A Literature Review on Train Motion Model Calibration

Alex Cunillera, *Member, IEEE*, Nikola Bešinović, Ramon M. Lentink, Niels van Oort, and Rob M. P. Goverde, *Member, IEEE*

***Abstract*—The dynamics of a moving train are usually described by means of a motion model based on Newton's second law. This model uses as input track geometry data and train characteristics like mass, the parameters that model the running resistance, the maximum tractive effort and power, and the brake rates to be applied. It can reproduce and predict train dynamics accurately if the mentioned train characteristics are carefully calibrated. The model constitutes the core element of a broad variety of railway applications, from timetabling tools to Driver Advisory Systems and Automatic Train Operation. Among the existing train motion model calibration techniques, those that use operational data are of particular interest, as they benefit from on-board recorded data, capturing the train dynamics during operation. In this literature review article we provide an overview of the train motion model calibration techniques that have been published in the scientific literature between January 2000 and December 2021 and either use operational data or can be minimally adapted to use it. To this end, we present a critical overview of the existing train motion model calibration approaches, distinguishing online calibration that analyzes data on-the-go and offline calibration that analyzes historical data batchwise. We propose a research agenda and highlight some potential goals to be tackled in the near future: from devising accurate online calibrators for eco-driving applications to quantitizing the physical sources of parameter variation. Last, we discuss practical recommendations for practitioners and scholars inferred from the current state of the art.**

***Index Terms*—Train motion model, parameter estimation, parameter identification, model calibration, vehicle dynamics, railways.**

List of Abbreviations

ATO	Automatic Train Operation.
ATP	Automatic Train Protection.
CFD	Computational Fluid Dynamics.
DAS	Driver Advisory Systems.
EKF	Extended Kalman Filter.
ETCS	European Train Control System.
GA	Genetic Algorithm.
GNSS	Global Navigation Satellite System.
ILI	Iterative Learning Identification.
LSR	Least Squares Regression.
MILS	Multi-Innovation Least Squares.
MLE	Maximum Likelihood Estimation.
PSO	Particle Swarm Optimization.
RLS	Recursive Least Squares.
SA	Simulated Annealing.
SQP	Sequential Quadratic Programming.
UKF	Unscented Kalman Filter.
V-MILS	Interval-Varying Multi-Innovation Least Squares.

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Alex Cunillera, Niels van Oort, and Rob M. P. Goverde are with the Department of Transport and Planning, Delft University of Technology, 2600 GA Delft, The Netherlands (e-mail: a.cunilleraperez@tudelft.nl).

Nikola Bešinović was with the Department of Transport and Planning, Delft University of Technology, 2600 GA Delft, The Netherlands. He is now with the Department of Transport and Planning, Delft University of Technology, 2600 GA Delft, The Netherlands, and also with the Chair of Rail and Public Urban Transport, Faculty of Transport and Traffic Sciences "Friedrich List", Technical University of Dresden, 01069 Dresden, Germany.

Ramon M. Lentink is with Nederlandse Spoorwegen, 3500 HA Utrecht, The Netherlands.

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I. Introduction

Most railway simulators and several railway applications like timetabling, eco-driving tools, and train trajectory generation algorithms in Driver Advisory Systems (DAS) and Automatic Train Operation (ATO) rely on a dynamical system that models the train dynamics: the train motion model [1]. This model is obtained from Newton's second law and requires as input several train and track characteristics, namely the maximum tractive and brake effort and power, the running resistances that affect its motion, the track geometry, the maximum acceleration rates, the brake rates to be used, the mass and the rotating mass factor [2]. The model can reproduce and predict a train's movement along a track accurately if the parameters that describe the mentioned train and track characteristics are carefully determined [3].

An accurate train motion model calibration is essential to guarantee the performance and effectiveness of the railway applications that include this model [4]. Simulators utilize the motion model to reproduce a train's dynamics in both real and hypothetical scenarios. For instance, they are used to perform capacity assessments of both existing and future railway lines [5], to calculate track section occupancy and arrival times that are used as input for timetabling tools [6], to estimate energy consumption [7], to support infrastructure planning [8], to test new signalling systems [9] and to train drivers [10]. However, a wrong calibration of the train motion model embedded in a simulator usually leads to erroneous calculations. An inaccurate capacity assessment can produce an increase of the number of disturbances and an overall decrease in the service reliability. An inaccurate calculation of track section occupation times may lead to more trains approaching yellow and red signals and to infeasible timetables. New infrastructure and signalling usually require large investments [11], and errors in the planning stage lower the effectiveness of the mentioned investments.

In train trajectory optimizers, the train motion model is utilized along the timetable and the allocated path in the track to generate energy-efficient train trajectories that fulfill the arrival and passing times described in the timetable. To this end, they describe the reference speed to be followed at every point of the path. Train trajectory optimizers require a precise model calibration, which, in the event of not fully capturing the train's behavior, could lower the drivability of the generated trajectories and cause deviations from the expected arrival times and reducing the energy savings. In turn, DAS and ATO generate respectively driving advice and the right amount of tractive and brake effort to be applied to drive the train. The train motion model is used in these applications to generate the reference trajectories to be followed, usually by means of an embedded train trajectory optimizer or a set of precalculated trajectories [12]. However, the accuracy of these applications may be affected by a wrong calibration. An imprecise driving advice is unlikely to be accepted by drivers, reducing their trust on DAS. Similarly, this impacts ATO by causing deviations from the expected arrival time and the expected stopping locations, and by increasing the probability of overspeeding, which triggers the intervention of Automatic Train Protection systems and drivers in the control process. As a consequence, DAS and ATO require a highly precise calibration, which usually hinders their accurate implementation.

Train motion model calibration is usually a difficult problem in the railway industry and academia due to the spatial and temporal parameter variability shown by some of the mentioned characteristics and the existence of several external factors that influence the train dynamics [13]. For instance, wear may alter the tractive effort performance of engines over time. Moreover, strong winds may increase the aerodynamic drag of the train, augmenting its running resistance and energy consumption. Computational Fluid Dynamics (CFD) and scaled wind tests are usually performed to estimate the running resistance parameters, but the results obtained are usually inaccurate [14]. Full-scale tests can also be used to calibrate train dynamics, although these tests are usually resource and capacity demanding [15]. The main issue regarding the mentioned procedures is that the calibration may not always be representative of the current operation conditions, which lowers the accuracy of the calibrated model and the applications that incorporate it. Consequently, the train motion model calibration has to be updated often or performed continuously. To address this, the available operational data recorded on board represents a resourceful alternative for calibrating the train motion model and to boost the performance of railway applications, as it allows to capture the current operation conditions and train characteristics.

Such calibration may be performed in two ways when using operational data as input: online, by processing on-board collected data on-the-go, and offline, by analyzing historical data. These online and offline model calibration procedures have complementary scopes that determine the kind of applications in which they are embedded. Online model calibration can be used to detect the variability of the parameters of the train motion model in real time and enhance its accuracy by accommodating these variations. Furthermore, it is highly suitable for freight trains with variable rolling stock composition, for which there is no historical data available [4]. However, the algorithms used need to be fast and computationally efficient. In this regard, the effects of varying electric power, weather, wind, adhesion, track, brakes and engine condition, and driving style can be assessed in real time and utilized to increase the accuracy of real-time applications such as online eco-driving algorithms, DAS and ATO [16]. In turn, offline model calibration can process a larger amount of data coming from several sources and may be easier to implement. Thus, more powerful but slow and computationally demanding techniques can be applied offline to support decision makers by enhancing the accuracy of simulation and timetabling applications [17], [18]. Usually, offline approaches produce a single estimate for each target parameter that constitutes the best fit to the considered dataset, as opposed to online calibration, that produces time-varying estimates.

In this article we review the current state of the art in online and offline train motion model calibration, addressing particularly those methods that utilize on-board measurements. This article is of special relevance in the railway industry and academia, since no recent literature review article on this topic is available. Rochard and Schmid [15] is the latest review on running resistance calculation for high speed trains, focusing on collecting and comparing several running resistance equations that have been developed by scholars and railway undertakings from different countries. However, most of these formulas constitute empirical or semi-empirical approximations that usually overestimate the running resistance of current trains. Despite its limited scope, it is one of the most influential articles on train motion model calibration. In turn, one of the most extensive lists of empirical running resistance formulas can be found in [19, Chapter 13.2.2.2]. However, we do not recommend using such formulas if either calibration is possible or, alternatively, if the manufacturer formulas are available. A recent review [20] focuses on a close topic: the estimation of adhesion, namely the ratio of the tangential force between wheel and rail, and the wheel load. Most of the techniques covered there require modelling the tractive wheels dynamics and measuring the wheels rotating speed with an encoder, which is compared to the train speed to gauge the wheel slip. Although adhesion estimation is essential for determining the applied tractive effort and brake, we do not review adhesion estimation techniques in this article, as we do not cover techniques that require wheels angular speed or traction torque measurements. Covering railway applications that benefit from train motion model calibration is also out of the scope of this article. Our article and [20] constitute a complete overview of train motion model estimation techniques. Moreover, model calibration is closely related to monitoring algorithms, which are reviewed in [21], focusing on rolling stock health monitoring and fault detection. Regarding other fields of transport, motion model calibration is also covered in the case of wheeled vehicles [22], flapping wing micro-aerial vehicles [23], and aircrafts [24].

Considering that no literature review on train motion model calibration based on on-board measurements can be found in the existing literature, we review articles published from January 2000 to December 2021. We selected 42 articles and the techniques found are classified in two main categories: online and offline algorithms. Then, the existing techniques are clustered according to the type of method used for calibration. Last, each technique is analyzed individually, outlining their main advantages and disadvantages. The main contributions of this literature review article are:

- A broad overview, description and critical discussion of the existing train motion model calibration methods.
- Highlighting the relevance of train motion model calibration in many areas of railway research and industry, which may ultimately lead to a broader and more effective calibration of railway applications and to boost their performance.
- A research agenda based on the current state of the art and research gaps.
- Practical recommendations for practitioners and scholars.

The remainder of this article is structured as follows. We illustrate in Section II the train motion model calibration problem. We present the methodology used for this literature review in Section III. We describe and discuss the train motion model calibration methods found in the existing literature in Section IV and outline the main findings of the overview in Section V. Next, we propose a research agenda for the coming years and practical recommendations in Section VI. Last, we outline the main conclusions of this literature review in Section VII.

II. The Train Motion Model Calibration Problem

In this section we introduce the train motion model calibration problem and outline the input parameters to be estimated. Furthermore, we describe the main sources of variation of each parameter, which highlight the difficulty of the calibration problem. We refer to [25] and [26] for a deeper discussion on train dynamics modelling.

The dynamics of a train along its movement on a track can be described according to Newton's second law [2], as

$$m\rho \frac{dv}{dt} = f(v) - r(v) - g(s), \tag{1}$$

where t represents the time, s the train location, v the train speed, m the train mass, ρ the rotating mass factor accounting for the train's rotating parts inertia, $f(v)$ the tractive and brake effort, $g(s)$ the influence on the track geometry on the train dynamics and $r(v)$ the running resistance, which is represented as a quadratic function of speed called the Davis equation [27],

$$r(v) = r_0 + r_1 v + T_f r_2 v^2, \tag{2}$$

where r_0, r_1 and r_2 are nonnegative running resistance parameters that characterize the resistance to motion of a train. T_f corresponds to the extra running resistance observed when running through tunnels. T_f takes unit value when the train is running in the open air and a value larger than one when running through a tunnel due to the extra aerodynamic drag. This tunnel factor depends on several factors, like the train head geometry, the rolling stock composition, the length of the train and the tunnel, the relative cross section of the train with respect to the tunnel and the surface of the tunnel's walls [28].

The resistance due to the track geometry, $g(s)$, accounts for the resistance due to the gradients and curves in the first and second term of Eq. (3), respectively,

$$g(s) = mg\sin(\alpha(s)) + m\frac{k}{R(s)}, \tag{3}$$

where g is the gravity acceleration, $\alpha(s)$ is the angle of the slope of the track at the location s. Factors like the curve radius and superelevation, the train speed and length, the condition and design of the train wheels, and the rail maintenance condition and lubrication are acknowledged to influence the curve resistance [19], although the influence of each factor on the resistance to motion is still not well understood [29]. The curve resistance is mainly modelled as the equivalent gradient that would produce the same resistance to motion, with a hyperbolic dependence on the radius of the curves $R(s)$, being k the associated proportional constant [30]. This means that higher resistances would be experienced on tighter curves. This equivalent-gradient description is usually called the Schmidt formula [31]. Although it is the most widely used for modelling the resistance due to the curves, it is still a simplification of the actual resistance and might not always be accurate, particularly on tight curves and uphill sections, where the Schmidt formula largely underestimates the curve resistance, leading sometimes to freight trains stalling on those sections [32], [33]. Moreover, the influence of the train on the curve resistance [29], [34] highlights the impact of curves on the dynamics of high speed trains, despite the typical large radii of the curves found in the infrastructure used by high speed trains. More sophisticated formulas have been proposed [35], although they are not used as widely as the Schmidt formula [34]. In spite of the relevance of the curve resistance on a train's dynamics, in this article we do not cover the current state of the art on curve resistance estimation, since an exhaustive theoretical and experimental study is required first to understand it better and model it accurately.

Moreover, the maximum effort that can be applied is limited by adhesion, and the engine and brake characteristics and wear,

$$-f_{\min}(v) \le f(v) \le f_{\max}(v), \tag{4}$$

where $f_{\max}(v)$ and $f_{\min}(v)$ are the maximum tractive and brake effort.

Furthermore, the train acceleration is usually bounded due to operational constraints like the predefined braking curves that describe a train's deceleration and comfort constraints to guarantee the integrity of freight rolling stock compositions and the comfort of passengers. Thus,

$$-a_{\min}(v) \le \frac{dv}{dt} \le a_{\max}(v), \tag{5}$$

where $a_{\min}(v)$ and $a_{\max}(v)$ are the maximum and minimum deceleration and acceleration, respectively. The jerk, namely the rate of change of the acceleration, is also another important factor that is usually taken into account to guarantee the comfort of passengers, since abrupt acceleration changes may be felt uneasy by passengers,

$$\left|\frac{d^2v}{dt^2}\right| \leq \gamma_{\max}. \tag{6}$$

with $\gamma_{\max}$ the maximum jerk that can be applied along a journey.

Moreover, besides the explicit parameter dependencies that are shown in Eqs. (1-6), several of the mentioned parameters may also show spatial and temporal variations. Some physical sources of train motion model parameter variation are the wind force and direction, the weather and temperature, the available power at the catenary, the wheel and rail adhesion condition, and the wear of wheels, rail, mechanical parts and engine. In the running resistance r_0 and r_1 describe the mechanical resistance to motion, while r_2 describes the aerodynamic drag resistance. r_0 accounts for the internal frictions and the contact ellipses between wheels and rails, while r_1 stands for the effect of the air momentum on the train dynamics and the flange friction between wheels and rails. The value of r_2 is determined by the train head geometry and cross section, and its value may increase when running through a tunnel due to the extra aerodynamic drag. In addition, adhesion may cap the maximum applicable tractive and brake effort. For instance, bad rail and wheel maintenance may lead to a lower adhesion condition. Moreover, water, snow and fallen leaves on rails may also lower the available adhesion, so these variations are also season-dependent. At high speeds, the maximum tractive effort depends on the tractive power, which in turn depends on the engine wear and temperature and the available catenary power.

Braking in railways is a complex process that is characterized by long braking distances, low friction and delays in the response times. Friction determines the effective maximum brake effort that can be applied, since wheels slip if the brake effort exceeds the friction limit. Friction is an uncertain factor limited by the mass of the train, adhesion, the current moisture and weather, the applied sanding and the wheels, brake and rail maintenance condition. In undisturbed normal operation conditions Automatic Train Protection (ATP) systems and predefined braking curves prevent from braking at the maximum capacity. Therefore, adhesion and manual driving driving variability constitute relevant sources of variations when braking, so the performed braking curves may be the target of calibration procedures in normal operation conditions instead of the maximum brake effort, while keeping the braking model simple. Furthermore, some braking procedures also consider reduced adhesion conditions, like the European Train Control System (ETCS). Among braking systems, those based on air brakes show a particularly complex behaviour. When the air brake is triggered, a difference in the pressure is produced by a compressor in the leading locomotive, propagating in the pneumatic system along the following cars at finite speed lower than the speed of sound [30]. This produces a delay in the response time of brakes of each car or wagon, which, in the extreme case of old and very long freight trains, may lead to differences of the order of a minute between the brake triggering at the locomotive and at the last wagon [36]. Thus, in these types of trains the brake has to be applied gently in order to reduce the risk of derailment. Electro-pneumatic brake systems can homogenize and reduce the response time along the train length by transmitting the brake command electronically to all cars and wagons [36]. Dynamic braking systems brake a train by turning its kinetic energy into electric energy that can be reused, returned into the catenary or simply dissipated as heat [36]. These systems help reducing the mechanical wear on wheels and brake shoes produced by air brake systems. However, dynamic brakes usually have a lower braking capacity, therefore, a smart brake blending strategy is needed to combine both braking systems [37]. Typically, dynamic brakes are applied first, and if the achieved brake rate is not sufficient, then air brake is applied. Still, the uncertainties on the air brake system must be considered in this scenario.

What is more, mass-specific parameters can be obtained by introducing the train mass in the remaining model parameters, which reduces in one the number of parameter to be estimated. The mass can be estimated from the rolling stock operational empty weight and the passengers or freight load and can be considered as constant between consecutive stops. Alternatively, weigh-in-motion systems can be used to estimate the mass of a train by measuring the load of each axle, thus also detecting uneven load distributions. These systems have been predominantly used by infrastructure managers to detect overloaded trains and to control uneven axle loads that may produce extra wear on rails and boost the risk of derailment, and to assess the load capacity on bridges [38]. To this end, the mass can be estimated by placing piezoelectric sensors, strain gauges or accelerometers on the rails, and measuring the movement and bending of the rails under the weight of the wheels when passing above the sensors. Weigh-in-motion can also be performed using on-board equipment, for instance, by placing accelerometers on axles, although this approach is scarcely used [39]. On-board mass measurements can be used along the estimated mass-specific parameters and the rotating mass factor in ATO to determine the exact amount of tractive and brake effort to be applied for driving the train.

In manually-driven trains parameter variations may arise due to variations in the driving process. These driving-induced parameter variations have to be carefully determined in order to improve the realism of railway simulators and eco-driving algorithms. We can distinguish up to four driving phases in a train trajectory between two stops [40], [41].

1) Acceleration to reach a target speed or the track speed limits.
2) Cruising at the given target or track maximum speed, which can be performed in several ways. A constant cruise speed can be kept by holding the right amount of tractive effort to compensate running and track geometry resistances, by braking to avoid overspeeding on downhill track sections, or approximated by performing traction and coast cycles around the target cruise speed.
3) Coasting, which means applying no traction or brake.
4) Braking, which is usually performed by following predefined braking curves or a constant brake rate.

In train motion model calibration, coasting takes a special role due to the absence of effort applied, so the running resistance can be determined from speed measurements provided that the track geometry description is accurate enough. Acceleration is the second most important driving phase for model calibration, since train drivers can be instructed to apply maximum traction to accelerate, which simplifies the calculations needed for the calibration. Then, several model parameters and bounds can be calibrated if the maximum tractive effort or the traction engine's efficiency and the consumed energy are known or measured. Moreover, estimating the train motion model parameters is more difficult for the cruising and braking phases, since cruising is a complex phase that can be performed in several ways, and since the combination of running resistance and applied brake may lead to overestimating the running resistance when braking [13].

Consequently, the running resistance parameters, the maximum tractive effort and power, the maximum brake effort, the train mass and rotating mass factor and the performed braking curves have to be carefully calibrated to minimize the impact of physical and driving-induced sources of parameter uncertainty on railway applications.

III. Methodology

We searched for articles in two well-known databases, Scopus and IEEE Xplore, to review the existing literature on train motion model calibration that utilizes on-board measured operational data as input and those techniques that can be adapted for utilizing such data. We performed a boolean search using combinations of the keywords "model", "calibr*", "rail*", "monitor*", "parameter*", "estimat*", "determin*" in title, abstract and keywords, filtering the topics not related to railways. We restricted the search scope to academic articles in English, including journal articles and conference proceedings published from January 2000 to December 2021 that are available online.

To refine the selection we checked the articles' titles, keywords and abstracts manually. This refinement, however, was found to be inefficient, since some of the articles that used perform train motion model calibration for enhancing the accuracy of another method did not state it in the mentioned parts of the article. Due to this secondary role, the title, abstract and keywords do not usually reflect the fact that a new model calibration technique is introduced in these articles, so this manual inspection was mainly used to remove those articles that do not perform model calibration on the train motion equation parameters. This highlights the need to emphasize the relevance of train motion model calibration within the scientific community, who should give more visibility to train motion model calibration by making its usage explicit in the abstract and keywords of their articles.

Then, forward and backward snowball research became crucial in order to find most of the articles that perform train motion model calibration. We checked the references and citations of the articles found and we also explored other articles published by the same authors. Last, we performed a final refinement of the selected articles, restricting to those

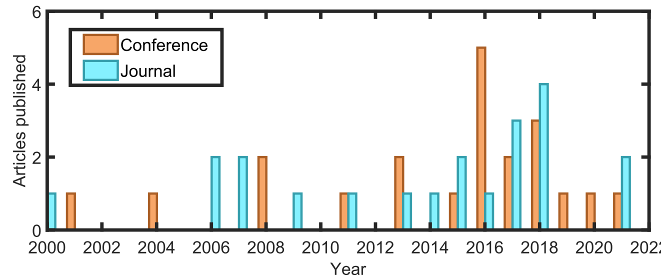

Fig. 1. Distribution of journal and conference per year of publication.

that present train motion model calibration approaches that do not require the usage of extra measurement equipment. In total, 42 articles were finally selected for this literature review: 21 conference and 21 journal articles. Figure 1 shows the number of journal and conference articles reviewed in this article per year of publication. We observe an increasing publication trend that might be due to the growing amount of data available and the increasing need for achieving higher accuracy levels by calibrating the algorithms that embed the train motion model.

IV. Literature Review

Train motion model calibration is a key step towards an accurate implementation and performance of railway applications. Train motion model calibration approaches can be encapsulated in three main groups depending on the stage in which the calibration is performed.

1) In early rolling stock design phases, scaled models, wind tests, Computational Fluid Dynamics CFD) and simulations are performed to estimate the train motion model parameters, particularly the running resistance [42].
2) Full scale tests are performed in controlled scenarios like test tracks or in level and straight track sections to estimate the input parameters of the motion model.
3) For trains in service, operational data is analyzed to perform model calibration.

In this literature review we cover train motion model calibration techniques that use operational data or can be minimally adapted to use it, since using operational data shows many advantages with respect to CFD and tests. For instance, they require less resources, benefit from a larger amount of data available and may lead to estimates that represent the current operating conditions more faithfully. Furthermore, test runs may not be possible for trains of variable composition, thus model calibration based on operational data may be crucial for such railway systems [43]. The archetypical example are freight wagons, since flat, open and closed wagons show radically different dynamics.

In full-scale tests, most efforts have been dedicated to estimating the running resistance. The European Committee for Standarization outlines the main full scale tests for determining the resistance parameters in a European standard [44]. One of these tests consists of pulling the train at a constant speed by means of a windlass and using a dynamometer to measure the resistance to motion. However, this test can only be performed at low speeds and requires the train mass as input. Coasting tests are widely used to explore the train dynamics in a broader speed interval. The rolling stock is accelerated until a target speed is reached and then it decelerates by coasting. This test requires to be repeated several times for statistical purposes. A third type of test can be performed by driving the train at a constant low speed while measuring the consumed tractive power [45]. This method requires an accurate knowledge of the traction efficiency and its accuracy is easily affected by unexpected accelerations. The main problems regarding the mentioned tests are that they are very demanding resource-wise and that they may not lead to accurate results. These tests generally require to be performed in level and straight track sections and are very sensitive to errors in the track geometry description. New full-scale tests that allow for running resistance and rotating mass factor estimation without requiring an accurate track geometry description have been proposed in [14] and [46]. Parameters r_1 and r_2 from Eq. (2) are obtained from three or more standard coasting tests at the same track section with different starting speeds and using Eq. (1) on the speed difference to triangulate the mentioned parameters. Meanwhile, r_0 and the rotating mass factor are obtained from swinging tests at low speeds, where the train is let to coast and move back in a steep uphill track section.

The tractive effort can be determined by setting load cells at the coupling between the locomotive and the next car [47]. To this end, the load cells measure the pulling force needed to move all the cars that succeed the measuring coupling. Alternatively, the applied tractive effort can also be determined by measuring the pulling force at the drawbar between the locomotive and a dynamometer car [48], which is a special measurement car that is coupled for measuring the mentioned pulling force, among other magnitudes. These methods underestimate the tractive effort, since the measurements do not include the tractive effort needed to accelerate the locomotive. Nevertheless, these measurements can be combined with power consumption measurements to enhance the accuracy of the tractive effort measurements. Furthermore, dynamometer cars are typically not suitable for measuring the tractive effort during real operation, so they are mostly used by manufacturers in full-scale tests to determine the characteristics of locomotives' engines.

In this literature review we classify operational data-based calibration techniques based on their way of analyzing data: online techniques that perform the calibration on-the-go and offline ones that analyze data from one or more trajectories at once. These two main types of calibration techniques show substantial differences and complement each other with respect to their applicability. Online model calibration techniques perform adaptive parameter estimation that allows to capture parameter variability along the train run, as opposed to offline techniques that usually produce static parameters that constitute the best fit for the entire analyzed trajectory. Online techniques may therefore be able to capture varying environmental conditions, like the typical increment of the running resistance due to the extra aerodynamic drag in tunnels, the effect of varying wind strength and direction on the resistance along a train's trajectory and the impact of varying adhesion conditions on the applied maximum tractive effort at different parts of the track. In turn, offline techniques capture the average behaviour of a train given the operating condition and their estimates. Offline techniques can analyze larger quantities of data in one go, benefitting from a larger amount of historical data available and more computational power. In contrast, online ones have to be suitable for on-board implementations, which requires them to be faster and less computationally-demanding. However, the smaller amount of data used by online techniques to generate the estimates makes them more sensitive to data anomalies that can affect their accuracy. Railway applications like simulators, timetabling tools and train trajectory optimizers use static train motion model parameters as input, which makes offline techniques more suitable for calibrating such applications than online ones that produce time-varying estimates. Nevertheless, online techniques are of special relevance for monitoring freight trains and trains with varying rolling stock composition in real time due to the lack of historical data available, and for assessing the impact of varying weather conditions.

Tables I and II show a classification of the train motion model calibration approaches covered in this article, clustered in terms of method and estimation technique used. Table I gathers offline model calibration techniques and Table II, the online ones. They include the data used as input, whether the techniques use data measured in real operation, the estimated parameters, whether the estimation is also performed in tunnels and the type of railway system targetted in each article. In Table III the main characteristics of each method are outlined, including the computation speed, the ease of implementation, whether the method is robust against measurement outliers and anomalous data, the driving regimes in which are usually applied and some additional comments. Figure 2 shows a schematic representation of the mentioned techniques, including classifications with respect to their real-time applicability and different technique-type clustering.

A. *Offline Train Motion Model Calibration*

1) Regression: Least Squares Regression (LSR) is the most widely used method to estimate the running resistance parameters and, particularly, the main method used for analyzing data from coasting tests. Speed and location measurements are used along with the track geometry description and the applied effort, if available, to calculate the value of the running resistance needed to match each individual pair of speed and location observations. In the case of a coasting train in a flat and straight track, the train decelerates only due to the running resistance. Then, this method estimates the optimal parameters by minimizing the sum of squared errors between the measurements and the model predictions.

This technique has been mainly used to estimate the running resistance parameters from coasting tests data. Particularly, low speed coasting and cruising tests data is analyzed in [50], where the drive-train efficiency is also estimated. r_2 is estimated in [51] and [52], and the latter included the relative train speed with respect to the wind speed in the quadratic term of the resistance. r_2 in tunnels is estimated in [49] and [53] to

TABLE I
DETAILS FROM SELECTED ARTICLES ON OFFLINE CALIBRATION, SORTED BY METHOD AND TECHNIQUE

Method	**Technique**	**Articles**	**Input data**	**Operational data**	**Parameters estimated**	**Tunnels considered**	**Railway system**
Regression	Least squares	[49]	Location, speed	Yes	r_2 in tunnels	Yes	Main line
		[50]	Location, speed, energy consumption	No	r_0, r_1, traction efficiency	No	Hybrid hydrogen locomotive
		[51]	Location, speed	No	r_0, r_1, r_2	Yes	High speed
		[52]	Location, speed	No	r_0, r_1, r_2	No	High speed
		[53]	Location, speed	No	r_2 in tunnels	Yes	High speed
		[54–56]	Location, speed	No	r_0, r_1, r_2	No	Freight trains
		[57]	Location, speed, traction and brake notches, weather data	Yes	r_0, r_1, r_2, m	Yes	Main line
		[58]	Location, speed, wind speed	No	r_0, r_1, r_2	No	High speed
		[43]	Location, speed	No	r_0, r_1, r_2	No	Freight trains
		[59]	Location, speed, electric power, traction and brake effort command	No	r_0, r_1, r_2	No	Monorail
		[60]	Location, speed, notch, voltage, brake effort	Yes	r_0, r_1, r_2	Yes	Main line
		[48]	Location, speed, tractive effort	No	r_0, r_1, r_2, $f_{\max}$	No	Diesel-electric locomotive
		[15]	Location, speed	No	r_0, r_1, r_2	No	High speed trains
Constrained optimization	Sequential Quadratic Programming	[45]	Location, speed, energy consumption	Yes	r_0, r_1, r_2, m, mechanical efficiency	No	Electric train
Maximum Likelihood Estimation	Expectation Maximization	[61]	Location, speed, voltage, current	No	m	No	Not stated
State observers	Iterative learning identification	[62]	Location, speed	No	r_0, r_1, r_2	No	Main line
	State observers & least squares	[63]	Location, speed, energy consumption, wind speed	No	r_0, r_1, r_2, electric power parameters	No	High speed
Simulation-based optimization	Search	[64]	Location, speed, energy consumption	Yes	r_0, r_1, r_2	No	Main line
		[65]	Location, acceleration	Yes	r_0, r_1, brake distance	No	Tram
		[49]	Location, speed, energy consumption	Yes	r_2 in tunnels	Yes	Main line
		[66]	Location, speed, energy consumption	No	r_0, r_1, r_2, traction and brake efficiencies, auxiliary power	No	High speed
		[67]	Location, speed, tractive effort	Yes	r_0, r_1, r_2	No	Freight trains
	Iteration-based search	[68]	Location, speed	Yes	brake rate, cruise speed	Yes	Metro
	Manual fit	[69]	Location, speed	Yes	traction and brake performance, cruise speed	No	Main line
Metaheuristics	Genetic Algorithm	[70, 71]	Track occupation data	Yes	r_0, r_1, r_2, train length, $f_{max}(v)$, brake rates, cruise speed	No	Main line
		[72, 73]	Location, speed, energy consumed and recovered	Yes	r_0, r_1, r_2, traction and brake performance, cruise speed	Yes	Freight trains
		[74]	Location, speed, energy consumption	Yes	r_0, r_1, r_2	No	Main line
		[75]	Average fuel consumption	Yes	r_0, r_1, r_2	No	Diesel trains
	Particle Swarm Optimization	[76]	Energy consumption	No	r_0, r_2, line resistivity, traction efficiency, rotating mass coefficient	Yes	Metro
	Simulated Annealing	[6, 77]	Location, speed	Yes	traction and brake performance, cruise speed	No	Main line

TABLE II
DETAILS FROM SELECTED ARTICLES ON ONLINE CALIBRATION, SORTED BY METHOD AND TECHNIQUE

Method	Technique	Articles	Input data	Operational data	Parameters estimated	Tunnels considered	Railway system
Regression	Recursive least squares	[78, 79]	Location, speed, applied effort	No	r_0, r_1, r_2, m	No	Monorail
		[80]	Location, speed	Yes	r_0, r_1, r_2	Yes	Metro
	Multi-innovation least squares	[80]	Location, speed	Yes	r_0, r_1, r_2	Yes	Metro
Kalman-like state observers	Unscented Kalman Filter	[81]	Location, speed	Yes	r_0, r_1, r_2, train length, $f_{\max}(v)$, brake rates, cruise speed	No	Main line
		[82]	Location, speed	No	r_0, r_2	No	Freight trains
	Extended Kalman Filter	[83]	Location, speed, applied effort	No	r_0, r_1, r_2	No	High speed
Bayesian methods	Particle filtering	[84]	Location, speed	No	r_0, r_1, r_2	No	High speed
Gradient descent	Multi-start gradient optimization	[4]	Location, speed, power consumed and generated	Yes	r_0, r_1, r_2	Yes	Main line

TABLE III
LIST OF CHARACTERISTICS OF EACH METHOD

Offline method	Articles	Computation time	Ease of implementation	Robust method	Driving regimes	Additional comments
Regression	[15, 43, 48–60]	Fast	Low	No	Coasting	A constrained regression method should be used to avoid obtaining wrong estimates of r_1
Constrained optimization	[45]	Medium	Medium	Yes	All	Accuracy sensitive to initialization values
Maximum likelihood estimation	[61]	Slow	High	Yes	All	Accuracy sensitive to initialization values
State observers	[62, 63]	Fast	High	No	All	Requires training. Calibration may fail if an input trajectory differs significantly from the ones used for training
Simulation-based optimization	[49, 64–69]	Medium	Low	No	All	Finds local optima. The simulator used may bias the estimates
Metaheuristics	[6, 70–77]	Slow	Low	No	All	Finds local optima. Explores the parameter space extensively
Online method	**Articles**	**Computation time**	**Ease of implementation**	**Robust method**	**Driving regimes**	**Additional comments**
Regression	[78–80]	Fast	Low	Yes*	All	*: robust if a measurement window is considered in each iteration
Kalman-like state observers	[81–83]	Fast	High	No	All	Accuracy highly sensitive to initialization values
Bayesian methods	[84]	Medium	Medium	Yes	All	Accuracy dependent on set of parameter candidates
Gradient descent	[4]	Slow	Low	No	All	Computation time too slow for real-time calibration if the measurement window is large

gauge the impact of the extra aerodynamic drag on the train dynamics. The speed measurements are weighted in [49] to compensate the uneven speed measurements distribution and to avoid underfitting r_2 in the least-visited speed intervals. In [54], the author concludes that the impact of the wind can be mitigated by coasting in both directions and averaging among all tests, that an inaccurate value of the mass and the rotating factor may bias the estimation, and that r_1 is the most uncertain and difficult parameter to be calibrated due to the predominance of the other two parameters in low and high speeds. Several empirical formulas that model individual running resistance parameters dependencies on train characteristics like number of cars, number of axles, train length or axle load are calibrated in [55] and [56]. r_0 depends linearly on axle load and number of axles and might be affected significantly by the state of the track. r_1 depends on the train length, but not on the train mass, and no effect from air momentum drag is distinguished, contradicting the general knowledge about r_1. In turn, r_2 has two parts: a constant one depending only on the front and rear of the train and a component that increases linearly with train length. This is supported by [58], where the impact of bogie fairings on the running resistance is evaluated, observing that they may reduce up to 10% of the total aerodynamic drag, including some impact on r_0. In [43], new running resistance formulas are estimated with the aim to update the phenomenological formulas used in the Czech Republic. The existing methodological formulas are found to be too conservative, being most of them not correct

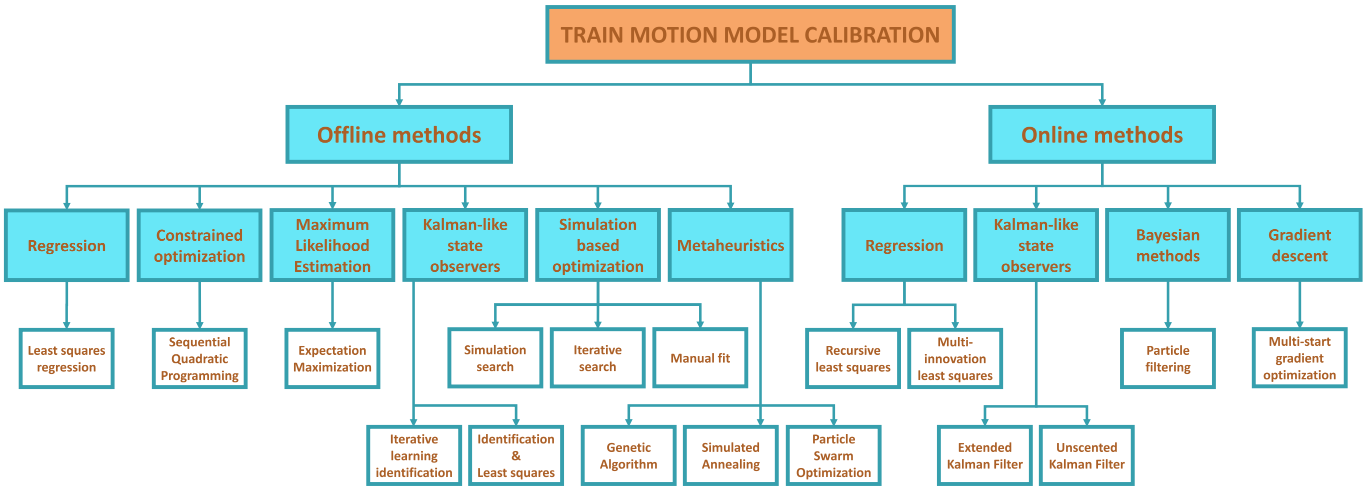

Fig. 2. Schematic representation of the train motion model calibration techniques included in this literature review.

physically. The results show that r_0 has two components: one mass-dependent and a constant one, while r_2 depends linearly on the number of new fronts of a train, namely loaded wagons behind empty ones. The running resistance parameters of a monorail are estimated in [59]. The estimates obtained from coasting tests are compared with well-known formulas in [15], finding that most of these formulas overestimate the running resistance. Among those, Armstrong and Swift's formula [85] is the most accurate. In [48], the maximum tractive effort and running resistance parameters are estimated from pulling and coasting tests, respectively. Regarding operational data, running resistance estimation under various circumstances is explored in [57], including tunnels and different weather conditions using monitoring data. The results obtained from operational data are found to be consistent with coasting tests in [60].

Some of the advantages of LSR are that it allows to analyze large amounts of data, it is fast and easy to be implemented. This made LSR the most frequently used offline calibration method, particularly for analyzing coasting data. However, errors in the track description may lower its accuracy. LSR often leads to negative values of r_1, which is not physically correct as the running resistance parameters should be nonnegative and standard LSR is not suited for constrained regressions. Moreover, speed measurement records usually show an uneven distribution over speed intervals, leading to overestimating parameters in the most frequently-visited speed intervals and underestimating them in the least-visited ones. Furthermore, ordinary least squares algorithms are not robust under the presence of outliers in the measurements.

2) Constrained Optimization Methods: Parameter estimation can be formulated as a constrained optimization problem by choosing an appropriate objective function to be minimized with respect to the target parameters, and by adding constraints to the optimization model like upper and lower bounds to the target parameters. Several techniques can be used to solve the resulting constrained optimization problem. For instance, Sequential Quadratic Programming (SQP) [86] models the constrained optimization problem as a quadratic progamming subproblem that is solved iteratively until the set of estimates converges to a solution.

SQP is utilized in [45] to estimate the mass, mechanical efficiency and running resistance parameters from speed, location and energy consumption data, distinguishing the individual contributions of the locomotive and cars to the total running resistance. The error in the accumulated energy consumption is used as the objective function. SQP is found to be robust, although the rotating mass factor was omitted and the running resistance was overfitted in the most visited speed interval.

Formulating the parameter estimation problem as a constrained optimization one allows for more flexibility in the model and the techniques to be applied for solving it than LSR. SQP can be used to avoid obtaining negative values of r_1 and shows more robustness against measurement noise than LSR. However, the computation time may be larger and may depend on parameter initialization.

3) Maximum Likelihood Estimation: Maximum Likelihood Estimation (MLE) aims to find the set of parameter values that maximizes the probability of the values of the target parameters with respect to the available observations. This maximum can be obtained by means of numerical methods like the Expectation-Maximization algorithm, where the maximum is calculated iteratively. At each iteration, the algorithm computes the expected value of the log-likelihood function of the estimates of the previous iteration conditioned to the speed measurements, and then a new set of estimates that maximize the mentioned log-likelihood is obtained. To this end, a particle filter has to be used to approximate the probability density functions required for the computation of the log-likelihood.

MLE is used in [61] to estimate the train mass, showing that estimating the mass may lead to a more accurate speed profile reconstruction. However, the algorithm is validated using only simulated data.

MLE may produce estimates robust under measurement noise and missing data, provided that a sufficiently large set of trajectories is used as input. However, MLE is computationally expensive and may be sensitive to the parameter initialization.

Furthermore, understanding and deriving the likelihood equations may be a complex task depending on the mathematical background of the user.

4) Kalman-Like State Observers: In control theory, state observers are used to estimate the internal state of a system from observations of a certain set of variables. These state observers may perform parameter estimation by introducing the target parameters as state variables.

An Iterative Learning Identification (ILI) that exploits the repeating characteristics of train trajectories is proposed in [62]. ILI is able to improve the estimation iteration after iteration. To this end, at each iteration a state observer is utilized to produce a new estimation of the target parameters based on the error between the predicted and measured variables. Several state observers are combined with LSR in [63] to estimate the running resistance and some electric power parameters. However, this method failed to calibrate r_0.

While ILI is guaranteed to converge if the dynamical system under consideration fulfills a certain set of requirements, this approach needs to be explored further to assess whether it is able to produce accurate estimates when using real data, particularly in the case of trajectories that deviate significantly from the ones used for training the model.

5) Simulation-Based Optimization: Simulators can also take a relevant role in parameter estimation. Given a set of input parameters, simulators produce data that can be compared with real measurements to assess the performance of the mentioned parameters. Search algorithms, gradient descent, iteration-based search and manual fit are among the different approaches that can be used to find the set of parameters that reproduces the real measurements more accurately.

Simulation-based optimization is used to estimate the running resistance parameters in the following articles. A commercial optimization toolbox is used in [64] to calculate the estimates that minimize the squared difference between the observed and the calculated energy consumption, provided that the speed profiles match. The braking distance of trams is also estimated in [65] using coasting operational data. Traction and brake efficiencies and auxiliary power are also estimated in [66], targeting to match energy consumption records, finding that the parameters that are most related to energy consumption are r_2 and the traction efficiency. Tractive effort measurements are used in [67] to generate speed profiles and calibrate r_0 first at low speeds, and then r_2 at high speeds. In [49], r_2 is estimated in tunnels by minimizing the squared deviation between the measured and the calculated speed time series subject to the measured energy consumption and speed. There, the tractive effort is obtained from the consumed power, assuming constant traction efficiency. This method is found to be more robust than LSR and the authors observed that theoretical values of r_2 clearly overestimate the running resistance. Furthermore, the mass factor is shown to influence the value of r_2, and the inclusion of energy consumption data may add an extra layer of precision to the estimates.

In [68], an iterative search is used to estimate deceleration rates and the cruise speed, targeting to match the observed running time and minimize the root mean squared error between the measured and calculated speed profiles. Performance parameters representing the actual value relative to the maximum of the traction, cruise speed and brake rates are calibrated manually in [69]. Although the simplified kinematic models used in the last two articles constitute a good first order calibration, they may not be able to capture complex driving behaviour.

Simulation-based optimization methods offer more modelling flexibility than other approaches and are generally easy to be implemented. However, these methods usually lead to local optima and the quality of the estimates is correlated to the level of realism of the simulator, which internal configuration may constitute a source of bias with respect to the model calibration.

6) Metaheurisitics: Metaheuristic algorithms are also a common way of solving simulation-based optimization problems in industry and academia due to their ease of implementation and practicality. In these algorithms, a set of parameters which performance is evaluated is called a candidate, and the set of candidates is called population. The initial population is randomly generated, and each algorithm proposes a different way of evolving the population.

In a Genetic Algorithm (GA), at each iteration the candidates that show the worst performance are eliminated, and new ones are generated by combining and modifying randomly the remaining candidates, so that the population is kept constant at each iteration. The running resistance is estimated in several articles using GA [70], [71], [72], [73], [74], [75]. Particularly, GA is combined in [75] with a SQP-based speed profile calculator, finding results consistent with coasting tests. A quadratic interpolation routine was used to assess whether the newly generated candidates are promising or not, allowing to save computation time from the SQP calculation. Furthermore, train lengths, maximum tractive effort and power, brake rates and the cruise speed are estimated in [70] and [71] from track occupation data and statistical distributions of those parameters are calculated. Acceleration and deceleration rates and the traction performance rate are also estimated in [72] and [73].

In Particle Swarm Optimization the candidates in the population move over the parameter space while sharing information between them, so that they are influenced by their individual and the population best-fit locations. PSO is used in [76] to calibrate the dynamics of a metro vehicle for a simulator. The track was divided in several segments, and the running resistance parameters, the rotating mass factor, the overhead wire resistivity and the power efficiency were calculated in each segment.

In Simulated Annealing (SA), the candidates move randomly in the parameter space, introducing a temperature parameter that controls the mobility of the particles through the parameter space. This temperature is gradually lowered through the iterations, reducing the candidates' speed and guaranteeing the algorithm convergence close to a local minimum. Two articles [6], [77] build on [69] by substituting the manual fit by SA, which simplifies the calibration procedure. However, the calibration performance is limited again by the simplified model used.

As simulation-based optimization methods, metaheuristic algorithms show similar pros and cons. However, they may cover the parameter space more extensively at the cost of showing the highest computation times of the considered methods.

B. Online Train Motion Model Calibration

1) Regression: LSR can be adapted to perform online estimation by adding recursively in each iteration the last set of measurements to the least squares procedure and calculating the resulting parameter update. The resulting method, Recursive Least Squares (RLS), is enhanced as Multi-Innovation Least Squares (MILS) by considering not only the error between the model predictions and the last set of measurements, but the one with respect to the last few observations, which constitute the observation window. MILS can also be implemented with a varying window size (V-MILS), which helps producing more robust estimates in the presence of measurement outliers or missing data.

RLS is used in [78] and [79] to estimate the train weight and the running resistance parameters r_0 and r_1. In both articles RLS is validated using simulation data with added noise representing the effect of wind. Several adaptive parameter estimation algorithms including RLS, a regularized version of RLS that guarantees the positivity of r_1, MILS and V-MILS are tested in [80]. These methods are verified using real data from the ATO system of a metro train and estimate the running resistance accurately. Both MILS and V-MILS outperform RLS.

RLS shows the same advantages and disadvantages as LSR. It is simple, easy to be implemented and computationally very efficient. The inherited sensitivity to measurement outliers may be particularly problematic for RLS due to the iterative data processing procedure. However, MILS and V-MILS may solve this issue. Moreover, V-MILS is robust under the presence of missing or anomalous measurements. Furthermore, RLS allows to weight less the oldest measurements by introducing a forgetting factor.

2) Kalman-Like State Observers: Several nonlinear versions of the Kalman Filter have been used for estimating train motion model parameters in real time. In the Unscented Kalman Filter (UKF) the state and noise statistics are sampled deterministically by means of a technique called Unscented Transformation and propagated through the nonlinear train motion model in order to determine the state estimates at each iteration. Another nonlinear version of the Kalman Filter is the Extended Kalman Filter (EKF). The main difference with resepct to UKF is that EKF considers the state distribution a Gaussian random variable, which is propagated through a linearization of the dynamical system.

In [82], a simulated case study shows that UKF can estimate the running resistance parameters r_0 and r_2 when a train is coasting on a level track. The results obtained are sensitive to the initialization value of the covariance matrix and the authors conclude that a rigorous filter initialization procedure is required for the successful implementation of the algorithm. Similar conclusions are drawn in [81], where UKF is applied in the four driving phases mentioned in Section II. There, UKF is combined with a driving phase identifier that determines in real time the current driving phase. Then, the remaining train motion model parameters and their maximum bounds are estimated by means of a post-processing module that analyzes the outputs of the UKF and the driving phase identifier. The authors conclude that tractive and brake effort measurements are an asset for enhancing the estimation accuracy. EKF is combined in [83] with Gaussian sum theory to model any non-Gaussian state and noise statistics as a sum of Gaussian random variables that can be processed by a set of EKFs to estimate the running resistance. There, the number of Gaussian terms is reduced after each iteration of the algorithm by selecting those with the largest weights to guarantee real-time performance.

Kalman-based filters are widely spread in the industry, as they can produce very accurate estimates, deal with non-Gaussian noise and are computationally efficient. However, their main drawback is that they are very difficult to be implemented and tuned. Furthermore, their performance depends critically on the covariance initialization value.

3) Bayesian Methods: Particle Filtering is a different online calibration technique that can deal with non-Gaussian dynamics, where the state statistics is evaluated by means of a random sampling.

In [84], the running resistance parameters are described as a weighted sum of a given set of predefined candidates. This procedure assumes that the real value of the parameters will lay within the mentioned set of predefined parameters. The estimation is performed by means of a Particle Filter that calculates the posterior probability of each parameter candidate subject to the most recent set of measurements and chooses the most likely parameter among the candidates as output if its probability is more than 90%, and it is computed as the weighted sum of all the probabilities associated to the parameter candidates otherwise.

Bayesian methods are conceptually simple, yet powerful and may deal with non-Gaussian noise. However, the major drawback of the implementation proposed in [84] is that it may be inaccurate if the set of predefined parameter candidate does not represent the current real value of the target parameters. Compared to the previous two online calibration methods, Particle Filtering is not deterministic and is more expensive computationally.

4) Gradient Descent: In optimization, gradient descent is a technique that aims to minimize a function iteratively, by evaluating the function in the direction where it decreases faster with respect to the current estimates.

In [4], a multi-start gradient optimization procedure is used to estimate the running resistance parameters from a finite window containing the last few measurements. The driving phase is determined first from the acceleration and net power consumed for every set of measurements, while the predicted tractive effort is used as the measure of performance of the proposed framework. Two different objective functions are explored: the root mean square error and the sum of absolute errors between the predicted and the measured tractive effort, being the latter faster, less sensitive to outliers and the one used in the remaining of the article.

However, this method can only perform online estimation for small measurement windows due to its high computation time.

V. Main Findings

In total, we have reviewed 42 articles. 34 on offline calibration techniques and 8 on online approaches. The reviewed techniques have been applied to most types of rolling stock: from monorail to high-speed trains. Most articles focus on determining a few parameters, being the running resistance parameters those that have received most attention due to their impact on the energy consumption.

Least Squares Regression (LSR) is the most frequently-used calibration method due to its simplicity and performance. The acceleration is usually calculated from speed measurements and the force balance in Eq. (1) is used to find the running resistance parameters that best fit the measurements. However, in [49], this procedure was found to be less consistent than fitting the squared error between the measured speed and the predicted speed calculated from the previous speed measurement. All the regression methods were explored using a squared error-based objective function. The sum of absolute errors has been explored in [4], showing that in some cases it can be more computationally efficient than a squared error objective while still producing similar results. Other offline techniques can be used to estimate more parameters at once, at the expense of more computation time. Regarding online calibration techniques, further research is needed to reach a reliable on-board implementation.

Several factors that limit the calibration accuracy are systematically overlooked:

- The non-negativity running resistance parameter constraint is usually introduced manually in LSR by either fixing r_0 and r_1 or by setting $r_1 = 0$. This latter parameter, r_1, is generally considered to be the most difficult running resistance parameter to be calibrated. Running resistance parameters estimation is a constrained optimization problem that should be addressed by using an appropriate constrained optimization method. In the case of LSR, this can be solved by using a modified version of LSR that allows for constrained optimization.
- Speed measurements are often distributed unevenly along a train's speed range, leading to overfitting the parameters in the most-visited speed intervals.
- So far, only a handful of authors have developed calibration techniques that are robust against anomalous data or outliers [4], [45], [80]. However, these articles mainly focus on estimating the running resistance parameters, and only [45] estimated the train mass and mechanical efficiency.
- All the reviewed methods require either an accurate track geometry description or to be used on a level and straight track section. This is not a hard constraint, since a test for determining running resistance and the rotating mass factor without requiring track geometry information has been designed in [14] and [46]. However, the train speed has to be measured at the exact same locations along different runs, so extending this method for using operational data may require further research.
- We have observed that validating and comparing the existing calibration techniques is generally difficult due to the lack of target parameters' ground-truth data and the wide variety of real and simulated case studies used. Some of the techniques are validated by means of data generated in simulations, although this does not guarantee their performance or accuracy in a real scenario.

Despite offline calibration has received more attention in the literature, some articles highlight the need for on-board calibration [55], [56], particularly for rolling stock of variable composition and freight trains. Some of the discussed articles challenge the general knowledge on the physical sources of train motion model parameter variation [56]. Furthermore, just a few articles have addressed determining the relevance of individual sources of variation [49], [55], [56] or driving-induced variations [4], [70], [71], [81]. Remarkably, only [49], [53] focus on estimating the extra running resistance in tunnels, despite its impact on the train dynamics and energy consumption.

Online calibration techniques show a wide variety of ways of analyzing the available measurements. Some of them use only the last measurement available to generate the estimates at each iteration of the algorithm [78], [79], [80], [81], [82], [83], which guarantees a faster computation time at the cost of more sensitivity of the estimates to anomalous data and noise. In contrast, some techniques consider all the available measurements at each iteration, and add the new ones as soon as they are generated [84]. This produces an increasingly larger computation time that might become an obstacle in the event of an onboard implementation. Other techniques include a measurement window consisting of a finite number of the most recently recorded measurements [4], [80]. The size of the window has to be fine-tuned for each technique to optimize the balance between computation time and robustness against measurement outliers and missing data. In [80], measurement windows including from 2 to 6 measurements are explored (with a sampling rate of 1s), showing that larger windows produce more accurate estimates than the standard Recursive Least Squares that only uses the latest measurement available. Moreover, a varying window size has also been explored [80]. In turn, in [4] very large windows that include 30 to 60 measurements were found not suitable for real-time calibration, so the measurement window has to be reduced to 10 measurements (sampling rate, 1s). However, the size of the window and the computation time depend on each technique, the efficiency of the implementation and the computational power available. Surprisingly, the addition of a forgetting factor that contributes to gradually give less weight to old measurements than to the newly generated ones is still to be explored in train motion model calibration.

The analysis of the literature shows another advantage of using operational data for train motion model calibration over full-scale tests: the applicable range of speed of the calibration techniques. On the one hand, pulling and cruising full-scale tests, including the new ones proposed in [14] and [46], have to be performed at low speeds. On the other hand, most techniques that analyze operational data could be applicable in any speed range. For instance, RLS has been historically used to analyze coasting data, which usually spans the upper speed interval of the train, however, this method can also be used to analyze data below 10km/h [50]. Similarly, the Unscented Kalman Filter has been used to analyze coast curves in [82] and has been extended to cover the whole trajectory in [81].

VI. Research Agenda and Recommendations

Based on the train motion model calibration literature review presented in Section IV and the findings outlined in the previous section, we hereby outline open research questions and practical recommendations.

A. Research Agenda

1) Online Calibration for on-Board Eco-Driving Algorithms: An adequate implementation and usage of on-board eco-driving algorithms like real-time train trajectory optimizers, DAS and ATO requires performing several iterative rounds of tests and measurements to calibrate such algorithms and achieve their target levels of precision. This is a difficult and resource-consuming task that is usually performed in early implementation stages and fine-tuned at the beginning of their operational life. However, as mentioned in Section II, the wear generated in the mechanical parts of the train and its engines, varying environmental and seasonal conditions may have an impact on the train dynamics. This can be modelled as parameter variability, which may lower in time the accuracy of the calibrations. Furthermore, in the case of freight trains with variable rolling stock composition, the usual calibration tests cannot be performed [55], [56]. This fact highlights the need for an online train motion model calibration framework for on-board energy-efficient applications, which is still missing in the current literature. Such framework could benefit from the large amount of data that is measured and stored on-board. It would have to calibrate accurately in real time the usual input parameters and bounds of energy-efficient applications mentioned in Section II, from mass-specific running resistance parameters to bounds on the tractive effort and power and, in the case of manually driven trains, the performed brake curves. Moreover, it should be computationally efficient in order to allow for real-time on-board calculations and easy to be configured to perform the estimations automatically. This latter characteristic would reduce the need for tests, lowering partially the economic, capacity and resource costs associated to the calibration procedures during the implementation phase. An initial step towards such a framework can be found in [81].

2) Calibration Frameworks Robust Against Anomalous Data and Varying Conditions: Nowadays, railways are fitted with a wide variety of sensors that produce the input data required by algorithms like train motion model calibrators. However, all measurements are affected by noise. This may affect the accuracy of the calibration, especially when outliers or wrong measurements are introduced as input. Moreover, coverage issues may lead to missing data. An archetypal example is the missing Global Navigation Satellite System (GNSS) location data at tunnels where the satellite coverage is lost. These data issues affect particularly online calibration algorithms that analyze the measured data on-the-go, as these errors may stand out from the limited amount of measurements used at each calculation, biasing the estimates. Train motion model calibration frameworks need to be robust under the presence of outliers and missing data, particularly those that are designed for on-board applications. The accuracy of the estimates must not be affected by such anomalous data in order to guarantee the reliability of the estimates in most operational scenarios. As shown in Section IV, online calibration algorithms add a layer of robustness to their estimations by analyzing several measurements at the same time, either by using the last few measurements [4], [80], or by using all the available measurements [84]. However, analyzing more than one set of measurements at each iteration is more demanding computationally, which might become an obstacle in the event of an onboard implementation. Most online calibration algorithms require using an initial guess of the estimates and a robust framework should be robust against the mentioned initial guess, although this feature is still to be explored in the scientific literature. Regarding offline calibration procedures, Least Squares Regression is the most popular technique among them, however, it may be highly sensitive to measurement outliers. Least absolute deviations, which is only explored in [4], constitutes a usual alternative to least squares when aiming for robustness. Furthermore, a robust calibration framework covering most train motion model parameters is still to be developed. What is more, the track geometry description is a simplification of the actual geometry of the track, constituting a significant source of uncertainty and error for the train motion model. Most calibration procedures either consider the track as flat, or assume that the track geometry description is perfectly accurate. Therefore, a calibration procedure that uses operational data as input and does not require knowing the track geometry description, in line with [46] and [14], would also be of practical interest. In conclusion, more research on robust train motion model calibration is needed to enhance the reliability of the estimates and to boost the performance of railway applications.

3) Quantifying Sources of Parameter Variation: A systematic study of the individual physical sources of parameter variability based on operational data is still missing in the scientific literature. In Section II we described several sources of parameter variation for the train motion model. Passing through tunnels and running under headwind may increase the running resistance and the energy consumption. The train mass may vary depending on the load, especially in the case of freight trains. Fluctuations in the catenary power may vary the maximum tractive effort that can be applied. These are just a few examples of the existing sources of variation affecting a train's dynamics. As we outlined in Section V, some individual sources of parameter variations have been studied in a small portion of the articles covered, but more research is required to quantify the individual contributions. A study of the impact of rolling stock characteristics and the main variations present in a fleet of trains with the same composition may emphasize the importance of individual train calibrations and train-tailored solutions. Particularly, energy-efficiency research, like eco-driving, could benefit from more studies on the impact of running through tunnels on the running resistance, building on [49] and [53]. Analogously, online calibration may contribute to understanding further the impact of curve resistance on the dynamics of a train. Moreover, weather may also influence train motion model parameter variations. Public weather data sources could be used for analyzing seasonal and weather-induced variations [57]. The impact of wind speed and direction could also be assessed, as it may also influence the train dynamics. For example, the Sauthoff formula is a widely used variation of Davis equation that accounts for the wind speed in its quadratic term, however, it assumes a wind speed of 15km/h [15]. This research could also validate running resistance formulas that aim to be more accurate, like the wind-dependent one proposed in [87]. The knowledge obtained could be of interest for rolling stock manufacturers, who could optimize their designs taking into account the observed variability. Moreover, the relevance of on-board calibration could be further highlighted by this research, as well as the need for robust algorithms against the initial guess of the estimates, so that their accuracy is not affected by uncertainties and variations of the operating conditions. Alternatively, driving-induced parameter variability could also be researched. The maximum applied tractive effort and power, the applied brake rates and switching points between driving strategies are affected by driving style variations, particularly in the case of manually-driving trains [6], [70], [81]. Comparing the results of manually and automatically-driven trains could also be of interest to further gauge the advantages and disadvantages of railway automation. Furthermore, driving-induced variations could be assessed for different schedule deviation scenarios, that is to say, whether trains run on-time, delayed or early.

B. Practical Recommendations

1) A Public Dataset for Model Validation and Benchmarking: We suggest the creation of a publicly available dataset for validating and benchmarking the performance of train motion model calibration frameworks. Data availability is an issue in railways research. Some of the techniques analyzed in this literature review are validated by means of simulations, although this does not guarantee the performance or accuracy of the proposed approaches in real life. Moreover, comparing the performance and results obtained by those techniques that are tested using real measurements is difficult due to the large variety of case studies found in the literature, spanning from monorail to high speed trains. Furthermore, usually no ground-truth data for the real value of the estimated parameters exists. These three facts constitute a remarkable hurdle for the validation and comparison of train motion model calibration frameworks. We recommend a minimum number of 50 runs of a train unit running on a line to be included in such dataset, including an accurate track description of the considered line, GNSS location, speed and applied effort measurements. Train mass, tractive wheels rotation speed and energy consumption measurements could be a plus that would allow for more flexibility in new calibration techniques. Up to the authors knowledge, there are no such publicly available datasets for operating trains [88].

2) Good Practices for Train Motion Model Calibration: Train motion model calibration is generally a difficult problem and we have detected some common pitfalls in the articles covered, which are outlined in Section V. Input data should be weighted to avoid underfitting the target parameters in the least-visited speed intervals, as speed measurements show an uneven distribution along a train's speed range. Moreover, r_1 is generally considered the most difficult running resistance parameter to be estimated. Calibration procedures usually lead to a negative value of r_1, which is not physically correct. This can be solved by applying a constrained optimization procedure like Constrained Least Squares, SQP, or by adding a regularization term to the cost function. Furthermore, the value of the rotating mass factor may affect the running resistance estimation, thus it must be taken into account in order to produce accurate estimates [49].

Regarding the data sampling rate for calibrating the train motion model, we suggest a sampling rate from 1Hz to 10Hz for all the mentioned variables except for the energy consumption, as higher sampling rates require more filtering, storage and subsampling, while lower sampling rates may not be suitable for most applications or may lead to inaccurate estimates. The European Committee for Standarization recommends a minimum sampling rate of 10Hz for train measurements [44], however, the mentioned inconveniences of higher sampling rates should be taken into account.

We do not recommend using accelerometers data for train motion model calibration, as they may produce noisy data, leading to estimates that are less accurate than those inferred from other data sources [51].

Train motion model calibration should be at least an essential preliminary step in the implementation of any railway application to guarantee its accuracy and effectiveness. We strongly suggest to use online techniques for calibrating continuously any on-board application that utilizes the train motion model. Particularly, train control algorithms, Driver Advisory Systems and Automatic Train Operation usually have very high precision requirements, so model calibration is vital for such applications. The performance and realism of simulators and train trajectory optimizers also require an exhaustive calibration, which may be performed offline using large amounts of data from historical measurement records.

3) Train Motion Railway Calibration and the Existing Railway Systems: Any of the presented online and offline calibration techniques are suitable for main line, freight and high speed trains. However, regression algorithms that analyze coasting data only are not suitable for monorail, urban/light and metro trains due to the absence of coasting phases or their typical short duration in such railway systems. Instead, any of the other calibration techniques presented in this article could be used for calibrating the dynamics of these latter systems.

4) The Role of Infrastructure Managers: In Section VI-A.2 we mentioned that the track geometry description is a source of error in the train dynamics due to the fact that it is a simplification of the real geometry. Moreover, no train motion model calibration framework that utilizes operational data and that is robust against errors in the track geometry description exists yet. Therefore, we request infrastructure managers to describe the track geometry as accurately as possible, as this can enhance the train motion model performance and benefit other railway applications, like ETCS continuous brake curves calculation.

5) Giving Visibility to Train Motion Model Calibration in Academia: We would like to demand researchers to state clearly in the title or abstract that train motion model calibration is performed in this article, as this will contribute to drawing more attention to this topic. When gathering the articles that cover train motion model calibration for preparing this literature review article we faced an unexpected challenge. Scholars tend not to mention explicitly that train motion model calibration is performed when it is used as a means to calibrate an algorithm that embeds the motion model. This hinders the task of learning new calibration techniques and good practices.

VII. Conclusion

An accurate and effective implementation of railway applications like timetabling tools to Driver Advisory Systems and Automatic Train Operation requires calibrating a train motion model that describes the dynamics of a train running on a track. In order to highlight the importance of train motion model calibration in the railway industry and academia, we have presented a critical review of the train motion model calibration techniques that are found in the scientific literature. Particularly, we have focused on those techniques that use operational data, since such techniques benefit from larger data availability, do not require scheduling resource-demanding tests, and may lead to estimates that represent better ordinary operation conditions than tests and simulations. To this end, we have clustered the existing calibration techniques in two categories: online calibration that analyzes the measured data on-the-go, and offline calibration that is able to analyze data from several trajectories in one go. We have described and analyzed each technique, identifying its pros and cons, as well as the most common problems and pitfalls.

Based on this analysis, we have proposed a research agenda for the coming years. We have highlighted the need for an accurate online calibration framework for on-board energy-efficient driving applications, as well as for frameworks that are robust under measurement outliers and missing data that may decrease the calibration accuracy. We have described this calibration problem and listed the main sources of parameter variation, including physical sources like wind and weather, and manual driving that produces driving-induced variations. In this regard, we also proposed a quantitative research of the parameter variability due to each individual source.

Last, we have suggested also several practical recommendations regarding train motion model calibration. Particularly, the railway community would benefit from a publicly available dataset for validating and benchmarking newly developed calibration frameworks. In addition, we have outlined the main pitfalls to avoid when calibrating the train motion model, recommended a sampling rate for the measurements used as input to the calibration frameworks and highlighted the role of infrastructure managers in describing the track geometry accurately for improving the performance of such calibration frameworks and the role of researchers in giving visibility to this topic within academia.

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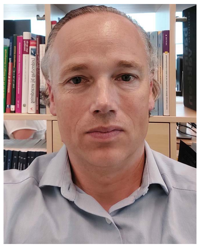
Alex Cunillera (Member, IEEE) was born in Palma de Mallorca, Spain, in 1995. He received the B.Sc. degree in mathematics and physics from the Complutense University of Madrid in 2017 and the M.Sc. degree in physics of complex systems from the Institute for Cross-Disciplinary Physics and Complex Systems, University of the Balearic Islands. He is currently pursuing the Ph.D. degree with the Delft University of Technology. He has been working on train eco-driving since 2019. His research interests include railways, optimal control theory, trajectory optimization, robust optimization and control, parameter estimation, driver advisory systems, and automatic train operation.

Nikola Bešinović was born in Serbia in 1985. He received the B.Sc. and M.Sc. degrees from the University of Belgrade, Serbia, in 2009 and 2011, respectively, and the Ph.D. degree in transport from the Delft University of Technology in 2017.

He worked at the Department of Transport and Planning, Delft University of Technology. He is currently the Head of the Chair of Rail and Public Urban Transport, TU Dresden. His research interests include AI, optimization, analytics in railway and public transport systems, and resilience of transport networks.

Dr. Bešinović received several scientific awards, including the IEEE ITS Dissertation Award and the Young Railway Operations Researcher Award from IAROR. He is also a Vice-Chair of the INFORMS Railway Application Section.



Ramon M. Lentink was born in The Netherlands in 1975. He received the M.Sc. degree in business econometrics from VU Amsterdam, The Netherlands, and the Ph.D. degree in railway transportation from the Rotterdam School of Management in 2006.

He is currently a Principal Researcher with Nederlandse Spoorwegen. He coauthored several papers in leading journals. He aims at making impact in railway practice with applied research. In addition, he improves railway operations by implementing several innovations for drivers, conductors, and passengers. His research interests include railway timetabling, railway traffic management, disruption management, train control, and rolling stock rescheduling.

Niels van Oort was born in The Netherlands in 1978. He received the master's degree in 2003 and the Ph.D. degree in traffic and transport from the Delft University of Technology in 2011.

He is currently the Co-Director of the Smart Public Transport Laboratory, Delft University of Technology, where approximately 25 researchers investigate how to increase effectiveness and efficiency of public transport and shared mobility, including access and egress modes. He has been working on public transport projects for over 15 years, for instance as a consultant and a strategic transit planner. In 2012, he started working part-time with the Delft University of Technology, where he has been working full-time since 2018. He teaches multiple B.Sc. and M.Sc. courses and has been supervising over 80 M.Sc. thesis students. His research interests include concern data driven design, multimodality, behavior, and assessment.

Rob M. P. Goverde (Member, IEEE) was born in The Netherlands in 1968. He received the M.Sc. degree in mathematics from Utrecht University, The Netherlands, in 1993, and the Professional Doctorate in Engineering (P.D.Eng.) degree in mathematical modeling and decision support and the Ph.D. degree in railway transportation from the Delft University of Technology in 1996 and 2005, respectively.

He is currently a Professor of railway traffic management and operations and the Director of the Digital Rail Traffic Laboratory, Delft University of Technology. He has cooperated in many international research projects, including the EU Projects ON-TIME, MOVINGRAIL, RAILS, PERFORMINGRAIL, and SORTEDMOBILITY. He coauthored over 70 journal articles. His research interests include railway timetabling, railway traffic management, disruption management, automatic train operation, train control, and railway signalling. He is a fellow of the Institution of Railway Signal Engineers (IRSE). He served as an Associate Editor for the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS and also the Editor-in-Chief of the *Journal of Rail Transport Planning and Management*. He is the President of the International Association of Railway Operations Research (IAROR).