore, criteria “efficiency” and “non-negativity” are satisfied. Moreover, the EBEs are recognized based on the PT method, so the criterion “consideration of network position” is satisfied as well.

If we assume an increment  $\Delta P_g$  in the generation of generator  $g$ , then the allocation proportion of the generator is changed to

$$f_{g+\Delta g,ij}^H = \sum_l \left( \left( f_{g+\Delta g,ij} \cdot \frac{EBX_{g+\Delta g,l}}{P_{g+\Delta P_g}} + f_{l,ij} \cdot \frac{EBX_{gl}}{P_l} \right) \cdot r \right). \quad (51)$$

Even if the proportion results by PT  $f_{g+\Delta g,ij}$  may be lower than  $f_{g,ij}$ , the generator still has to be responsible for parts of proportions of loads, which are increased correspondingly. Therefore, the criterion “monotone non-decreasing to amount of power” is satisfied.

Assume a transaction  $g_1-l$  between load  $l$  and generator  $g_1$ , and a transaction  $g_2-l$  between load  $l$  and generator  $g_2$ . The incremental uses to branch  $ij$  induced by the two transactions meet

$$P_{g_1+\Delta l,ij} < P_{g_2+\Delta l,ij}. \quad (52)$$

Similar to the analysis of criterion “monotone non-decreasing to amount of power,” even if the proportion results by PT that the load  $l$  is responsible for may be decreased, it still has to be responsible for parts of proportions of generators, which are increased correspondingly. Therefore, the criterion “signal to trade” is satisfied.

Even though the PT may result in a zero value of  $f_{g,ij}$ , the generator  $g$  still has to be responsible for part of the uses by the EBEs with loads whose allocation factors are  $f_{l,ij}$ . As a result, the criterion “less fluctuant to different scenarios” is satisfied.

The PTEBX method treats all transactions as bilateral exchanges; it satisfies the criterion “equal treatment on different transaction types.”

Table III illustrates conclusions made in this section on whether the methods satisfy the criteria.

TABLE III  
CRITERIA VS. METHODS

Criteria	Pro-rata	MP	MP (abs)	WW	WW (abs)	PT	EBX	$Z_{bus}$	AS	PTEBX
1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2	✓	×	✓	×	✓	✓	✓	✓	✓	✓
3	×	✓	✓	✓	✓	×	✓	✓	✓	✓
4	✓	×	✓	×	✓	×	✓	✓	✓	✓
5	×	×	×	×	×	×	×	×	×	✓
6	✓	×	✓	×	✓	×	✓	✓	✓	✓
7	✓	×	×	×	×	×	✓	×	×	✓

✓ = Satisfied; × = Not satisfied.

(abs) = Power flow variance is calculated by the absolute value.

Criterion: 1 = Efficiency; 2 = Non-negativity; 3 = Consideration of network position; 4 = Monotone non-decreasing to amount of power; 5 = Signal to trade; 6 = Less fluctuant to different scenarios; 7 = Equal treatment on different transaction types.

## V. NUMERICAL EXAMPLES

The eight methods theoretically analyzed in Section IV are further compared using the IEEE-30 system [22]. Data of this fundamental case are acquired from [22]. Besides the fundamental case, three more cases are tried:

*Case 5.1:* Fundamental case.

*Case 5.2:* Incremental trade from generator at Node 5 to load at Node 5 with an amount of 20 MW.

*Case 5.3:* Incremental trade from generator at Node 2 to load at Node 5 with an amount of 20 MW.

*Case 5.4:* Incremental trade from wind power generator at Node 5 to load at Node 2 with an amount of 20 MW.

Since the tariff methods justify whether the criteria “efficiency,” “non-negativity” and “consideration of network position” are satisfied, we focus on the criteria “monotone non-decreasing to amount of power,” “signal to trade,” “less fluctuant to different scenarios” and “equal treatment on different transaction types.”

### A. EBE Recognition

With (17), all EBEs are recognized, as seen in Table IV. The EBEs are decomposed depending on the electric distance of the generator and the load. The generator usually has EBEs

of larger amount of power with electrically closer loads. This result is consistent with the PT results. In particular, since the amount of power of the load at Node 5 is larger than that of the generator at Node 5, it is viewed that Node 5 acts as a net load. Therefore, the generator at Node 5 only has an EBE with the load at Node 5.

### B. Cases Involving Incremental Purchase by Load at Node 5

In Case 5.1, the real power flow of branch 2–5 is 57.11 MW, from Node 2 to Node 5. In Case 5.2, the incremental transaction between generator at Node 5 and load at Node 5 has no impact on the power flow of the system. In Case 5.3, the incremental transaction between generator at Node 2 and load at Node 5 increases the power flow of branch 2–5 to 69.19 MW. The allocation results of branch 2–5 in Case 5.1, 5.2, and 5.3 using the eight methods are illustrated in Table V. The following conclusions are made.

- 1) As the power flow of branch 2–5 is larger in Case 5.3 than in Case 5.2, the load at Node 5 should be responsible for a larger proportion of transmission cost in branch 2–5 in Case 5.3. However, the load at Node 5 is responsible for a smaller proportion of transmission cost in branch 2–5 using the pro-rata,  $Z_{bus}$ , and AS methods. Therefore, these three methods do not satisfy the criterion “signal to trade.”

TABLE IV  
EQUIVALENT BILATERAL EXCHANGES (MW)

Load	Generator at Node 1	Generator at Node 2	Generator at Node 5	Generator at Node 8	Generator at Node 11	Generator at Node 13
Load at 2	11.6652	217	0	0	0	0
Load at 3	24	0	0	0	0	0
Load at 4	75.6536	7.1403	0	0	0	0
Load at 5	548.0063	149.2751	215.2000	29.5186	0	0
Load at 7	157.2959	28.8191	0	43.5559	0	0
Load at 8	0	0	0	300.0000	0	0
Load at 12	75.3311	6.7311	0	0	0	31.1677
Load at 14	41.2480	3.6856	0	0	0	17.0660
Load at 15	56.4541	5.0443	0	0	0	23.3579
Load at 16	23.2850	2.0806	0	0	0	9.6340
Load at 17	33.4178	13.3591	0	11.3909	11.3909	20.5042
Load at 18	21.2893	1.9022	0	0	0	8.80832
Load at 19	30.8643	15.4269	0	13.9122	13.9122	20.9260
Load at 23	21.2893	1.9022	0	0	0	8.8083
Load at 24	27.4170	14.2382	0	15.5989	12.6794	17.7101
Load at 26	15.1157	2.7694	0	17.1148	0	0
Load at 29	10.3650	1.8990	0	11.7359	0	0
Load at 30	45.9232	8.4138	0	51.9969	0	0
Total generation	1403.41	482.52	215.20	518.53	122.50	120.00

TABLE V  
ALLOCATION RESULTS OF OF BRANCH 2–5

Generator/load in branch 2–5 in different cases		Pro-rata	MP (abs)	WW (abs)	PT	EBX	$Z_{bus}$	AS	PTEBX
Generator at Node 5	Case 5.1	0.0375	0.0972	0.0081	0	0.0525	0.0087	0.0182	0.0571
	Case 5.2	0.0676	0.1589	0.0404	0	0.0816	0.0412	0.0495	0.0909
	Case 5.3	0.0350	0.0887	0.0017	0	0.0416	0.0014	0.0134	0.0471
Load at Node 5	Case 5.1	0.1641	0.4255	0.3991	0.5000	0.3358	0.3985	0.6750	0.3424
	Case 5.2	0.1860	0.4370	0.4314	0.5000	0.3284	0.4347	0.7260	0.3424
	Case 5.3	0.1860	0.4707	0.4411	0.5000	0.3540	0.4203	0.7225	0.3550
Generator at Node 2	Case 5.1	0.0851	0.0101	0.0218	0.1138	0.1265	0.0225	0.0330	0.0964
	Case 5.2	0.0795	0.0086	0.0062	0.1138	0.1158	0.0077	0.0241	0.0895
	Case 5.3	0.1118	0.0130	0.0373	0.1699	0.1604	0.0364	0.0484	0.1463



- 2) Under the PT method, the generator at Node 5 is not responsible for any proportion of transmission cost in branch 2–5, because Node 5 is considered as a net load and the power flow direction is from Node 2 to Node 5. At the same time, the load at Node 5 is responsible for 0.5 proportion of transmission cost in branch 2–5. Therefore, the PT method does not satisfy both criterion “monotone non-decreasing to amount of power” and criterion “signal to trade.”
- 3) In Case 5.2, the generator and load at Node 5 are the only network users that increase their traded power. According to criterion “monotone non-decreasing to amount of power,” the load at Node 5 should take more responsibility for the proportion of transmission cost in branch 2–5. Apparently, the EBX method does not satisfy the criterion “monotone non-decreasing to amount of power.”
- 4) Under the PTEBX method, the generator and load at Node 5 take on more proportion in transmission costs in Case 5.2 than in Case 5.1. The load at Node 5 takes more proportion in transmission cost in Case 5.3 than in Case 5.2. These results satisfy the criteria “monotone non-decreasing to amount of power” and “signal to trade.”

Assume that the transmission cost is calculated by [16]

$$C_{ij} = 1000 \times z_{ij} (\$/h). \quad (53)$$

Comparisons of the proportion of transmission cost by the generator and load at Node 5 in Case 5.1 and 5.2 are shown in Table VI. In view of bilateral exchanges, the closer the transmission cost of transaction parties are, the better the method is. From Table VI, the PTEBX method has the best performance for its lowest value of difference in incremental transmission cost between the generator and the load at Node 5. Results of the PT method are not comparable due to its dependence on the power flow, which is not changed.

### C. Cases Involving Incremental Transaction Between Wind Power Generator at Node 5 and Load at Node 2

Assume the generator at Node 5 is a wind power generator. Wind data in [24] is adopted. We use an ARIMA model to

generate 2000 scenarios of wind power output and fast forward selection algorithm to reduce to five scenarios [23]. Assume all the wind power output at Node 5 is traded by bilateral transaction with the load at Node 2. Table VII shows the variance value of the transmission cost proportion of branch 5–7 of the wind power generator in the five scenarios.

From Table VII, the PT method results in the largest value of variance, while the Pro-rata method results in the smallest. This is because the Pro-rata method only uses the power amount proportion to determine the transmission cost proportion and ignores the power flow. Apart from the Pro-rata method, the PTEBX method has the smallest value of variance. This result certifies that the PTEBX method satisfies the criterion “less fluctuant to different scenarios” and is suitable to be applied to systems with large-scale renewable energy.

## VI. CONCLUSION

This paper proposes the power tracing based equivalent bilateral exchanges method for allocating the transmission cost to network users. The PTEBX method decomposes the generations and loads to multiple bilateral exchanges based on the results of power tracing. Network users are responsible for not only the power flows that they induce, but also those induced by other network users with whom they have equivalent bilateral exchanges. Seven criteria are put forward to evaluate the performance of methods for allocating transmission cost while taking into consideration the characteristics of the electricity market. A theoretical analysis is conducted to test whether the seven widely used methods and the proposed method satisfy the criteria. The analysis shows that the proposed PTEBX method is the only one that satisfies all the seven criteria. The PTEBX method leads to efficient and non-negative results with consideration of network positions. The results are monotone non-decreasing to amount of power and provide signals to trade. The PTEBX method also performs well with systems with large-scale renewable energy and treats different transaction types in a fair manner. Numerical examples based on the IEEE-30 system further validate the efficiency of the proposed method.

TABLE VI  
COMPARISON OF TREATMENT ON BILATERAL EXCHANGES (\$)

Generator/Load at Node 5 in Different Cases		Pro-rata	MP (abs)	WW (abs)	PT	EBX	$Z_{bus}$	AS	PTEBX
Generator at Node 5	Case 5.1	307.34	72.36	71.16	0	226.72	71.16	54.56	19.86
	Case 5.2	554.35	127.07	133.26	0	396.27	133.26	168.85	31.61
Load at Node 5	Case 5.1	1345.6	316.77	888.74	173.89	867.66	888.71	2064.6	677.15
	Case 5.2	1524.7	349.50	1030.3	173.89	942.95	1030.3	2523.5	677.15
Difference in incremental transmission cost between generator at load		67.61	21.95	79.47	0	94.26	79.47	144.62	11.75

TABLE VII  
VARIANCE OF PROPORTIONS OF WIND POWER GENERATOR

Item	Pro-rata	MP (abs)	WW (abs)	PT	EBX	$Z_{bus}$	AS	PTEBX
Variance	0.001845	0.00908	0.009962	0.012617	0.006781	0.008962	0.008226	0.006676

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