# Minimum Time Headway in Platooning Systems Under the MPF Topology for Different Wireless Communication Scenario

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***Abstract*— The multiple-predecessor following (MPF) topology is used in vehicle platoons to make it robustly string stable and reduce the minimum employable time headway. It has been demonstrated that communication imperfections such as time delays coming from wireless communications can affect string stability as well as the minimum time headway required to guarantee string stability. Specifically, it was shown that the larger the time delay, the longer the minimum time headway will be. However, by utilizing on-board vehicle sensors, such as radar, lidar and cameras, the distance and speed of nearby vehicles can be measured almost instantaneously, i.e., with almost no delay. Another effective parameter on string stability and minimum time headway is the heterogeneity of the vehicles. Due to the immense complexity of the MPF topology, string stability analysis of this topology in literature has been confined to homogeneous platoons. In this paper, we consider the case of heterogeneous platoons under the MPF topology with the use of the combination of sensors and wireless communications for receiving information. Following that, we find conditions to guarantee the internal and string stability for the heterogeneous case and propose the minimum time headway required to guarantee string stability. Finally, we provide a table, in which we propose the minimum time headway for two other wireless communication scenarios as well: *(i)* having no communication delay and *(ii)* having fully-delayed information, i.e., all information, whether it comes from the ego vehicle or its predecessors, is delayed. In addition to exploring the analysis of string stability for the vehicles with more possible connections (vehicles after the $r^{th}$ vehicle, when information from $r$ immediate vehicles is used), we study the string stability conditions (with which we aim at avoiding collisions) and find the minimum time headway for the first few vehicles (vehicle $r$ and all its predecessors). Numerical results clearly show the effectiveness of the proposed lower bounds.**

***Index Terms*— Platooning, multiple-predecessor following, communication delays, sensors, string stability, time headway, heterogeneous platoons.**

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## I. Introduction

VEHICLE platooning is a promising method that can significantly increase traffic throughput while simultaneously reducing fuel consumption [2], [3], [4]. When setting up a vehicle platoon we seek for two main properties; namely, internal stability and string stability. Internal stability is the general stability required for each system, and refers to the ability of the vehicles to maintain the desired inter-vehicle distance from their predecessors as well as moving at a desired speed. By having internal stability, the spacing error between vehicles will converge to zero. However, during acceleration, deceleration or any other disturbances acting on the preceding vehicles, the spacing error can propagate along the string and cause a traffic jam or stop-and-go traffic, which reduces the benefits of platooning [5], [6], [7]. Therefore, the ability of attenuating the effects of disturbances along the string, called string stability, is essential for vehicle platoons.

The desired inter-vehicle distance is determined by the spacing policy, which greatly impacts string stability. The most common spacing policies are the Constant Spacing Policy (CSP) [8], [9] and the Constant Time Headway Spacing Policy (CTHP) [10], [11], [12]. A constant value defines the desired inter-vehicle distance in CSP, while for CTHP, the desired distance is a linear function of the speed, with the proportional gain called time headway. There are also more complex spacing policies in the literature, such as delay-based spacing policy [13], semi-constant spacing policy [14] and nonlinear spacing policies [15]. For CTHP, which is the spacing policy under consideration in this paper, the inter-vehicle distance increases with the speed, which results in a reduction in traffic capacity. For that reason, a small value for the time headway is required to maximize the road capacity. On the other hand, a very small time headway can result in string instability. Therefore, finding the minimum acceptable time headway, which can guarantee string stability while improving the use of road capacity, carries great significance in platooning. Many studies have considered the problem of finding the minimum time headway and showed that this value is significantly influenced by the information flow topology within the platoon as well as the communication delay. Early-stage platoons could only gather information through radar or onboard sensors, known as predecessor following (PF) topology. For the platoons under this topology, the relationship between the minimum time headway and the time lag (it will be defined in the system model) is proposed in [16]. This relationship is further elaborated in [17], for both homogeneous and heterogeneous platoons with time delays. By using Vehicle-to-Vehicle (V2V) communications, a form of wireless communication, vehicles can receive information and communicate with many vehicles in the platoon and thus, various topologies have emerged, including predecessor leader following (PLF), bidirectional leader (BDL) and MPF topology. The minimum employable time headway for platoons with time lags under the MPF topology, the topology considered in this work, is presented in [18] and [19], using two different definitions of the desired inter-vehicle distance. The work [19] was expanded in [20], by considering the effects of communication delays on the minimum time headway. All of the aforementioned works show that larger time lags and time delays lead to a larger minimum time headway, as it is expected. Moreover, it has been proven that by increasing the number of connected vehicles, as in the case with the MPF topology, a smaller minimum time headway can be achieved compared to the basic PF topology and, therefore, platoons can greatly benefit from the MPF topology. The work [21] provides a complete analysis of the effects of information flow topology on the platoon's ability to reject disturbances, detect cyber-attacks, and resist them. The MPF topology, referred to as $k$-nearest neighbor topology in [21], may well have the desired security and performance levels while topologies that just employ information from the nearest predecessor or follower are demonstrated to be weak. Given all of this, this paper will focus on the MPF topology.

All of the works mentioned heretofore, which consider the MPF topology, analyze string stability and find the minimum time headway for homogeneous platoons, where the time lag and time headway are the same for all vehicles. In reality, platoons on the roads are formed by different vehicles and therefore time lags vary. Also, a higher road throughput can be achieved by considering non-identical time headways rather than using the most conservative time headway. A heterogeneous vehicle platoon is unquestionably a more realistic scenario, and hence, analyzing string stability and computing the minimum time headway in this scenario is more important.

In this work, we focus on the MPF topology and analyze internal stability and string stability of heterogeneous platoons, where time lags and time headways are non-identical and also, we propose the minimum acceptable time headway to guarantee string stability. String stability in heterogeneous platoons has been analyzed and widely studied in the literature, although not with the spacing policy and information flow topology examined in this work. For example, string stability was studied in [22] for CSP under predecessor-successor following topology, in [23] for nonlinear model of vehicles with CSP under two-predecessor-leader following topology, [24] for CTHP and the PF topology equipped with Cooperative Adaptive Cruise Control (CACC), in [25] for nonlinear vehicle models with CTHP under predecessor-successor following topology using asymmetric bidirectional control algorithms, and in [26] for multiple-predecessor-leader following (MPLF) topology with CTHP, while leader's velocity, which is constant, is used to define the desired inter-vehicle distance and the signal of interest for analyzing string stability is the error between the follower and the leader.

The MPF topology has been made possible by using V2V wireless communication, which, due to its wireless nature, inevitably causes time delays (due to, e.g., encoding and decoding of messages, packet losses that trigger retransmissions, etc). In [20], it is assumed that the controller of each vehicle only has access to delayed information, i.e., its own states as well as all the predecessors' states come through wireless communication, which has a time delay. To reduce the effects of communication delays, sensors like radar, lidar or cameras can be used together with V2V communications [27], [28]. Therefore, assuming that each vehicle has access to its own position, velocity and acceleration with no delays, by utilizing onboard sensors, the position and velocity of its immediate predecessor can be obtained easily without any time delay. Only the acceleration of the immediate predecessor [29] as well as the position, velocity and acceleration of vehicles farther along the string have to be received through wireless communication. In this work, we focus on such systems (which combine sensors and V2V communications) and analyze their internal and string stability.

Another important issue, that is not usually addressed in the literature, is the minimum employable time headway of the first few vehicles of a platoon. In fact, all the related aforementioned works that consider a platoon under the MPF topology, in which each vehicle is connected to '$r$' immediate predecessors, find the relationship between the minimum time headway and the number of connections '$r$'. However, for the first few vehicles, i.e., before the $r^{th}$ vehicle, there are obviously fewer possible connections and as a result, the minimum acceptable time headway of these vehicles is different than the vehicles after the $r^{th}$ vehicle. While one would argue that string stability is a property associated with a long string, it is also associated indirectly with the collision of vehicles. Establishing a different time headway for the first $r$ vehicles (instead of having one for all vehicles), *i)* the collision between them is more likely to be avoided (since the computed time headway is larger), and *ii)* the error propagated through the string will be eventually smaller. This is demonstrated via illustrative examples in the numerical evaluations. The main contributions of this paper are listed below:

- We consider a platoon, under the MPF topology, when both on-board sensors and V2V communication are used. We have proven that, regardless of the size of the delay or heterogeneity of the vehicles, this type of platoon can be internally stable.
- String stability for the homogeneous platoons are analyzed and a lower bound on time headway is proposed.
- String stability and the minimum time headway have been investigated in the case of heterogeneous platoons for all

vehicles, including those before and following the $r^{th}$ vehicle.

- Finally, a table is provided that summarizes the minimum time headway for heterogeneous platoons, under the MPF topology in different scenarios: *(i)* with no communication delay, *(ii)* with fully-delayed information, and *(iii)* with partially-delayed information, as explained above.

The remainder of this paper is structured as follows. Section II gives the notation and some necessary mathematical preliminaries. In Section III, vehicle model and control structure are presented and the problem is formulated. Internal stability is analyzed in Section IV. Section V analyzes string stability for homogeneous as well as heterogeneous platoons. Numerical results are provided in Section VI to show the effectiveness of the proposed theorems. We conclude the paper in Section VII.

## II. Notation and Mathematical Preliminaries

### A. Notation

Vectors and matrices are denoted by lowercase and uppercase letters, respectively. Integer and natural numbers sets are denoted by $\mathbb{Z}$ and $\mathbb{N}$, respectively. $\mathbb{Z}_0 \triangleq \{0, 1, 2, \ldots\}$, $\mathbb{Z}_i^n \triangleq \{i, i+1, i+2, \ldots, n\}$, and $\mathbb{N}^n \triangleq \{1, 2, \ldots, n\}$. Real and nonnegative real numbers sets are denoted by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. $m \times n$ real matrices are denoted by $\mathbb{R}^{m\times n}$. For any matrix $A \in \mathbb{R}^{m\times n}$, $(m, n) \in \mathbb{N} \times \mathbb{N}$, we denote its transpose by $A^T$ and its entries by $a_{ij}$, $i \in \mathbb{N}^m$, $j \in \mathbb{N}^n$ (i.e., $A = [a_{ij}]$). The $n \times n$ identity matrix is denoted by $I_n$.

### B. Mathematical Preliminaries

*Lemma 1 ([30]):* Suppose $A$, $B$, $C$ and $D$ are matrices of dimension $n \times n$, $n \times m$, $m \times n$ and $m \times m$, respectively. Then, if $A$ is invertible, for the block matrix we have

$$\det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A)\det(D - CA^{-1}B). \tag{1}$$

In this paper, we consider the vehicle platoon as a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is a set of nodes representing all the following vehicles and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges representing the connections between each pair of following vehicles. The following matrices characterize some properties of $\mathcal{G}$. First, the Laplacian matrix associated with $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]$, $i, j \in \mathbb{N}^N$, with

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{k=1}^{N} a_{ik}, & i = j, \end{cases} \tag{2}$$

where $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. Also, the connections between the vehicles and the leader can be modeled by $\mathcal{P} = \mathrm{diag}\{p_{11}, p_{22}, \ldots, p_{NN}\}$, where $p_{ii} = 1$ when vehicle $i$ receives information from the leader and $p_{ii} = 0$, otherwise. Then, a new information topology matrix can be defined as

$$\mathcal{L}_p := \mathcal{L} + \mathcal{P}. \tag{3}$$

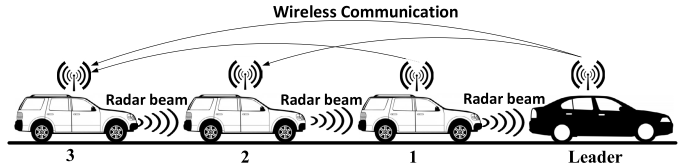



Fig. 1. A platoon under MPF topology, using V2V communications and onboard sensors.

## III. Problem Statement

### A. Vehicle Model

Consider a platoon of $N$ vehicles following a leader. The longitudinal dynamics of vehicle $i$ is described by the following simplified nonlinear model, as in, e.g., [31]

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = \frac{1}{m_{i,veh}}\left(\eta_{T,i}\frac{T_i(t)}{R_i} - C_{A,i}v_i^2(t) - m_{i,veh}gf\right), \\ \tau_i\dot{T}_i(t) + T_i(t) = T_{i,des}(t), \end{cases} \tag{4}$$

where $p_i(t)$ and $v_i(t)$ are the position and velocity of the $i$th vehicle, $m_{i,veh}$ is the vehicle mass, $C_{A,i}$ is the lumped aerodynamic drag coefficient, $g$ is the acceleration due to gravity, $f$ is the coefficient of rolling resistance, $T_i(t)$ denotes the actual driving/braking torque, $T_{i,des}(t)$ is the desired driving/braking torque, $\tau_i > 0$ is the inertial delay of vehicle longitudinal dynamics or the time lag in the powertrain, $R_i$ denotes the tire radius, and $\eta_{T,i}$ is the mechanical efficiency of driveline. The work [31] shows that the third-order linear model below can be derived from (4)

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = a_i(t), \\ \tau_i\dot{a}_i(t) + a_i(t) = u_i(t), \end{cases} \tag{5}$$

where $a_i(t)$ and $u_i(t)$ are the acceleration and control input of the $i$th vehicle, respectively. Also, $p_0(t)$, $v_0(t)$, $a_0(t)$ and $u_0(t)$ are the position, velocity, acceleration and control input of the lead vehicle, respectively.

It is assumed that vehicle $i$ can use information from multiple predecessor vehicles. An example is shown in Fig. 1, where vehicle $i$, $i \in \mathbb{Z}_3^N$, is connected to three predecessor vehicles. For this topology, the desired distance $d_{i,i-l}(t)$ between vehicle $i$ and the $l$-th vehicle ahead of it is considered as [19]

$$d_{i,i-l}(t) = \sum_{k=i-l+1}^{i}\left(h_k v_k(t) + d_k\right), \tag{6}$$

where $h_k \geq 0$ is the time headway of vehicle $k$ and $d_k > 0$ is the desired standstill gap between vehicle $k$ and $k-1$.

### B. Control Structure

The following linear feedback controller is used in [19], for vehicle $i$ when there are no time-delays:

$$\begin{aligned} u_i(t) = -\sum_{l=1}^{r_i}\Bigg(k_{pi}\Big(p_i(t) - p_{i-l}(t) \\ + \sum_{k=i-l+1}^{i}\left(h_k v_k(t) + d_k\right)\Big) \\ + k_{vi}\Big(v_i(t) - v_{i-l}(t)\Big) + k_{ai}\Big(a_i(t) - a_{i-l}(t)\Big)\Bigg), \end{aligned} \tag{7}$$

where $r_i \leq i$ is the number of the vehicles directly ahead of vehicle $i$ that send their information to it. Control parameters $k_{pi}$, $k_{vi}$ and $k_{ai}$ are positive tunable gains for feeding back distance, velocity and acceleration errors between vehicle $i$ and its $l$-th vehicle ahead.

### C. Problem Formulation

We assume that the controller of each vehicle knows its own states (position, velocity and acceleration) as well as the position and velocity of its immediate predecessor with no time delays, i.e., $\{p_i(t), v_i(t), a_i(t), p_{i-1}(t), v_{i-1}(t)\}$. However, the acceleration of the immediate predecessor as well as information from all other connected vehicles come through the wireless network and thus, have time delays, i.e., $\{a_{i-1}(t-\Delta), p_{i-l}(t-\Delta), v_{i-l}(t-\Delta), a_{i-l}(t-\Delta)\}$, $\forall 2 \leq l \leq r_i$, where $\Delta$ is the homogeneous time-delay. With this type of information, controller (7) therefore changes to

$$\begin{aligned} u_i(t) = &-k_{pi}\Big(p_i(t) - p_{i-1}(t) + h_i v_i(t) + d_i\Big) \\ &-k_{pi}\sum_{l=2}^{r_i}\Big(p_i(t) - p_{i-l}(t-\Delta) - \Delta v_0 \\ &+ \sum_{k=i-l+1}^{i}\big(h_k v_k(t-\beta_{ki}\Delta) + d_k\big)\Big) \\ &-k_{vi}\Big(v_i(t) - v_{i-1}(t)\Big) \\ &-k_{vi}\sum_{l=2}^{r_i}\Big(v_i(t) - v_{i-l}(t-\Delta)\Big) \\ &-k_{ai}\sum_{l=1}^{r_i}\Big(a_i(t) - a_{i-l}(t-\Delta)\Big), \end{aligned} \tag{8}$$

where

$$\beta_{ki} = \begin{cases} 0, & k = i \quad \text{or} \quad k = i-1 \\ 1, & \text{otherwise.} \end{cases} \tag{9}$$

The communication time delay can be computed in automated vehicles, and thus $\Delta v_0$ as a supplement is proposed in the literature, such as [32] and [33], in order to mitigate the negative impact of communication delay and reduce the difference between $p_{i-l}(t-\Delta)$ and $p_{i-l}(t)$. The speed of the lead vehicle $v_0$ is assumed to be constant and also available for all vehicles. Also, the controller of each vehicle receives information on the value of its predecessors' time headways via V2V communications.

In order for vehicles to keep track of the desired inter-vehicle distance and speed, the control parameters must be prudently selected. This objective is explored in Section IV. In addition to that, the controllers must be able to prevent the propagation of disturbances along the vehicle string. This goal is analyzed in Section V.

## IV. Internal Stability Analysis

In this section, we first find the closed loop platoon dynamics and then we provide necessary and sufficient conditions, which guarantee the asymptotic stability of the vehicle platoon.

Position, velocity and acceleration error with respect to the lead vehicle's states are defined as

$$\begin{cases} \bar{p}_i(t) = p_i(t) - p_0(t) + \sum_{k=1}^{i}(h_k v_k(t) + d_k), \\ \bar{v}_i(t) = v_i(t) - v_0, \\ \bar{a}_i(t) = a_i(t) - a_0. \end{cases} \tag{10}$$

Since the leader's velocity is constant, we have $u_0(t) = 0$ and $a_0(t) = 0$. Using (5), the error dynamics is obtained as

$$\begin{cases} \dot{\bar{p}}_i(t) = \bar{v}_i(t) + \sum_{k=1}^{i} h_k \bar{a}_k(t), \\ \dot{\bar{v}}_i(t) = \bar{a}_i(t), \\ \dot{\bar{a}}_i(t) = -\frac{1}{\tau_i}\bar{a}_i(t) + \frac{1}{\tau_i}u_i(t). \end{cases} \tag{11}$$

Also, after algebraic manipulations, the control input (8) can be rewritten as

$$u_i(t) = -k_{pi}\big(\bar{p}_i(t) - \bar{p}_{i-1}(t)\big) \tag{12a}$$
$$-k_{pi}\sum_{l=2}^{r_i}\big(\bar{p}_i(t) - \bar{p}_{i-l}(t-\Delta)\big) \tag{12b}$$
$$+k_{pi}\sum_{l=2}^{r_i}\Big(\sum_{k=1}^{i-2} h_k\big(\bar{v}_k(t) - \bar{v}_k(t-\Delta)\big)\Big) \tag{12c}$$
$$-k_{vi}\big(\bar{v}_i(t) - \bar{v}_{i-1}(t)\big) \tag{12d}$$
$$-k_{vi}\sum_{l=2}^{r_i}\big(\bar{v}_i(t) - \bar{v}_{i-l}(t-\Delta)\big) \tag{12e}$$
$$-k_{ai}\sum_{l=1}^{r_i}\big(\bar{a}_i(t) - \bar{a}_{i-l}(t-\Delta)\big). \tag{12f}$$

Term (12c) can be rewritten as

$$k_{pi}(r_i - 1)\Big(\sum_{k=1}^{i-2} h_k\big(\bar{v}_k(t) - \bar{v}_k(t-\Delta)\big)\Big). \tag{13}$$

Then, let $\bar{p} = \big[\bar{p}_1, \ldots, \bar{p}_N\big]^\top$, $\bar{v} = [\bar{v}_1, \ldots, \bar{v}_N]^\top$, $\bar{a} = [\bar{a}_1, \ldots, \bar{a}_N]^\top$ and $\xi = [\bar{p}^\top, \bar{v}^\top, \bar{a}^\top]^\top$. Using (12), the closed loop platoon dynamics can be obtained as

$$\begin{aligned} \dot{\xi}(t) &= A\xi(t) + A_\Delta \xi(t-\Delta), \\ \xi(t) &= \Phi(t), \quad t \in [-\Delta, 0], \end{aligned} \tag{14}$$

where $\Phi(\cdot) \in \mathcal{C}([-\Delta, 0], \mathbb{R}^\nu)$ represents the initial state of the system, and $A$ and $A_\Delta \in \mathbb{R}^{\nu\times\nu}$, $\nu = 3N$, are given as

$$A = \begin{bmatrix} 0 & I_N & H \\ 0 & 0 & I_N \\ -\mathrm{T}K_p\mathcal{L}_{p_1} & -\mathrm{T}(K_v\mathcal{L}_{v_1} + K_pFH) & -\mathrm{T}-\mathrm{T}K_a\mathcal{L}_{a_1} \end{bmatrix},$$
$$A_\Delta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\mathrm{T}K_p\mathcal{L}_{p_2} & -\mathrm{T}(K_v\mathcal{L}_{v_2} - K_pFH) & -\mathrm{T}K_a\mathcal{L}_{a_2} \end{bmatrix},$$

with

$$\begin{aligned} K_m &= \mathrm{diag}\{k_{m1}, \ldots k_{mN}\}, \qquad m \in \{p, v, a\}, \\ \mathrm{T} &= \mathrm{diag}\{1/\tau_1, \ldots 1/\tau_N\}, \end{aligned}$$

and

$$H = \begin{bmatrix} h_1 & 0 & \dots & 0 \\ h_1 & h_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ h_1 & h_2 & \dots & h_N \end{bmatrix}.$$

Moreover, we have

$$\mathcal{L}_{p_1} = \mathcal{L}_{v_1} = \begin{bmatrix} r_1 & 0 & 0 & \dots & 0 \\ \text{-1} & r_2 & 0 & \dots & 0 \\ 0 & \text{-1} & r_3 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \text{-1} & r_N \end{bmatrix},$$

$$\mathcal{L}_{a_1} = \begin{bmatrix} r_1 & 0 & 0 & \dots & 0 \\ 0 & r_2 & 0 & \dots & 0 \\ 0 & 0 & r_3 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & r_N \end{bmatrix},$$

and then, $\mathcal{L}_{p_2}$, $\mathcal{L}_{v_2}$ and $\mathcal{L}_{a_2}$ can be computed using

$$\mathcal{L}_{p_1} + \mathcal{L}_{p_2} = \mathcal{L}_{v_1} + \mathcal{L}_{v_2} = \mathcal{L}_{a_1} + \mathcal{L}_{a_2} = \mathcal{L}_p, \tag{15}$$

where $\mathcal{L}_p$ is defined in (3). Also, we have

$$F = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ f_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & f_4 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & f_N & 0 & 0 \end{bmatrix},$$

where $f_i = r_i - 1$.

Now, we present necessary and sufficient conditions, which guarantee the asymptotic stability of the vehicle platoon (14). More specifically, we show that the internal stability of the system does not depend on the magnitude of the delays.

*Theorem 1:* Closed loop system (14) with $\Delta \geq 0$, is internally stable if and only if

$$\frac{1}{\tau_i}(1 + k_{ai} r_i)(k_{vi} + k_{pi} h_i) > k_{pi}, \quad \forall i \in \mathbb{N}^N. \tag{16}$$

*Proof*: See Appendix A.

*Remark 1:* As a result of Theorem 1, a controller that guarantees internal stability can always be found by simply selecting the parameters such that (16) holds. In this way, communication delays, regardless of their magnitude, do not have an impact on internal stability. It is important to note that internal stability conditions might change if the leader's speed is time varying. This case is beyond the scope of this paper.

## V. String Stability Analysis

In this section, we analyze the string stability of the vehicle platoon for two different cases: first, we assume a homogeneous platoon, in which all parameters are the same for all vehicles; second, all the parameters are the same, except for the time headway $h_i$ and the time lag $\tau_i$ that can be different for each vehicle.

### A. Homogeneous Platoon

We assume a homogeneous platoon, in which $\tau_i = \tau > 0$, $r_i = r$, $k_{pi} = k_p$, $k_{vi} = k_v$ and $k_{ai} = k_a$, while the time headway is also identical for all vehicles $h_i = h$. Since we assume $r_i = r$, we can only analyze the string stability for the vehicle $i$, $i > r$.

*1) String Stability Analysis for Vehicle $i$, $i > r$:* From the vehicle dynamics (5) we have

$$\tau \dddot{p}_i(t) + \ddot{p}_i(t) = u_i(t) \tag{17}$$

and

$$\tau \dddot{p}_{i-1}(t) + \ddot{p}_{i-1}(t) = u_{i-1}(t). \tag{18}$$

The time derivative of (17) is

$$\tau \dddot{v}_i(t) + \ddot{v}_i(t) = \dot{u}_i(t). \tag{19}$$

Then, after calculating (17) − (18) + h × (19), substituting from (8) and (21), and considering $h_i = h$, we obtain

$$\begin{aligned} \tau \dddot{e}_i(t) &+ (1 + rk_a)\ddot{e}_i(t) + r\left(k_v + k_p h\right)\dot{e}_i(t) + rk_p e_i(t) \\ &= k_a \ddot{e}_{i-1}(t - \Delta) + \left(k_v - k_p h(r-1)\right)\dot{e}_{i-1}(t) + k_p e_{i-1}(t) \\ &+ \sum_{l=2}^{r} \Big(k_a \ddot{e}_{i-l}(t - \Delta) + \left(k_v - k_p h(r-l)\right)\dot{e}_{i-l}(t - \Delta) \\ &+ k_p e_{i-l}(t - \Delta)\Big), \end{aligned} \tag{20}$$

where

$$e_i(t) = p_i(t) - p_{i-1}(t) + h_i v_i(t) + d_i. \tag{21}$$

After taking the Laplace transform, we have

$$E_i(s) = H_1(s)E_{i-1}(s) + \sum_{l=2}^{r} H_l(s)E_{i-l}(s), \tag{22}$$

where $E_i(s)$ is the Laplace transformation of $e_i(t)$ and

$$H_1(s) = \frac{k_a s^2 e^{-\Delta s} + \left(k_v - k_p h(r-1)\right)s + k_p}{\tau s^3 + (1 + rk_a)s^2 + r\left(k_v + k_p h\right)s + rk_p} \tag{23}$$

and

$$H_l(s) = \frac{k_a s^2 e^{-\Delta s} + \left(k_v - k_p h(r-l)\right)s e^{-\Delta s} + k_p e^{-\Delta s}}{\tau s^3 + (1 + rk_a)s^2 + r\left(k_v + k_p h\right)s + rk_p}. \tag{24}$$

In order to determine string stability under the MPF topology, we adopt the following definition from [19],

*Definition 1:* A platoon under the MPF topology is strictly $\mathcal{L}_2$ string stable if

$$\|e_i(t)\|_2^2 \leq \frac{1}{r}\sum_{l=1}^{r} \|e_{i-l}(t)\|_2^2, \tag{25}$$

where $\|e_i(t)\|_2^2 = \int_{-\infty}^{+\infty} |e_i(t)|^2 \, dt$.

Based on (25), the $\mathcal{L}_2$ spacing error must be attenuated so that it is less than the average of its predecessors' $\mathcal{L}_2$ spacing errors. It is proved in [19] that the following string stability specification proposed by [34] is a sufficient condition for (25)

$$\sum_{l=1}^{r} \|H_l(j\omega)\|_\infty \leq 1, \tag{26}$$

where $\|H(j\omega)\|_\infty = \sup_{\omega>0} |H(j\omega)|$. Then, since $\lim_{\omega\to 0^+} |H_l(j\omega)| = \frac{1}{r}$, condition (26) holds if and only if

$$\|H_l(j\omega)\|_\infty \leq \frac{1}{r}, \forall 1 \leq l \leq r. \tag{27}$$

Using (27), the platoon (5) with controller (8) is string stable if both of the following conditions hold:

$$\|H_1(j\omega)\|_\infty \leq \frac{1}{r}, \tag{28a}$$

$$\|H_l(j\omega)\|_\infty \leq \frac{1}{r}, \quad \forall 2 \leq l \leq r, \tag{28b}$$

where $H_1(j\omega)$ and $H_l(j\omega)$ are derived by substituting $s = j\omega$ in (23) and (24). It should be noted that by having condition (16), $H_1(s)$ and $H_l(s)$ will be stable, i.e., their poles will have negative real parts.

*Theorem 2:* Consider the vehicle platoon (5) with the homogeneous structure described in Section V-A, the control input (8) and the control parameters that satisfy internal stability condition (16). Then, the string stability specification (26) holds if for all $2 \leq l \leq r$, all the following conditions hold

$$r\big(1-(l-r)^2\big)h^2k_p + 2r(1+r-l)hk_v - 2 \geq 0, \tag{29a}$$

$$k_v - k_ph(r-1) \geq 0, \tag{29b}$$

$$rk_a\Delta \leq \tau, \tag{29c}$$

$$2r^2hk_v \geq 2(1+2\,rk_a) + r^3k_ph^2 - 2r^2k_ph^2, \tag{29d}$$

$$1 + 2\,r(k_a - \tau(k_v + k_ph)) \geq 2\,r^2k_a\big(k_v - k_ph(r-1)\big)\Delta. \tag{29e}$$

The minimum acceptable time headway that holds in the region defined by conditions (29) is

$$h \geq h_{\min} = \max\left\{h_{\min,1}, h_{\min,l}\right\}, \tag{30}$$

where

$$h_{\min,1} = \frac{2(\tau + rk_a\Delta)}{r} \quad , \quad h_{\min,l} = \frac{2\tau}{2rk_a+1}. \tag{31}$$

$h_{\min,1}$ and $h_{\min,l}$ are the minimum time headways that guarantee (28a) and (28b), respectively.

*Proof*: See Appendix B.

It can be seen from (30) and (31) that the minimum time headway is dependent on the number of connected vehicles. By using V2V communications and increasing the number of connected vehicles $r$, a smaller minimum time headway and thus a higher road throughput can be achieved.

*Remark 2:* The minimum time headway proposed in [20] and [35], when all the information only comes from V2V communications and is subject to time delay, is

$$h_{\min} = \frac{2(\tau + \Delta)}{2rk_a + 1}. \tag{32}$$

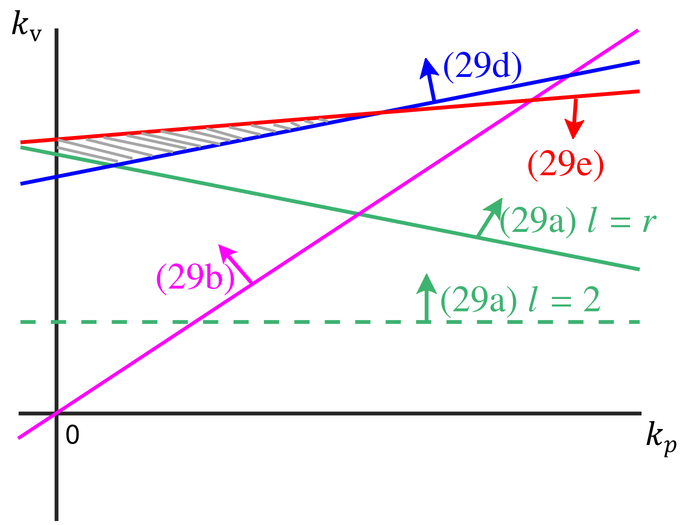



Fig. 2. The feasible region for $(kp, kv)$, based on (29).

It is clear that when in (30), $h_{\min} = \frac{2\tau}{2rk_a+1}$, this minimum time headway is smaller than (32) and hence, the use of onboard sensors has resulted in a higher throughput. If $h_{\min} = \frac{2(\tau + rk_a\Delta)}{r}$, the values of time delay $\Delta$ and also $\tau$, $k_a$ and $r$ will define whether (32) is smaller or (30). The reason that having access to some delay-free information is not necessarily preferable to having only delayed information could be due to the fact that to find the control input (8), some delayed information are compared with delay-free information, while in [20] and [35], each piece of information that forms the control input has the same value of time delay.

*Remark 3:* Given a set of parameters $\{k_a = 0.3, r = 3, \tau = 0.4s, \Delta = 0.3\ s\}$ that satisfies (29c) and also considering $h = 0.5\ s$, the feasible region for $(k_p, k_v)$, that satisfies conditions in (29), is shown shaded in Fig. 2. It can be seen that, while considering the given set of parameters, which are also used in the Numerical Results section, conditions (29a) and (29b) are inactive constraints and do not cause any limitation. These conditions, however, can be active constraints with a different set of parameters.

### *B. Heterogeneous Platoon*

We assume $k_{pi} = k_p$, $k_{vi} = k_v$ and $k_{ai} = k_a$, while the time headway of vehicle $i$ is $h_i$ and the time lag in the powertrain is $\tau_i$.

*1) String Stability Analysis of Vehicle i, $1 \leq i \leq r$ :* Because analyzing string stability of the general case for arbitrary $r$ will be very lengthy, we first look at the specific case $r = 3$ as an example and find string stability conditions and then, we generalize the results.

Now, our objective is to find the minimum time headway for vehicles 1, 2 and 3, when $r = 3$. First, similar to Section V-A.1, we find the relationship between $e_1(t)$ and $e_2(t)$. We have

$$\tau_2\,\dddot{p}_2(t) + \ddot{p}_2(t) = u_2(t), \tag{33a}$$

$$\tau_1\,\dddot{p}_1(t) + \ddot{p}_1(t) = u_1(t), \tag{33b}$$

$$\tau_2\,\dddot{v}_2(t) + \ddot{v}_2(t) = \dot{u}_2(t). \tag{33c}$$

Similar to what we did to obtain (20), we calculate (33a) − (33b) + $h_2$ × (33c), substituting from (8) and also defining the spacing errors $e_1(t)$ and $e_2(t)$ from (21). Then, we obtain

$$\begin{aligned}\tau_2 \dddot{e}_2(t) &+ (1+rk_a)\ddot{e}_2(t) + \left(k_v + 2k_p h_2\right)\dot{e}_2(t) + 2k_p e_2(t)\\ &= k_a \ddot{e}_1(t-\Delta) + \left(k_v - k_p h_2\right)\dot{e}_1(t)\\ &\quad - k_v\Big(v_2(t) - v_0(t-\Delta) + h_2 a_2(t) - h_2 a_0(t-\Delta)\Big)\\ &\quad - k_a\Big(a_2(t) - a_0(t-\Delta) + h_2 \dot{a}_2(t) - h_2 \dot{a}_0(t-\Delta)\Big)\\ &\quad - \lambda_{12} k_v a_1(t)\\ &\quad - \lambda_{12} k_a \dot{a}_1(t-\Delta)\\ &\quad - \mu_{12} \dddot{p}_1(t),\end{aligned} \tag{34}$$

where $\mu_{12} = \tau_1 - \tau_2$ and $\lambda_{12} = h_1 - h_2$. After taking the Laplace transform, we have

$$E_2(s) = H_{2,1}(s)E_1(s) + \Psi_2(s), \tag{35}$$

where

$$\begin{aligned}\Psi_2(s) &= -sk_v\Big(P_2(s) - P_0(s)e^{-\Delta s} + h_2 V_2(s) - h_2 V_0(s)e^{-\Delta s}\Big)\\ &\quad - s^2 k_a\Big(P_2(s) - P_0(s)e^{-\Delta s} + h_2 V_2(s) - h_2 V_0(s)e^{-\Delta s}\Big)\\ &\quad - \lambda_{12}\Big(k_v s^2 P_1(s) + k_a s^3 P_1(s)e^{-\Delta s}\Big) + \mu_{12}s^3 P_1(s),\end{aligned} \tag{36}$$

and $P_i(s)$, $V_i(s)$, $A_i(s)$ are the Laplace transformations of $p_i(t)$, $v_i(t)$, $a_i(t)$, respectively, and

$$H_{2,1}(s) = \frac{k_a s^2 e^{-\Delta s} + \left(k_v - k_p h_2\right)s}{\tau_2 s^3 + (1+k_a)s^2 + \left(k_v + 2k_p h_2\right)s + 2k_p}. \tag{37}$$

Knowing that the string stability transfer functions, i.e., the transfer function from each vehicle's error to the error of its interconnected predecessors, such as $H_1(s)$ and $H_l(s)$ in (22) or $H_{2,1}(s)$ in (35), are likely to have their largest magnitude at low frequencies while their magnitude decreases as $s$ rises, we need to check $\Psi_2(s)$ only at low frequencies [36], [37], [38]. Then, in (36), because of the zero located at the origin, we have that

$$|\Psi_2(j\omega)| \ll 1, \quad \omega \to 0, \tag{38}$$

and therefore we only need to have the following condition to guarantee string stability

$$\left\|H_{2,1}(j\omega)\right\|_\infty \le 1, \tag{39}$$

where $H_{i,j}(j\omega)$ and $\Psi_i(j\omega)$ can be derived by substituting $s = j\omega$. Indeed, since the purpose of establishing string stability is to have a platoon in which the spacing errors defined in (21) do not propagate, we use (39) as the definition of string stability in this case.

*Remark 4:* It can be seen that (39) is independent of $h_1$. Moreover, since string stability cannot be defined only between vehicle 1 and the leader, finding the minimum time headway for vehicle 1 is not meaningful. Therefore, for the sake of simplicity, we assume $h_1 = h_2$.

*Proposition 1:* String stability specification (39) holds if

$$k_v - k_p h_2 \ge 0, \tag{40a}$$
$$k_a \Delta \le \tau_2, \tag{40b}$$
$$6k_v h_2 \ge 4(1+k_a) - 3k_p h_2^2, \tag{40c}$$
$$1 + 2\big(k_a - \tau_2(k_v + 2k_p h_2)\big) \ge 2\, k_a\big(k_v - k_p h_2\big)\Delta. \tag{40d}$$

The minimum acceptable time headway that holds in the region defined by conditions (40), is

$$h_1 = h_2 \ge \frac{4(\tau_2 + k_a\Delta)(1+k_a)}{3(1+2k_a)}. \tag{41}$$

*Proof*: In order to prove Proposition 1, we exactly follow the proof of Theorem 2 in Appendix B.

Now, we assume that the time headway of vehicle 1 and 2 is $h_1 = h_2 = \frac{4(\tau_2+k_a\Delta)(1+k_a)}{3(1+2k_a)}$ and the time headway of vehicle 3 is $h_3$. The next step is analyzing the string stability for vehicle 3. Similar to what we did for vehicle 2, we find the relationship among $e_1(t)$, $e_2(t)$ and $e_3(t)$ (the calculations are omitted due to their similarity to the previous subsections) and then after taking the Laplace transform, we obtain

$$E_3(s) = H_{3,1}(s)E_2(s) + H_{3,2}(s)E_1(s) + \Psi_3(s), \tag{42}$$

where

$$\begin{aligned}\Psi_3(s) &= -sk_v\Big(P_3(s) - P_0(s)e^{-\Delta s} + h_3 V_3(s)e^{-\Delta s} - h_3 V_0(s)\Big)\\ &\quad - s^2 k_a\Big(P_3(s) - P_0(s)e^{-\Delta s} + h_3 V_3(s)e^{-\Delta s} - h_3 V_0(s)\Big)\\ &\quad - \lambda_{23} k_p s\big(P_1(s)e^{-\Delta s} - P_1(s)\big)\\ &\quad - \lambda_{23} k_v s^2\big(P_2(s) + P_1(s)e^{-\Delta s}\big)\\ &\quad - \lambda_{23} k_a s^3\big(P_2(s)e^{-\Delta s} + P_1(s)e^{-\Delta s}\big)\\ &\quad + \mu_{23}s^3 P_2(s),\end{aligned} \tag{43}$$

and $\mu_{23} = \tau_2 - \tau_3$, $\lambda_{23} = h_2 - h_3$ and

$$H_{3,1}(s) = \frac{k_a s^2 e^{-\Delta s} + \left(k_v - 2k_p h_3\right)s}{\tau_3 s^3 + (1+2k_a)s^2 + \left(2k_v + 3k_p h_3\right)s + 3k_p} \tag{44a}$$

$$H_{3,2}(s) = \frac{k_a s^2 e^{-\Delta s} + \left(k_v - k_p h_3\right)s e^{-\Delta s}}{\tau_3 s^3 + (1+2k_a)s^2 + \left(2k_v + 3k_p h_3\right)s + 3k_p}, \tag{44b}$$

There is a zero located at the origin in (43) and similar to (38), we can neglect the effects of $|\Psi_3(j\omega)|$ at low frequencies, and hence to guarantee string stability, we require

$$\left\|H_{3,1}(j\omega)\right\|_\infty \le \frac{1}{2} \quad , \quad \left\|H_{3,2}(j\omega)\right\|_\infty \le \frac{1}{2}. \tag{45}$$

It can be seen that (45) does not depend on the time headway of the predecessors of vehicle 3, i.e., $h_1$ and $h_2$. Similar to Proposition 1, sufficient conditions can be found to guarantee (45) and the minimum time headway that corresponds to the region defined by those conditions can be derived. However, due to the similarity to Proposition 1, we skip this part and we proceed to generalize studying string stability for vehicle $i$, $1 \le i \le r$ for any arbitrary value of $r$. Similarly to how we obtained (39) and (45), we need to have the following conditions, for vehicle $i$, $i \le r$, to guarantee string stability

$$\left\| H_{i,1}(j\omega) \right\|_\infty \le \frac{1}{i-1}, \tag{46a}$$

$$\left\| H_{i,l}(j\omega) \right\|_\infty \le \frac{1}{i-1}, \quad \forall 2 \le l \le i-1, \tag{46b}$$

where

$$H_{i,1}(s) = \frac{k_a s^2 e^{-\Delta s} + \big(k_v - k_p h_i (i-1)\big) s}{\tau_i s^3 + \big(1 + (i-1)k_a\big)s^2 + \big((i-1)k_v + i k_p h_i\big)\, s + i k_p} \tag{47}$$

and

$$H_{i,l}(s) = \frac{k_a s^2 e^{-\Delta s} + \big(k_v - k_p h_i (i-l)\big) s e^{-\Delta s}}{\tau_i s^3 + \big(1 + (i-1)k_a\big)s^2 + \big((i-1)k_v + i k_p h_i\big)\, s + i k_p}. \tag{48}$$

It can be seen that (47) and (48) do not depend on the time headway of the predecessors of vehicle $i$.

*Theorem 3:* Consider system (5) with the heterogeneous structure described in Section V-B and the control input (8) that is internally stable. String Stability specification (46) holds for vehicle $i$, $i \le r$, if

$$\Big(2(i-1)^3 + 2i(i-1)\Big) k_v h_i \ge 2i\big(1 + (i-1)k_a\big) - \Big(i^2 - (i-1)^4\Big) k_p h_i^2, \tag{49a}$$

$$k_v - (i-1)k_p h_i \ge 0, \tag{49b}$$

$$(i-1)k_a \Delta \le \tau_i, \tag{49c}$$

$$\Big(2i(i-1) + 2(i-1)^2(i-l)\Big) k_v h_i \ge 2i\big(1 + (i-1)k_a\big) - \Big(i^2 - (i-1)^2(i-l)^2\Big) k_p h_i^2, \quad \forall 2 \le l \le i-1, \tag{49d}$$

$$1 + 2\Big((i-1)k_a - \tau_i \big((i-1)k_v + i k_p h_i\big)\Big) \ge 2(i-1)^2 \times k_a \big(k_v - (i-1)k_p h_i\big)\Delta. \tag{49e}$$

The minimum acceptable time headway that holds in the region defined by conditions (49), is

$$h_i \ge \max\left\{h_{i,min1}, h_{i,min2}\right\}, \quad 1 < i \le r. \tag{50}$$

where

$$h_{i,min1} = \frac{(2i)\Big(\tau_i + (i-1)k_a\Delta\Big)\Big(1 + (i-1)k_a\Big)}{(i^2 - i + 1)\Big(1 + 2(i-1)k_a\Big)}, \tag{51a}$$

$$h_{i,min2} = \frac{2i\tau_i\Big(1 + (i-1)k_a\Big)}{(2i-1)\Big(1 + 2(i-1)k_a\Big)}, \tag{51b}$$

*Remark 5:* To obtain (40), (41), (49), and (50), we follow exactly what we did to find Theorem 2 (more information can be found in Appendix B). Therefore, here we skip presenting the proof.

*Remark 6:* It can be easily verified that the string stability conditions as well as the minimum acceptable time headway that are found for the specific case $r = 3$ in Proposition 1 are compatible with the results in Theorem 3.

*Remark 7:* By studying the string stability of vehicle $i$, $1 \le i \le r$ in addition to the the string stability of vehicle $i$, $i > r$, we establish string stability for all vehicles in order to reduce collision avoidance for each vehicle to the maximum extent possible.

*2) String Stability Analysis of Vehicle $i$, $i > r$ :* We again assume $r_i = r = 3$, and we want to find the time headway for vehicle $i$, $i > r$. Then, we will generalize the results for arbitrary $r$.

Fist, we analyze vehicle 4. Similar to what we did in V-B.1, we obtain

$$E_4(s) = H_{4,1}(s)E_3(s) + \sum_{l=2}^{r} H_{4,l}(s)E_{4-l}(s) + \Psi_4(s), \tag{52}$$

where

$$\begin{aligned}\Psi_4(s) = &-2k_p(\alpha+\beta)s\big(P_2(s)e^{-\Delta s} - P_2(s)\,\big)\\ &- \lambda_{34}(k_v s^2 + k_a s^3)\big(P_3(s) + P_3(s)e^{-\Delta s}\big)\\ &- (\lambda_{23} + \lambda_{34})(k_v s^2 + k_a s^3)\big(P_2(s) + P_1(s)\big)e^{-\Delta s}\\ &+ \mu_{34} s^3 P_3(s),\end{aligned} \tag{53}$$

where $\mu_{34} = \tau_3 - \tau_4$ and $\lambda_{34} = h_3 - h_4$. After taking the Laplace transform, we obtain

$$H_{4,1}(s) = \frac{k_a s^2 e^{-\Delta s} + \big(k_v - k_p h_4 (r-1)\big)s + k_p}{\tau_4 s^3 + (1 + r k_a)s^2 + r\left(k_v + k_p h_4\right) s + r k_p}, \tag{54a}$$

$$H_{4,l}(s) = \frac{k_a s^2 e^{-\Delta s} + \big(k_v - k_p h_4 (r-l)\big)s e^{-\Delta s} + k_p e^{-\Delta s}}{\tau_4 s^3 + (1 + r k_a)s^2 + r\left(k_v + k_p h_4\right) s + r k_p}. \tag{54b}$$

There is a zero located at the origin in (53) and similar to the (38) and (43), we can neglect the effects of $|\Psi_4(j\omega)|$ at low frequencies, and hence to guarantee string stability, it is required to have

$$\left\| H_{4,1}(j\omega) \right\|_\infty \le \frac{1}{r}, \tag{55a}$$

$$\left\| H_{4,l}(j\omega) \right\|_\infty \le \frac{1}{r}, \quad \forall 2 \le l \le r, \tag{55b}$$

It can be seen that (54a) and (54b) are similar to (23) and (24). Also, after analyzing the string stability for vehicle 5 and all the following vehicles, we find the similar string stability transfer functions as (23) and (24). This result can also be obtained for any arbitrary value of $r$. Therefore, here we propose a theorem to generalize the minimum time headway for vehicle $i$, $i > r$.

*Theorem 4:* Consider system (5) with the heterogeneous structure described in Section V-B and the control input (8) that is internally stable. To guarantee string stability for vehicle

TABLE I
MINIMUM TIME HEADWAY FOR VEHICLES OF A HETEROGENEOUS PLATOON IN DIFFERENT WIRELESS COMMUNICATION SCENARIOS

| | Vehicle i, $1 < i \leq r$ | Vehicle i , $i > r$ |
|---|---|---|
| No Delay | $\frac{2i\tau_i\left(1+(i-1)k_a\right)}{(2i-1)\left(1+2(i-1)k_a\right)}$ | $\frac{2\tau_i}{2rk_a+1}$ |
| Fully-Delayed | $\frac{2i(\tau_i+\Delta)\left(1+(i-1)k_a\right)}{(2i-1)\left(1+2(i-1)k_a\right)}$ | $\frac{2(\tau_i+\Delta)}{2rk_a+1}$ |
| Partially-Delayed (Theorems 3 and 4) | $\max\left\{\frac{2i\left(\tau_i+(i-1)k_a\Delta\right)\left(1+(i-1)k_a\right)}{(i^2-i+1)\left(1+2(i-1)k_a\right)}, \frac{2i\tau_i\left(1+(i-1)k_a\right)}{(2i-1)\left(1+2(i-1)k_a\right)}\right\}$ | $\max\{\frac{2(\tau_i+rk_a\Delta)}{r}, \frac{2\tau_i}{2rk_a+1}\}$ |

$i$, $i > r$, we need

$$r\left(1-(l-r)^2\right)h_i^2k_p + 2r(1+r-l)h_ik_v - 2 \geq 0, \tag{56a}$$

$$k_v - k_ph_i(r-1) \geq 0, \tag{56b}$$

$$rk_a\Delta \leq \tau_i, \tag{56c}$$

$$2r^2h_ik_v \geq 2(1+2\ rk_a) + r^3k_ph_i^2 - 2r^2k_ph_i^2, \tag{56d}$$

$$1+2\ r\left(k_a - \tau_i(k_v+k_ph_i)\right) \geq 2\ r^2k_a\left(k_v - k_ph_i(r-1)\right)\Delta. \tag{56e}$$

The minimum acceptable time headway that holds in the region defined by conditions (56) is

$$h_i \geq \max\{\frac{2(\tau_i + rk_a\Delta)}{r}, \frac{2\tau_i}{2rk_a+1}\}. \tag{57}$$

*Proof*: Similar to the proof of Theorem 2 in Appendix B.

A summary of the minimum time headway for different vehicles in a heterogeneous platoon with different wireless communication scenarios are provided in TABLE I. In the first scenario, there is no communication delay, while in the second scenario, vehicles receive all information only via communication links suffering the time delay $\Delta$, as in [20]. The third scenario is the scenario assumed and studied in this work. The control structures used are (7), [20]-(8) and (8) for the first, second and third scenario, respectively. When the time lag $\tau_i$ is the same for all vehicles, the results in the last column degenerates to the results of [19] and [20] and Theorem 2 of this work, for no delay, fully-delayed and partially-delayed case, respectively.

## VI. NUMERICAL RESULTS

To demonstrate the validity of the proposed theorems, numerical simulations are presented in this section. We consider two cases: a homogeneous platoon, and a heterogeneous platoon, where in both, vehicles have the linear model as (5), controller (8) is used to make the platoon internally and string stable and the number of connected predecessors is $r_i = r = 3$. Vehicles start from rest and move to reach the desired velocity, which is the leader's velocity, and also the desired inter-vehicle distance, based on (6). At $t = 60$, when the platoon has reached its stable equilibrium, an external disturbance in the form of a sinusoidal perturbation $u_0(t) = A_0\sin(\omega_0 t)$ with the duration of one cycle, acts on the leader. The numerical values for simulation parameters are given in Table II.

TABLE II
MODEL PARAMETERS

| $N$ | $r$ | $d$ | $v_0$ | $A_0$ | $\omega_0$ |
|---|---|---|---|---|---|
| 5 | 3 | $5\ m$ | $20\ m/s$ | $10\ m/s^2$ | $1\ rad/s$ |

TABLE III
MODEL PARAMETERS FOR DIFFERENT SCENARIOS

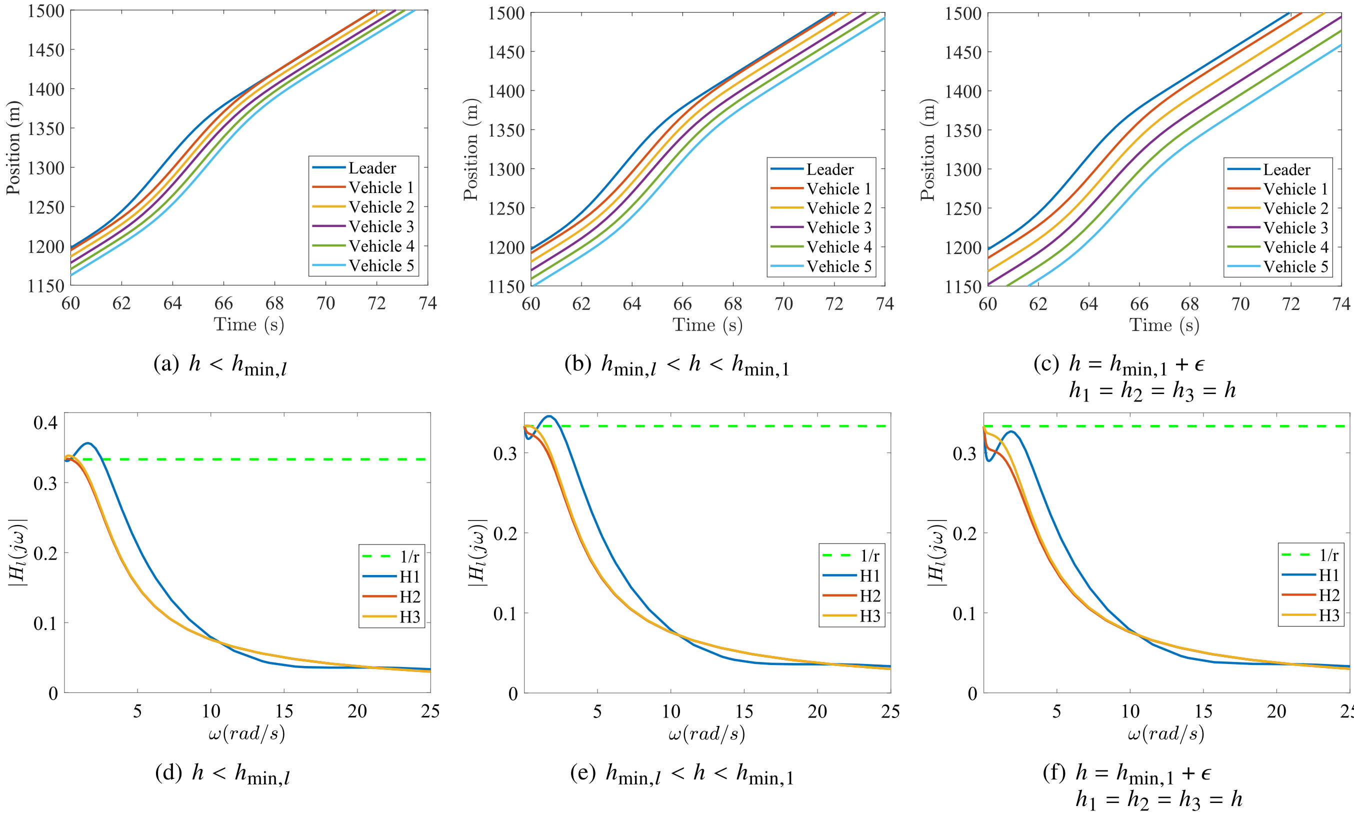
| | $\tau$ | $\Delta$ | $k_a$ | $h_{\min,1}$ | $h_{\min,l}$ |
|---|---|---|---|---|---|
| Scenario 1 | $0.4\ s$ | $0.3\ s$ | 0.3 | $0.45\ s$ | $0.29\ s$ |
| Scenario 2 | $0.5\ s$ | $0.1\ s$ | 0.18 | $0.37\ s$ | $0.48\ s$ |

### A. Homogeneous Platoon

The numerical values of a homogeneous platoon for two different scenarios are considered as Table III and the control parameters are chosen in such a way that the internal stability condition (16) holds for the both scenarios.

In the first scenario, there is a relatively long communication delay between the vehicles and based on the values in Table III, $h_{\min,1} > h_{\min,l}$ and according to Theorem 2, $h_{\min} = h_{\min,1} = 0.45\ s$. Therefore, in this scenario the magnitude of $H_1(j\omega)$ defines the acceptable time headway. To corroborate these results, simulations have been done for three different values of the time headway, as it is shown in Fig. 3. In the first case, when $h < h_{\min,l}$, it is demonstrated in Fig. 3(d) that all the string stability functions surpass $1/r$. By increasing the time headway and having $h_{\min,l} < h < h_{\min,1}$, Fig. 3(e) shows that although the behavior of $|H_2(j\omega)|$ and $|H_3(j\omega)|$ is acceptable, there are some frequencies at which $|H_1(j\omega)|$ is larger than $1/r$ and hence, the platoon becomes string unstable after acting of the external disturbance. In the last case, when $h > h_{\min,1}$, Fig. 3(f) depicts that the platoon is string stable.

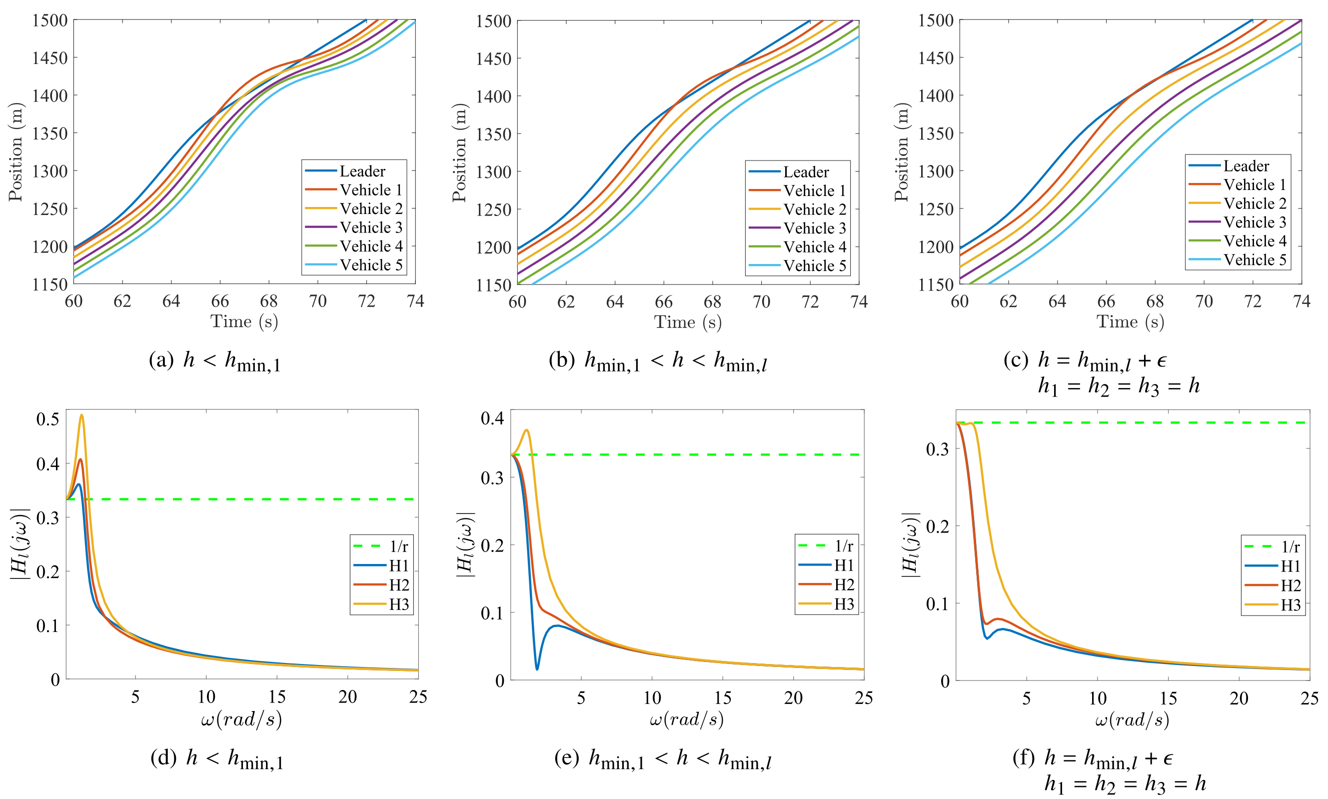
A smaller value for the time delay is considered in the second scenario and unlike the previous scenario, we obtain $h_{\min,1} < h_{\min,l}$ and based on Theorem 2, the final minimum time headway will be $h_{\min} = h_{\min,l} = 0.48\ s$. To verify this result, three different values for the time headway are considered in Fig. 4. It can be seen in Fig. 4(d) that when $h < h_{\min,1}$, the magnitudes of all string stability functions surpass $1/r$. In Fig. 4(e), when $h_{\min,1} < h < h_{\min,l}$, although the magnitudes of $H_1(j\omega)$ and $H_2(j\omega)$ are smaller than $1/r$, the magnitude of $H_3(j\omega)$ is not acceptable for string stability. Finally, when $h > h_{\min,l}$, the magnitudes of all string stability




(a) $h < h_{\min,l}$ (b) $h_{\min,l} < h < h_{\min,1}$ (c) $h = h_{\min,1} + \epsilon$, $h_1 = h_2 = h_3 = h$

(d) $h < h_{\min,l}$ (e) $h_{\min,l} < h < h_{\min,1}$ (f) $h = h_{\min,1} + \epsilon$, $h_1 = h_2 = h_3 = h$

Fig. 3. Simulation results for a vehicle platoon, with $h_{\min,1} > h_{\min,l}$.




(a) $h < h_{\min,1}$ (b) $h_{\min,1} < h < h_{\min,l}$ (c) $h = h_{\min,l} + \epsilon$, $h_1 = h_2 = h_3 = h$

(d) $h < h_{\min,1}$ (e) $h_{\min,1} < h < h_{\min,l}$ (f) $h = h_{\min,l} + \epsilon$, $h_1 = h_2 = h_3 = h$

Fig. 4. Simulation results for a vehicle platoon, with $h_{\min,l} > h_{\min,1}$.

functions are smaller than $1/r$ and the platoon is string stable, as it is shown in Fig. 4(f).

*Remark 8:* If we calculate the minimum time headway introduced in [20] and [35], we will have $h_{\min} = 0.58\ s$ and

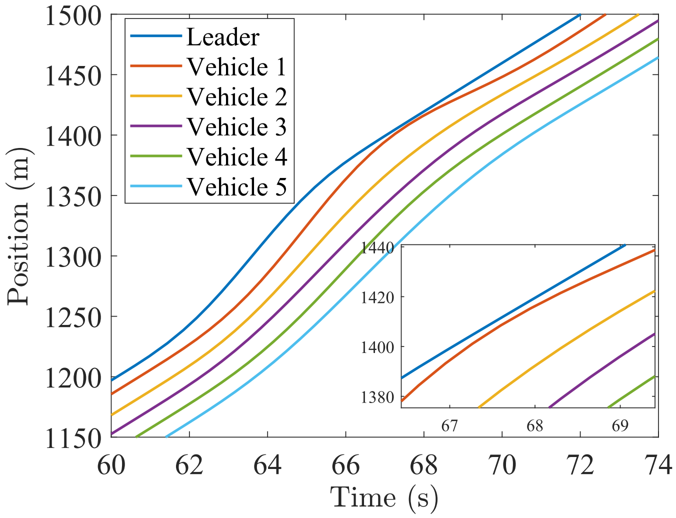



Fig. 5. $h = h_{\min,l} + \epsilon\ h_1$, $h_2$ and $h_3$ based on TABLE I.

TABLE IV
MODEL PARAMETERS OF A HETEROGENEOUS PLATOON

| | veh 1 | veh 2 | veh 3 | veh 4 | veh 5 | veh 6 | veh 7 |
|---|---|---|---|---|---|---|---|
| $\tau$ | 0.5 $s$ | 0.48 $s$ | 0.55 $s$ | 0.51 $s$ | 0.4 $s$ | 0.49 $s$ | 0.58 $s$ |
| $h$ | 0.58 $s$ | 0.58 $s$ | 0.52 $s$ | 0.49 $s$ | 0.38 $s$ | 0.47 $s$ | 0.56 $s$ |

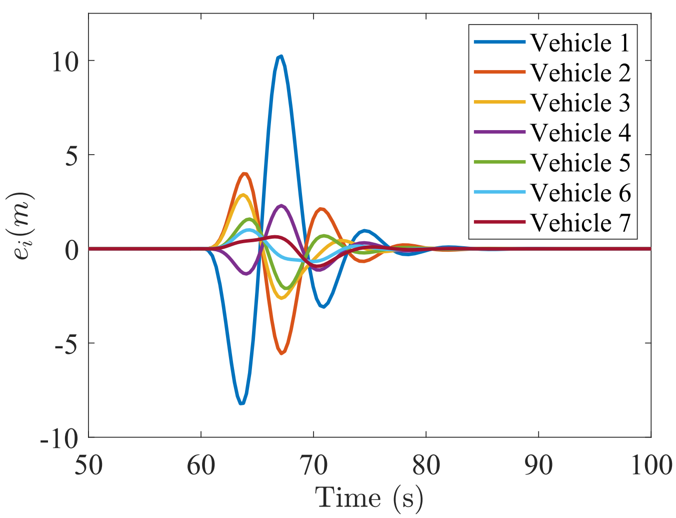



Fig. 6. Platoon response for the heterogeneous case.

$h_{\min} = 0.5\ s$ for the first and the second scenario respectively, which are larger than the values obtained from Theorem 2. Therefore, for these two scenarios, the proposed lower bound in (30) is more beneficial.

*Remark 9:* It can be seen that by using an appropriate time headway, we obtain string stability in both scenarios, but Fig. 4(c) depicts a collision in the first scenario. Fig. 5 shows that this problem can be solved by selecting time headway value of the first few vehicles, i.e., vehicle $i$, $i \leq r$, from Table I.

### B. Heterogeneous Platoon

We assume the same platoon as in scenario 2, but with different time lags $\tau_i$ for different vehicles, as in TABLE IV. Using TABLE I, we can find appropriate time headway $h_i$, as in TABLE IV. By having non-identical time headways instead of having a large identical time headway, the platoon takes the advantage of having smaller time headways for those vehicles with smaller $\tau_i$. Fig. 6 shows the internal stability and string stability of the system.

## VII. CONCLUSION AND FUTURE DIRECTIONS

### A. Conclusions

In this paper, we consider vehicle platoons under the MPF topology when both on-board sensors (which give delay-free information) and V2V communications (which give delayed information) are used. First we analyzed internal stability for the heterogeneous case and we showed that the system is internally stable irrespective of the size of the communication delays. Then, we investigated string stability for the homogeneous case, which has less complexity, and found the minimum time headway that guarantees string stability. Next, we focused again on the heterogeneous case, in which time lag and time delay are non-identical and we found the minimum time-headway for three different wireless communication scenarios, i.e., delay-free, fully-delayed and partially-delayed information. The proposed theorems were investigated through simulations, where it showed the importance of these lower bounds on the time headway.

### B. Future Directions

We plan to analyze internal and string stability in platoons with time-varying communication delays. Also, we plan to investigate the effects of packet drops on internal and string stability of platoons. Furthermore, the assumption of having a constant speed leader and knowing its value by all followers is not perfectly possible and hence, this can be a useful direction for further research.

## APPENDIX A
## PROOF OF THEOREM 1

To find the stability conditions of the platoon, we take the Laplace transform of (14) and we obtain

$$\Xi(s) = (sI_{3N} - A - A_{\Delta}e^{-\Delta s})^{-1}\xi(0). \tag{58}$$

The characteristic equation of the platoon can be written as

$$\begin{aligned} G(s) &= \det(sI_{3N} - A - A_{\Delta}e^{-\Delta s}) \\ &= \det\begin{bmatrix} sI_N & -I_N & -H \\ 0 & sI_N & -I_N \\ \tilde{G}_1 & \tilde{G}_2 & sI_N + \tilde{G}_3 \end{bmatrix} \end{aligned} \tag{59}$$

where

$$\tilde{G}_1 = \mathrm{T}K_p(\mathcal{L}_{p_1} + \mathcal{L}_{p_2}e^{-\Delta s}), \tag{60a}$$

$$\tilde{G}_2 = \mathrm{T}K_v(\mathcal{L}_{v_1} + \mathcal{L}_{v_2}e^{-\Delta s}) + \mathrm{T}K_pFH(1 - e^{-\Delta s}), \tag{60b}$$

$$\tilde{G}_3 = \mathrm{T} + \mathrm{T}K_a(\mathcal{L}_{a_1} + \mathcal{L}_{a_2}e^{-\Delta s}). \tag{60c}$$

By using Lemma 1, the determinant in (59) can be obtained as

$$\begin{aligned} G(s) &= \det(sI_N)\det\left(\begin{bmatrix} sI_N & -I_N \\ \tilde{G}_2 & sI_N+\tilde{G}_3 \end{bmatrix} - \begin{bmatrix} 0 \\ \tilde{G}_1 \end{bmatrix}(sI_N)^{-1}\begin{bmatrix} -I_N & -H \end{bmatrix}\right) \\ &= \det(sI_N)\det\begin{bmatrix} sI_N & -I_N \\ \tilde{G}_2+\frac{1}{s}\tilde{G}_1 & sI_N+\tilde{G}_3+\frac{1}{s}\tilde{G}_1 H \end{bmatrix}. \end{aligned} \tag{61}$$

Again, by using Lemma 1, we have

$$\begin{aligned} G(s) &= \det(sI_N)\det(sI_N)\det\Big[sI_N+\tilde{G}_3+\frac{1}{s}\tilde{G}_1 H \\ &\quad +(\tilde{G}_2+\frac{1}{s}\tilde{G}_1)(sI_N)^{-1}\Big] \\ &= \det\Big[s^3 I_N + s^2\tilde{G}_3 + s(\tilde{G}_1 H+\tilde{G}_2)+\tilde{G}_1\Big]. \end{aligned} \tag{62}$$

Substituting from (60), one can see that $s^3 I_N + s^2\tilde{G}_3 + s(\tilde{G}_1 H + \tilde{G}_2) + \tilde{G}_1$ is a lower triangular matrix and hence, its determinant equals the product of diagonal entries. Also, since the diagonal elements of $\mathcal{L}_{p_1}$, $\mathcal{L}_{v_1}$, $\mathcal{L}_{a_1}$ are same as the diagonal elements in $\mathcal{L}_p$, based on (15), the diagonal elements of $\mathcal{L}_{p_2}$, $\mathcal{L}_{v_2}$, $\mathcal{L}_{a_2}$ are zero. Moreover, matrix $F$ has zero diagonal and hence, it is clear that $\tilde{G}_1$, $\tilde{G}_2$ and $\tilde{G}_3$ do not have any delay terms on their diagonals and as a result, the diagonal elements of (62) do not include the delay term. Eventually, (62) can be decoupled to $N$ subsystems in the form of

$$\begin{aligned} G(s) &= \prod_{i=1}^{N}\Big(s^3 + s^2\frac{1}{\tau_i}(1+k_{ai}l_{ii}) + s\frac{1}{\tau_i}l_{ii}\left(k_{vi}+k_{pi}h_i\right) \\ &\quad + \frac{1}{\tau_i}l_{ii}k_{pi}\Big). \end{aligned} \tag{63}$$

For $i \geq r$, the final polynomial is defined as

$$\begin{aligned} g_i(s) &:= s^3 + s^2\frac{1}{\tau_i}(1+k_{ai}r_i) + s\frac{1}{\tau_i}r_i\left(k_{vi}+k_{pi}h_i\right) \\ &\quad + \frac{1}{\tau_i}r_i k_{pi}. \end{aligned} \tag{64}$$

Then, the closed loop system (14) is stable if and only if the roots of (64) lie in the left half plane. As proposed in [19], using the Routh Hurwitz stability criterion and also knowing that all the control parameters are positive, $g_i(s)$ is stable if and only if

$$\left(\frac{1}{\tau_i}(1+k_{ai}r_i)\right)\left(\frac{1}{\tau_i}r_i(k_{vi}+k_{pi}h_i)\right) > \frac{1}{\tau_i}r_i k_{pi}, \tag{65}$$

which is equivalent to (16).

## Appendix B
## Proof of Theorem 2

Inequalities (28a) and (28b) can be re-written as

$$\sup_{\omega>0}|H_1(j\omega)|^2 \leq \frac{1}{r^2}, \tag{66a}$$

$$\max_{2\leq l\leq r}\|H_l(j\omega)\|_\infty^2 = \max_{2\leq l\leq r}\sup_{\omega>0}|H_l(j\omega)|^2 \leq \frac{1}{r^2}. \tag{66b}$$

From [19], it can be easily seen that in order to satisfy condition (66b) and accordingly (28b), the following conditions should hold, for all $2 \leq l \leq r$:

$$1 + 2\,r(k_a - \tau(k_v + hk_p)) \geq 0, \tag{67a}$$

$$r(1-(l-r)^2)h^2 k_p + 2r(1+r-l)hk_v - 2 \geq 0, \tag{67b}$$

Then, it is proved in [19] that the minimum employable time headway that can guarantee string stability, is

$$h \geq h_{\min,l} = \frac{2\tau}{2rk_a+1}. \tag{68}$$

Condition (68) is obtained by analyzing the region defined in (67). The next step is to find control parameters and their corresponding minimum time headway, which guarantee condition (66a). We define

$$|H_1(j\omega)|^2 \triangleq \frac{N_1}{D_1}, \tag{69}$$

where

$$\begin{aligned} N_1 &= \left(k_p - k_a\omega^2\cos(\Delta\omega)\right)^2 \\ &\quad + \Big(\big(k_v - k_p h(r-1)\big)\omega + k_a\omega^2\sin(\Delta\omega)\Big)^2, \end{aligned} \tag{70}$$

and

$$D_1 = \left(rk_p - (1+rk_a)\omega^2\right)^2 + \left(r(k_v+k_p h)\omega - \tau\omega^3\right)^2. \tag{71}$$

Inequality (66a) is equivalent to $D_1 - r^2 N_1 \geq 0$ . After some algebraic manipulations, we obtain

$$D_1 - r^2 N_1 = M_6\omega^6 + M_4\omega^4 + M_3\omega^3 + M_2\omega^2, \tag{72}$$

where

$$M_6 = \tau^2, \tag{73a}$$

$$M_4 = 1 + 2\,rk_a - 2r\tau(k_v + k_p h), \tag{73b}$$

$$M_3 = -2r^2 k_a\big(k_v - k_p h(r-1)\big)\sin(\Delta\omega), \tag{73c}$$

$$\begin{aligned} M_2 &= -2rk_p(1+rk_a) + r^2(k_v+k_p h)^2 \\ &\quad + 2r^2 k_p k_a\cos(\Delta\omega) - r^2(k_v - k_p h(r-1))^2. \end{aligned} \tag{73d}$$

Considering the fact that $\sin(\Delta\omega) \leq \Delta\omega$ for $\omega \geq 0$ and $\cos(\Delta\omega) \geq -1$, if

$$k_v - k_p h(r-1) \geq 0, \tag{74}$$

then it follows from (72) that we can lower-bound $D_1 - r^2 N_1$ as

$$D_1 - r^2 N_1 \geq \omega^2(\overline{M}_4\omega^4 + \overline{M}_2\omega^2 + \overline{M}_0), \tag{75}$$

where

$$\overline{M}_4 = \tau^2, \tag{76a}$$
$$\overline{M}_2 = 1 + 2\, rk_a - 2r\tau(k_v + k_p h) - 2r^2 k_a\big(k_v - k_p h(r-1)\big)\Delta, \tag{76b}$$
$$\overline{M}_0 = -2rk_p(1 + rk_a) + r^2(k_v + k_p h)^2 - 2r^2 k_p k_a - r^2(kv - k_p h(r-1))^2. \tag{76c}$$

If $\overline{M}_2, \overline{M}_0 \geq 0$, the right-hand side of (75) is non-negative. This implies that $D_1 - r^2 N_1 \geq 0$ and the platoon is string stable. Conditions (29d) and (29e) are equivalent to $\overline{M}_0 \geq 0$ and $\overline{M}_2 \geq 0$, respectively.

Now, we intend to evaluate whether or not $\overline{M}_0 \geq 0$ and $\overline{M}_2 \geq 0$ can form a feasible region in parameter space. We have

$$\overline{M}_2 \geq 0 \Longleftrightarrow k_v \leq \frac{1 + 2\, rk_a - 2rk_p h\big(\tau - r\Delta k_a(r-1)\big)}{2r(\tau + rk_a\Delta)}, \tag{77a}$$

$$\overline{M}_0 \geq 0 \Longleftrightarrow k_v \geq \frac{2(1 + 2\, rk_a) + r^3 k_p h^2 - 2r^2 k_p h^2}{2r^2 h}. \tag{77b}$$

The upper bound of $k_v$ in (77a) should be larger than its lower bound in (77b), which is equivalent to the condition

$$(1 + 2rk_a)\big(rh - 2(\tau + rk_a\Delta)\big) + r^3 k_p h^2(rk_a\Delta - \tau) \geq 0. \tag{78}$$

In particular, if we choose $k_a$ to satisfy

$$rk_a\Delta \leq \tau, \tag{79}$$

then

$$rh - 2(\tau + rk_a\Delta) \geq 0 \tag{80}$$

ensures that (79) holds. Note that a smaller value of $k_a$ can guarantee that both conditions (79) and (80) hold. Nevertheless, this has an impact on $h_{\min,l}$ in (68).

From (80), the minimum acceptable time headway can be found as

$$h \geq h_{\min,1} = \frac{2(\tau + rk_a\Delta)}{r}. \tag{81}$$

Finally, the minimum time headway which guarantees the string stability of platoon (5) controlled by (8) is

$$h \geq h_{\min} = \max\{h_{\min,1}, h_{\min,l}\}, \tag{82}$$

where $h_{\min}$ is the minimum time headway that holds in the region defined by conditions (29) and hence, it can guarantee string stability.

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