Nonfragile Quantitative Prescribed Performance Control of Waverider Vehicles With Actuator Saturation

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The existing prescribed performance control (PPC) strategies exhibit the fragility and nonguarantee of the prescribed performance when they are applied to dynamic systems with actuator saturation, and moreover, all of them are unable to quantitatively design prescribed performance. This article aims at remedying those deficiencies by proposing a new nonfragile PPC method for waverider vehicles (WVs) such that the quantitative prescribed performance can be guaranteed for tracking errors in the presence of actuator saturation. First, readjusting performance functions are developed to achieve quantitative prescribed performance and prevent the fragile problem. Then, low-complexity fuzzy neural control protocols are presented for velocity subsystem and altitude subsystem of WVs, while there is no need of recursive back-stepping design. Furthermore, auxiliary systems are designed to generate effective compensations on control constraints, which contributes to the guarantee of the desired prescribed performance, being proved via Lyapunov synthese. Finally, compared simulation results are given to validate the superiority.

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I. INTRODUCTION

Recently, considerable efforts have been made in the research field of waverider vehicles (WVs) for the sake of providing a technology basis for the development of hypersonic flight in near-space [1]–[3]. One of the critical issues in the development of WVs is represented by the flight control system design that poses challenging problems owing to the unique features caused by airframe-integrated scramjet engine, hypersonic flight speed, extreme flight conditions, and large flight envelope. Hence, an emphasis should be placed on the requirement for adequate control and performance indices including transient performance and steady-state performance as well as the adequate compromise between them [4]–[6].

The prescribed performance control (PPC), first proposed by Bechlioulis *et al.* [7], provides a suitable tool for pursuing both transient performance (i.e., overshoot and convergence time) and steady-state performance (i.e., steady-state tracking error) for control systems. By the prescribed performance, it means that the tracking error $e(t)$ should always evolve within a prescribed funnel "$-\rho(t) < e(t) < \rho(t)$" to satisfy some transient and steady-state specifications [8]–[12], where $\rho(t) : \Re_{\geq 0} \rightarrow \Re_{>0}$ is a performance function [13]. On this basis, a transformed error $\varepsilon(t) \in \Re$ is further defined by equivalently transforming the "constrained" system "$-\rho(t) < e(t) < \rho(t)$" into an "unconstrained" one. By developing an appropriate formulation for $\rho(t)$, $e(t)$ will exhibit a satisfactory prescribed performance including the transient performance and the steady-state performance. It has been proved that the guarantee of prescribed performance (i.e., $-\rho(t) < e(t) < \rho(t)$) is equivalent to the boundedness of $\varepsilon(t)$. Consequently, the transformed error $\varepsilon(t)$, instead of the initial tracking error $e(t)$, is used for feedback control development. Finally, the design objective becomes to stabilize $\varepsilon(t)$ via Lyapunov synthese.

In recent years, intense interests have been shown in the investigations of PPC, and considerable research works have been made to extend the applications of PPC [7]–[15]. The main concerned issue focuses on the formulation of performance function, which will be used to construct the prescribed funnel. Up to now, several new types of performance functions have been proposed in order to handle the inherent deficiencies such as singular problem, too large overshoot, and unfixed convergence time. In [8], the proposed PPC strategy did not require the accurate value of initial tracking error, which is difficult to obtain in practice, by constructing a new performance function whose initial funnel can be set to any size. As to this strategy, however, the overshoot of tracking error may be out of control. To tackle this problem, novel prescribed funnels, being able to guarantee tracking errors with small overshoots, were investigated and applied to hypersonic flight control [10]. Furthermore, a PPC scheme that guarantees finite-time convergence of control error was proposed for WVs [15], achieving an improvement, in comparison with some other existing PPC methodologies that the convergence time can be set as needed. Besides, other concerns are on reducing the complexity of PPC structure, and promoting the applications of PPC in various dynamic systems [6], [11]–[15].

Despite of the obtained improvements by the abovementioned PPC methods, there still exist some challenging problems that have not been mentioned in the existing literature. *The first defect of PPC is the fragile problem arising from actuator saturation*. As to WVs, large manipulations are needed for maneuvering flight in near space. However, the manipulation efficiency of control actuator declines significantly owing to the flight height. Thus, actuator saturation becomes a common problem [1], [3] for WVs. Actuator saturation will inevitably cause an increase in tracking error $e(t)$ such that $e(t) \rightarrow \rho(t)$ or $e(t) \rightarrow -\rho(t)$, resulting in the unboundedness of $\varepsilon(t)$ [14], [15]. In this case, the spurred prescribed performance cannot be guaranteed for $e(t)$ any more. Thereby, the current PPC shows an obvious fragility to actuator saturation. *Another weakness is represented by the nonguarantee of the prescribed performance when employing traditional approaches to compensate saturated actuators*. The existing compensated system developed in [16] is a common selection to deal with actuator saturation problem by using its states to modify the transformed error $\varepsilon(t)$. Although the convergence of modified error can be guaranteed via Lyapunov synthese, the boundedness of $\varepsilon(t)$ cannot be strictly proved. As a result, the spurred prescribed performance cannot be achieved under actuator saturation. *The third deficiency denotes the computational complexity stemming from online learning burden associated with fuzzy/neural-based PPC approaches* [17]–[21]. Fuzzy/neural approximations are common selections to reject unknown system dynamics by adaptively regulating online learning parameters. However, too many online learning/adaptive parameters result in computation burden and further harm the real-time performance of control system. This is not conducive to the realization of hypersonic and maneuver flight for WVs. Besides, it is also worthy to point out that few of the existing PPC methodologies can effectively guarantee tacking errors with quantitative prescribed performance (i.e., convergence time, overshoot, and steady-state error).

To sum up, it is the purpose of the proposed paper to consider the abovementioned deficiencies associated with the existing PPC, and to exploit a new nonfragile PPC scheme for WVs for the sake of quantitatively guaranteeing tracking errors with the desired prescribed performance under actuator saturation. The special contributions are summarized as follows:

1) A new type of performance functions, being able to actively readjust the constructed prescribed funnels according actuator saturation, are developed to tackle the fragile problem associated with the existing PPC strategies [13]–[15].
2) In comparison with traditional qualitative PPC schemes [4]–[7], in this article, an improvement is achieved such that improved prescribed performance, including convergence time, overshoot, and steady-state accuracy, can be quantitatively designed for tracking errors.
3) Auxiliary systems are constructed to solve the nonguarantee problem of the prescribed performance.
4) By exploiting low-computational algorithms for online learning parameters, a low-complexity design without back-stepping is adopted to devise fuzzy neural control protocols for velocity subsystem and altitude subsystem.

The rest of this article is outlined as follows. Section II presents vehicle model and preliminaries. The controller is developed in Section III. Simulation results are presented in Section IV. Finally, Section V concludes this article.

II. VEHICLE MODEL AND PRELIMINARIES

A. Motion Equations

The motion equations of a typical WV are given by [22]

$$m\dot{V} = T\cos\left(\theta-\gamma\right) - D - g\sin\gamma \tag{1}$$

$$\dot{h} = V\sin\gamma \tag{2}$$

$$mV\dot{\gamma} = L + T\sin\left(\theta-\gamma\right) - mg\cos\gamma \tag{3}$$

$$\dot{\theta} = Q \tag{4}$$

$$I_{yy}\dot{Q} = M + \tilde{\psi}_1\ddot{\eta}_1 + \tilde{\psi}_2\ddot{\eta}_2 \tag{5}$$

$$\ddot{\eta}_1 = -\frac{2\zeta_1\omega_1\dot{\eta}_1}{k_1} - \frac{\omega_1^2\eta_1}{k_1} + \frac{N_1}{k_1} - \frac{\tilde{\psi}_1}{k_1 I_{yy}}(M + \tilde{\psi}_2\ddot{\eta}_2) \tag{6}$$

$$\ddot{\eta}_2 = -\frac{2\zeta_2\omega_2\dot{\eta}_2}{k_2} - \frac{\omega_2^2\eta_2}{k_2} + \frac{N_2}{k_2} - \frac{\tilde{\psi}_2}{k_2 I_{yy}}(M + \tilde{\psi}_1\ddot{\eta}_1). \tag{7}$$

Equations (1)– (5) describe the rigid-body state motion, and (6) and (7) are the motion equations of the flexible states. Rigid-body states include the following: velocity $V \in \Re_{>0}$, altitude $h \in \Re_{>0}$, flight-path angle $\gamma \in \Re$, pitch angle $\theta \in \Re$, and pitch rate $Q \in \Re$. Flexible states are $\eta_1 \in \Re$ and $\eta_2 \in \Re$. The control inputs (fuel equivalence ratio $\Phi \in \Re_{>0}$ and elevator angular deflection $\delta_e \in \Re$) do not appear directly in the above equations, instead, they are implied in trust force T, drag force D, lift force L, pitching moment M, and generalized forces N_1 and N_2 [22], given by

$$\begin{cases} T \approx \beta_1\Phi\alpha^3 + \beta_2\alpha^3 + \beta_3\Phi\alpha^2 + \beta_4\alpha^2 \\ \quad + \beta_5\Phi\alpha + \beta_6\alpha + \beta_7\Phi + \beta_8 \\ D \approx \left(C_D^{\alpha^2}\alpha^2 + C_D^{\delta_e^2}\delta_e^2 + C_D^{\alpha}\alpha + C_D^{\delta_e}\delta_e + C_D^0\right) \\ \quad \times \frac{1}{2}\exp\left(\frac{h_0-h}{h_s}\right)\rho_0 V^2 S \\ L \approx \exp\left(\frac{h_0-h}{h_s}\right)\frac{\rho_0 V^2 S}{2}\left(C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^0\right) \\ M \approx \exp\left(\frac{h_0-h}{h_s}\right)\frac{\rho_0 V^2 S\bar{c}}{2}\begin{bmatrix} C_{M,\alpha}^0 + c_e\delta_e + \\ C_{M,\alpha}^{\alpha^2}\alpha^2 + C_{M,\alpha}^{\alpha}\alpha \end{bmatrix} \\ \quad + z_T T \\ N_1 = N_1^{\alpha^2}\alpha^2 + N_1^{\alpha}\alpha + N_1^0 \\ N_2 = N_2^{\alpha^2}\alpha^2 + N_2^{\alpha}\alpha + N_2^{\delta_e}\delta_e + N_2^0. \end{cases}$$

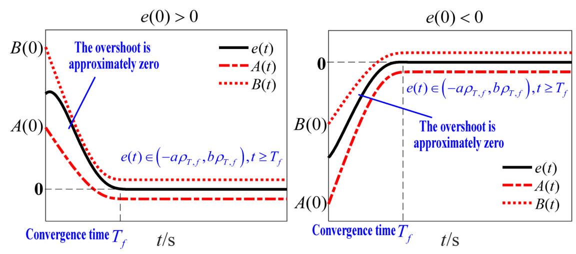

Fig. 1. Proposed prescribed funnels without actuator saturation.

B. Quantitative Prescribed Performance

By the quantitative prescribed performance, we mean that the transient performance (overshoot and convergence time) and the steady-state performance (steady-state error) of tracking error should be quantitatively set as needed. To achieve this objective, we propose the following new performance functions $A(t)$ and $B(t)$

$$A(t) = \begin{cases} \left(\frac{T_f - t}{T_f}\right)^p \left[\text{sign}(e(0)) - a\right](\rho_0 - \rho_{T,f}) \\ -a\rho_{T,f} - R, \text{ if } \ t \in \left[0, T_f\right) \\ -a\rho_{T,f} - R, \text{ if } \ t \in \left[T_f, \infty\right) \end{cases} \tag{8}$$

$$B(t) = \begin{cases} \left(\frac{T_f - t}{T_f}\right)^p \left[\text{sign}(e(0)) + b\right](\rho_0 - \rho_{T,f}) \\ +b\rho_{T,f} + R, \text{ if } \ t \in \left[0, T_f\right) \\ b\rho_{T,f} + R, \text{ if } \ t \in \left[T_f, \infty\right) \end{cases} \tag{9}$$

where $p > 1$, $T_f \in \Re_{>0}$, $\rho_0 (\in \Re_{>0}) > \rho_{T,f} (\in \Re_{>0})$, $a \in (0, 1)$, and $b \in (0, 1)$ are design parameters, and $R \in \Re_{\geq 0}$ is an readjusting term that will be designed to handle the fragile problem caused by actuator saturation.

The tracking error $e(t)$ should evolve within the prescribed funnel

$$A(t) < e(t) < B(t). \tag{10}$$

To promote the control design procedure, the "constrained" system (10) should be relaxed as an "unconstrained" one. Thus, we define a transformed error $\eta_e \in \Re$

$$\eta_e = \ln\left(\frac{e(t) - A(t)}{B(t) - e(t)}\right). \tag{11}$$

LEMMA 1: The boundedness of η_e is equivalent to that the tracking error $e(t)$ satisfies prescribed performance (10).

REMARK 1: The proof of Lemma 1 can be easily found in [15]. It is concluded from the boundedness of η_e that the tracking error $e(t)$ will evolve within the prescribed funnel "$A(t) < e(t) < B(t)$." If the actuator is saturated, the term R will adaptively increase $A(t)$ and decrease $B(t)$ to avoid the fragile problem. Otherwise, $R \equiv 0$. When the actuator is not saturated, the proposed prescribed performance is clearly shown in Fig. 1. It can be seen that the convergence time and the steady-state tracking error ($e(t) \in (-a\rho_{T,f}, b\rho_{T,f})$, if $t \geq T_f$) can be quantitatively designed. Moreover, the overshoot of $e(t)$ is approximately zero.

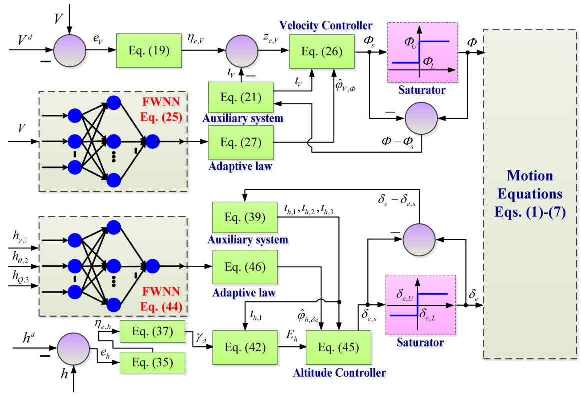

Fig. 2. Control structure diagram.

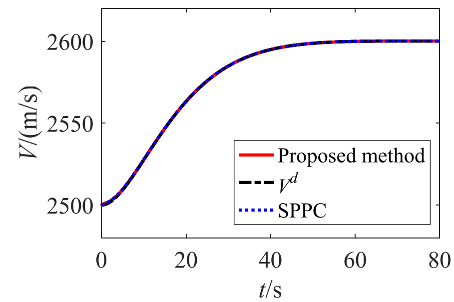

Fig. 3. Velocity tracking in Case 1.

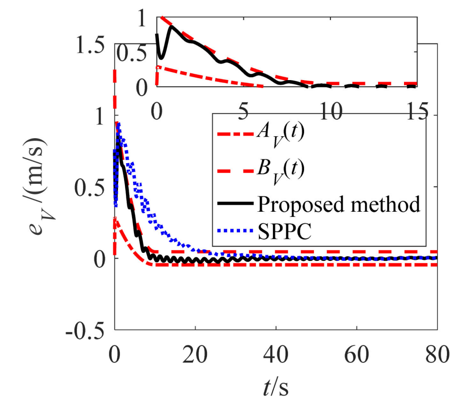

Fig. 4. Velocity tracking error in Case 1.

C. Fuzzy Neural Approximation

In the motion equations of WVs, there exists unknown dynamics, which restricts the feasibility of controller. Fuzzy systems and neural networks are two typical approximators with an excellent performance, and a combination of them, called fuzzy wavelet neural network (FWNN) [15], [23], [24], will be used in the article to approximate the unknown dynamics of WVs. FWNN has been proved to be an excellent approximator due to that the estimation performance of wavelet neural network is strengthened via fuzzy logics.

A typical FWNN, containing a singleton fuzzifier, product inference, and weighted average defuzzifier, can be used to approximate any continuously unknown function $F(x) : \Re^n \rightarrow \Re$

$$F(x) = U_F^T \gamma_F(x) + \varepsilon_F, |\varepsilon_F| \leq \varepsilon_F^M \tag{12}$$

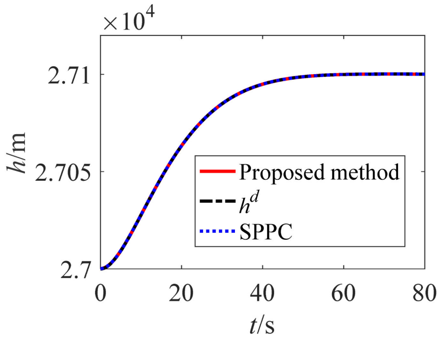

Fig. 5. Altitude tracking in Case 1.

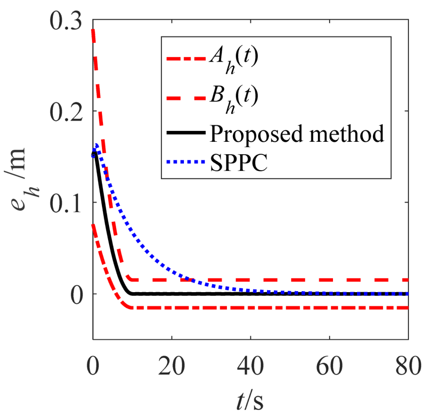

Fig. 6. Altitude tracking error in Case 1.

where $\varepsilon_F^M \in \Re_{\geq 0}$ is the upper bound of the approximation error $\varepsilon_F \in \Re, x = [x_1, x_2, \ldots, x_n]^T \in \Re^n$ means the input vector, $U_F = [u_F^1, u_F^2, \ldots, u_F^N]^T \in \Re^N$ denotes the weight matrix, $\gamma_F(x) = [\gamma_{F,1}(x), \gamma_{F,2}(x), \ldots, \gamma_{F,N}(x)]^T \in \Re^N$ is the basis function vector, and $\gamma_{F,j}(x), j = 1, 2, \ldots, N$ are given by

$$\gamma_{F,j}(x) = \frac{\prod_{i=1}^{n} g_{ji}(x_i)\varpi_j}{\sum_{j=1}^{N} \varpi_j}, i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, N \tag{13}$$

with $g_{ji}(x_i) = 1 - b_{ji}^2(x_i - c_{ji})^2$ and $\varpi_j = \prod_{i=1}^{n} \mu_{A_{ji}}(x_i)$, where $\mu_{A_{ji}}(x_i) = exp(-b_{ji}^2(x_i - c_{ji})^2)$ are membership functions of fuzzy logic, b_{ji} are dilation parameters, and c_{ji} are translation parameters.

D. Control Objective

The design objective is to devise velocity controller Φ and altitude controller δ_e based on fuzzy neural approximation for velocity subsystem (1) and altitude subsystem (2)–(5) under actuator saturation, such that V tracks its command $V^d \in \Re_{>0}$ and h tracks its command $h^d \in \Re_{>0}$. Meanwhile, the tracking errors satisfy the quantitative prescribed performance. Hereon, $V^d \in \Re_{>0}$, $h^d \in \Re_{>0}$, and their time derivatives are assumed to be bounded.

III. Controller Design

A. Velocity Controller

Velocity subsystem (1) can be rewritten as [14]

$$\dot{V} = f_{V,\Phi}(V, \Phi) = F_{V,\Phi} + k_\Phi \Phi \tag{14}$$

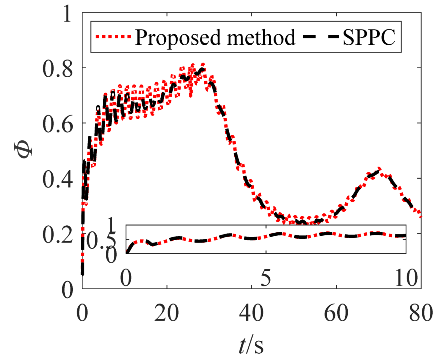

Fig. 7. Velocity control input in Case 1.

with $k_\Phi \in \Re_{>0}$ and $F_{V,\Phi} := f_{V,\Phi}(V, \Phi) - k_\Phi \Phi$, where $F_{V,\Phi}$ and $f_{V,\Phi}(V, \Phi)$ are continuously unknown functions.

Velocity control input Φ is assumed to be constrained by $\Phi = \mathrm{sat}(\Phi_s) : \Re \to [\Phi_L, \Phi_U]$, where $\Phi_L \in \Re_{>0}$ and $\Phi_U \in \Re_{>0}$ are the lower bound and the upper bound of Φ, that is, $\Phi \in [\Phi_L, \Phi_U]$, and Φ_s is the ideal value of Φ. $\mathrm{sat}(\Phi_s)$ is defined as

$$\mathrm{sat}(\Phi_s) = \begin{cases} \Phi_U, \Phi_s \in (\Phi_U, +\infty) \\ \Phi_s, \Phi_s \in [\Phi_L, \Phi_U] \\ \Phi_L, \Phi_s \in (-\infty, \Phi_L). \end{cases} \tag{15}$$

Velocity control objective: Devise Φ_s under (15) for (14) such that velocity tracking error $e_V = V - V^d$ satisfies the following prescribed performance

$$A_V(t) < e_V < B_V(t) \tag{16}$$

with

$$A_V(t) = \begin{cases} \left(\frac{T_{V,f}-t}{T_{V,f}}\right)^{p_V} [\mathrm{sign}(e_V(0)) - a_V] (\rho_{V,0} - \rho_{V,T,f}) \\ -a_V \rho_{V,T,f} - R_\Phi, \text{if } t \in [0, T_{V,f}) \\ -a_V \rho_{V,T,f} - R_\Phi, \text{if } t \in [T_{V,f}, \infty) \end{cases} \tag{17}$$

$$B_V(t) = \begin{cases} \left(\frac{T_{V,f}-t}{T_{V,f}}\right)^{p_V} [\mathrm{sign}(e_V(0)) + b_V] (\rho_{V,0} - \rho_{V,T,f}) \\ +b_V \rho_{V,T,f} + R_\Phi, \text{if } t \in [0, T_{V,f}) \\ b_V \rho_{V,T,f} + R_\Phi, \text{if } t \in [T_{V,f}, \infty) \end{cases} \tag{18}$$

where $p_V > 1$, $T_{V,f} \in \Re_{>0}$, $\rho_{V,0} (\in \Re_{>0}) > \rho_{V,T,f} (\in \Re_{>0})$, $a_V \in (0, 1)$, and $b_V \in (0, 1)$ are design parameters, $R_\Phi = \frac{r_\Phi |\Phi - \Phi_s|}{|\Phi - \Phi_s| + \delta_\Phi}$ is an readjusting term with $\delta_\Phi \in \Re_{>0}$ and $r_\Phi \in \Re_{>0}$.

Define transformed error $\eta_{e,V} \in \Re$

$$\eta_{e,V} = \ln\left(\frac{e_V - A_V(t)}{B_V(t) - e_V}\right). \tag{19}$$

Invoking (14) and (19), the time derivative of $\eta_{e,V}$ is given by

$$\begin{aligned} \dot{\eta}_{e,V} &= \sigma_V \dot{e}_V + \sigma_V \frac{\dot{A}_V(t) - \dot{B}_V(t)}{B_V(t) - A_V(t)} e_V \\ &+ \sigma_V \frac{\dot{B}_V(t) A_V(t) - B_V(t) \dot{A}_V(t)}{B_V(t) - A_V(t)} \\ &= \sigma_V \left(F_{V,\Phi} + k_\Phi \Phi - \dot{V}^d + \Delta_V\right) \end{aligned} \tag{20}$$

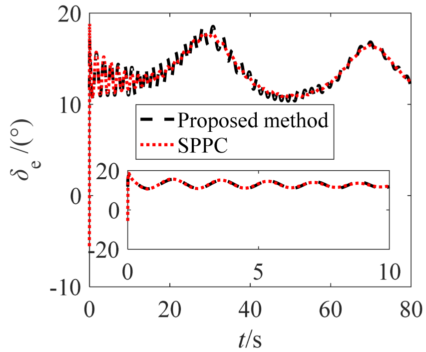

Fig. 8. Altitude control input in Case 1.

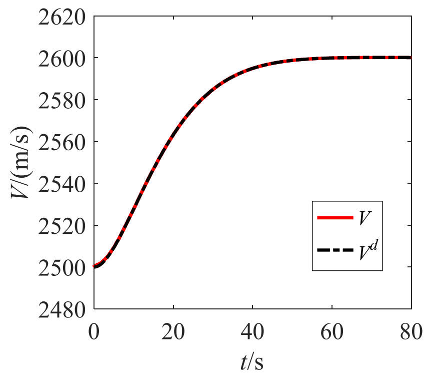

Fig. 9. Velocity tracking by the proposed method in Case 2.

with

$$\begin{cases} \sigma_V := \frac{B_V(t)-A_V(t)}{(B_V(t)-e_V)(e_V-A_V(t))} \\ \quad = \frac{1}{(1-S_V(\eta_{e,V}))S_V(\eta_{e,V})(B_V(t)-A_V(t))} > 0 \\ S_V(\eta_{e,V}) = \frac{e^{\eta_{e,V}}}{1+e^{\eta_{e,V}}} \in (0,1) \\ \Delta_V := \frac{\dot{A}_V(t)-\dot{B}_V(t)}{B_V(t)-A_V(t)} e_V + \frac{\dot{B}_V(t)A_V(t)-B_V(t)\dot{A}_V(t)}{B_V(t)-A_V(t)} \end{cases}.$$

REMARK 2: We note that $0 < 1 - S_V(\eta_{e,V}) < 1$ since $S_V(\eta_{e,V}) \in (0, 1)$. Furthermore, $B_V(t) - A_V(t)$ is positive and bounded. Finally, we conclude that σ_V is positive and bounded, that is, $0 < \sigma_{V,m} \leqslant \sigma_V \leqslant \sigma_{V,M} \in \Re_{>0}$, where $\sigma_{V,m}$ and $\sigma_{V,M}$ are constants.

To cope with the saturation on Φ, we give the following auxiliary system:

$$\dot{\iota}_V = -\frac{k_{V,\iota}\sigma_V \iota_V}{|\iota_V| + \delta_V} + k_\Phi \sigma_V(\Phi - \Phi_s) \tag{21}$$

where $\iota_V \in \Re$ is a system state, and $\delta_V \in \Re_{>0}$ and $k_{V,\iota} \in \Re_{>0}$ are design parameters.

THEOREM 1: The auxiliary system (21) is the bounded-input-bounded-state system, that is, if Φ_s is bounded, then ι_V also is bounded.

PROOF OF THEOREM 1: Define Lyapunov function $W_{\iota,V} = \iota_V^2/2$ whose time derivative is derived as by utilizing (21)

$$\begin{aligned} \dot{W}_{\iota,V} &= \iota_V \dot{\iota}_V \\ &\leq -\sigma_V \left(\frac{k_{V,\iota}|\iota_V|}{|\iota_V|+\delta_V} - k_\Phi |\Phi - \Phi_s| \right) |\iota_V|. \end{aligned} \tag{22}$$

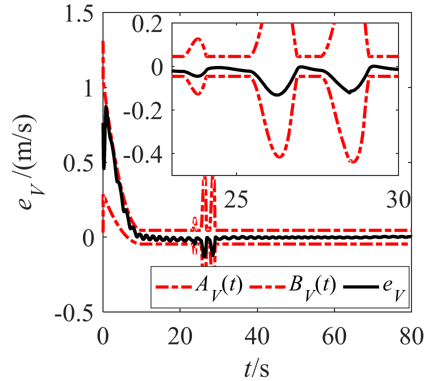

Fig. 10. Velocity tracking error by the proposed method in Case 2.

Let $k_{V,\iota} > k_{V,\iota}|\iota_V|/(|\iota_V| + \delta_V) > k_\Phi|\Phi - \Phi_s|$ and then we have $\dot{W}_{\iota,V} < 0$. Thereby, ι_V will keep bounded as long as $|\Phi - \Phi_s|$ does not diverge. Noting that $\Phi \in [\Phi_L, \Phi_U]$, this only requires that Φ_s is bounded.

Employing ι_V to modify $\eta_{e,V}$, we have

$$z_{e,V} = \eta_{e,V} - \iota_V. \tag{23}$$

Considering (20)–(23), we get

$$\begin{aligned} \dot{z}_{e,V} &= \dot{\eta}_{e,V} - \dot{\iota}_V \\ &= \sigma_V \left[k_\Phi \Phi_s + F_{V,\Phi} - \dot{V}^d + \Delta_V + \frac{k_{V,\iota}\iota_V}{|\iota_V|+\delta_V} \right]. \end{aligned} \tag{24}$$

In (24), the unknown function $F_{V,\Phi}$ is approximated by one FWNN

$$F_{V,\Phi} = U_{V,\Phi}^T \gamma_{V,\Phi}(V) + \varepsilon_{V,\Phi}, |\varepsilon_{V,\Phi}| \leq \varepsilon_{V,\Phi}^M \tag{25}$$

where $U_{V,\Phi} \in \Re^{N_V}$ is the weight vector, $\gamma_{V,\Phi}(V) \in \Re^{N_V}$ is the basis function vector, and $\varepsilon_{V,\Phi} \in \Re$ is the approximation error whose upper bound is $\varepsilon_{V,\Phi}^M \in \Re_{\geqslant 0}$.

We design the following control protocol for Φ_s

$$\begin{aligned} k_\Phi \Phi_s = &-l_{V,\Phi,1} z_{e,V} - l_{V,\Phi,2} \int_0^t z_{e,V} d\tau \\ &- \frac{1}{2} z_{e,V} \hat{\varphi}_{V,\Phi} \gamma_{V,\Phi}^T(V) \gamma_{V,\Phi}(V) \\ &+ \dot{V}^d - \Delta_V - k_{V,\iota}\iota_V/(|\iota_V| + \delta_V) \end{aligned} \tag{26}$$

where $l_{V,\Phi,1} \in \Re_{>0}$ and $l_{V,\Phi,2} \in \Re_{>0}$ are design parameters, and $\hat{\varphi}_{V,\Phi}$ is the estimation of $\varphi_{V,\Phi} = ||U_{V,\Phi}||^2$.

The adaptive law for $\hat{\varphi}_{V,\Phi}$ is developed as

$$\dot{\hat{\varphi}}_{V,\Phi} = \frac{\kappa_{V,\Phi}\sigma_V z_{e,V}^2 \gamma_{V,\Phi}^T(V)\gamma_{V,\Phi}(V)}{2} - 2\sigma_V \hat{\varphi}_{V,\Phi} \tag{27}$$

with $\kappa_{V,\Phi} \in \Re_{>0}$.

THEOREM 2: Consider the closed-loop system consisting of plant (14) with controller (26) and adaptive law (27). Then, all the signals involved in (30) are ultimately bounded, and e_V satisfies the prescribed performance (16).

PROOF OF THEOREM 2: Define

$$\tilde{\varphi}_{V,\Phi} = \hat{\varphi}_{V,\Phi} - \varphi_{V,\Phi}. \tag{28}$$

Substituting (25) and (26) into (24), we obtain

$$\begin{aligned}\frac{\dot{z}_{e,V}}{\sigma_V} &= -l_{V,\Phi,1} z_{e,V} - l_{V,\Phi,2}\int_0^t z_{e,V}\mathrm{d}\tau \\ &\quad -\tfrac{1}{2} z_{e,V}\hat{\varphi}_{V,\Phi}\gamma_{V,\Phi}^T(V)\gamma_{V,\Phi}(V) \\ &\quad +U_{V,\Phi}^T\gamma_{V,\Phi}(V) + \varepsilon_{V,\Phi}.\end{aligned} \tag{29}$$

Define Lyapunov function

$$W_V = z_{e,V}^2 + \frac{d_\sigma l_{V,\Phi,2}}{2}\left(\int_0^t z_{e,V}\mathrm{d}\tau\right)^2 + \frac{\tilde{\varphi}_{V,\Phi}^2}{2\kappa_{V,\Phi}}. \tag{30}$$

with

$$d_\sigma = \begin{cases}\sigma_{V,m}, \text{ if } \ z_{e,V}\int_0^t z_{e,V}\mathrm{d}\tau \ge 0 \\ \sigma_{V,M}, \text{ if } \ z_{e,V}\int_0^t z_{e,V}\mathrm{d}\tau < 0\end{cases}.$$

Taking time derivative along (30) and using (28) and (29), it leads to

$$\begin{aligned}\frac{\dot{W}_V}{\sigma_V} &= z_{e,V}\dot{z}_{e,V} + d_\sigma l_{V,\Phi,2} z_{e,V}\int_0^t z_{e,V}\mathrm{d}\tau + \frac{\tilde{\varphi}_{V,\Phi}\dot{\tilde{\varphi}}_{V,\Phi}}{\kappa_{V,\Phi}} \\ &\le -l_{V,\Phi,1} z_{e,V}^2 + z_{e,V}U_{V,\Phi}^T\gamma_{V,\Phi}(V) + z_{e,V}\varepsilon_{V,\Phi} \\ &\quad -\frac{z_{e,V}^2\varphi_{V,\Phi}\gamma_{V,\Phi}^T(V)\gamma_{V,\Phi}(V)}{2} - \frac{2\tilde{\varphi}_{V,\Phi}\hat{\varphi}_{V,\Phi}}{\kappa_{V,\Phi}}.\end{aligned} \tag{31}$$

Noticing that $-\frac{2\tilde{\varphi}_{V,\Phi}\hat{\varphi}_{V,\Phi}}{\kappa_{V,\Phi}} \le \frac{\varphi_{V,\Phi}^2}{\kappa_{V,\Phi}} - \frac{\tilde{\varphi}_{V,\Phi}^2}{\kappa_{V,\Phi}}, z_{e,V}U_{V,\Phi}^T\gamma_{V,\Phi}(V) \le \frac{1}{2}z_{e,V}^2\varphi_{V,\Phi}\gamma_{V,\Phi}^T(V)\gamma_{V,\Phi}(V) + \frac{1}{2}$, and $z_{e,V}\varepsilon_{V,\Phi} \le \frac{1}{2}z_{e,V}^2(\varepsilon_{V,\Phi}^M)^2 + \frac{1}{2}$, (31) becomes

$$\frac{\dot{W}_V}{\sigma_V} = -\left[l_{V,\Phi,1} - \frac{1}{2}\left(\varepsilon_{V,\Phi}^M\right)^2\right]z_{e,V}^2 - \frac{\tilde{\varphi}_{V,\Phi}^2}{\kappa_{V,\Phi}} + 1 + \frac{\varphi_{V,\Phi}^2}{\kappa_{V,\Phi}}. \tag{32}$$

Let $l_{V,\Phi,1} > \frac{1}{2}(\varepsilon_{V,\Phi}^M)^2$ and define compact sets

$$\Omega_z = \left\{ z_{e,V} \,\middle|\, \left|z_{e,V}\right| \le \sqrt{\frac{1+\varphi_{V,\Phi}^2/\kappa_{V,\Phi}}{l_{V,\Phi,1} - \left(\varepsilon_{V,\Phi}^M\right)^2/2}} \right\}$$

$$\Omega_{\tilde{\varphi},V} = \left\{ \tilde{\varphi}_{V,\Phi} \,\middle|\, \left|\tilde{\varphi}_{V,\Phi}\right| \le \sqrt{\left(1 + \frac{\varphi_{V,\Phi}^2}{\kappa_{V,\Phi}}\right)\Big/\frac{1}{\kappa_{V,\Phi}}} \right\}.$$

If $z_{e,V} \notin \Omega_z$ or $\tilde{\varphi}_{V,\Phi} \notin \Omega_{\tilde{\varphi},V}$, we know $\dot{W}_V < 0$. Thereby, $z_{e,V}$ and $\tilde{\varphi}_{V,\Phi}$ are ultimately bounded. We can further conclude that Φ_s is bounded based on (26). Hence, the input (i.e., $|\Phi - \Phi_s|$) of auxiliary system (21) also is bounded. Moreover, Theorem 1 has proved that auxiliary system (21) is a bounded-input-bounded-state system. Then, the transformed error $\eta_{e,V} = z_{e,V} + \iota_V$ also is bounded, and the desired prescribed performance can be guaranteed for e_V according to Lemma 1.

REMARK 3: The proposed auxiliary system provides a bounded compensation "$\frac{k_{V,\iota}\iota_V}{|\iota_V|+\delta_V}$" on Φ_s in (26). This guarantees prescribed performance in the presence of actuator saturation. However, the compensation strategy addressed in [16] cannot ensure the guarantee of the prescribed performance owing to the unboundedness of Φ_s.

B. Altitude Controller

Altitude control input $\delta_e \in \Re$ is assumed to be constrained by $\delta_e = sat(\delta_{e,s}) : \Re \to [\delta_{e,L}, \delta_{e,U}]$, where $\delta_{e,L} \in \Re$ and $\delta_{e,U} \in \Re$ are the lower bound and the upper bound of δ_e, that is, $\delta_e \in [\delta_{e,L}, \delta_{e,U}]$, and $\delta_{e,s}$ is the ideal value of δ_e. $sat(\delta_{e,s})$ is defined as

$$\operatorname{sat}\left(\delta_{e,s}\right) = \begin{cases}\delta_{e,U}, \delta_{e,s} \in \left(\delta_{e,U}, +\infty\right) \\ \delta_{e,s}, \delta_{e,s} \in \left[\delta_{e,L}, \delta_{e,U}\right] \\ \delta_{e,L}, \delta_{e,s} \in \left(-\infty, \delta_{e,L}\right).\end{cases} \tag{33}$$

Altitude control objective: Develop a control protocol for $\delta_{e,s}$ under (33) for altitude subsystem (2)–(5) such that altitude tracking error $e_h = h - h^d$ satisfies the following prescribed performance:

$$A_h(t) < e_h < B_h(t) \tag{34}$$

with

$$A_h(t) = \begin{cases}\left(\frac{T_{h,f}-t}{T_{h,f}}\right)^{p_h}\left[\operatorname{sign}(e_h(0)) - a_h\right](\rho_{h,0} - \rho_{h,T,f}) \\ -a_h\rho_{h,T,f} - R_{\delta e}, \text{if} t \in \left[0, T_{h,f}\right) \\ -a_h\rho_{h,T,f} - R_{\delta e}, \text{if} t \in \left[T_{h,f}, \infty\right)\end{cases}$$

$$B_h(t) = \begin{cases}\left(\frac{T_{h,f}-t}{T_{V,f}}\right)^{p_h}\left[\operatorname{sign}(e_h(0)) + b_h\right](\rho_{h,0} - \rho_{h,T,f}) \\ +b_h\rho_{h,T,f} + R_{\delta e}, \text{if} t \in \left[0, T_{h,f}\right) \\ b_h\rho_{h,T,f} + R_{\delta e}, \text{if} t \in \left[T_{h,f}, \infty\right)\end{cases}$$

where $p_h > 1$, $T_{h,f} \in \Re_{>0}$, $\rho_{h,0}(\in \Re_{>0}) > \rho_{h,T,f}(\in \Re_{>0})$, $a_h \in (0,1)$, and $b_h \in (0,1)$ are design parameters, $R_{\delta e} = \frac{r_{\delta e}|\delta_e - \delta_{e,s}|}{|\delta_e - \delta_{e,s}| + \delta_{\delta e}}$ is an readjusting term with $\delta_{\delta e} \in \Re_{>0}$ and $r_{\delta e} \in \Re_{>0}$.

Define transformed error $\eta_{e,h} \in \Re$

$$\eta_{e,h} = \ln\left(\frac{e_h - A_h(t)}{B_h(t) - e_h}\right). \tag{35}$$

Using (2) and (35), $\dot{\eta}_{e,h}$ is derived as

$$\begin{aligned}\dot{\eta}_{e,h} &= \sigma_h\left[\dot{e}_h + \frac{\dot{A}_h(t) - \dot{B}_h(t)}{B_h(t) - A_h(t)}e_h + \frac{\dot{B}_h(t)A_h(t) - B_h(t)\dot{A}_h(t)}{B_h(t) - A_h(t)}\right] \\ &= \sigma_h\left(V\sin\gamma - \dot{h}^d + \Delta_h\right)\end{aligned} \tag{36}$$

with

$$\begin{cases}\sigma_h := \frac{B_h(t) - A_h(t)}{(B_h(t) - e_h)(e_h - A_h(t))} \in \Re_{>0} \\ \Delta_h := \frac{\dot{A}_h(t) - \dot{B}_h(t)}{B_h(t) - A_h(t)}e_h + \frac{\dot{B}_h(t)A_h(t) - B_h(t)\dot{A}_h(t)}{B_h(t) - A_h(t)}.\end{cases}$$

The command of γ is chosen as

$$\gamma_d = \operatorname{asin}\left(\frac{-l_{h,\gamma}\eta_{e,h} + \dot{h}^d - \Delta_h}{V}\right) \tag{37}$$

with $l_{h,\gamma} \in \Re_{>0}$. If $\gamma \to \gamma_d$, we have $\eta_{e,h}\dot{\eta}_{e,h} = -l_{h,\gamma}\sigma_h\eta_{e,h}^2 \le 0$. Thus, $\eta_{e,h}$ is bounded and the spurred prescribed performance can be guaranteed according to Lemma 1.

Based on the obtained results [23], the rest part of altitude subsystem [i.e., (3)–(5)] can be transformed into the following equivalent system:

$$\begin{cases}\dot{h}_{\gamma,1} = h_{\theta,2} \\ \dot{h}_{\theta,2} = h_{Q,3} \\ \dot{h}_{Q,3} = F_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) + k_{\delta e}\delta_e\end{cases} \tag{38}$$

where $h_{\gamma,1} = \gamma$, $k_{\delta e} \in \Re_{>0}$, and $F_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) : \Re \times \Re \times \Re \to \Re$ is a continuously unknown function.

Then, the control objective becomes to devise $\delta_{e,s}$ for (38) such that $\gamma \to \gamma_d$.

To handle the saturation on δ_e, we design the following auxiliary system:

$$\begin{cases} \dot{\iota}_{h,1} = \iota_{h,2} \\ \dot{\iota}_{h,2} = \iota_{h,3} \\ \dot{\iota}_{h,3} = -\frac{k_{h,1}\iota_{h,1}}{|\iota_{h,1}|+\delta_h} - \frac{k_{h,2}\iota_{h,2}}{|\iota_{h,2}|+\delta_h} \\ \qquad -\frac{k_{h,3}\iota_{h,3}}{|\iota_{h,3}|+\delta_h} + k_{\delta e}\left(\delta_e - \delta_{e,s}\right) \end{cases} \tag{39}$$

where $\iota_{h,1} \in \Re$, $\iota_{h,2} \in \Re$, and $\iota_{h,3} \in \Re$ are system states, and $k_{h,1} \in \Re_{>0}, k_{h,2} \in \Re_{>0}, k_{h,3} \in \Re_{>0}$, and $\delta_h \in \Re_{>0}$ are design parameters.

THEOREM 3: The auxiliary system (39) is a bounded-input-bounded-state system, that is, if $\delta_{e,s}$ is bounded, then $\iota_{h,1}$, $\iota_{h,2}$, and $\iota_{h,3}$ are also bounded.

PROOF OF THEOREM 3: There must exist a neighborhood near the equilibrium point $\left(\frac{k_{\delta e}(\delta_e-\delta_{e,s})\delta_{h,1}}{k_{h,1}-k_{\delta e}(\delta_e-\delta_{e,s})}, 0, 0\right)$ such that $0 \le \iota_{h,1} \le \iota_{h,2}$ and $\iota_{h,3} \ge 0$.

Define Lyapunov function in the neighborhood defined earlier

$$\begin{aligned} W_{\iota,h} = k_{h,2}\int_0^{\iota_{h,1}} \frac{\tau_1}{|\tau_1|+\delta_h} d\tau_1 + \frac{\iota_{h,3}^2}{2} \\ +k_{h,2}\int_{h,1}^{\iota_{h,2}} \frac{\tau_1}{|\tau_1|+\delta_h} d\tau_1. \end{aligned} \tag{40}$$

Substituting (39) into (40) yields

$$\begin{aligned} \dot{W}_{\iota,h} &= k_{h,2}\frac{\iota_{h,2}}{|\iota_{h,2}|+\delta_h}\dot{\iota}_{h,2} + \iota_{h,3}\dot{\iota}_{h,3} \\ &= -\frac{k_{h,1}\iota_{h,3}\iota_{h,1}}{|\iota_{h,1}|+\delta_h} - \frac{k_{h,3}\iota_{h,3}\iota_{h,3}}{|\iota_{h,3}|+\delta_h} + k_{\delta e}\iota_{h,3}(\delta_e - \delta_{e,s}) \\ &\le -\begin{bmatrix} \frac{k_{h,1}\iota_{h,1}}{|\iota_{h,1}|+\delta_h} + \frac{k_{h,3}|\iota_{h,3}|}{|\iota_{h,3}|+\delta_h} - \\ k_{\delta e}\left(\delta_e - \delta_{e,s}\right) \end{bmatrix} |\iota_{h,3}|. \end{aligned} \tag{41}$$

Let $k_{h,3} > k_{h,3}\frac{|\iota_{h,3}|}{|\iota_{h,3}|+\delta_h} \ge -k_{\delta e}(\delta_e - \delta_{e,s})$. Then, we know $\dot{W}_{\iota,h} \le 0$. Only when $\iota_{h,3} = 0$, we have $\dot{W}_{\iota,h} = 0$. If $\iota_{h,2} \ne 0$ and $\iota_{h,1}$ does not satisfy "$\frac{k_{h,1}\iota_{h,1}}{|\iota_{h,1}|+\delta_h} + \frac{k_{h,3}\iota_{h,3}}{|\iota_{h,3}|+\delta_h} - k_{\delta e}(\delta_e - \delta_{e,s}) = 0$," then $\dot{\iota}_{h,3} \not\equiv 0$, that is, "$\iota_{h,3} = 0$" is unsustainable. Moreover, when $\iota_{h,1} \to \infty$, $\iota_{h,2} \to \infty$ and $\iota_{h,3} \to \infty$, we know $W_{\iota,h} \to \infty$. Therefore, based on the discussions of [25], we know that $\iota_{h,1} \in \Re$, $\iota_{h,2} \in \Re$, and $\iota_{h,3} \in \Re$ are globally bounded.

Define

$$E_h = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_{h,s}\right)^3 \int_0^t S_h \mathrm{d}\tau \tag{42}$$

with $\lambda_{h,s} \in \Re_{>0}$, $S_h = s_h - \iota_{h,1}$, and $s_h = h_{\gamma,1} - \gamma_d$. Since the polynomial $(s + \lambda_{h,s})^3$ is Hurwitz, we have that "E_h is bounded" $\Rightarrow$ "S_h is bounded."

Taking time derivative along (42) and substituting (38) and (39), we get

$$\begin{aligned} \dot{E}_h &= S_h^{(3)} + 3\lambda_{h,s}\ddot{S}_h + 3\lambda_{h,s}^2\dot{S}_h + \lambda_{h,s}^3 S_h \\ &= F_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) + k_{\delta e}\delta_{e,s} - \gamma_d^{(3)} \\ &\quad + \frac{k_{h,1}\iota_{h,1}}{|\iota_{h,1}|+\delta_h} + \frac{k_{h,2}\iota_{h,2}}{|\iota_{h,2}|+\delta_h} + \frac{k_{h,3}\iota_{h,3}}{|\iota_{h,3}|+\delta_h} \\ &\quad + 3\lambda_{h,s}\ddot{S}_h + 3\lambda_{h,s}^2\dot{S}_h + \lambda_{h,s}^3 S_h. \end{aligned} \tag{43}$$

The unknown function $F_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3})$ of (43) is approximated by one FWNN

$$\begin{aligned} F_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) = U_{h,\delta e}^T\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ +\varepsilon_{h,\delta e}, |\varepsilon_{h,\delta e}| \le \varepsilon_{h,\delta e}^M \end{aligned} \tag{44}$$

where $\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \in \Re^{\mathrm{N}_h}$ is a basis function vector, $U_{h,\delta e} \in \Re^{\mathrm{N}_h}$ is a weight vector, and $\varepsilon_{h,\delta e} \in \Re$ is the approximation error whose upper bound is $\varepsilon_{h,\delta e}^M \in \Re_{\ge 0}$.

The altitude controller is designed as

$$\begin{aligned} k_{\delta e}\delta_{es} = &-l_{h,\delta e}E_h + \gamma_d^{(3)} \\ &-\tfrac{1}{2}E_h\hat{\varphi}_{h,\delta e}\gamma_{h,\delta e}^T(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ &\times\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ &-\frac{k_{h,1}\iota_{h,1}}{|\iota_{h,1}|+\delta_h} - \frac{k_{h,2}\iota_{h,2}}{|\iota_{h,2}|+\delta_h} - \frac{k_{h,3}\iota_{h,3}}{|\iota_{h,3}|+\delta_h} \\ &-3\lambda_{h,s}\ddot{S}_h - 3\lambda_{h,s}^2\dot{S}_h - \lambda_{h,s}^3 S_h \end{aligned} \tag{45}$$

where $l_{h,\delta e} \in \Re_{>0}$ is a design parameter and $\hat{\varphi}_{h,\delta e}$ is the estimation of $\varphi_{h,\delta e} = ||U_{h,\delta e}||^2$.

The adaptive law for $\hat{\varphi}_{h,\delta e}$ is selected as

$$\begin{aligned} \dot{\hat{\varphi}}_{h,\delta e} = \tfrac{1}{2}\kappa_{h,\delta e}E_h^2\gamma_{h,\delta e}^T(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ \times \gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) - 2\hat{\varphi}_{h,\delta e} \end{aligned} \tag{46}$$

with $\kappa_{h,\delta e} \in \Re_{>0}$.

THEOREM 4: Consider the closed-loop system consisting of plant (38) with controller (45) and adaptive law (46). Then, all the signals involved in (49) are ultimately bounded, and e_h satisfies the prescribed performance (34).

PROOF OF THEOREM 4: Define

$$\tilde{\varphi}_{h,\delta e} = \hat{\varphi}_{h,\delta e} - \varphi_{h,\delta e}. \tag{47}$$

Substituting (44) and (45) into (43), we obtain

$$\begin{aligned} \dot{E}_h = &-l_{h,\delta e}E_h - \tfrac{1}{2}E_h\hat{\varphi}_{h,\delta e}\gamma_{h,\delta e}^T(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ &\times \gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ &+ U_{h,\delta e}^T\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) + \varepsilon_{h,\delta e}. \end{aligned} \tag{48}$$

Define Lyapunov function

$$W_h2 = \frac{1}{2}E_h^2 + \frac{\tilde{\varphi}_{h,\delta e}^2}{2\kappa_{h,\delta e}}. \tag{49}$$

Combining (48) with (49), it yields

$$\begin{aligned} \dot{W}_h &= E_h\dot{E}_h + \frac{\tilde{\varphi}_{h,\delta e}\dot{\hat{\varphi}}_{h,\delta e}}{\kappa_{h,\delta e}} \\ &= -l_{h,\delta e}E_h^2 - \frac{2\tilde{\varphi}_{h,\delta e}\hat{\varphi}_{h,\delta e}}{\kappa_{h,\delta e}} + E_h\varepsilon_{h,\delta e} \\ &\quad + E_hU_{h,\delta e}^T\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ &\quad - \tfrac{1}{2}E_h^2\varphi_{h,\delta e}\gamma_{h,\delta e}^T(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \\ &\quad \times \gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}). \end{aligned} \tag{50}$$

Since $E_hU_{h,\delta e}^T\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) \le \frac{1}{2}E_h^2\varphi_{h,\delta e} \times\gamma_{h,\delta e}^T(h_{\gamma,1}, h_{\theta,2}, h_{Q,3})\gamma_{h,\delta e}(h_{\gamma,1}, h_{\theta,2}, h_{Q,3}) + \frac{1}{2}$, $E_h\varepsilon_{h,\delta e} \le \frac{1}{2}E_h^2(\varepsilon_{h,\delta e}^M)^2 + \frac{1}{2}$ and $-\frac{2\tilde{\varphi}_{h,\delta e}\hat{\varphi}_{h,\delta e}}{\kappa_{h,\delta e}} \le \frac{\varphi_{h,\delta e}^2}{\kappa_{h,\delta e}} - \frac{\tilde{\varphi}_{h,\delta e}^2}{\kappa_{h,\delta e}}$, we have

$$\dot{W}_h = -\left(l_{h,\delta e} - \frac{1}{2}\left(\varepsilon_{h,\delta e}^M\right)^2\right)E_h^2 - \frac{\tilde{\varphi}_{h,\delta e}^2}{\kappa_{h,\delta e}} + \frac{\varphi_{h,\delta e}^2}{\kappa_{h,\delta e}} + 1. \tag{51}$$

Let $l_{h,\delta e} > \frac{1}{2}(\varepsilon_{h,\delta e}^M)^2$ and define compact sets

$$\Omega_E = \left\{ E_h \middle| |E_h| \leq \sqrt{\frac{\varphi_{h,\delta e}^2/\kappa_{h,\delta e}+1}{l_{h,\delta e}-\left(\varepsilon_{h,\delta e}^M\right)^2/2}} \right\}$$

$$\Omega_{\tilde{\varphi},h} = \left\{ \tilde{\varphi}_{h,\delta e} \middle| |\tilde{\varphi}_{h,\delta e}| \leq \sqrt{\left(\frac{\varphi_{h,\delta e}^2}{\kappa_{h,\delta e}}+1\right)/\frac{1}{\kappa_{h,\delta e}}} \right\}.$$

Once $E_h \notin \Omega_E$ or $\tilde{\varphi}_{h,\delta e} \notin \Omega_{\tilde{\varphi},h}$, we conclude $\dot{W}_h < 0$. Then, E_h and $\tilde{\varphi}_{\tilde{\varphi},h}$ are ultimately bounded. Due to the Hurwitz polynomial $(s+\lambda_{h,s})^3$, S_h also is bounded. We further have that δ_{es} is bounded according to (45). Hence, the input (i.e., $|\delta_e - \delta_{e,s}|$) of auxiliary system (39) is bounded. Furthermore, Theorem 3 shows that auxiliary system (39) is a bounded-input-bounded-state system. Then, $s_h = S_h + \iota_{h,1}$ is bounded, that is, $\gamma \to \gamma_d$. Finally, we conclude that the spurred prescribed performance is guaranteed for e_h according to Lemma 1.

REMARK 4: The existing compensation approach [16] will result in the nonguarantee problem of the prescribed performance due to the unbounded compensation. While such defect is effectively handled via developing the auxiliary systems (21) and (39).

REMARK 5: Different from the methodologies developed via back-stepping [13], [26], the design complexity of this study is reduced significantly owing to the avoidance of complex recursive back-stepping design. Moreover, for each subsystem, only one online learning parameter is needed for fuzzy neural approximation, as depicted in Fig. 2 . The computational burden is satisfactory.

REMARK 6: The existing PPC methodologies [7]–[15] are fragile to actuator saturation since they will encounter the control singular problem when actuator saturation causes tracking errors to increase and cross the prescribed funnels. In this article, the developed performance functions contain two readjusting terms "$R_\Phi = \frac{r_\Phi|\Phi-\Phi_s|}{|\Phi-\Phi_s|+\delta_\Phi}$" and "$R_{\delta e} = \frac{r_{\delta e}|\delta_e-\delta_{e,s}|}{|\delta_e-\delta_{e,s}|+\delta_{\delta e}}$". When actuators are not saturated (i.e., $\Phi - \Phi_s = 0$ and $\delta_e - \delta_{e,s} = 0$), we have $R_\Phi = 0$ and $R_{\delta e} = 0$. Otherwise, if actuators are saturated (i.e., $\Phi - \Phi_s \neq 0$ and $\delta_e - \delta_{e,s} \neq 0$), we know $R_\Phi > 0$ and $R_{\delta e} > 0$. In this case, R_Φ and $R_{\delta e}$ are able to play the role of increasing the upper prescribed funnels $B_V(t), B_h(t)$and decreasing the lower prescribed funnels $A_V(t), A_h(t)$ such that tracking errors are always within prescribed funnels to satisfy the spurred prescribed performance in the presence of actuator saturation. As a result, the fragile problem associated with PPC is tackled.

IV. SIMULATION STUDY

In this section, the proposed method is compared with an existing small-overshoot PPC (SPPC) [10], [27] strategy via numerical simulation to validate the effectiveness and advantage. Design parameters are chosen as follows: $k_\Phi = 4$, $p_V = 2, \rho_{V,0} = 2.5, \rho_{V,T,f} = 0.3, a_V = 0.5, b_V = 0.5, \delta_\Phi = 0.1, r_\Phi = 3, k_{V,\iota} = 1.3, \delta_V = 0.1, l_{V,\Phi,1} = 0.3, l_{V,\Phi,2} = 0.8$, $\kappa_{V,\Phi} = 0.05$, $p_h = 2$, $T_{h,f} = 10$, $\rho_{h,0} = 0.7$, $\rho_{h,T,f} = 0.1$,

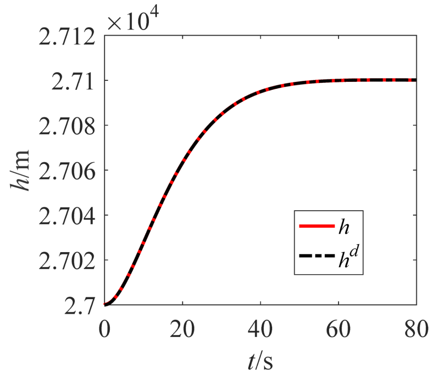

Fig. 11. Altitude tracking by the proposed method in Case 2.

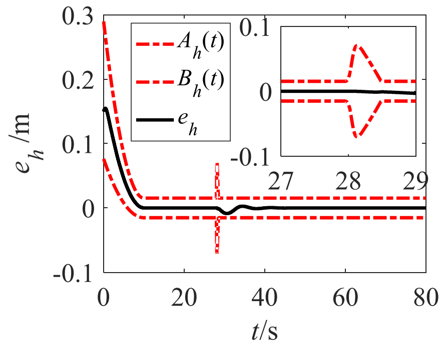

Fig. 12. Altitude tracking error by the proposed method in Case 2.

$a_h = 0.5, b_h = 0.5, \delta_{\delta e} = 0.5, \delta_{\delta e} = 50, l_{h,\gamma} = 2, k_{\delta e} = 0.9$, $k_{h,1} = 0.5, k_{h,2} = 1, k_{h,3} = 1, \delta_h = 0.8, \lambda_{h,s} = 80, l_{h,\delta e} = 7$, $\kappa_{h,\delta e} = 0.05$. The quantitative transient performance for e_V and e_h is: convergence time is not more than 10 s, and the overshoot is approximately zero. The steady-state value of e_V is not more than 0.05 m/s, and the steady-state value of e_h is less than 0.02 m.

Two different cases are considered. In Case 1, we suppose that the actuators are not constrained, and in Case 2, actuators are constrained by $\Phi \in [0.05, 0.8]$ and $\delta_e \in [0 \text{ deg}, 18.5 \text{ deg}]$.

As to Case 1, the compared simulation results are shown in Figs. 3–8. It can be seen from Figs. 3–6 that the spurred quantitative transient performance mentioned above is guaranteed for e_V and e_h. Moreover, the transient performance provided by the proposed method is better in comparison with SPPC, as shown in Figs. 4 and 6. Figs. 7 and 8 reveal that the control inputs of both methods are bounded and without high frequency chattering.

In case 2, the actuator saturation problem is considered, and the simulation results are presented in Figs. 9–21. Figs. 9–12 infer that the spurred prescribed performance still is guaranteed in the presence of actuator saturation (see Figs. 13 and 14) owing to timely compensations (see Figs. 15 and 16) and the readjusting prescribed performance (see Figs. 10 and 12). Hence, the fragile problem of PPC is effectively handled. Finally, the boundedness of transformed errors is shown in Fig. 17. Besides, we can clearly find the fragility defect of SPPC from Figs. 18–21. If actuators are saturated (see Fig. 20), the velocity tracking

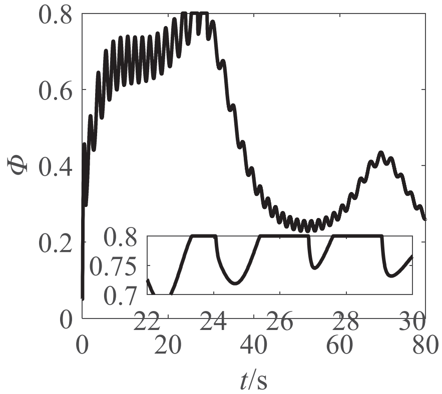

Fig. 13. Velocity control input by the proposed method in Case 2.

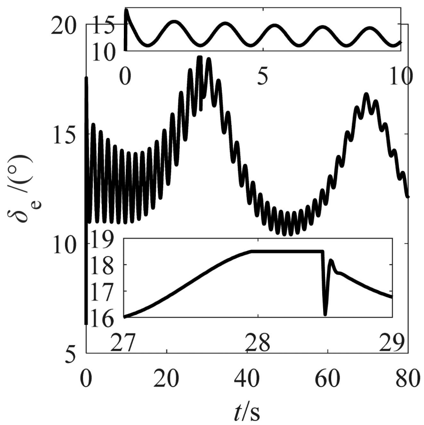

Fig. 14. Altitude control input by the proposed method in Case 2.

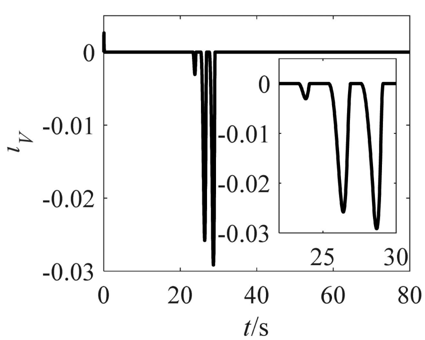

Fig. 15. The response of ι_V by the proposed method in Case 2.

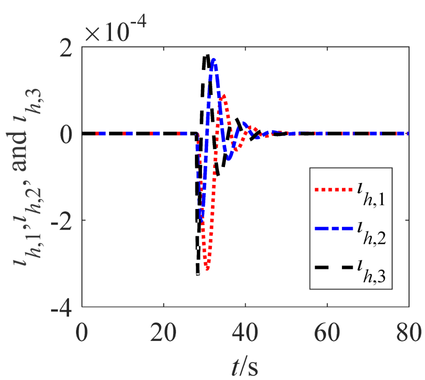

Fig. 16. The responses of $\iota_{h,1}$, $\iota_{h,2}$, and $\iota_{h,3}$ by the proposed method in Case 2.

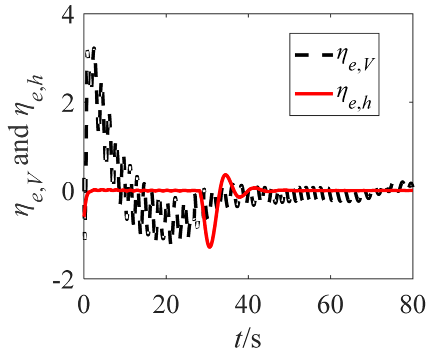

Fig. 17. Transformed errors by the proposed method.

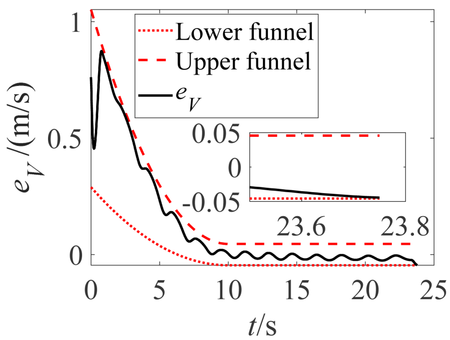

Fig. 18. Velocity tracking error by SPPC in case 2.

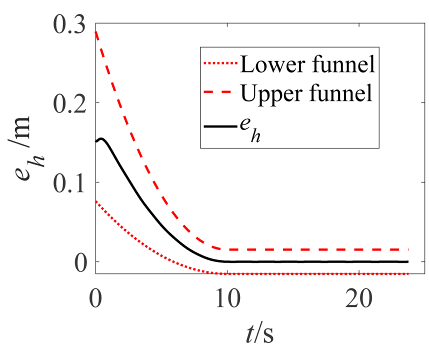

Fig. 19. Altitude tracking error by SPPC in case 2.

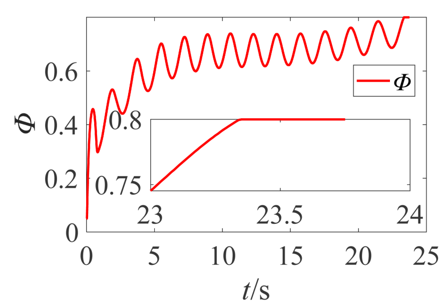

Fig. 20. Velocity control input by SPPC in case 2.

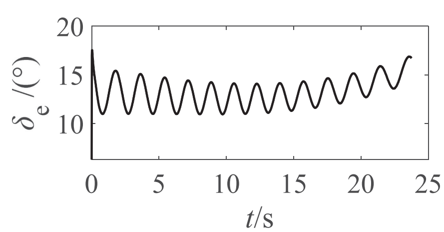

Fig. 21. Altitude control input by SPPC in case 2.

error tends to cross the prescribed funnel (see Fig. 18), resulting the control singular problem.

V. CONCLUSION

A nonfragile PPC approach is addressed for WVs subject to actuator saturation. A new type of performance functions containing readjusting terms are developed to guarantee tracking errors with quantitative prescribed performance. Fuzzy neural controllers with simple structure are devised for velocity subsystem and altitude subsystem, and they are further compensated by the designed auxiliary systems such that the problem of actuator saturation is handled. The guarantee of the spurred prescribed performance in the presence of actuator saturation is proved via Lyapunov synthese. Finally, the superiority is verified by the presented compared simulation results. Our future work will focus on overcoming the fragility of PPC to states constraints and actuator faults [28]–[30].

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