Utilization Management and Pricing of Parking Facilities Under Uncertain Demand and User Decisions

Amir Mirheli and Leila Hajibabai

***Abstract*— Excessive search for parking spots in congested areas contributes to additional travel delays and negative socio-economic impacts. While managing parking utilization to address the agency's objectives, it is often very beneficial to reflect the diversity of users' behaviors and their travel choices. This paper develops a stochastic dynamic parking management model, under competitive user-agency perceptions and uncertain user demand and parking occupancy, to simultaneously minimize the total travelers' costs and maximize the parking agency's revenue. The problem is formulated into a dynamic programming model and solved using a stochastic look-ahead technique based on the Monte Carlo tree search algorithm to determine optimal actions on parking price assignment and spot utilization over time. The numerical experiments on a hypothetical and empirical case study are conducted to show the performance of the proposed algorithm and to draw managerial insights. The results are compared with those of benchmark algorithms, which indicate that the proposed methodology can determine near-optimal solutions efficiently.**

***Index Terms*— Parking utilization, dynamic pricing, Monte Carlo tree search, look-ahead model, stochastic, bi-level, integer program.**

I. Introduction

RAPID development of individual modes of transportation in modern cities has caused insufficient solutions to urban mobility and yielded bolder congestion problems in downtown areas. A study on 11 major cities shows that cruising for each parking spot generates extra 4,927 vehicle-miles-traveled annually [43]. Besides, the additional congestion caused by cruising, wastes an average of 8.1 minutes per vehicle [44] and leads to even more local circling by 10% when spending 20 minutes search for an available parking spot [41]. It, thus, enforces excessive fuel consumption, i.e., burning extra 3.87 million gallons of gasoline [5], carbon emissions, as well as driver frustration. Therefore, parking and congestion problems are inter-related, sharing significant socio-economic impacts, especially, in congested urban areas. Smart parking strategies offer considerable savings in time, energy, and dollar amounts, which lead to improvements in the livability of modern societies. However, under- and over-utilized parking areas are the immediate consequences of inefficient parking spot management that significantly affect the parking industry, businesses, and people (e.g., parking users, general population) in terms of economy and health. For example, offering unreasonably-priced (i.e., too high or free/inexpensive) parking spots worsens parking utilization and generates additional congestion as users attempt to cruise to avoid expensive parking areas due to economic reasons. Similarly, distant parking lots from businesses or shopping centers encourages users to circle the neighboring streets to find closer parking spots. Consequently, effective parking management strategies shall reliably account for a number of parking agency- and user-related factors, including parking price, location, demand, and available supply reliably (see Figure 1).

Recent advancements in technologies offer higher levels of flexibility to users as well as the decision-makers to improve parking management strategies. Such flexibilities include (i) obtaining information about spot availability on neighboring parking areas in advance, (ii) adjusting rates based on the level of occupancy or stay meter times, and (iii) accepting various payment options from credit cards to smart-phone payments among others. Hence, decision-makers can more effectively maximize their revenue by setting prices for parking garages based on the incoming demand. Besides, information on price and occupancy provides users with the opportunity to choose available parking areas with acceptable costs (i.e., close to their desired destination at a reasonable price). However, achieving an optimal parking utilization, even with real-time information is still complex. Demands for parking areas in the network change over time, which enforce parking agencies to acquire a dynamic pricing scheme to maximize the parking utilization and revenue. On the other hand, the decisions made by parking agencies (e.g., on pricing) affect the users' decisions regarding parking spot selections. This user-agency decision-making scheme can be represented by a game theoretical model, where parking agencies and users compete until they reach an equilibrium condition.

This paper presents a dynamic parking management problem that accounts for an optimal price assignment by parking location over time that aims to drive parking utilization towards a target occupancy. The objective is to simultaneously (i) minimize the total user costs (including driving from origins to parking spots and walking from the parking locations to

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The authors are with the Department of Civil Engineering, The State University of New York at Stony Brook, Stony Brook, NY 11794 USA (e-mail: amir.mirheli@stonybrook.edu; leila.hajibabaidizaji@stonybrook.edu).

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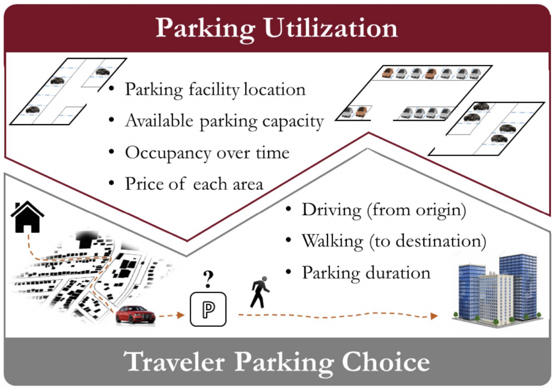


Fig. 1. Inter-relationships between parking utilization and travelers' parking choices.

their final destinations, both in terms of travel time), (ii) maximize the parking agency's revenue (mainly by minimizing the difference between the actual parking occupancy and its target level); see Figure 1. The problem is formulated into a bi-level optimization program with the upper-level focusing on maximizing parking utilization and parking agency's revenue, while the lower-level aims to minimize total user costs. To find an optimal solution, the bi-level optimization program is converted into an equivalent single-level formulation by applying Karush-Kuhn-Tucker (KKT) conditions and transforming user-equilibrium (UE) constraints in the lower-level problem to complementary equations. The dynamic user behavior and stochastic demand are captured by formulating the problem as a dynamic program. A Monte Carlo tree search (MCTS) based on a stochastic look-ahead model [1], [7], [36] is implemented to determine the optimal occupancy of parking areas and user assignment to spots at each time period. The proposed model and solution technique are applied to a hypothetical and an empirical case study. Besides, the proposed methodology is numerically compared to two benchmarks: (i) a greedy algorithm and (ii) an exact method to solve the proposed bi-level program. Numerical results show that the proposed algorithm solves the problem effectively and outperforms the benchmark methodologies.

The remainder of this paper is organized as follows. Section II highlights the existing research on parking spot management and relevant literature. Section III introduces the mathematical formulation developed for dynamic parking management. Section IV details the proposed approach to solve the problem and Section V presents the numerical results. Finally, Section VI summarizes the concluding remarks and trends for future research.

II. Literature Review

Advanced technologies with the support of the Internet-of-Things enable parking agencies to obtain real-time demand requests from parking neighborhoods. While parking agencies compete with their peers on the market share and aim to absorb higher number of parking users, there is a trade-off between price adjustment to gain more revenue and user attraction to maximize parking utilization. Users, on the other hand, are involved in a competition to find the best available parking spot fitting their needs, that is frustrating when the capacity of the parking areas is limited. Such problems can be translated into bi-level (e.g., [10], [15]) game theoretical (e.g., [28], [48]) optimization problems. Despite its importance, dynamic parking management under uncertain future demand and occupancy over time has not been thoroughly studied in the literature. This section summarizes relevant research efforts on model development and methodology.

Preliminary research on parking management neglects pricing strategies and solely accounts for the game theoretical methods that address the competition among users and parking agencies. For instance, Ayala *et al.* [4], [5] have developed a congestion game model for parking assignment that minimizes the extra cruising for available parking lots. Mejri *et al.* [30] have further studied a full cooperation between parking agencies to optimize the parking lots' distribution and account for a congestion game among drivers in search for the best parking area. Their goal is to minimize the overall network cost, which outperforms the centralized as well as greedy heuristics. Furthermore, Levy *et al.* [26] have formulated the spatial autocorrelation of occupied on-street parking areas. They have shown that for occupancy rate above 85%, the contiguity of space has a significant impact on parking dynamics. This is obtained based on the comparison of a micro simulation-based solution, PARKAGENT,[1] to that of a non-spatial analytical solution, PARKANALYST.[2]

Another school of research focuses on dynamic parking pricing strategies using game theoretical solutions. For example, Qian and Rajagopal [39], [40] have formulated the relationship between dynamic parking pricing and information sharing. The study results in optimal UE as well as system-optimal (SO) parking flow patterns and pricing schemes per lot that minimize total users' cruising time. Their experiments suggest a balance between the parking congestion and convenience of chosen lots in terms of terminal occupancy to set the parking prices and distribute drivers via a UE condition. Besides, Kokolaki *et al.* [24] have proposed an un-coordinated information-assisted parking search procedure under different categories: (i) low-priced but limited-capacity public facilities versus (ii) high-priced yet un-limited private parking spots. They assume that drivers aim to minimize their cruising cost under public or private parking choices following a game theoretical method using information on prices, total parking capacities, and demand. The results lead to SO demand assignments by controlling the price of the parking facilities. Another competition game formulation is proposed by He *et al.* [22] that addresses the parking space assignment to a limited number of vehicles under complete information on the parking status, with the objective of minimizing total system cost. The study results in SO pricing schemes under UE assignment of parking spaces to vehicles. Later, Du and Gong [12] have developed a stochastic Poisson game theoretical model to

[1]PARKAGENT is an agent-based model for parking in urban areas that simulates the drivers' parking choices on street networks ([6]).

[2]PARKANALYST introduces an aggregate analytical perspective (i.e., average parking conditions for every driver) to represent the temporal dynamics of cruising; see [2] for more information on PARKANALYST and PARKAGENT.

indicate the competitions among drivers in multiple parking areas in a decentralized and coordinated online parking mechanism. However, the dynamic pricing impact on the demand allocation is not discussed. Problems solved with game theoretical methods are frequently formulated by the Stackelberg leader-follower model: the market situation is described by a hierarchical model, where a set of stakeholders aim to achieve their objectives under various individual optimal decisions (e.g., see [45], [49]). Similarly, Mackowski *et al.* [27] have developed a dynamic non-cooperative Stackelberg leader-follower game that determines the real-time parking prices and optimizes the parking utilization. A bi-level mathematical program with equilibrium constraints (MPEC) is formulated that accounts for demand variations due to the parking pricing. Then, a single-level mixed-integer quadratic program (MIQP) reformulation of the original problem is implemented in a rolling-horizon structure. However, the rolling horizon approach is substituted with a myopic sequential solution technique to account for the convexity restriction of the objective function. Further, Lei and Ouyang [25] have applied the approximate dynamic programming (ADP) approach on MIQP formulation developed by Mackowski *et al.* [27] to include the information of future parking demand and available spots into the model. In this study, the value function is approximated using concave separable piece-wise linear functions and updated by dual sub-gradient information in each iteration of the algorithm. Their numerical results show that implementing future information via ADP outperforms the myopic solution. However, the existing stochasticities (e.g., cruising time) has not been addressed in this research.

Dynamic parking utilization problem, on the other hand, deals with uncertainty over time due to unknown future demand and occupancy. Such problems are often formulated into dynamic programming (DP) models under discrete state and action spaces [37], [38]. However, such methods commonly suffer from size limitations to estimate the impact of decisions on future outcomes. To overcome the computational burden, possible outcomes in problems accompanying uncertainty can be simulated using Monte Carlo methods [8], [46]. For example, Al-Kanj *et al.* [1] have applied sampled Monte Carlo tree search (MCTS) algorithm in a stochastic look-ahead policy to create a partial tree to handle the unknown events in vehicle routing problem in the emergency storm response context. It turns out that decision trees are also effective in reinforcement learning and prioritizing the actions. Despite all the efforts in the related areas, the problem of dynamic parking utilization management under future uncertainties has not been fully addressed. As such, Sections III - IV propose a methodology to bridge the gap.

III. Model Formulation

This section introduces the optimization model based on Mackowski *et al.* [27] for managing parking spots,[3] in a dynamic programming framework. The physical and temporal elements of the problem are first introduced, as follows. Let T be the number of discrete time periods in the planning horizon and $\Gamma = \{0, 1, \cdots, T-1\}$ present the times at which parking decisions are made. Υ denotes the set of physical parking areas in an urban neighborhood, where each area provides a set of available parking spots over time. Each $j \in \Upsilon$ has a parking capacity c_j.

For each $t \in \Gamma,\ j \in \Upsilon$, let $\widehat{\mathcal{J}}_j^t$ denote the number of parking spots that first become available at time t. Accordingly, $\widehat{\mathcal{J}}^t = (\widehat{\mathcal{J}}_j^t)_{j\in\Upsilon}$ presents the spatial distribution of all newly realized parking spots at time t. Let $\mathcal{J}_j^t$ be the number of parking spots already available in parking lot j at time t before any new spot availability; similarly, $\mathcal{J}^t = (\mathcal{J}_j^t)_{j\in\Upsilon}$ represents the total number of parking spots that are already available at time t. Hence, $\mathcal{J}^{t^+}$ denotes the total number of available parking spots in time period t, where $\mathcal{J}^{t^+} = \mathcal{J}^t + \widehat{\mathcal{J}}^t$ that includes all existing as well as newly realized parking spots. Similarly, to monitor the available users, $\widehat{\mathcal{D}}^t$ is defined for each $t \in \Gamma$ to present the set of users that first become available (i.e., cruise for available parking spots) in time period t. Then, $\mathcal{D}^t$ denotes the available users at time t before the new arrivals in $\widehat{\mathcal{D}}^t$ are included in the system. Again, $\mathcal{D}^{t^+}$ indicates the set of available users at time t, including the new users have just arrived, i.e., $\mathcal{D}^{t^+} = \mathcal{D}^t \cup \widehat{\mathcal{D}}^t$. Therefore, $\mathcal{D}_j^{t^+}$ denotes the set of users that must be served by parking spots in area j at time t, i.e., $\mathcal{D}^{t^+} = \bigcup_{j\in\Upsilon} \mathcal{D}_j^{t^+}$. Each user has to spend some time to search for a free spot based on the occupancy of the preferred parking lot at the beginning of time t. Therefore, $\widehat{\mathcal{L}}_j^t$ denotes the expected average cruising time for users going to parking j at time t and $\widehat{\mathcal{L}}^t = \bigcup_{j\in\Upsilon} \widehat{\mathcal{L}}_j^t$ represents the set of newly realized cruising times at time t.

New information at time t is denoted by $\mathcal{W}^t = (\widehat{\mathcal{D}}^t, \widehat{\mathcal{J}}^t, \widehat{\mathcal{L}}^t)$, where $\left(\mathcal{W}^t\right)_{t=0}^T$ represents the stochastic information process with realization $\mathcal{W}^t(\omega) = \omega^t = (\widehat{\mathcal{D}}^t(\omega), \widehat{\mathcal{J}}^t(\omega), \widehat{\mathcal{L}}^t(\omega))$, [similar to 14, 19]. Sample realization ω depends on the policy selection, e.g., provide more information about the real-time occupancy of each parking lot to users to decide the preferred parking lot and reduces their cruising time. Therefore, we let Ω^π be the set of outcomes that depend on policy π used to inform users about available parking spots. The number of available users and average cruising duration over time t follow random processes. The state of the system is captured by the distribution of available users and available spots in parking area j, i.e., $\mathcal{S}^t = \left\{\mathcal{D}^t, \mathcal{J}^t, \mathcal{L}^t\right\}$.

At each time period t, the parking agency makes a decision on the price p_j^t of each parking lot. The parking demand is defined by sets of origins $\mathcal{O}$ and destinations Δ, arrival time, and parking duration: user i from origin $o \in \mathcal{O}$ to destination $\delta \in \Delta$ may arrive at a parking neighborhood at time $t \in \Gamma$ and park for n time periods, where $n \in \{1, 2, \ldots, \mathcal{N}\}$; see Figure 2. Users going to parking j experience a driving cost μ_{oj} from origin o to parking spot in j (i.e., travel time and average cruising time in j), walking cost $\rho_{j\delta}$ from the parking location in j to final destination δ, and parking price p_j^t. We define $\mathcal{C}_{j,o\delta}^{t,n} = n\, p_j^t + \theta\, \mu_{oj} + \theta' \rho_{j\delta}$ to represent the total user cost from $o \in \mathcal{O}$ and $\delta \in \Delta$ to park in lot j for n time periods at time period t, where θ and θ' convert travel time to monetary cost. Similar to Mackowski *et al.* [27], we let $h_{j,o\delta}^{t,n}$ denote

[3]Mackowski *et al.* [27] have used a dynamic performance-based pricing approach to formulate the problem.

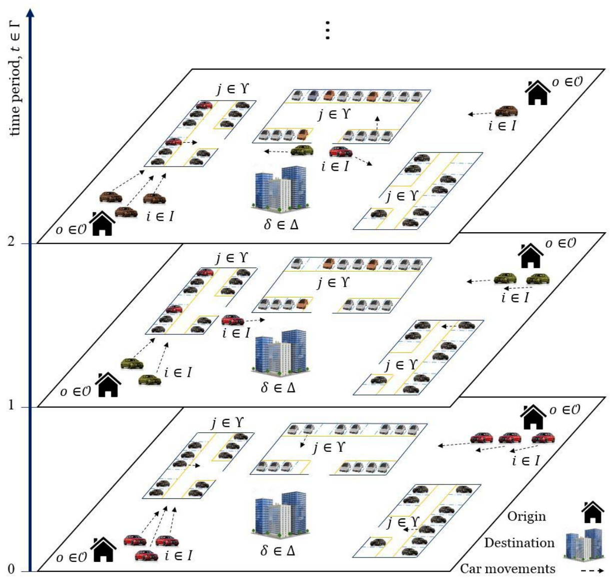


Fig. 2. Hypothetical illustration of user arrivals and parking spot availability over time.

the number of users cruising for parking spot in j. Besides, we let parameter σ_j^t indicate the occupancy of parking area j, where κ_j denote the target occupancy rate of parking area j. The impacts of the occupancy of each parking lot on drivers cruising time are studied in the literature ([3], [23], [26], [35]). The average expected cruising time $\mathcal{F}_j^t$ is a function of ψ_j^t for parking j at time t, where ψ_j^t includes the number of occupied parking spots σ_j^t at the beginning of time period t and the number of users $h_{j,o\delta}^{t,n}$ choose to park in parking lot j during time period t. User behaviors with respect to the occupancy information (provided by the parking agency) [3] can be represented by

$$\mathcal{F}_j^t(\psi_j^t) = e_j\, Z_j (1 - c_j^{-1}(\sigma_j^t + \sum_{n\in\{1,2,\ldots,\mathcal{N}\}} \sum_{o\delta\in\mathcal{O}\Delta} h_{j,o\delta}^{t,n}))^{-1}, \quad \forall j \in \Upsilon,\, t \in \Gamma, \quad (1)$$

where e_j represents the average cruising time for empty parking lot j. Besides, Z_j is a constant that represents the reaction of drivers to the provided information, where $Z_j = 1$ when they are completely aware of the updated occupancies. According to (1), when the parking lot occupancy reaches to the lot's capacity, cruising time goes to infinity. This indicates that there will be no chance for the new users, arriving during the next time period, to find a parking spot, unless some users leave the parking lot. In the UE condition, parking j is chosen at time t for n time periods by users traveling from $o \in \mathcal{O}$ to $\delta \in \Delta$, only when j's total user cost $u_{o\delta}^{t,n}$ is the minimum amongst all parking choices [40], mathematically defined as $h_{j,o\delta}^{t,n} > 0$ if $\mathcal{C}_{j,o\delta}^{t,n} = u_{o\delta}^{t,n}$ or 0 if $\mathcal{C}_{j,o\delta}^{t,n} > u_{o\delta}^{t,n}$, for all $j \in \Upsilon$, $o \in \mathcal{O}$, $\delta \in \Delta$, $n \in \{1, 2, \ldots, \mathcal{N}\}$, $t \in \Gamma$. Furthermore, Γ_i denotes the time window for user $i \in \mathcal{D}_t^+$ to find a parking spot in j. In case a user i is not served (i.e., cannot find a parking spot) within Γ_i, it is assumed lost in that time period but backlogged for the future periods. A dummy lot ζ is considered with enough capacity to tolerate all additional demand lost from the system. $\mathcal{D}_t$ is relatively small since it represents the set of cruising users left unserved from a previous time period. Hence, when the time window is tight (e.g., special cases), it can be eliminated from the state definition. The user-specific demand follows a $\mathcal{H}(u_{o\delta}^{t,n})$ form that is an inverse demand function of the $u_{o\delta}^{t,n}$ of such demand type. Thus, $\sum_{j\in\Upsilon\cup\{\zeta\}} h_{j,o\delta}^{t,n} = \mathcal{H}(u_{o\delta}^{t,n})$, $\forall j \in \Upsilon$, $o \in \mathcal{O}$, $\delta \in \Delta$, $t \in \Gamma$, $n \in \{1, 2, \ldots, \mathcal{N}\}$. Then, similar to Mackowski *et al.* [27], the number of arrivals and departures are respectively calculated by $q_j^{t,n} = \sum_{o\delta\in O\Delta} h_{j,o\delta}^{t,n}$ and $g_j^t = \sum_{m=\max(1,t-\mathcal{N})}^{t-1} q_j^{m,t-m}$. To capture the equilibrium condition for the parking users, a binary decision variable $\mathbf{x}_i^t = \{x_{ij}^t, \forall j \in \Upsilon \cup \{\zeta\}\} \in \{0, 1\}^{|\Upsilon\cup\{\zeta\}|}$ is defined, where $x_{ij}^t = 1$ if user i selects a parking spot at area j at time t or $x_{ij}^t = 0$ otherwise. We let $|\lambda_{o\delta}^{t,n}| = \mathcal{H}(u_{o\delta}^{t,n})$ present the set of users travel from $o \in \mathcal{O}$ to $\delta \in \Delta$ for parking duration n at time period t. The mathematical optimization model including parking agency's decisions and users' parking equilibrium choice can be formulated into a bi-level problem, as follows,

$$\begin{aligned} f^t&(p_j^t(\omega),\, h_{j,o\delta}^{t,n}(\omega),\, u_{o\delta}^{t,n}(\omega)) \\ &= \beta \sum_{j\in\Upsilon} \Big|(\kappa_j c_j - \sigma_j^t) - \sum_{n\in\{1,2,\ldots,\mathcal{N}\}} \sum_{o\delta\in\mathcal{O}\Delta} h_{j,o\delta}^{t,n}(\omega)\Big| \\ &\quad - \alpha \sum_{n\in\{1,2,\ldots,\mathcal{N}\}} \sum_{o\delta\in\mathcal{O}\Delta} \sum_{j\in\Upsilon} n\, p_j^t(\omega)\, h_{j,o\delta}^{t,n}(\omega), \qquad (2a) \end{aligned}$$

while the feasible region for a given ω at t is subject to

$$l_j \le p_j^t(\omega) \le u_j, \quad \forall j \in \Upsilon,\ t \in \Gamma, \qquad (2b)$$

and

$$h_{j,o\delta}^{t,n}(\omega) \in \underset{x_{ij}^t\in\{0,1\}^{|\Upsilon\cup\zeta|}}{\operatorname{argmin}} \sum_{i\in\lambda_{o\delta}^{t,n}} \sum_{j\in\Upsilon\cup\{\zeta\}} \mathcal{C}_{j,o\delta}^{t,n}\, x_{ij}^t \qquad (2c)$$

$$x_{ij}^t(\omega) \in \{0, 1\}, \quad \forall j \in \Upsilon \cup \{\zeta\},\ i \in \lambda_{o\delta}^{t,n},\ t \in \Gamma, \qquad (2d)$$

$$\sum_{j\in\Upsilon\cup\{\zeta\}} x_{ij}^t(\omega) = 1, \quad \forall i \in \lambda_{o\delta}^{t,n}, \qquad (2e)$$

$$\begin{aligned} h_{j,o\delta}^{t,n}(\omega) &= \sum_{i\in\lambda_{o\delta}^{t,n}} x_{ij}^t(\omega), \quad \forall j \in \Upsilon \cup \{\zeta\},\ o\delta \in \mathcal{O}\Delta, \\ n &\in \{1, 2, \ldots, \mathcal{N}\},\ t \in \Gamma, \qquad (2f) \end{aligned}$$

$$\begin{aligned} &\sum_{o\delta\in\mathcal{O}\Delta, n\in\{1,2,\ldots,\mathcal{N}\}} h_{j,o\delta}^{t,n}(\omega) \le c_j - \sigma_j^t, \\ &\forall j \in \Upsilon \cup \{\zeta\},\ t \in \Gamma, \qquad (2g) \end{aligned}$$

where $\mathcal{H}(u_{o\delta}^{t,n}) = a'^{t,n}_{o\delta} - b\, u_{o\delta}^{t,n}$, and $a'^{t,n}_{o\delta}$ denotes the linear demand curve's intercept for each user with $o\delta \in \mathcal{O}\Delta$ to park at time t for $n \in \{1, 2, \ldots, \mathcal{N}\}$ time steps.

The upper-level objective function (2a) defines the parking agency's revenue as well as the difference between the target occupancy and actual parking utilization, with respective weights of α and β. Constraints (2b) define a minimum l_j and a maximum u_j for the parking price at a spot in area j. The lower level problem (2c) captures the drivers' behaviors to find the parking spots at minimum costs. Constraints (2d) and (2e) denote that each user can park in only one parking spot.

Constraints (2f) present the relationship between the parking demand and user decisions. Finally, constraints (2g) ensure that the number of users parking in lot j is less than the number of available spots.

To further capture the dynamics of the system over time, let $\mathcal{D}_e^t(h_{j,o\delta}^{t,n})$ denote the set of "expired" users that are either served or never served by the end of their time window Γ_i. Hence, the dynamics of the users and parking areas are defined as

$$\mathcal{D}^{t+1} = \mathcal{D}^{t^+} \setminus \mathcal{D}_e^t, \quad \forall t \in \Gamma, \text{ and} \tag{3a}$$

$$\mathcal{J}_j^{t+1}(\omega) = c_j - \sigma_j^t(\omega) - q_j^t(\omega) + g_j^t(\omega), \quad \forall j \in \Upsilon \cup \{\zeta\}, \quad t \in \Gamma, \tag{3b}$$

where (3b) counts the number of available parking spaces in area j at time $t+1$ and can be translated into the occupancy of parking area j at time $t+1$, as

$$\sigma_j^{t+1}(\omega) = \sigma_j^t(\omega) + q_j^t(\omega) - g_j^t(\omega), \quad \forall j \in \Upsilon \cup \{\zeta\}, \ t \in \Gamma. \tag{4}$$

In addition, the price changes between consecutive time periods is defined within an upper bound ε' and a lower bound ε, as follows.

$$-\varepsilon' \le p_j^t(\omega) - p_j^{t-1}(\omega) \le \varepsilon, \quad \forall j \in \Upsilon, \ t \in \Gamma. \tag{5}$$

The objective is to minimize the expected costs (i.e., the difference between the target occupancy rate and actual parking utilization minus the agency's revenue[4]) of the system over the planning horizon, given an initial state $\mathcal{S}_0$, as follows:

$$\underset{(p_j^0,\, h_{j,o\delta}^{0,n},\, u_{o\delta}^{0,n})}{\text{minimize}} \ f^0(p_j^0, h_{j,o\delta}^{0,n}, u_{o\delta}^{0,n}) + \mathbb{E}\Big\{\sum_{t\in\Gamma\setminus\{0\}} \underset{(p_j^t,\, h_{j,o\delta}^{t,n},\, u_{o\delta}^{t,n})}{\text{minimize}} \ f^t(p_j^t, h_{j,o\delta}^{t,n}, u_{o\delta}^{t,n})\Big\}. \tag{6}$$

IV. Solution Technique

A. Single-Level Conversion

Problem formulated in (2a)-(2g) is a bi-level optimization problem with non-linear terms. We first reformulate it into an equivalent single-level model using KKT conditions. The UE conditions in the lower-level problem (2c)-(2g) are transformed into complementary equations by implementing minimum total user cost $u_{o\delta}^{t,n}$ and ν_j^t as dual variables of constraints (2g) [see 16, 27]. For notation simplicity, we focus on one generic sample realization and omit ω everywhere beginning from this section. Therefore, our problem becomes

$$f_t(p_j^t, h_{j,o\delta}^{t,n}, u_{o\delta}^{t,n}) = \beta \sum_{j\in\Upsilon} \Big|(\kappa_j c_j - \sigma_j^t) - \sum_{n\in\{1,2,\ldots,\mathcal{N}\}} \sum_{o\delta\in\mathcal{O}\Delta} h_{j,o\delta}^{t,n}\Big| - \alpha \sum_{n\in\{1,2,\ldots,\mathcal{N}\}} \sum_{o\delta\in\mathcal{O}\Delta} \sum_{j\in\Upsilon} n\, p_j^t\, h_{j,o\delta}^{t,n}, \tag{7a}$$

$$l_j \le p_j^t \le u_j, \quad \forall j \in \Upsilon, \ t \in \Gamma, \tag{7b}$$

[4]The terms of the objective function are converted into costs using respective coefficient β per time.

and

$$0 \le h_{j,o\delta}^{t,n} \perp \mathcal{C}_{j,o\delta}^{t,n} + \nu_j^t - u_{o\delta}^{t,n} \ge 0, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ o\delta \in \mathcal{O}\Delta, \ n \in \{1,2,\ldots,\mathcal{N}\}, \ t \in \Gamma, \tag{7c}$$

$$0 \le \nu_j^t \perp (c_j - \sigma_j^t) - \sum_{o\delta\in\mathcal{O}\Delta, n\in\{1,2,\ldots,\mathcal{N}\}} h_{j,o\delta}^{t,n} \ge 0, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ t \in \Gamma, \tag{7d}$$

where constraints (7c) are equivalent to

$$h_{j,o\delta}^{t,n} \le \gamma_{j,o\delta}^{t,n}(c_j - \sigma_j^t), \quad \forall j \in \Upsilon \cup \{\zeta\}, \ o\delta \in \mathcal{O}\Delta, \ n \in \{1,2,\ldots,\mathcal{N}\}, \ t \in \Gamma, \tag{8a}$$

$$\mathcal{C}_{j,o\delta}^{t,n} + \nu_j^t - u_{o,\delta}^{t,n} \le M(1 - \gamma_{j,o\delta}^{t,n}), \quad \forall j \in \Upsilon \cup \{\zeta\}, \ o\delta \in \mathcal{O}\Delta, \ n \in \{1,2,\ldots,\mathcal{N}\}, \ t \in \Gamma, \tag{8b}$$

$$\mathcal{C}_{j,o\delta}^{t,n} + \nu_j^t - u_{o,\delta}^{t,n} \ge 0, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ o\delta \in \mathcal{O}\Delta, \ n \in \{1,2,\ldots,\mathcal{N}\}, \ t \in \Gamma, \tag{8c}$$

$$\gamma_{j,o\delta}^{t,n} \in \{0,1\}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ o\delta \in \mathcal{O}\Delta, \ n \in \{1,2,\ldots,\mathcal{N}\}, \ t \in \Gamma. \tag{8d}$$

Similarly, constraints (7d) can be reformulated as

$$\nu_j^t \le M\eta_j^t, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ t \in \Gamma, \tag{9a}$$

$$(c_j - \sigma_j^t) - \sum_{o\delta\in\mathcal{O}\Delta, n\in\{1,2,\ldots,\mathcal{N}\}} h_{j,o\delta}^{t,n} \le M(1 - \eta_j^t), \quad \forall j \in \Upsilon \cup \{\zeta\}, \ t \in \Gamma, \tag{9b}$$

$$(c_j - \sigma_j^t) - \sum_{o\delta\in\mathcal{O}\Delta, \ n\in\{1,2,\ldots,\mathcal{N}\}} h_{j,o\delta}^{t,n} \ge 0, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ t \in \Gamma, \tag{9c}$$

$$\eta_j^t \in \{0,1\}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ t \in \Gamma, \tag{9d}$$

where $h_{j,o\delta}^{t,n}$ and p_j^t are defined as integer decision variables and M is a sufficiently large constant value. The objective function (7a) includes a bi-linear term which makes the problem non-convex. We apply the M-method to linearize that term. The value of M has a significant impact on the dimension of the problem, which makes it difficult to solve using commercial solvers, e.g., [11]. To reformulate the objective function, we re-introduce p_j^t and $h_{j,o\delta}^{t,n}$ as integer decision variables using $b_{j,k}^t \in \{0,1\}$, as follows.

$$p_j^t = \sum_{k\in K} 2^k\, b_{j,k}^t, \quad \forall j \in \Upsilon, \ t \in \Gamma, \tag{10}$$

where $K = \lfloor \log_2 u \rfloor : u = \max(u_j), \forall j \in \Upsilon$. Similarly, we substitute the integer variable $h_{j,o\delta}^{t,n}$ by binary variables $b'^{t,n}_{j,o\delta} \in \{0,1\}$ as

$$h_{j,o\delta}^{t,n} = \sum_{k'\in K'} 2^{k'}\, b'^{t,n}_{j,o\delta}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \ o\delta \in \mathcal{O}\Delta, \ n \in \{1,2,\ldots,\mathcal{N}\}, \ t \in \Gamma, \tag{11}$$

where $K' = \lfloor \log_2 h \rfloor : h = \max(h_{j,o\delta}^{t,n}), \forall j \in \Upsilon \cup \{\zeta\}$, $o\delta \in \mathcal{O}\Delta$, $n \in \{1,2,\ldots,\mathcal{N}\}$, $t \in \Gamma$. We then define a continuous auxiliary variable $z_{j,o\delta,k,k'}^{t,n}$ to represent the multiplication of the defined binary variables. Finally, we substitute the second term of objective function (7a) by

$$\sum_{n\in\{1,2,\ldots,\mathcal{N}\}}\sum_{o\delta\in\mathcal{O}\Delta}\sum_{j\in\Upsilon} n\, p_j^t\, h_{j,o\delta}^{t,n} = \sum_{n\in\{1,2,\ldots,\mathcal{N}\}}\sum_{o\delta\in\mathcal{O}\Delta}\sum_{j\in\Upsilon}\sum_{k\in K}\sum_{k'\in K'} 2^{k'}2^{k}\, z_{j,o\delta,k,k'}^{t,n}, \quad (12)$$

where

$$z_{j,o\delta,k,k'}^{t,n} \le b_{j,k}^t, \quad \forall j \in \Upsilon \cup \{\zeta\},\ o\delta \in \mathcal{O}\Delta,\ t \in \Gamma,\ k \in K,\ k' \in K',\ n \in \{1,2,\ldots,\mathcal{N}\}, \quad (13a)$$

$$z_{j,o\delta,k,k'}^{t,n} \le b_{j,k}^{\prime t}, \quad \forall j \in \Upsilon \cup \{\zeta\},\ o\delta \in \mathcal{O}\Delta,\ t \in \Gamma,\ k \in K,\ k' \in K',\ n \in \{1,2,\ldots,\mathcal{N}\}, \quad (13b)$$

$$z_{j,o\delta,k,k'}^{t,n} \ge b_{j,k}^t + b_{j,k}^{\prime t} - 1, \quad \forall j \in \Upsilon \cup \{\zeta\},\ o\delta \in \mathcal{O}\Delta,\ t \in \Gamma,\ k \in K,\ k' \in K',\ n \in \{1,2,\ldots,\mathcal{N}\}, \quad (13c)$$

$$z_{j,o\delta,k,k'}^{t,n} \ge 0, \quad \forall j \in \Upsilon \cup \{\zeta\},\ o\delta \in \mathcal{O}\Delta,\ t \in \Gamma,\ k \in K,\ k' \in K',\ n \in \{1,2,\ldots,\mathcal{N}\}. \quad (13d)$$

B. Dynamic Programming Procedure

This section applies a dynamic programming technique to represent the decisions made by the parking agency and users over time. Similar to Al-Kanj *et al.* [1] and Mirheli *et al.* [31], a stochastic look-ahead approach based on MCTS is used to solve problem (7a)-(7b), (8a)-(8d), (9a)-(9d), (10)-(12), and (13a)-(13d) considering current states and newly available information in each time t. To do so, we first rewrite the objective function (6) into equivalent Bellman's equation using the converted single-level formulation, i.e., $V^t(\mathcal{S}^t) = \mathbb{E}\{\text{minimize } f_t(p^t,\, h^t,\, u^t,\, \gamma^t,\, \eta^t,\, \nu^t,\, b^t,\, b^{\prime t},\, z^t) + V^{t+1}(\mathcal{S}^{t+1})|\mathcal{S}^t\}$.

DP-driven methodologies may provide solutions to the problem by gathering accurate information on state variables and actions at each time period t. However, the combination of available parking spots, number of users cruising for vacant parking spots, average cruising time spent at each parking lot, over all times $t \in \Gamma$ in all parking lots $j \in \mathcal{J} \cup \{\zeta\}$ makes the problem computationally intractable. To overcome the curses of dimensionality, approximation methods and adaptive techniques are used in the literature, e.g., [1], [13], [21], [29], [32] that provide initial estimations and improve them through iterative techniques. In this paper, the number of users cruising in parking system at each time t follows a linear inverse demand function of minimum parking cost. We apply a function to compute the average cruising time in each parking lot $j \in \Upsilon \cup \{\zeta\}$ that partially reduces the state space and simplifies the look-ahead model. We then estimate the value function by $\widetilde{V}^{t+1}(\mathcal{D}^{t+1},\, \mathcal{J}^{t+1})$, which is only a function of available parking spots and users. For notation simplicity, we let a^t represent all actions (i.e., decision variables) in each time $t \in \Gamma$. Our optimization model at each parking lot j in time t can be presented as

$$\mathcal{A}^{*^t}(\mathcal{S}^t) = \underset{a^t\in\mathcal{A}^t(\mathcal{S}^t)}{\operatorname{argmin}} \{f^t(\mathcal{D}^t, \mathcal{J}^t,\, a^t) + \widetilde{V}^{t+1}(\mathcal{D}^{t+1},\, \mathcal{J}^{t+1})\} \quad (14a)$$

subject to (7b), (8a)-(8d), (9a)-(9d), (10)-(12), and (13a)-(13d), where $\mathcal{A}^{*^t}$ denotes the near-optimal actions from the set of feasible actions $\mathcal{A}^t$.

Tree search algorithms are shown to be effective in approximating the value function in each t. However, exploration in each branch of the tree created by a feasible action as well as the inclusion of exogenous information at each time t, make the tree to grow exponentially. Therefore, the MCTS algorithm [1] is applied in the look-ahead policy to (i) efficiently provide the estimation of the value function and (ii) effectively introduce uncertainties to the problem. In each iteration of the MCTS algorithm, the following steps are repeated [9], [34]: selection, expansion, simulation, and back-propagation. In the selection step, the best action is selected based on the proposed actions and their updated values from previous iterations until we reach an expandable state. In each leaf node, one or more states are added to the tree. Then, in the simulation step, the value of the added state is calculated and finally, the back-propagation step updates the value functions of predecessor states based on the estimated value of recently added states. Our MCTS algorithm framework includes a (i) tree policy, (ii) value function estimation, and (iii) back-propagation steps.

In this paper, the exogenous information on unexpected average cruising time is acquired at the beginning of $t \in \Gamma$, where the stochastic information is added to the tree to handle the uncertainties. In the look-ahead model, all variables are indexed with $t,\, t'$ to identify the time iteration t in the main model and $t' = t,\, \ldots,\, t + H - 1$ in the look-ahead model, where H represents a limited time horizon as a threshold for tree expansion (i.e., inner tree iterations). Then, the value function estimation is divided into pre-decision $\widetilde{V}^{t,t'}(\widetilde{\mathcal{S}}^{t,t'})$ and post-decision $\widetilde{V}_a^{t,t'}(\widetilde{\mathcal{S}}_a^{t,t'})$ value functions that include the effects of adding exogenous information, where $\widetilde{\mathcal{S}}^t = \{\mathcal{D}^t,\, \mathcal{J}^t\}$. Besides, at state $\widetilde{\mathcal{S}}^{t,t'}$, we let $\widetilde{\mathcal{A}}^{t,\,t'}(\widetilde{\mathcal{S}}^{t,t'})$ denote the set of decisions, where the explored decisions are defined by $\widetilde{\mathcal{A}}_e^{t,\,t'}(\widetilde{\mathcal{S}}^{t,t'})$ that have been explored in the tree at time t' and its complement set $\widetilde{\mathcal{A}}_u^{t,\,t'}(\widetilde{\mathcal{S}}^{t,t'})$ presents the set of unexplored decisions. For the possible outcomes, we let $\widetilde{\Omega}^{t,\,t'+1}(\widetilde{\mathcal{S}}_a^{t,t'})$ present all possible random events that can take place at time $t' + 1$, where $\widetilde{\Omega}_e^{t,\,t'+1}(\widetilde{\mathcal{S}}_a^{t,t'})$ and $\widetilde{\Omega}_u^{t,\,t'+1}(\widetilde{\mathcal{S}}_a^{t,t'})$ represent the explored and unexplored possible outcomes, respectively. In the tree policy step, optimal decisions of the two-stage look-ahead policy $\widetilde{a}^{t,\,t'}$, convert the pre-decision states $\mathcal{S}^{t,t'}$ to post-decision states $\mathcal{S}_a^{t,t'}$. Once the parking prices and choices of users are set for the time t', a sample of possible outcomes $\widetilde{\omega} \in \widetilde{\Omega}^{t,\,t'+1}(\widetilde{\mathcal{S}}_a^{t,t'})$ will be generated and fed into (1) to compute the average cruising time of available users for each parking lot, which was unknown prior to time t.

Our algorithm framework follows a stochastic MCTS with the computational budget of N iterations: given a current state $\mathcal{S}^t$, we create a state $\widetilde{\mathcal{S}}^{t,\,t'}$ as a root node of the tree, generate the MCTS algorithm, and build a look-ahead model at each time t to estimate the value functions $\widetilde{V}^{t+1}$ and return the vector of near-optimal actions $\mathcal{A}^{*^t}$ in time period t. In each level of the tree, there is a trade-off between exploiting the high-reward states and exploring the states ignored more during the search, until we reach the threshold of sufficient possible actions d^{thr}. Therefore, in the selection step, we follow upper confidence-bounding for trees (UCT)[5] [1], [9] to make decisions as $\widetilde{a}^{*^{t,t'}} = \text{argmax}_{\widetilde{a}^{t,t'} \in \widetilde{\mathcal{A}}_e^{t,t'}(\widetilde{\mathcal{S}}^{t,t'})} \Big(-(\widetilde{f}^t(\widetilde{\mathcal{S}}^{t,t'}, \widetilde{a}^{t,t'}) + \widetilde{V}_a^{t,t'}(\widetilde{\mathcal{S}}_a^{t,t'})) + \iota \sqrt{\frac{\ln \mathcal{N}(\widetilde{\mathcal{S}}^{t,t'})}{\mathcal{N}(\widetilde{\mathcal{S}}^{t,t'}, \widetilde{a}^{t,t'})}} \Big)$, where ι is a tunable parameter to balance exploration and exploitation, $\mathcal{N}(\widetilde{\mathcal{S}}^{t,t'})$ represents the number of visiting states $\widetilde{\mathcal{S}}^{t,t'}$, and $\mathcal{N}(\widetilde{\mathcal{S}}^{t,t'}, \widetilde{a}^{t,t'})$ identifies the number of times a decision $\widetilde{a}^{t,t'}$ is taken from state $\widetilde{\mathcal{S}}^{t,t'}$ during the tree search process. Once decisions $\widetilde{a}^{t,t'}$ are made, the parking state will be $\widetilde{\mathcal{S}}_a^{t,t'}$, where we add a sample realization of exogenous information to reach the next pre-decision state $\widetilde{\mathcal{S}}^{t,t'} = \mathcal{S}^{\mathcal{T},a}(\widetilde{\mathcal{S}}_a^{t,t'}, \widetilde{\mathcal{W}}^{t,t'+1})$. $\mathcal{S}^{\mathcal{T},a}$ represents the transition function between the evolution of each two consecutive state variables [36]. In the simulation step, we develop a simulation/optimization approach to provide an initial estimate of the newly added node to the tree. In this step, we first generate a sample path $\widetilde{\omega} \in \widetilde{\Omega}^{t,t'}(\widetilde{\mathcal{S}}^{t,t'})$ to determine the level of information provided for the users at time t. Then, we modify the demand for the next time t based on the updated information on the parking costs and cruising times. Finally, we reformulate using the log arc elasticity coefficient χ [47] to efficiently solve the problem.

$$\min_{\sigma,p,q,h,g \geq 0} \Big\{ \beta \sum_{t''=t'}^{t''=t'+H} \sum_{j \in \Upsilon} (\kappa_j c_j - \sigma_j^t) - \alpha \sum_{t''=t'}^{t''=t'+H} \sum_{n \in \{1,2,\ldots,\mathcal{N}\}} \sum_{o\delta \in \mathcal{O}\Delta} \sum_{j \in \Upsilon} n p_j^{t''} h_{j,o\delta}^{t'',n} \Big\} \tag{15a}$$

subject to

$$-\varepsilon' \leq p_j^{t''}(\omega) - p_j^{t''-1}(\omega) \leq \varepsilon, \quad \forall j \in \Upsilon, \; t'' \in \{t', \ldots, t'+H\}, \tag{15b}$$

$$l_j \leq p_j^{t''} \leq u_j, \quad \forall j \in \Upsilon, \; t'' \in \{t', \ldots, t'+H\}, \tag{15c}$$

$$\sigma_j^{t''} = \sigma_j^{t''-1} - g_j^{t''} + \sum_{n \in \{1,2,\ldots,\mathcal{N}\}} q_j^{t'',n}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \; t'' \in \{t', \ldots, t'+H\}, \tag{15d}$$

$$q_j^{t'',n} = \sum_{o\delta \in O\Delta} h_{j,o\delta}^{t'',n}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \; n \in \{1,2,\ldots,\mathcal{N}\}, \; t'' \in \{t', \ldots, t'+H\}, \tag{15e}$$

$$g_j^{t''} = \sum_{m=\max(1,t''-\mathcal{N})}^{t''-1} q_j^{m,t''-m}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \; t'' \in \{t', \ldots, t'+H\}, \tag{15f}$$

$$h_{j,o\delta}^{t'',n} \leq \frac{(\chi-1) p_j^{t''-1} h_{j,o\delta}^{t''-1,n} - (\chi+1) p_j^{t''} h_{j,o\delta}^{t''-1,n}}{(\chi-1)\, p_j^{t''} - (\chi+1)\, p_j^{t''-1}}, \quad \forall j \in \Upsilon \cup \{\zeta\}, \; o \in \mathcal{O}, \; \delta \in \Delta, \; n \in \{1,2,\ldots,\mathcal{N}\}, \; t'' \in \{t', \ldots, t'+H\}, \tag{15g}$$

$$\sigma_j^{t''} \leq c_j, \quad \forall j \in \Upsilon \cup \{\zeta\}, t'' \in \{t', \ldots, t'+H\}, \tag{15h}$$

where constraints 15g capture the effect of dynamic prices on travel demand. The general algorithm framework is presented in Appendix A. In the MCTS algorithm, the tree policy step explores feasible actions to add to the set of explored actions until the threshold is reached. The value function estimation step provides an initial approximation to newly added nodes and finally, the back-propagation step updates the value function estimations for the predecessor nodes. Once the tree search terminates, the near-optimal actions will be selected, which correspond to the best value from the root node. Algorithm 1 is adopted from Al-Kanj *et al.* [1], where the tree search termination criteria are modified and minimum total user costs are updated.

[5]UCT is widely used in computer science literature and established for solving maximization problems [9]

V. Numerical Experiments

The proposed solution algorithm in Section IV is coded in JAVA and run on a desktop computer with quad-core 3.6 GHz CPU and 16 GB of memory. A Poisson distribution is applied to generate the initial demand pattern for five different time periods during a business day, defined by SFCTA [42], i.e., early AM, AM peak, mid-day, PM peak, and evening time-of-day.

A. Hypothetical Dataset

To evaluate the effectiveness of the proposed methodology, we first apply our problem formulated in (7b)-(7c) and (8)-(14a) and the MCTS solution framework to a hypothetical parking network under various capacities. The dataset contains 10 parking lots distributed between users' origin $\mathcal{O}$ and destination Δ sets. Driving time μ_{oj} from an origin o to a parking spot j and walking time $\rho_{j,\delta}$ from a parking location j to a final destination δ are assumed in a way so as to provide an equilibrium condition based on parking prices. In other words, total parking cost (i.e., combination of the aforementioned travel times and parking cost) provides a cooperative game environment for users to compete on available parking spots in an equilibrium condition, where each user aims to minimize its own total cost. Table I summarizes the capacity of each parking lot, driving time from origins to parking lots, and walking time between parking lots and destinations. Trips to and from dummy lot ζ with parking price of \$0 get high enough values to be chosen only when all parking lots are full. Two travel origin regions have been selected and it is assumed that 70% of demand enters the parking neighborhood from origin 1 and the rest from origin 2. Users are assumed to park for four different parking durations: 15 min, 30 min, 45 min, and 1 hr. The average vehicle arrivals over different time periods in a day is assumed to be 200, 520, 240, 480, and 200 for early AM, AM peak, mid-day, PM peak, and evening time-of-day for a medium demand level. Accordingly, the low and high demand levels are assumed to be half and twice the medium demand, respectively. A penalty value of $\beta = 10$ is considered to avoid parking lots with \$0 price in the solutions to minimize the number of lost users. Besides network-related parameters, values selected for model parameters follow. To balance the objective function terms, the value of α is set to 0.25. Note that a set of sensitivity analyses has been conducted to show the impact of α and β on the objective value (see Section V-A2). Furthermore, the time horizon threshold within the inner tree iteration is set to $H = 8$. Finally, at most four actions

TABLE I

PARKING SYSTEM PARAMETERS

Parking lot	Capacity	$\mu_{1,j}$ (min)	$\mu_{2,j}$ (min)	$\rho_{j,1}$ (\$)
j_1	140	15	18	2
j_2	120	12	17	3
j_3	140	17	17	4
j_4	120	17	12	5
j_5	40	17	15	5
j_6	40	18	9	6
j_7	100	17	10	6
j_8	40	18	10	7
j_9	100	19	11	7
j_{10}	100	20	12	8

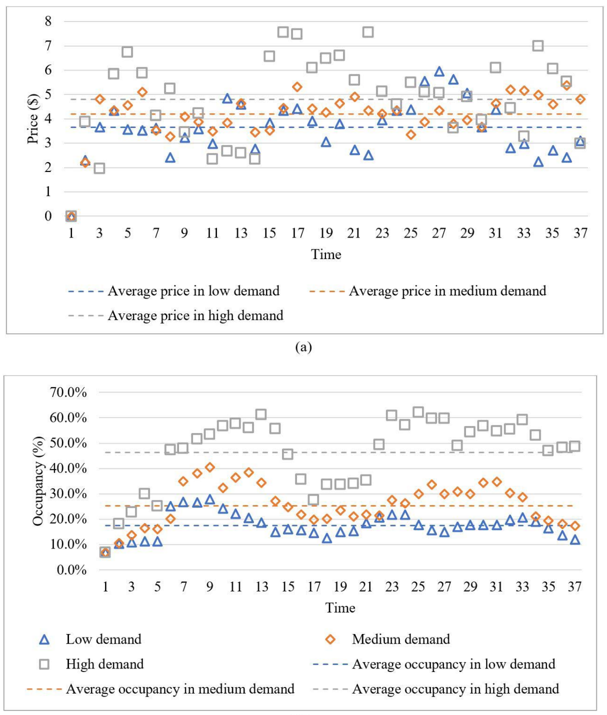


Fig. 3. Trends of low, medium and high demand for the hypothetical dataset for (a) price, and (b) occupancy.

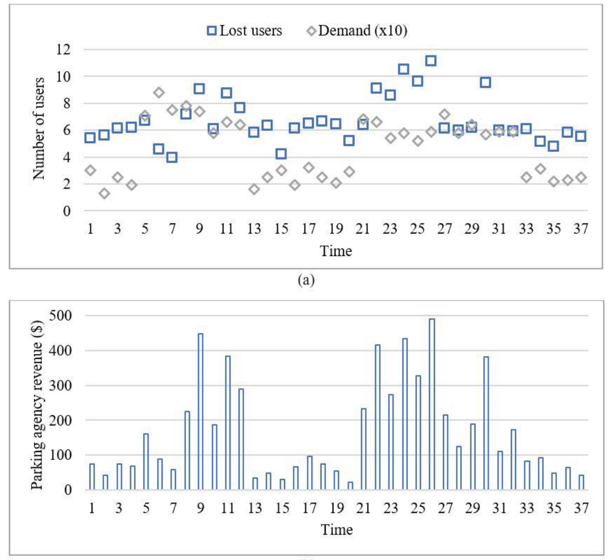


Fig. 4. (a) Number of lost users and (b) parking revenue for the low demand in the hypothetical dataset.

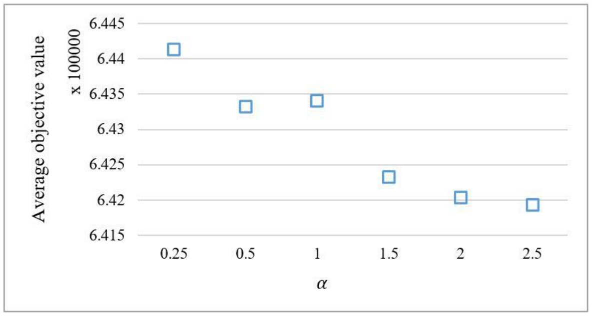
are considered in each search level to expand the tree (within the tree policy) that impose the price variations from \$2.0 – \$5.0 with \$1.0 increments. We assume that the minimum and maximum price of parking lots are \$0.2 and \$20.0, respectively.

1) Results: Figure 3 presents the average price and occupancy of parking lots within a day starting from 8 AM to 5 PM for low, medium, and high demand cases in the hypothetical dataset.

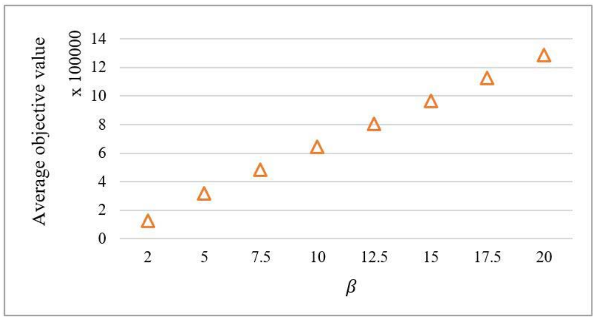
Parking prices are changing with respect to users arrival demand and occupancy of parking lots within the defined range. At the beginning of the study horizon, the prices are close to the minimum value where the parking lots are not fully occupied and then they increase as users from previous time steps as well as new incoming users occupy the available spots. The average occupancy over time follows the relative demand and remains less than the defined target occupancy by applying the dynamic price and cruising time information.

As indicated in Figure 3, the proposed dynamic pricing policy increases the average parking prices in high demand cases. It can be observed that the average and standard deviation of parking price is increased by 17.1% and 82.5% in the high demand case compared to the medium demand. The standard deviation captures the reaction of parking agencies to the number of users who search for available parking spots over time. The number of cruising users (including un-served and new users) increases when no available parking spot is realized, which significantly affects the average parking price in several time steps. On the other hand, in low demand cases, by decreasing the number of cruising users, the parking prices elevate at some time periods. In such cases, the ratio of available spots to user cruising (i.e., seeking for parking) is higher, which leads to lower values of dual multipliers in constraints (2g). Therefore, average parking prices can increase to maximize the parking agency's revenue, while the values of $u_{o\delta}^{t,n}$ are kept as low as possible, according to constraint (7c), to minimize the number of lost users.

Figure 4 presents the trade-off between parking agency revenue and the number of users who are not served by a parking spot at time t. The number of users varies due to price variations over time. While agencies may set high parking prices to increase their revenue, according to the figure, users are not willing to park at high prices and sometimes prefer to leave the parking system rather than paying extra dollars for available spots. Through our proposed methodology, the optimal price can provide a balance between the parking agency's revenue and users' costs and minimize the number of lost users. Uncertain relationship between parking price and demand, prevent using point elasticity for available functions. Therefore, we applied the arc elasticity coefficient, i.e., set to $b = -0.3$, to control the impact of minimum parking price on users who decide to use the parking system [47]. In other words, arc elasticity captures the recent impacts of changing the parking price on demand in the computations of the next time period. The CPU time for the low, medium, and high demand scenarios are $420\,sec$, $2,054\,sec$, and $16,537\,sec$, respectively.



(a)



(b)

Fig. 5. Objective values with respect to parking utilization and agency revenue: (a) α and (b) β.

2) Sensitivity Analyses: To evaluate the impact of parking utilization and parking agency revenue on the objective value (captured by (7a)), a set of sensitivity analyses is conducted on various parameter values, i.e., α and β, as shown in Figure 5. When the value of α increases, so does the objective function term on parking revenue, which imposes a negative impact on absorbing users as it increases the total cost (see Figure 5 (a)). According to Figure 5 (b), when the value of β increases, the impact of utilization improvement will be increased. Note that, according to Figure 5 (a) and (b), the choice of coefficient β is very significant as maximizing the utilization has a higher impact on the agency's pricing decisions compared to minimizing the total cost.

B. Benchmark

The proposed algorithm provides an accurate estimation of parking utilization and pricing in each time period and handles the computation complexity over the entire planning horizon. To evaluate the performance of the proposed algorithm, two benchmarks: (i) a greedy algorithm, and (ii) an exact method to solve the bi-level program, are introduced. The first benchmark consists of a rolling-horizon heuristic algorithm that solves the problem with known information at each t. In other words, our model (7b)-(7c) and (8)-(14a) is solved for $t = 0, 1, \ldots, T-1$, without a look-ahead policy at each iteration of the algorithm. Thus, the uncertainties involved in the problem, e.g., cruising time, are not considered at any time t. Table II specifically presents the parking agency's revenue and the number of lost users on average obtained from the proposed approach versus the greedy algorithm. This experiment illustrates that the proposed algorithm outperforms the greedy approach by an average improvement of 40.83% in the parking agency's revenue at the cost of losing 9.13% more users on average over the entire planning horizon.

TABLE II

COMPARISON WITH THE BENCHMARK SOLUTION OBTAINED BY THE GREEDY APPROACH

	Dynamic programming with look-ahead policy	Greedy approach	Difference (%)
Average parking agency revenue (\$)	167.69	119.07	40.83
Average number of lost users	6.68	6.12	9.13

The second benchmark is an alternative solution technique to solve the bi-level problem (2a)-(2g) to exact optimality over the entire planning horizon (see [33]). This approach theoretically evaluates the quality of our algorithm proposed in Section IV. The benchmark method generates theoretical lower- and upper-bounds to the proposed problem. To find the lower-bound, a global optimization problem is solved that includes the upper-level objective function (2a), constraints of both upper-level and lower-level problems (2b), (2d)-(2g), and parametric upper-bound of the optimal solution to the lower-level problem. The parametric upper-bound can be improved by adding new constraints to the global optimization problem through the following steps. First, the lower-level problem (2c)-(2g) is solved to global optimality given the value of upper-level decision variables p_j^t. Then, admissible values of the lower-level decisions x_{ij}^t, given the optimal objective value of lower-level problem (2d)-(2g), developed in the first step, are found. Finally, a subset within the bounds of p_j^t is obtained that satisfy the constraints of the lower-level problem. To find the upper-bound, similar global optimization problem is solved under constraints of the lower-level problem (2d)-(2g), given the value of upper-level decision variables p_j^t found in the lower-bound procedure.

Due to the computational burden, a small hypothetical dataset is used to compute the upper- and lower-bounds in the exact benchmark approach. The dataset contains two parking lots with two origins and one destination. The parking duration can be either 15 min or 30 min with a planning horizon of four time steps. The dataset includes an average demand of 11 users, where the capacity of parking lots is limited to 30 and 20 spots. The remaining parameters are the same as the hypothetical dataset introduced in Section V-A. The proposed algorithm in Sections III-IV is also applied to this case study for a comparison to the benchmark. Figure 6 shows the convergence of upper- and lower-bound with a gap of 2.5% for the bi-level optimization program, while the objective value obtained from the proposed methodology is 105.8, i.e., within the tight gap. The CPU time for the exact benchmark approach and the proposed algorithm is 132.4 hr and 0.8 sec, respectively, which indicates the computational efficiency of the proposed algorithm.

C. Real-World Dataset

The proposed formulation and solution technique are applied to a real-world case study in Pullman, Washington. The network includes 11 parking lots on Washington State

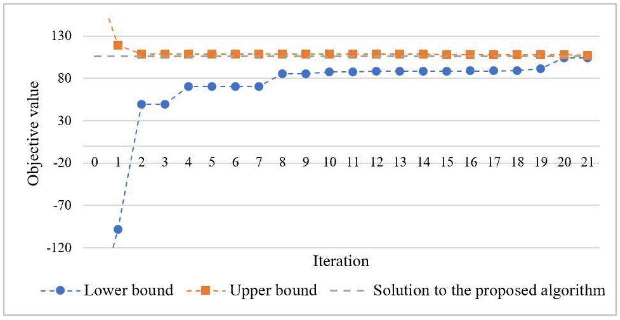


Fig. 6. Convergence of upper bound and lower bound for the solution of bi-level optimization model.

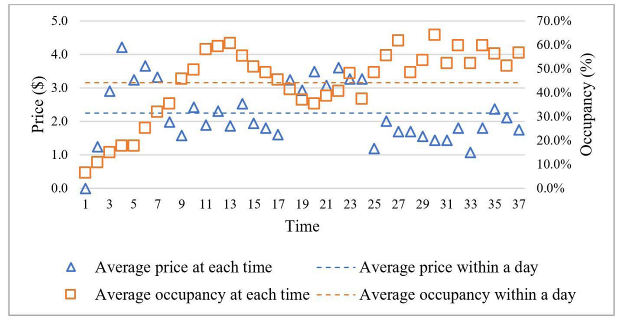


Fig. 8. The relationship between price and occupancy for medium demand case in the real-world dataset.

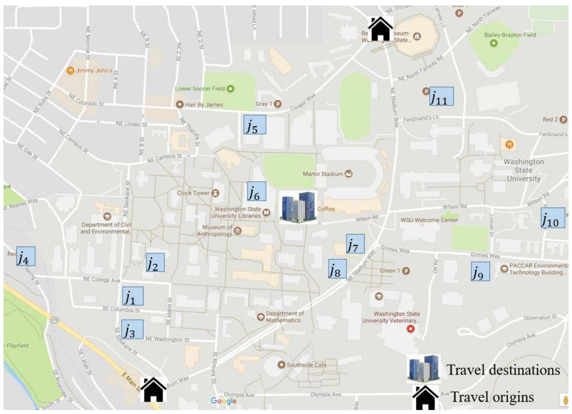


Fig. 7. Pullman, WA campus.

TABLE III

PARKING SYSTEM PARAMETERS IN PULLMAN, WA CAMPUS DATASET

Parking lot	Capacity	$\mu_{1,j}$ (min)	$\mu_{2,j}$ (min)	$\rho_{j,1}$ ($)
j_1	140	4	4	9
j_2	115	4.5	2.5	7
j_3	142	4	2	10
j_4	131	5	3	15
j_5	14	3	5	9
j_6	300	3.5	5.5	2
j_7	300	3	2	7
j_8	240	3	2	8
j_9	81	4	3	11
j_{10}	220	5	4	14
j_{11}	120	2	4	11

University campus as shown in Figure 7. Users include faculty, staff, students, and visitors who tend to park for more than one time step. Therefore, the parking durations of 15 min, 45 min, 1.5 hr, and 3 hr are considered to contain several parking options. The main library of the campus is considered as a destination where users enter the campus from two origins in the north and south point of the campus. Demand is assumed to be twice as high compared to the theoretical dataset for the medium demand level and $\alpha = 0.25$. The low and high demand levels are assumed to be 0.5 and 1.5 times of the medium demand level, respectively. Table III presents the capacity and travel time from each two points on the Pullman campus to the parking lots with minimum price of \$0.2 and maximum price of \$20.0.

Similar to the hypothetical dataset scenario, Figure 8 presents the average price and occupancy of parking lots within a day starting from 8 AM to 5 PM for medium demand case in the real-world dataset. Similar trends can be observed in the real-world case study, e.g., higher prices when the demand increases. However, as the users are able to park in the lots up to 3 hours, the results slightly differ from the hypothetical case study in terms of the occupancy and price variations over time. For example, the average parking price decreases by 46.3% while the average occupancy increases from 25.8% to 44.1% in the medium demand cases compared to the hypothetical dataset for all time steps. These trends are observed due to the accumulated revenue obtained by longer parking durations. Besides, the slight difference of standard deviations, realized in the average parking price (e.g., 6.1%), reports a similar trend in cruising time due to the additional demand and available parking spots at the same time. Unlike the hypothetical dataset, the average occupancy does not decrease by approaching the end of the day as users have parked for more time steps, i.e., longer durations. The CPU time for the low, medium, and high demand scenarios are $1,298\,sec$, $11,501\,sec$, and $22,710\,sec$, respectively.

VI. CONCLUSIONS

This paper studies a parking utilization and pricing scheme that aims to maximize the parking agency's revenue and minimize total travelers' costs simultaneously. Parking prices are assigned over time, while drivers choose their preferred spots considering parking prices, travel time for each trip, and cruising time needed to find a spot in each parking lot, given the information on the current occupancy. The problem is formulated into a mixed-integer bi-level problem, where the agency determines parking prices in the upper level while drivers' decisions are captured in the lower level. The bi-level problem is converted into an equivalent single-level dynamic programming model and solved using a stochastic look-ahead technique based on Monte Carlo tree search algorithm to determine the near-optimal actions (i.e., parking price and spot utilization over time). The proposed algorithm is applied to two datasets including a (i) hypothetical and (ii) real-world case study in Washington State University campus and compared with two benchmark solutions. The computational results show that the proposed algorithm is able to solve the problem effectively and outperforms the benchmark greedy solution by an average benefit of 40.83% (for parking agency revenue) at the cost of 9.13% increase in losing parking users over the planning horizon. Besides, the solution to the proposed algorithm falls within a tight theoretical gap (found from the exact solution technique to the bi-level problem), which indicates its capability in determining the near-optimal solution very efficiently (i.e., with a CPU time of 0.8 *sec* compared to 132.4 *hr* of the exact algorithm on a small dataset). Future research can be conducted in a few directions. It is very interesting to include a multi-agency competition to absorb users to respective parking lots under highly variable demand regions. Strategic facility location, similar to Hajibabai and Ouyang [18], [20], and capacity design is another interesting future research direction for this study. Furthermore, it is very interesting to study the integration of network route planning, e.g., [17], and parking facility management for service trucks. Besides, it is worthwhile to analyze the impact of real-time traffic data on network link travel times to draw more realistic insights on parking utilization management.

Algorithm 1 The Proposed Algorithm

procedure MCTS($\mathcal{S}^t$)
Create root node $\widetilde{\mathcal{S}}^{t,t}$ with state $\mathcal{S}^t$
set the iteration number $n = 0$ and $t \leftarrow t'$
while $n < N$
Tree policy
while $t' < t + H$
if $|\widetilde{\mathcal{A}}_e^{t,t'}(\widetilde{\mathcal{S}}^{t,t'})| < d^{thr}$
choose $\widetilde{a}^{*^{t,t'}}$ by solving the converted single-level problem
$\widetilde{\mathcal{S}}_a^{t,t'} = \mathcal{S}^{\mathcal{T}}(\mathcal{S}^{t,t'}, \widetilde{a}^{*^{t,t'}})$
$\widetilde{\mathcal{A}}_e^{t,t'}(\widetilde{\mathcal{S}}^{t,t'}) \leftarrow \widetilde{\mathcal{A}}_e^{t,t'}(\widetilde{\mathcal{S}}^{t,t'}) \cup \{\widetilde{a}^{*^{t,t'}}\}$
$\widetilde{\mathcal{A}}_u^{t,t'}(\widetilde{\mathcal{S}}^{t,t'}) \leftarrow \widetilde{\mathcal{A}}_u^{t,t'}(\widetilde{\mathcal{S}}^{t,t'}) - \{\widetilde{a}^{*^{t,t'}}\}$
else
$$\widetilde{a}^{*^{t,t'}} = \text{argmax}_{\widetilde{a}^{t,t'} \in \widetilde{\mathcal{A}}_e^{t,t'}(\widetilde{\mathcal{S}}^{t,t'})} \Big(-(\widetilde{f}^t(\widetilde{\mathcal{S}}^{t,t'}, \widetilde{a}^{t,t'}) + \widetilde{V}_a^{t,t'}(\widetilde{\mathcal{S}}_a^{t,t'})) + \iota \sqrt{\frac{\ln \mathcal{N}(\widetilde{\mathcal{S}}^{t,t'})}{\mathcal{N}(\widetilde{\mathcal{S}}^{t,t'}, \widetilde{a}^{t,t'})}} \Big)$$
$\widetilde{\mathcal{S}}_a^{t,t'} = \mathcal{S}^{\mathcal{T}}(\mathcal{S}^{t,t'}, \widetilde{a}^{*^{t,t'}})$
end if
Add exogenous information of average cruising time based on updated occupancies
$\widetilde{\mathcal{S}}^{t,t'+1}_{(\widetilde{a}^{t,t'}, \widetilde{\mathcal{W}}^{t,t'+1})} = \mathcal{S}^{\mathcal{T},a}(\widetilde{\mathcal{S}}_a^{t,t'}, \widetilde{\mathcal{W}}^{t,t'+1})$
Update the minimum parking cost $(u_{o,\delta}^{t',n})$
$t' \leftarrow t' + 1$
end while
Value function estimation
choose a sample path $\widetilde{\omega} \in \widetilde{\Omega}^{t,t'}(\widetilde{\mathcal{S}}^{t,t'})$
Update the demand for next time step based on $(u_{o,\delta}^{t',n})$
Solve the optimization model (15) to find the value of added node
Back-propagation
while $\widetilde{\mathcal{S}}^{t,t'} \neq \emptyset$
$N(\widetilde{\mathcal{S}}^{t,t'}) \leftarrow N(\widetilde{\mathcal{S}}^{t,t'}) + 1$
$\widetilde{\mathcal{S}}_a^{t,t^*} \leftarrow$ *predecessor of* $\widetilde{\mathcal{S}}^{t,t'}$
$N(\widetilde{\mathcal{S}}_a^{t,t^*}, \widetilde{a}^{t,t^*}) \leftarrow N(\widetilde{\mathcal{S}}_a^{t,t^*}, \widetilde{a}^{t,t^*}) + 1$
$\Xi \leftarrow \widetilde{f}^t(\widetilde{\mathcal{S}}^{t,t^*}, \widetilde{a}^{t,t^*}) + \widetilde{V}_a^{t,t^*}(\widetilde{\mathcal{S}}_a^{t,t^*})$
$$\widetilde{V}^{t,t^*}(\widetilde{\mathcal{S}}^{t,t^*}) \leftarrow \widetilde{V}^{t,t^*}(\widetilde{\mathcal{S}}^{t,t^*}) + \frac{\Xi - \widetilde{V}^{t,t^*}(\widetilde{\mathcal{S}}^{t,t^*})}{N(\widetilde{\mathcal{S}}^{t,t^*})}$$
$t' \leftarrow t^*$
end while
end while
return $a^{*^t} = \text{argmin}_{\widetilde{a}^{t,t} \in \widetilde{\mathcal{A}}_e^{t,t}(\widetilde{\mathcal{S}}^{t,t})} \widetilde{f}^t(\widetilde{\mathcal{S}}^{t,t}, \widetilde{a}^{t,t}) + \widetilde{V}_a^{t,t}(\widetilde{\mathcal{S}}_a^{t,t})$
end procedure

APPENDIX A

See Algorithm 1.

APPENDIX B

TABLE IV

DEFINITIONS OF SETS, DECISION VARIABLES, STATE VARIABLES, AND PARAMETERS

Sets	
$\Gamma = \{0, 1, \ldots, T-1\}$	Set of all time steps
Υ	Set of physical parking areas in an urban neighborhood
$\mathcal{O}$	Set of origins
Δ	Set of destinations
$\lambda_{o\delta}^{t,n}$	Set of users travel from $o \in \mathcal{O}$ to $\delta \in \Delta$ for parking duration n at time period t
Ω^{π}	Set of outcomes that depend on policy π
Decision Variables	
p_j^t	Parking price of each parking lot $j \in \Upsilon$ at time t
$c_{j,o\delta}^{t,n}$	Total user cost from $o \in \mathcal{O}$ and $\delta \in \Delta$ to park in lot j for n time periods at time t
$h_{j,o\delta}^{t,n}$	Number of users cruising for parking spot in j
ψ_j^t	Total number of occupied parking spots and users cruising in parking lot j at time t
$u_{o\delta}^{t,n}$	Minimum total user cost amongst all parking choices
$q_j^{t,n}$	Number of arrivals with parking duration n in parking lot j at time t
x_{ij}^t	Equals 1 if user i choose to park in parking lot j at time t, and 0 otherwise
State Variables	
$\widehat{\mathcal{J}}_j^t$	Number of parking spots that first become available at time t
$\widehat{\mathcal{J}}^t$	Spatial distribution of all newly realized parking spots at time t
$\mathcal{J}_j^t$	Number of parking spots already available in parking lot j at time t before any new spot availability
$\mathcal{J}^t$	Total number of parking spots that are already available at time t
$\mathcal{J}^{t^+}$	Total number of available parking spots in time period t
$\widehat{\mathcal{D}}^t$	Set of users that first become available at time t
$\mathcal{D}^t$	Set of available users at time t before the new arrivals in the system
$\mathcal{D}^{t^+}$	Set of available users at time t, including the new users just arrived
$\mathcal{D}_j^{t^+}$	Set of users that must be served by parking spots in area j at time t
$\widehat{\mathcal{L}}_j^t$	Expected average cruising time for users going to parking j at time t
$\widehat{\mathcal{L}}^t$	Set of newly realized cruising times at time t
$\mathcal{W}^t$	Stochastic information process with realization $\mathcal{W}^t(\omega) = \omega^t$ at each t
$\mathcal{S}^t$	State of the system at each time t
g_j^t	Number of departures from parking lot j at time t
σ_j^t	Occupancy of parking area j at time t
Parameters	
c_j	Capacity of parking $j \in \Upsilon$
μ_{oj}	Driving cost from origin $o \in \mathcal{O}$ to parking spot in $j \in \Upsilon$
$\rho_{j\delta}$	Walking cost from the parking location in $j \in \Upsilon$ to final destination $\delta \in \Delta$
θ, θ'	Coefficients to convert travel time to monetary cost
κ_j	Target occupancy rate of parking area j
e_j	Average cruising time for parking lot j
Z_j	Reaction of drivers to the provided information, $Z_j = 1$ when they are completely aware of the updated occupancies
M	A sufficiently large number
$\mathcal{N}$	Maximum number of consecutive time periods to stay in parking lots
l_i	Minimum parking price for parking $j \in \Upsilon$
u_i	Maximum parking price for parking $j \in \Upsilon$
$\varepsilon, \varepsilon'$	Lower and upper bound of price variation between two consecutive time steps
${a'}_{o\delta}^{t,n}$	Linear demand curve's intercept for users with $o\delta \in \mathcal{O}\Delta$ to park at time t for n time steps
b	Demand elasticity coefficient
d^{thr}	Maximum number of actions in the set of possible actions

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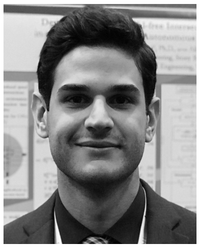

Amir Mirheli received the B.Sc. degree in civil engineering from the Sharif University of Technology in 2013 and the M.Sc. degree in construction engineering and management from the University of Tehran in 2015. He is currently pursuing the Ph.D. degree with the Department of Civil Engineering, The State University of New York (SUNY) at Stony Brook. His research interests include transportation and logistics systems optimization, with a particular focus on developing management strategies for transportation facilities involving uncertainties. He is a member of the Institute for Operations Research and the Management Sciences (INFORMS).

Leila Hajibabai received the M.Sc. degrees in industrial engineering and in civil engineering and the Ph.D. degree in civil engineering, transportation systems, from the University of Illinois at Urbana–Champaign in 2014. She is currently an Assistant Professor with the Department of Civil Engineering, The State University of New York (SUNY) at Stony Brook. Her research program focuses on operations research applications in transportation systems with a specific emphasis on resilient and economic city logistics concerning human-made decisions and emerging technologies. She is a member of the Transportation Research Board (TRB)'s Standing Committee on Maintenance Equipment (AHD60) and the Section on Maintenance and Preservation (AHD00). She is also a member of the Institute for Operations Research and the Management Sciences (INFORMS). She is a Co-Chair of the Operations and Preservation Group, TRB Young Members Council. She has actively participated in various professional activities of the ADB30 Transportation Network Modeling Standing Committee and is a member of the ADB30-5 Emerging Technologies in Network Modeling Sub-Committee of TRB.