Impact of Grid Connection of Large-Scale Wind Farms on Power System Small-Signal Angular Stability

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***Abstract*—The grid connection of a large-scale wind farm could change the load flow/configuration of a power system and introduce dynamic interactions with the synchronous generators (SGs), thus affecting system small-signal angular stability. This paper proposes an approach for the separate examination of the impact of those affecting factors, i.e., the change of load flow/configuration and dynamic interactions brought about by the grid connection of the wind farm, on power system small-signal angular stability. Both cases of grid connection of the wind farm, either displacing synchronous generators or being directly added into the power system, are considered. By using the proposed approach, how much the effect of the change of load flow/configuration brought about by the wind farm can be examined, while the degree of impact of the dynamic interaction of the wind farm with the SGs can be investigated separately. Thus, a clearer picture and better understanding of the power system small-signal angular stability as affected by grid connection of the large-scale wind farm can be achieved. An example of the power system with grid connection of a wind farm is presented to demonstrate the proposed approach.**

***Index Terms*—Double fed induction generator (DFIG), electromechanical oscillation modes, power system low-frequency oscillations, power system small-signal angular stability, wind farms.**

I. INTRODUCTION

LOW frequency power oscillations, which can occur as the result of inter-connection of local power networks and the installation of fast-acting automatic voltage regulators (AVRs), threaten the safe operation of power systems. It is generally understood that the occurrence of the oscillations is normally due to the lack of damping of power system electromechanical oscillation modes. Low-frequency power oscillations are the main concern of power system small-signal angular stability.

Connecting a large-scale wind farm with a great number of variable speed wind generators (VSWGs) into a power system can affect the systems small-signal angular stability. To examine the damping of power system low-frequency oscillations as affected by the large-scale grid-connected wind farm, a first step may be the investigation in the change of the damping of power system electromechanical oscillation modes brought about by the grid connection of the wind farm. The change can be computed by comparing the damping of the modes before and after the wind farm is connected to the power system, as illustrated by an example of the power system shown in Fig. 1 and Fig. 2.

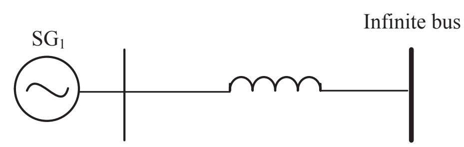

Fig. 1. A single-machine infinite-bus power system.

Fig. 1 is a single-machine infinite-bus power system. With a wind farm connected, the system is transformed as illustrated in Fig. 2. Note that $\Delta\lambda$ changes the electromechanical oscillation mode of the power system after the wind farm is connected. $\Delta\lambda$ can be easily obtained by modal computation of the system in Fig. 1 and Fig. 2 to arrive at $\Delta\lambda = \lambda - \lambda_{\rm w}$, where λ is the oscillation mode of the system of Fig. 1 and $\lambda_{\rm w}$ is that of the system in Fig. 2.

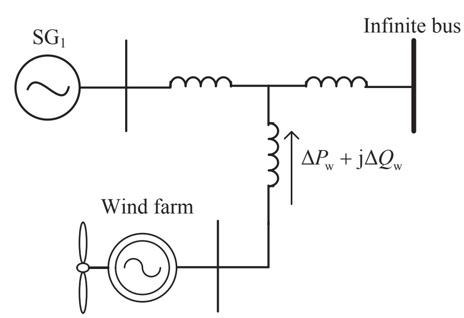

Fig. 2. Addition of a wind farm into the system of Fig. 1.

However, $\Delta\lambda$ is caused by two changes by adding wind. The first change is the load flow and system configuration introduced by the wind farm, which is obviously different between the system in Fig. 1 and Fig. 2. The second is the addition of the dynamic interaction between the wind farm an-

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d the synchronous generator. A simple comparison between the oscillation mode of the systems in Fig. 1 and Fig. 2 does not allow for a distinction of these two factors, namely, that the wind farm affects the system small-signal angular stability. Thus, exactly how and why the addition of the wind farm into the power system in Fig. 1 causes the change of system oscillation modes remains ambiguous by the comparison.

Fig. 2 and Fig. 3 show the difference that occurs in the power system when the affecting factors of the load flow and system configuration brought by the wind farm are excluded. When the synchronous generator SG_2 in the system is displaced by the wind farm (Fig. 3), the system then becomes Fig. 2. This comparison keeps the load flow and system configurations unchanged. Thus $\Delta\lambda = \lambda_{\mathrm{g}} - \lambda_{\mathrm{w}}$, where λ_g is the oscillation mode of the system of Fig. 3 and λ_{w} is that of the system in Fig. 2.

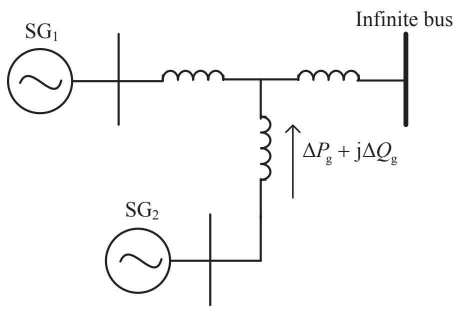

Fig. 3. Power system with SG_2 to be displaced by a wind farm.

However, the change of electromechanical oscillation mode $\Delta\lambda$ in this case is still caused by two affecting factors: 1) withdrawal of the dynamic interaction between SG_1 and SG_2 from the system; 2) addition of the dynamic interaction between the wind farm and SG_2 into the system. A simple comparison between the oscillation mode of the system in Fig. 2 and 3 does not indicate exactly how much in $\Delta\lambda$ is caused by withdrawing SG_2 and how much by adding the wind farm. Thus how the wind farm affects the system small-signal angular stability still needs further clarification.

For over a decade, the impact of the grid connection of large-scale wind farms on power system small-signal angular stability has been a topic of investigation with many published papers. Methods used thus far have been to examine the change of electromechanical oscillation modes by either adding a wind farm into a power system or with the wind displacing synchronous generators in the power system. For example, the addition of wind farms into a power system to meet the load requirement without displacing any synchronous generators has been investigated in [1]–[3]. Investigation by modal computation in [1]–[3] compares the results with and without wind farms connected to the power system. The case of wind displacing synchronous generators is examined in [4]–[8].

Despite significant research devoted to this important topic, how the power system small-signal angular stability is affected by the grid connection of large-scale wind farms has remained unclear. Perhaps the main reason is that the case of both the "addition" and the "displacement" outlined above contains two factors by which the wind farm affects the electromechanical oscillation modes of the power system. Separation of the affecting factors may be the key to or at least the first step towards an unambiguous examination of power system small-signal angular stability as affected by the grid connection of the large-scale wind farm.

The physical reasons for the existence of the dynamic interaction between a wind farm or a displaced synchronous generator with the rest of the power system is in fact the dynamic variation of power exchange $\Delta P_{\mathrm{w}} + \mathrm{j}\Delta Q_{\mathrm{w}}$ (Fig. 2) or $\Delta P_{\mathrm{g}} + \mathrm{j}\Delta Q_{\mathrm{g}}$ (Fig. 3). If $\Delta P_{\mathrm{w}} + \mathrm{j}\Delta Q_{\mathrm{w}} = 0$ (Fig. 2) or $\Delta P_{\mathrm{g}} + \mathrm{j}\Delta Q_{\mathrm{g}} = 0$ (Fig. 3), the dynamic interaction does not exist. In this assumed case, the wind farm or the displaced synchronous generator becomes a constant power source. Hence when the wind farm or the displaced synchronous generator is modelled as the constant power source, the affecting factor of the dynamic interaction between the wind farm, or the displaced synchronous generator in which the rest of the power system is excluded. In this way, affecting factors of adding the wind farm or displacing the synchronous generator by the wind farm may be examined separately.

Based on the idea outlined above, this paper proposes an approach to examine the impact of large-scale wind farms on power system small-signal angular stability. In establishing the linearized model of a power system connected to a large-scale wind farm, the dynamic power exchange of the wind farm, $\Delta P_{\mathrm{w}} + \mathrm{j}\Delta Q_{\mathrm{w}}$ (Fig. 2), or the displaced synchronous generator, $\Delta P_{\mathrm{g}} + \mathrm{j}\Delta Q_{\mathrm{g}}$ (Fig. 3), with the rest of the power is modelled as the input to the system. Thus the established model clearly indicates that the effect of the load flow and system configuration introduced by the wind farm and the displaced synchronous generator on the system small-signal angular stability can be examined by modelling the wind farm and the displaced synchronous generator as the constant power source.

The proposed approach enables the separation of three factors by which the wind farm affects the electromechanical oscillation modes. Using this approach, a clearer picture and deeper insight can be arrived at to understand how and why the wind farm can impact the power system small-signal angular stability.

This paper is organized as follows. In Section II, the proposed approach to examine the power system small-signal angular stability as affected by the grid connection of large-scale wind farms is introduced. Section III presents an example of a multi-machine power system with a large-scale wind farm. Results of modal computation demonstrate and validate the application of the proposed approach are also discussed in this section. Section IV provides a conclusion that summarizes contributions of this paper.

II. Proposed Approach to Examine the Effect of Wind Farms on Power System Small-Signal Angular Stability

A. Addition of A Wind Farm

Fig. 4 shows a power system where a wind farm is connected at the PCC (point of common connection). The power injection from the wind farm to the power system is denoted by $P_\mathrm{w} + \mathrm{j}Q_\mathrm{w}$, and the magnitude and phase of the voltage at the PCC by U_w and φ_w respectively. The following linearized state-space model of the power system is established:

$$\begin{aligned} s\Delta \boldsymbol{X}_\mathrm{G} &= \boldsymbol{A}_\mathrm{G}\Delta \boldsymbol{X}_\mathrm{G} + \boldsymbol{b}_{\mathrm{G}1}\Delta P_\mathrm{w} + \boldsymbol{b}_{\mathrm{G}2}\Delta Q_\mathrm{w} \\ \begin{bmatrix} \Delta U_\mathrm{w} \\ \Delta \varphi_\mathrm{w} \end{bmatrix} &= \boldsymbol{C}_\mathrm{G}\Delta \boldsymbol{X}_\mathrm{G} + \boldsymbol{d}_{\mathrm{G}1}\Delta P_\mathrm{w} + \boldsymbol{d}_{\mathrm{G}2}\Delta Q_\mathrm{w}, \end{aligned} \tag{1}$$

$$\begin{aligned} s\Delta \boldsymbol{X}_\mathrm{W} &= \boldsymbol{A}_\mathrm{W}\Delta \boldsymbol{X}_\mathrm{W} + \boldsymbol{b}_{\mathrm{W}1}\Delta U_\mathrm{w} + \boldsymbol{b}_{\mathrm{W}2}\Delta \varphi_\mathrm{w} \\ \begin{bmatrix} \Delta P_\mathrm{w} \\ \Delta Q_\mathrm{w} \end{bmatrix} &= \boldsymbol{C}_\mathrm{W}\Delta \boldsymbol{X}_\mathrm{W} + \boldsymbol{d}_{\mathrm{W}1}\Delta U_\mathrm{w} + \boldsymbol{d}_{\mathrm{W}2}\Delta \varphi_\mathrm{w}. \end{aligned} \tag{2}$$

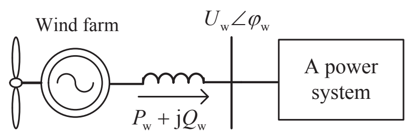

Fig. 4. Addition of a wind farm into a system.

Equation (1) is the linearized model of the power system where $\Delta \boldsymbol{X}_\mathrm{G}$ is the state variable vector of the synchronous generators (SGs). Equation (2) is the linearized model of the wind farm where $\Delta \boldsymbol{X}_\mathrm{W}$ is the state variable vector of the wind farm. Seeing the SGs as the open-loop plant and the wind farm as the feedback controller, the power system with the wind farm connected forms a closed-loop system, as shown in Fig. 5.

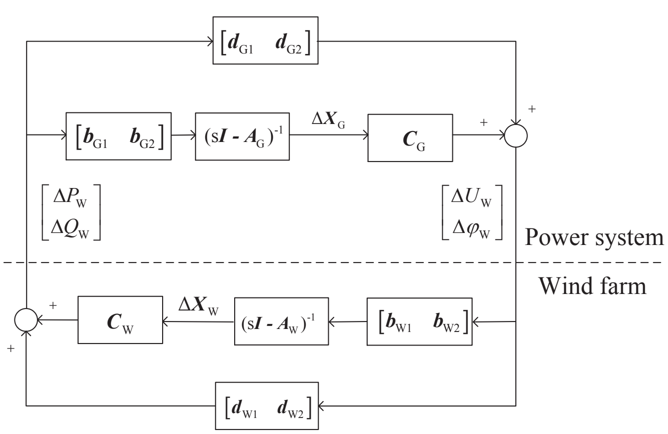

Fig. 5. Linearized model of the power system with the wind farm.

Fig. 5 describes the two-way dynamic interactions between the SGs and the wind farm. One way is the response of the wind farm to $\Delta U_\mathrm{w}\angle\Delta\varphi_\mathrm{w}$ to inject a variable power $\Delta P_\mathrm{w} + \mathrm{j}\Delta Q_\mathrm{w}$ into the power system. The other way is the response of the power system to $\Delta P_\mathrm{w} + \mathrm{j}\Delta Q_\mathrm{w}$ to generate a variation of the voltage at the PCC of the wind farm $\Delta U_\mathrm{w}\angle\Delta\varphi_\mathrm{w}$ during the electromechanical transient of the power system. If $\Delta P_\mathrm{w} + \mathrm{j}\Delta Q_\mathrm{w} = 0$, the wind farm is decoupled dynamically with the power system and there is no dynamic interaction between the wind farm and the power system. This is the case where the effect of the dynamic interaction of the wind farm with the power system on system small-signal angular stability is zero. In this assumed case, the influence of the wind farm on the system small-signal angular stability only exists in the way of the load flow and system configuration change it introduces. This influence is represented in the state matrix of the open-loop plant $\boldsymbol{A}_\mathrm{G}$ and can be determined by the modal computation of $\boldsymbol{A}_\mathrm{G}$. Since $\Delta P_\mathrm{w} + \mathrm{j}\Delta Q_\mathrm{w} = 0$ means that the wind farm is degraded into a constant power source, electromechanical oscillation modes computed from $\boldsymbol{A}_\mathrm{G}$ must have included the effect of the load flow and change of system configuration brought about by the wind farm on the small-signal angular stability.

From (1) and (2), the state-space model of the closed-loop system in Fig. 5 can be obtained to be

$$\begin{bmatrix} s\Delta \boldsymbol{X}_\mathrm{G} \\ s\Delta \boldsymbol{X}_\mathrm{W} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{X}_\mathrm{G} \\ \Delta \boldsymbol{X}_\mathrm{W} \end{bmatrix} = \boldsymbol{A}_{\mathrm{G+W}} \begin{bmatrix} \Delta \boldsymbol{X}_\mathrm{G} \\ \Delta \boldsymbol{X}_\mathrm{W} \end{bmatrix}, \tag{3}$$

where

$$\begin{aligned} \boldsymbol{A}_{11} =& \boldsymbol{A}_\mathrm{G} + [\boldsymbol{b}_{\mathrm{G}1} \ \boldsymbol{b}_{\mathrm{G}2}]\left(\boldsymbol{I} - [\boldsymbol{d}_{\mathrm{W}1} \ \boldsymbol{d}_{\mathrm{W}2}][\boldsymbol{d}_{\mathrm{G}1} \ \boldsymbol{d}_{\mathrm{G}2}]\right)^{-1} \\ & [\boldsymbol{d}_{\mathrm{W}1} \ \boldsymbol{d}_{\mathrm{W}2}]\boldsymbol{C}_\mathrm{G} \\ \boldsymbol{A}_{12} =& [\boldsymbol{b}_{\mathrm{G}1} \ \boldsymbol{b}_{\mathrm{G}2}]\left(\boldsymbol{I} - [\boldsymbol{d}_{\mathrm{W}1} \ \boldsymbol{d}_{\mathrm{W}2}][\boldsymbol{d}_{\mathrm{G}1} \ \boldsymbol{d}_{\mathrm{G}2}]\right)^{-1}\boldsymbol{C}_\mathrm{W} \\ \boldsymbol{A}_{21} =& [\boldsymbol{b}_{\mathrm{W}1} \ \boldsymbol{b}_{\mathrm{W}2}]\boldsymbol{C}_\mathrm{G} + [\boldsymbol{b}_{\mathrm{W}1} \ \boldsymbol{b}_{\mathrm{W}2}][\boldsymbol{d}_{\mathrm{G}1} \ \boldsymbol{d}_{\mathrm{G}2}] \\ & \left(\boldsymbol{I} - [\boldsymbol{d}_{\mathrm{W}1} \ \boldsymbol{d}_{\mathrm{W}2}][\boldsymbol{d}_{\mathrm{G}1} \ \boldsymbol{d}_{\mathrm{G}2}]\right)^{-1}[\boldsymbol{d}_{\mathrm{W}1} \ \boldsymbol{d}_{\mathrm{W}2}]\boldsymbol{C}_\mathrm{G} \\ \boldsymbol{A}_{22} =& \boldsymbol{A}_\mathrm{W} + [\boldsymbol{b}_{\mathrm{W}1} \ \boldsymbol{b}_{\mathrm{W}2}][\boldsymbol{d}_{\mathrm{G}1} \ \boldsymbol{d}_{\mathrm{G}2}] \\ & \left(\boldsymbol{I} - [\boldsymbol{d}_{\mathrm{W}1} \ \boldsymbol{d}_{\mathrm{W}2}][\boldsymbol{d}_{\mathrm{G}1} \ \boldsymbol{d}_{\mathrm{G}2}]\right)^{-1}\boldsymbol{C}_\mathrm{W}. \end{aligned}$$

The electromechanical oscillation modes computed from $\boldsymbol{A}_{\mathrm{G+W}}$ includes the effect of the load flow and change of system configuration introduced by the wind farm and the dynamic interaction between the wind farm and the power system.

Therefore, modal analysis to examine the effect of adding the wind farm into the power system on the system small-signal angular stability can be carried out in two steps as follows.

Step 1: Model the wind farm as a constant power and establish the linearized model of the power system of Fig. 4 in the form of (1) and (2) and compute the electromechanical oscillation modes of the power system from the open-loop state matrix $\boldsymbol{A}_\mathrm{G}$ as $\bar{\lambda}_{0i}, i = 1, 2, \cdots$.

Step 2: Derive the closed-loop state-space model of the power system of (3) and compute the electromechanical oscillation modes of the power system from the closed-loop state matrix $\boldsymbol{A}_{\mathrm{G+W}}$ as $\bar{\lambda}_i, i = 1, 2, \cdots$.

Thus $\bar{\lambda}_{0i}, i = 1, 2, \cdots$ indicates the effect of the load flow and system configuration change introduced by the wind farm on the system small-signal angular stability. $\Delta\bar{\lambda}_i = \bar{\lambda}_i - \bar{\lambda}_{0i}, i = 1, 2, \cdots$ is the effect of the dynamic interaction between the wind farm and the power system on the system small-signal angular stability. Two affecting factors of adding the wind farm into the power system on the system small-signal angular stability are successfully separated.

B. Wind Displacing Synchronous Generators

Fig. 6 shows a power system where a synchronous generator (SG) is displaced by a wind farm represented by a VSWG. The power injection from the SG to the power system is denoted by $P_{\mathrm{g}} + \mathrm{j}Q_{\mathrm{g}}$, the magnitude and phase of the terminal voltage by U_{g} and φ_{g} respectively. The following linearized state-space model of the power system can be established:

$$s\Delta\boldsymbol{X}_{\mathrm{G-1}} = \boldsymbol{A}_{\mathrm{G-1}}\Delta\boldsymbol{X}_{\mathrm{G-1}} + \boldsymbol{b}_{(\mathrm{G-1})1}\Delta P_{\mathrm{g}} + \boldsymbol{b}_{(\mathrm{G-1})2}\Delta Q_{\mathrm{g}}$$
$$\begin{bmatrix}\Delta U_{\mathrm{g}}\\ \Delta\varphi_{\mathrm{g}}\end{bmatrix} = \boldsymbol{C}_{\mathrm{G-1}}\Delta\boldsymbol{X}_{\mathrm{G-1}} + \boldsymbol{d}_{(\mathrm{G-1})1}\Delta P_{\mathrm{g}} + \boldsymbol{d}_{(\mathrm{G-1})2}\Delta Q_{\mathrm{g}}, \tag{4}$$

$$\begin{aligned} s\Delta\boldsymbol{X}_{\mathrm{g}} &= \boldsymbol{A}_{\mathrm{g}}\Delta\boldsymbol{X}_{\mathrm{g}} + \boldsymbol{b}_{\mathrm{g}1}\Delta U_{\mathrm{g}} + \boldsymbol{b}_{\mathrm{g}2}\Delta\varphi_{\mathrm{g}} \\ \begin{bmatrix}\Delta P_{\mathrm{g}}\\ \Delta Q_{\mathrm{g}}\end{bmatrix} &= \boldsymbol{C}_{\mathrm{g}}\Delta\boldsymbol{X}_{\mathrm{g}} + \boldsymbol{d}_{\mathrm{g}1}\Delta U_{\mathrm{g}} + \boldsymbol{d}_{\mathrm{g}2}\Delta\varphi_{\mathrm{g}}. \end{aligned} \tag{5}$$

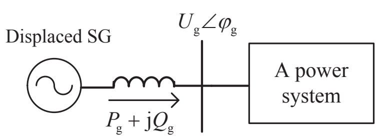

Fig. 6. A VSWG displacing a synchronous generator in a power system.

Eq. (4) is the linearized model of the power system without the displaced SG, and $\Delta\boldsymbol{X}_{\mathrm{G-1}}$ is the state variable vector of the SGs remaining in the system. Eq. (5) is the linearized model, and $\Delta\boldsymbol{X}_{\mathrm{g}}$ is the state variable vector of the displaced SG.

As stated above regarding the case of adding a wind farm into a power system, the electromechanical oscillation modes computed from the state matrix $\boldsymbol{A}_{\mathrm{G-1}}$ should include the effect of the load flow and system configuration change introduced by the displaced SG on the system small-signal angular stability. Similar to the derivation of Eq. (3), the closed-loop state-space model of the power system with the displaced SG remaining in the system can be established and the closed-loop state matrix $\boldsymbol{A}_{(\mathrm{G-1})+\mathrm{g}}$ can be obtained. The electromechanical oscillation modes computed from $\boldsymbol{A}_{(\mathrm{G-1})+\mathrm{g}}$ should include the influence of the displaced SG on the system small-signal angular stability from both the load flow and system configuration change introduced by the displaced SG and the dynamic interaction between the displaced SG and the rest of the SGs in the power system.

With the displaced SG being displaced by a wind farm, the linearized model of the power system becomes the combination of Eq. (2) and (4), that is

$$s\Delta\boldsymbol{X}_{\mathrm{G-1}} = \boldsymbol{A}_{\mathrm{G-1}}\Delta\boldsymbol{X}_{\mathrm{G-1}} + \boldsymbol{b}_{(\mathrm{G-1})1}\Delta P_{\mathrm{g}} + \boldsymbol{b}_{(\mathrm{G-1})2}\Delta Q_{\mathrm{g}}$$
$$\begin{bmatrix}\Delta U_{\mathrm{g}}\\ \Delta\varphi_{\mathrm{g}}\end{bmatrix} = \boldsymbol{C}_{\mathrm{G-1}}\Delta\boldsymbol{X}_{\mathrm{G-1}} + \boldsymbol{d}_{(\mathrm{G-1})1}\Delta P_{\mathrm{g}} + \boldsymbol{d}_{(\mathrm{G-1})2}\Delta Q_{\mathrm{g}}, \tag{6}$$

$$\begin{aligned} s\Delta\boldsymbol{X}_{\mathrm{W}} &= \boldsymbol{A}_{\mathrm{g}}\Delta\boldsymbol{X}_{\mathrm{W}} + \boldsymbol{b}_{\mathrm{W}1}\Delta U_{\mathrm{w}} + \boldsymbol{b}_{\mathrm{W}2}\Delta\varphi_{\mathrm{w}} \\ \begin{bmatrix}\Delta P_{\mathrm{w}}\\ \Delta Q_{\mathrm{w}}\end{bmatrix} &= \boldsymbol{C}_{\mathrm{W}}\Delta\boldsymbol{X}_{\mathrm{W}} + \boldsymbol{d}_{\mathrm{W}1}\Delta U_{\mathrm{w}} + \boldsymbol{d}_{\mathrm{W}2}\Delta\varphi_{\mathrm{w}}, \end{aligned} \tag{7}$$

where

$$\begin{bmatrix}\Delta P_{\mathrm{w}}\\ \Delta Q_{\mathrm{w}}\end{bmatrix} = \begin{bmatrix}\Delta P_{\mathrm{g}}\\ \Delta Q_{\mathrm{g}}\end{bmatrix}, \quad \begin{bmatrix}\Delta U_{\mathrm{w}}\\ \Delta\varphi_{\mathrm{w}}\end{bmatrix} = \begin{bmatrix}\Delta U_{\mathrm{g}}\\ \Delta\varphi_{\mathrm{g}}\end{bmatrix}. \tag{8}$$

Similar to the derivation of (3), the closed-loop system state-space model of the power system with the SG being displaced by the wind farm can be established. The closed-loop state matrix can be obtained to be $\boldsymbol{A}_{(\mathrm{G-1})+\mathrm{W}}$ from (6)–(8). Modal analysis to check the power system small-signal angular stability as affected by the wind generator displacing the SG can be conducted in three steps as follows.

Step 1: Model the displaced SG as a constant power and establish the linearized model of the power system with the displaced SG in the form of (4) and (5) and compute the electromechanical oscillation modes of the power system from the open-loop state matrix $\boldsymbol{A}_{\mathrm{G-1}}$ as $\bar{\lambda}_{0i}, i = 1, 2, \cdots$.

Step 2: Derive the closed-loop state-space model of the power system with the displaced SG remaining in the system from (4) and (5) and compute the electromechanical oscillation modes of the power system from the closed-loop state matrix $\boldsymbol{A}_{(\mathrm{G-1})+\mathrm{g}}$ as $\bar{\lambda}_{\mathrm{g}i}, i = 1, 2, \cdots$. Then $\Delta\bar{\lambda}_{\mathrm{g}i} = \bar{\lambda}_{\mathrm{g}i} - \bar{\lambda}_{0i}, i = 1, 2, \cdots$ gives the quantity of the effect of dynamic interaction between the displaced SG and the rest of the SGs on the system small-signal angular stability. The effect of withdrawing the dynamic interaction from the power system obviously is $-\Delta\bar{\lambda}_{\mathrm{g}i} = -\bar{\lambda}_{\mathrm{g}i} + \bar{\lambda}_{0i}, i = 1, 2, \cdots$.

Step 3: Derive the closed-loop state-space model of the power system with the displaced SG being displaced by the wind farm from (6), (7), and (8) and compute the electromechanical oscillation modes of the power system from the closed-loop state matrix $\boldsymbol{A}_{(\mathrm{G-1})+\mathrm{W}}$ as $\bar{\lambda}_{\mathrm{w}i}, i = 1, 2, \cdots$. Then $\Delta\bar{\lambda}_{\mathrm{w}i} = \bar{\lambda}_{\mathrm{w}i} - \bar{\lambda}_{0i}, i = 1, 2, \cdots$ indicates the range of the effect of dynamic interaction between the wind farm and the rest of the SGs on the system small-signal angular stability.

Displacing the SG by the wind farm does not change the load flow and system configuration. The effect of displacement on system small-signal angular stability includes two aspects: 1) withdrawing the dynamic interaction between the displaced SG and the rest of the SGs in the power system; 2) adding the dynamic interaction between the wind farm and the rest of the SGs. Thus the total effect of the displacement is

$$\begin{aligned} \Delta\bar{\lambda}_i &= -\Delta\bar{\lambda}_{\mathrm{g}i} + \Delta\bar{\lambda}_{\mathrm{w}i} = -\bar{\lambda}_{\mathrm{g}i} + \bar{\lambda}_{0i} + \bar{\lambda}_{\mathrm{w}i} - \bar{\lambda}_{0i} \\ &= \bar{\lambda}_{\mathrm{w}i} - \bar{\lambda}_{\mathrm{g}i}, i = 1, 2, \cdots. \end{aligned} \tag{9}$$

The conventional way of examining the effect of the VSWG displacing the SG is to compute the difference between the electromechanical oscillation modes before and after the displacement, that is, to obtain $\Delta\bar{\lambda}_i = \bar{\lambda}_{\mathrm{w}i} - \bar{\lambda}_{\mathrm{g}i}, i = 1, 2, \cdots$. The proposed approach above in three steps can successfully separate two factors that the system small-signal angular stability is affected by the displacement: 1) withdrawing the displaced SG, that is $-\Delta\bar{\lambda}_{\mathrm{g}i} = -\bar{\lambda}_{\mathrm{g}i} + \bar{\lambda}_{0i}, i = 1, 2, \cdots$; 2) adding the displacing wind farm, that is $\Delta\overline{\lambda}_{wi} = \overline{\lambda}_{wi} - \overline{\lambda}_{0i}, i = 1, 2, \cdots$.

III. Example

Fig. 7 shows the configuration of a New England 10-machine 39-bus example power system, which has been used for studying power system low-frequency oscillations on many occasions. Parameters of the system and generators provided in [9] are used.

A. Addition of a Wind Farm

A wind farm is represented by a DFIG wind generator. Its model is given in [10] and [11] and parameters are presented in the Appendix. When the wind farm is connected at node 16 without displacing any synchronous generators, the computational results of electromechanical oscillation modes of the power system are presented in Table I. The 1^{st} to the 8^{th} oscillation modes are local electromechanical oscillation modes and the 9^{th} is the inter-area mode of G_{10} to $G_1 - G_9$.

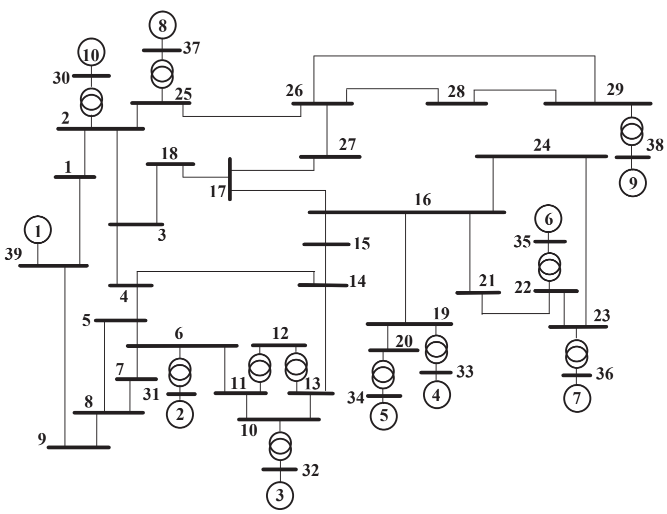

Fig. 7. Configuration of New England power system.

1) By comparing results in column B (without the wind farm added) and D (with the wind farm added) in Table I, it can be seen that the maximum impact of adding the wind farm is on the inter-area oscillation mode. The impact is detrimental to the system small-signal angular stability as the inter-area mode becomes less damped due to the addition of the wind farm with total change of the real part of the mode to be 0.0494.

2) From column E in Table I it can be seen that the impact of dynamic interaction between the wind farm and the system is also biggest on the inter-area oscillation mode. This impact benefits the system small-signal angular stability as the impact moves the inter-area mode further left by -0.0045 on the complex plane.

3) Hence the impact of load flow and system configuration change introduced by adding the wind farm moves the inter-area oscillation mode on the complex plane by 0.0494−0.0045=0.0449. It is detrimental to the system small-signal angular stability and much greater than that brought about by the dynamic interaction between the wind farm and the system.

TABLE I
COMPUTATIONAL RESULTS OF ELECTROMECHANICAL OSCILLATION MODES WHEN WIND FARM IS ADDED ON NODE 16 IN THE NEW ENGLAND POWER SYSTEM

A	B	C($\overline{\lambda}_{0i}$)	D($\overline{\lambda}_i$)	E($\Delta\overline{\lambda}_i$)	F
1	-0.4254 + 7.7704j	-0.4397 + 7.8457j	-0.4396 + 7.8457j	0.0001 + 0.0000j	-0.0143 + 0.0753j
2	-0.3796 + 7.7454j	-0.3889 + 7.7762j	-0.3891 + 7.7762j	-0.0002 + 0.0000j	-0.0093 + 0.0308j
3	-0.4034 + 7.6132j	-0.4024 + 7.6670j	-0.4020 + 7.6670j	0.0004 + 0.0000j	0.0010 + 0.0538j
4	-0.1546 + 6.4331j	-0.1589 + 6.4350j	-0.1588 + 6.4351j	0.0001 + 0.0001j	-0.0043 + 0.0019j
5	-0.2784 + 6.5730j	-0.2841 + 6.2278j	-0.2846 + 6.2286j	-0.0005 + 0.0008j	-0.0057 - 0.3452j
6	-0.2858 + 6.1251j	-0.2513 + 5.8822j	-0.2495 + 5.8811j	0.0018 - 0.0011j	0.0345 - 0.2429j
7	-0.2375 + 5.3408j	-0.2273 + 5.3448j	-0.2278 + 5.3450j	-0.0005 + 0.0002j	0.0102 + 0.0040j
8	-0.2184 + 5.6232j	-0.2155 + 4.4747j	-0.2154 + 4.4761j	0.0001 + 0.0014j	0.0029 - 1.1485j
9	-0.0592 + 3.2629j	-0.0098 + 3.1483j	-0.0143 + 3.1567j	-0.0045 + 0.0084j	0.0494 - 0.1146j

A: number of electromechanical oscillation modes;
B: oscillation modes without the wind farm added;
C: ($\overline{\lambda}_{0i}$): oscillation modes with the wind farm modelled as a constant power;
D: ($\overline{\lambda}_i$): oscillation modes with the full dynamics of the wind farm included;
E: ($\Delta\overline{\lambda}_i = \overline{\lambda}_i - \overline{\lambda}_{0i}$): effect of dynamic interaction between the wind farm and the SGs on the oscillation modes;
F(C-B): effect of system load flow and configuration change on the oscillation modes.

B. Wind Displacing a Synchronous Generator

The wind displacing a synchronous generator is examined by assuming that a wind farm is connected at node 35 to replace the synchronous generator G_6. The displacement does not change the original power flow and system configuration. Computational results of the electromechanical oscillation modes of the power system are presented in Table II. The 8^{st} oscillation mode is the inter-area electromechanical oscillation mode and the others are local ones.

1) Comparing column C with D it can be seen that the DFIG displacing G_6 introduces a mixture of beneficial and detrimental effect on the electromechanical oscillation modes as the real part of some modes becomes more negative and some more positive. The biggest impact is on the 3^{rd} local mode, which indicates a beneficial impact on the system small-signal angular stability. Further examination of the 3^{rd} mode is that the impact is mainly brought about by the withdrawal of the dynamic interaction between G_6 and other SGs in the system, and with the movement of the mode on the complex plane by -0.0996. Though the dynamic interaction between the DFIG and the rest of the SGs (without G_6) affects the mode positively with a movement by -0.0013, its effect is obviously much less

TABLE II
COMPUTATIONAL RESULTS OF ELECTROMECHANICAL OSCILLATION MODES WHEN WIND FARM IS ADDED ON NODE 16 IN THE NEW ENGLAND POWER SYSTEM

A	B($\overline{\lambda}_{0i}$)	C($\overline{\lambda}_{gi}$)	D($\overline{\lambda}_{wi}$)	E($-\Delta\overline{\lambda}_{gi}$)	F($\Delta\overline{\lambda}_{wi}$)
1	-0.3828 + 7.7404j	-0.3796 + 7.7454j	-0.3827 + 7.7405j	-0.0032 - 0.0050j	0.0001 + 0.0001j
2	-0.4336 + 7.7145j	-0.4034 + 7.6132j	-0.4345 + 7.7152j	-0.0302 + 0.1013j	-0.0009 + 0.0007j
3	-0.3750 + 6.8919j	-0.2784 + 6.5730j	-0.3763 + 6.8992j	-0.0966 + 0.3189j	-0.0013 + 0.0073j
4	-0.1614 + 6.3938j	-0.1546 + 6.4331j	-0.1606 + 6.3923j	-0.0068 - 0.0393j	0.0008 - 0.0015j
5	-0.2791 + 6.5721j	-0.2858 + 6.1251j	-0.2791 + 6.5722j	0.0067 + 0.4470j	0.0000 - 0.0001j
6	-0.2135 + 5.6320j	-0.2184 + 5.6232j	-0.2129 + 5.6315j	0.0049 + 0.0088j	0.0006 - 0.0005j
7	-0.2494 + 5.3431j	-0.2375 + 5.3408j	-0.2502 + 5.3436j	-0.0119 + 0.0023j	-0.0008 + 0.0005j
8	-0.0511 + 3.3394j	-0.0592 + 3.2629j	-0.0589 + 3.3477j	0.0081 + 0.0765j	-0.0078 + 0.0083j
9		-0.4254 + 7.7704j			

A: number of electromechanical oscillation modes;
B ($\overline{\lambda}_{0i}$): oscillation modes with G_6 modelled as a constant power;
C: ($\overline{\lambda}_{0i}$): oscillation modes with dynamic model of G_6 included;
D: ($\overline{\lambda}_i$): oscillation modes with G_6 displaced by the wind farm and dynamic model of wind farm being included;
E: ($-\Delta\overline{\lambda}_{gi} = \overline{\lambda}_{0i} - \overline{\lambda}_{gi}$): effect of withdrawing the dynamic interaction between G_6 and rest of the SGs on the oscillation modes;
F: ($\Delta\overline{\lambda}_{wi} = \overline{\lambda}_{wi} - \overline{\lambda}_{0i}$): effect of withdrawing the dynamic interaction between G_6 and rest of the SGs on the oscillation modes.

than that of the dynamic interaction between G_6 and the rest of the SGs.

2) It is very interesting to see from the 8^{th} row of Table II that the impact of withdrawing G_6 from the system is detrimental as far as the inter-area oscillation mode is concerned. While adding the wind farm is beneficial, scale of the impact from both aspects is almost equal and hence the total result is that the DFIG displacing G_6 is of small impact on the inter-area oscillation mode.

IV. CONCLUSION

The major contribution of this paper is the proposal of an approach for the separate examination of factors that grid connection of a large-scale wind farm affects the power system small-signal angular stability. When the wind farm is added to a power system, the total effect of the addition includes that from the change of load condition/system configuration and the dynamic interactions between the added wind farm and the synchronous generators in the power system. By using the proposed approach, those two aspects of effect can be examined separately. When the wind farm is connected to the power system by displacing a synchronous generator, the total effect of displacement is also in two aspects: 1) withdrawing the dynamic interactions between the displaced synchronous generator and the rest of the remaining synchronous generators in the power system; 2) adding the dynamic interactions between the wind farm and the rest of remaining synchronous generators in the power system. By using the proposed approach, those two aspects can be computed separately. Thus a clearer picture and better understanding is provided by using the proposed approach on the power system small-signal angular stability as affected by the grid connection of the wind farm. Application of the proposed approach is demonstrated by an example power system in the paper.

Modal analysis and computation has been used in examining the effect of grid connection of the large-scale wind farm on power system small-signal angular stability. However, all the methods proposed and results obtained so far can only provide the assessment of the total effect of the grid-connected wind farm. The main contribution of this paper is the proposed method based on a modal analysis that can separately examine individual effects brought about by the grid-connected wind farm for gaining deeper insight into the issue. The proposed method can be applied to the study case when multiple wind farms are considered. This is because the method is based on the linearized model of power systems and thus the principle of superimposition is applicable. It is expected that by applying the proposed method in practical power systems, useful conclusions can be obtained as the method can provide more detailed examination. For the same reason, the proposed method would also be helpful in the selection of grid connection locations of wind farms, an area of future work.

The effect of the grid connection of wind farms is an important research area. There are many significant issues that need to be addressed and investigated including 1) the possibility that the converter-controlled wind farms may introduce new oscillation modes, and studying their interaction with conventional electromechanical oscillation modes, along with the elimination of detrimental interactions; 2) removal of the harmful effect of grid-connected wind farms by applying power system damping control. These are also future topics of research planned by the authors.

Power system small-signal angular stability is a long-standing, challenging, and complex problem. Many aspects of the problem have not been completely understood and solved, such as the numerical difficulty when the scale of a power system is extremely large. This is also an obstacle when the proposed method is applied, and again represents a future area of research.

APPENDIX

A. Parameters of the Wind Farm Used in the Example Power System

Dynamic model of a DFIG can be written as [10], [11]:

$$pE'_{\mathrm{wd}} = \omega_0\left(-\frac{R_{\mathrm{r}}}{X_{\mathrm{rr}}}E'_{\mathrm{wd}} + sE'_{\mathrm{wq}} + \frac{R_{\mathrm{r}}X_{\mathrm{m}}^2}{X_{\mathrm{rr}}^2}I_{\mathrm{sq}} - \frac{X_{\mathrm{m}}}{X_{\mathrm{rr}}}U_{\mathrm{rq}}\right)$$
$$pE'_{\mathrm{wq}} = \omega_0\left(-\frac{R_{\mathrm{r}}}{X_{\mathrm{rr}}}E'_{\mathrm{wq}} - sE'_{\mathrm{wd}} - \frac{R_{\mathrm{r}}X_{\mathrm{m}}^2}{X_{\mathrm{rr}}^2}I_{\mathrm{sd}} + \frac{X_{\mathrm{m}}}{X_{\mathrm{rr}}}U_{\mathrm{rd}}\right)$$
$$ps = \frac{1}{J}(T_{\mathrm{e}} - T_{\mathrm{wm}}).$$

Configuration of rotor-side converter control system [10], [11] is shown by Fig. A1.

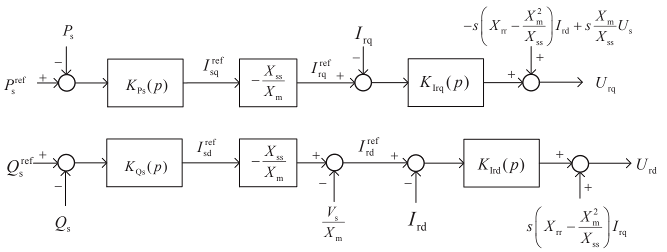

Fig. AI. Configuration of rotor-side converter control system.

Parameters of the DFIG used in the example in p.u. are as follows:

$T_{\rm J}=8s, R_{\rm s}=0, R_{\rm r}=0.0145, X_{\rm m}=2.4012, X_{\rm s}=0.1784, X_{\rm r}=0.1225$

$$K_{\rm Ps}(p)=0.2+\frac{12.56}{p}, K_{\rm Qs}(p)=0.2+\frac{12.56}{p}$$
$$K_{\rm Ird}(p)=1+\frac{62.5}{p}, K_{\rm Irq}(p)=1+\frac{62.5}{p}.$$

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