Distributed MPC-Based String Stable Platoon Control of Networked Vehicle Systems

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***Abstract*— In this paper, the string stable platoon control problem of discrete-time networked vehicle systems is considered by using distributed model predictive control (MPC) based method. An optimization problem is established to minimize the cost function associated to the system trajectories. The last-step shifting method is applied to set the local optimal solution as the assumed solution and send it to the neighbor vehicles. By using the sum of the cost function as Lyapunov function, the stability of the closed-loop platoon system is studied. Comparing with existing results, the string stability, which is the unique characteristics of the platoon system, is guaranteed under the bidirectional-based structure as well as the predecessor-follower-based information flow structure. Finally, several simulations are presented to demonstrate the effectiveness of the proposed algorithms.**

***Index Terms*— Platoon control, MPC-based method, string stable, networked vehicle.**

I. Introduction

In the past decades, traffic jam has become increasingly serious [1], [2]. How to ensure the vehicle runs fast and safely on the road has attracted the attention of many researchers. Platoon control is one of the most promising solutions to this issue and has achieved many remarkable results, see [3], [4], [5] and the references therein. Vehicle platoon control aims to ensure the specified distance between each vehicle, with all vehicles in the system having the same speed and acceleration as the (virtual) leader vehicle's [6], [7], [8].

The platoon system includes four elements, including vehicle dynamics (VD), information flow topology (IFT), formation geometry (FG) [9], [10] and distributed controller (DC). The IFT structures are mainly divided into predecessor-following-based [9], [11], [12], [13] and bidirectional-based structure [5], [6]. FG represents the distance strategy between the vehicles in the platoon [14]. Intuitively, the design of DC is closely related to VD, IFT, and FG. In recent years, some advanced control methods, which can improve the performance of the connected vehicle platoon system are proposed, such as adaptive control method [15], sliding mode control method [16], and robust control method [17]. Specifically, a two-layer distributed control scheme is proposed to ensure the closed-loop stability for the platoon system moving in one-dimension space with constant vehicle spacing being guaranteed [15]. In [16], the sliding mode control is introduced into the vehicle platoon control to solve the distributed trajectory optimization problem for the heterogeneous platoon system. A cooperative adaptive cruise control scheme is proposed for the vehicle platoon system and the distributed observer is used to estimate the lump disturbance in [17]. However, most of these methods fail to consider the discrete vehicle model as well as the string stability, which is a peculiar property of vehicle platoon.

To solve the trajectory optimization problem with discrete vehicle dynamics, model predictive control (MPC) is introduced into vehicle platoon systems by solving the formulated optimization problems within the predictive horizon to update the control signals. By setting current states as the initial states in each sampling moment, the MPC method can predict the next-step state by using the current one [18]. Since the MPC method can solve complex problems such as disturbance reduction and delay tolerance issues, it is popular in practical applications. In [19], the traffic control problem on highways is considered, but the centralized MPC's calculation time scales poorly. Due to the limitations of collecting all vehicle information and the challenges of calculation complexity of the optimization problems, the centralized MPC is not suitable for an actual vehicle platoon system [20], [21], [22]. However, the majority of the applications are used in a centralized approach, which means the controllers are designed by assuming that all of the states in the system are known. For this issue, the distributed MPC method is proposed for the vehicle platoon systems, which means only the local information can be used to each local controller to solve the MPC problem [23]. In [24], a distributed iteration control approach based on the feasible direction method is proposed for multi-agent systems

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to solve the distributed MPC optimization problems. The distributed MPC method removes the requirement on all states of the vehicles are known and, however, how to ensure the string stability of the platoon system is also one of the most challenging problems and still not well explored.

String stability is a special characteristic of the vehicle platoon system [25], [26], [27], [28]. Generally, there are two different stability definitions in vehicle platoon, i.e., individual stability and string stability. The former one, which is also known as Lyapunov stability, means that each vehicle in platoon system can be stabilized on some specific trajectories, while the latter one means the disturbances are not being magnified downstream along the vehicle platoon [28], [29], [30]. If the string stability is not guaranteed, a small disruption can be amplified and produce traffic congestion as the platoon length grows, such as stop-and-go phenomenon [31], [32]. Intuitively, VDs, FGs as well as IFTs of the platoon system will all affect the string stability [13], [29], [33]. In [29], the authors proved that since the complementary sensitivity integral constraint, the system is string unstable under the PF topology and CD policy. Under BD structure, the string stability of the platoon system is limited under the linear controller by some assumptions [33]. Some detailed comprehensive comparison between different types of string stability definitions under different analysis methods can be found in [26] and the reference therein. The existing results show that changing the forward communication structure of IFTs or the FGs between vehicles does not affect the string stability of the system. It should be noted that, to our best knowledge, how to ensure the string stability for a discrete platoon system by using distributed MPC method is still an open problem and not well explored.

In this paper, the distributed MPC string stable platoon control of discrete networked vehicle systems is considered. An optimization problem is considered to minimize the cost function associated to the system trajectories. A distributed MPC scheme is proposed, including the last-step shifting method, which is applied to set the local optimal solution as the assumed solution and send it to its neighbor vehicles. By using the sum of the cost function as Lyapunov function, the individual stability as well as the string stability of the closed-loop platoon system are studied. The effective of the proposed method is verified by the simulation results and the simulation results show that the proposed distributed MPC method can be applied to many different topology structures of vehicle platoon system. The contributions of the paper are summarized as follows:

1) A distributed MPC-based method is proposed for a realistic discrete vehicle platoon systems by using the last-step shifting method. Comparing with [34], the proposed algorithm removes the requirement on all states of the platoon system be known.
2) Both the individual stability and the string stability are considered for predecessor-follower-based and bidirectional-based vehicle platoon structure. Comparing with [35] and [36], not only the individual stability but also the string stability of the closed-loop system are considered under the proposed controller. Also unlike [37], rigorous proof is presented to guarantee the string stability under the bidirectional-based structure as well as the predecessor-follower-based vehicle platoon structure.

The outline of this paper is given as follows: the preliminaries and problem formulation are given in Section II. The distributed MPC-based method for the discrete vehicle platoon system and the prove processing is given in Section III. Section V presents the simulation results and the conclusion of the paper is given in Section VI.

II. Preliminaries

A. Vehicle Model for Platoon Control

The dynamics of the networked vehicle is the basis for designing a DC. Many existing results on vehicle platoon systems focus on the linear system model, such as the first-order integral model [3] and the second-order integral model [38]. In recent years, the third-order integral model has been studied, but these continuous models are harsh for real vehicle systems.

The vehicle model is defined as follows:

$$\begin{aligned} p_i(t+1) &= p_i(t) + v_i(t)\tau, \\ v_i(t+1) &= v_i(t) + a_t(t)\tau, \\ a_i(t+1) &= a_i(t) + (f_i(t) + q_i(t)c_i(t))\tau, \end{aligned} \tag{1}$$

where $p_i(t)$, $v_i(t)$, $a_i(t)$ represents the i-th vehicle's position, velocity and acceleration, respectively. c_i is the engine input representing the desired driving or braking torque. τ represents the sampling time. $f_i(t) = -\frac{1}{\Delta_s}(a_i + m_a c_a c_g v_i^2/2m_i + f_{c_g}/m_i) - m_a c_a c_g v_i a_i/m_i$ and $q_i(t) = \frac{1}{\Delta_s m_i}$ with m_a, m_i, Δ_s, c_a and c_g being the specific air mass, the vehicle mass, the time lag, the area of cross-sectional, and the drag coefficient, respectively. Based on [39], the engine input is given as follows:

$$c_i(t) = m_i u_i + m_a c_a c_g v_i^2/2 + f_{c_g} + m_a c_a c_g v_i a_i, \tag{2}$$

where u_i is the designed control input. For each vehicle, the state is defined as $\Phi_i(t) = [p_i(t), v_i(t), a_i(t)]^T \in \mathbb{R}^{3\times 1}$. Then, the vehicle model can be rewritten as follows:

$$\Phi_i(t+1) = A_i(\Phi_i(t)) + B_i u_i(t), \tag{3}$$

where

$$A_i(\Phi_i(t)) = \begin{bmatrix} p_i(t) + v_i(t)\tau \\ v_i(t) + a_i(t)\tau \\ a_i(t) - \frac{1}{\Delta s}a_i(t)\tau \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\Delta s}\tau \end{bmatrix}.$$

B. Control Objectives

As mentioned above, the stability definition for the platoon system includes individual stability and string stability. The control objective is to design a DC for a platoon system modeled by (1) and to ensure system stability in both senses, where the distances between vehicles are identical to the desired distance, and the vehicle velocities and accelerations in the platoon system are equal to the leader's ones. The control objective are described as follows. The desired position of i_{th} vehicle is $\lim_{t\to\infty} p_i(t) = p_i^* = p_0 + l_{i,0}$, the desired velocity is $\lim_{t\to\infty} v_i(t) \to v_i^* = v_0$, the desired acceleration is $\lim_{t\to\infty} a_i(t) \to a_i^* = a_0$, where $l_{i,0}$ is the desired distance between the leader vehicle and the i-th vehicle, p_0, v_0 and a_0 are the (virtual) leader's position, velocity and acceleration, respectively. Based on [37], the definition of the platoon system's string stability is that all state errors of the system are converged to zero and the following condition should also be satisfied:

$$\max \left\| p_i - p_0 - l_{i,0} \right\| \leq k_i \max \left\| p_{i-1} - p_0 - l_{i-1,0} \right\|, \quad (4)$$

where $0 < k_i < 1$, $i = 1, 2, \ldots, N$. This equation means that the disturbance is propagate downstream along the vehicle platoon system without being magnified, then the system is string stability.

Remark 1: The desired distance among the vehicles in platoon system can be either constant or time-varying, which is referred as the CD or CTH policy in a platoon system [26]. In [40], the result shows that the vehicle platoon systems with CTH policy by using MPC algorithm can be converged. In this paper, we focus on the networked vehicle platoon system's stability by using the distributed MPC-based approach and the CD policy is considered in platoon system.

C. Communication Topology

In this paper, both the BD information flow structure in platoon system are considered. The communication topology can be described by a undirected graph $\mathcal{G}_N = \{\mathcal{V}_N, \mathcal{E}_N\}$, where $\mathcal{V}_N = \{1, 2, \ldots, N\}$ is the vehicle index set, and $\mathcal{E}_N \in \mathcal{V}_N \times \mathcal{V}_N$ is the set of edges between vehicles. Note that the leader is indexed by 0. The vehicle i can communicate with the vehicles j if $e_{i,j} = (i, j) \in \mathcal{E}_N$. The adjacency matrix is given as $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{N\times N}$, where $a_{i,j} = 1$ if $(i, j) \in \mathcal{E}_N$, otherwise $a_{i,j} = 0$. The neighbor set of vehicle i is defined as $\mathcal{N}_i = \{j \in \mathcal{V}_N, a_{i,j} = 1\}$. The Lapiacian matrix $\mathcal{L}$ of $\mathcal{G}_N$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where the degree matrix $\mathcal{D}$ is given as $\mathcal{D} = diag\{d_1, d_2, \ldots, d_N\}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. If i-th vehicle can communicate with the leader, the pinning gain $q_i = 1$. The pinning matrix is given as $\mathcal{P} = diag\{q_1, q_2, \ldots, q_N\}$. The leader accessible set for vehicle i is given as $Q = \{0\}$ if $q_i = 1$, otherwise $Q = \emptyset$. Obviously only the information of vehicle j in the set $\mathcal{I}_i = \mathcal{N}_i \cup Q$ can be used for the designed controller of i-th vehicle. The following assumption is given for the communication topology.

Assumption 1: The platoon system communication topology $\mathcal{G}_N$ contains at least one spanning tree rooting at the leader vehicle and the self-loop does not exist in $\mathcal{G}_N$.

The following well-known lemma for graph $\mathcal{G}_N$ is also given.

Lemma 1: [41] Under the Assumption 1, if $\mathcal{G}_N$ contains a spanning tree rooting at the leader, then $\mathcal{D} + \mathcal{P}$ is invertible and $\lambda_{(\mathcal{D}+\mathcal{P})^{-1}\mathcal{A}} < 1$.

III. Distributed Model Predictive Based Method

In this section, the distributed MPC-based method for networked vehicle platoon systems is proposed. The local optimization problem is firstly proposed and a solution algorithm is given. Then rigorous proof is given to demonstrate that both the Lyapunov stability and string stability can be guaranteed.

A. Distributed Local Optimization Control Problem

Although all the information for vehicle j in set $\mathcal{I}_N$ can be used for control design of the i_{th} vehicle, the local optimal problem is established only used the information from the $(i-1)_{\text{th}}$ and $(i+1)_{\text{th}}$ vehicle in the platoon system. In this subsection, the following definitions are used. N_p is defined as the predictive horizon, δ is the updating period in distributed MPC algorithm, s represents the predicted time at time t, the predicted state is $\Phi_i^p(s,t)$ with $s = 0, 1, 2, \ldots, Np$, i.e., $s = 1$ means $s = t_0 + \delta$, which represent the predicted value of the state Φ_i^p of the time $t + \delta$ at the time t. The optimal predicted state is defined as $\Phi_i^*(s,t)$, the assumed state is denoted by $\hat{\Phi}_i(s,t)$, the predicted control input is $u_i^p(s,t)$, the optimal control input is $u_i^*(s,t)$, the assumed control input is $\hat{u}_i(s,t)$. The state variable $\Phi_i^p(s,t)$ is used to parameterizes the optimal control problem, $\Phi_i^*(s,t)$ is defined as an optimal solution to the optimal control problems. $\hat{\Phi}_i(s,t)$ is the trajectory communicated to its neighbours $(i+1) \in \mathcal{I}_i$ and $(i-1) \in \mathcal{I}_i$. In the proposed algorithm, the shifted last-step optimal trajectory of vehicle i is used and it will be implemented within the update period $[t, t+\delta]$. It should be noted that $\Phi_i^p(s,t) = \Phi_i^*(s,t) = \hat{\Phi}_i(s,t)$ at the initial moment.

The optimization control problem for vehicle $i \in \{1, 2, \ldots, N\}$ in platoon system at time t is established as follows:

$$\begin{aligned}\Gamma_i(t) : J_i(t) &= \min_{u_i^p(s,t)} J_i(\Phi_i^p, u_i^p, \hat{\Phi}_i, \hat{\Phi}_j) \\ &= \sum_{s=0}^{N_p-1} g_i(\Phi_i^p(s,t), u_i^p(s,t), \hat{\Phi}_i(s,t), \hat{\Phi}_j(s,t))\end{aligned} \quad (5)$$

subject to

$$\Phi_i^p(s+1,t) = A(\Phi_i^p(s,t)) + Bu_i^p(s,t), s < N_p, \quad (6)$$

where $u_i^p(s,t) \in U_i$ is the control input with U_i being the feasible set of $u_i^p(s,t)$, $\Phi_i^p(s_{N_p},t) = \frac{1}{\|\mathcal{I}_i\|}(\sum_{j\in\mathcal{I}_i, j>i}(\hat{\Phi}_j(s_{N_p},t) + L_{i,j}) + \sum_{j\in\mathcal{I}_i, j<i}(\hat{\Phi}_j(s_{N_p},t) - L_{j,i}))$ with s_{N_p} means $s = N_p$. The cost function $g_i(\Phi_i^p, u_i^p, \hat{\Phi}_i, \hat{\Phi}_j)$ in (5) is given as (7), shown at the bottom of the next page, where $L_{i,i-1} = [l_{i,i-1}, 0, 0]^T$, $L_{i,i+1} = [l_{i,i+1}, 0, 0]^T$. $K_i \geqslant 0$ denotes the penalized strength from the desired trajectory to the predicted state trajectory. For the vehicles that can not access to the leader, $K_i = 0$. $R_i \geq 0$ denotes the control input weight. $F_i \geqslant 0$ is the weight coefficient for the moving suppression term, in which the assumed state trajectory is the shifted last-step optimal state. $G_i \geqslant 0$ denotes the weight of predecessor relative error term, which means the vehicle i try to ensure the predicted trajectory as close to the assumed trajectory of the neighbor vehicle $i-1$ and $i+1$. In addition, this paper doesn't consider the effects of the communication and actuation delays in the platoon control. Theoretically, the proof is still valid in the presence of communication delays, since the last-step shifting method is applied to set the local optimal

solution as the assumed solution, where the optimal predictive process is independent of the communication delay. However, as mentioned in [42], the communication and actuation delays are detrimental to string stability. How to ensure the string stability under both the communication and actuation delays in a distributed MPC scheme for networked vehicle systems is still an open problem, which will be considered in the future.

Remark 2: Note that Problem $\Gamma_i(t)$ only needs the neighboring information thus be fully distributed. Comparing with the results in [34], where the leader information being used in each subsystems, this paper removes the requirements on any prior information of the leaders. In addition, the proposed distributed MPC based method in this paper can ensure both the asymptotic stability and string stability, which is also different the one in [36], where the string stability isn't considered. Rigorous proof and discussions will be given in Section IV.

B. Distributed MPC-Based Algorithm

In this subsection, the distributed MPC-based algorithm is summarized as follows:

IV. Stability Analysis for the Vehicle Platoon System

In this section, since the particularity of the vehicle platoon system, both the closed-loop stability and the string stability of the system are analyzed by using the Lyapunov function theory and mathematical induction, respectively. The terminal equality constraint is also proved to ensure that the predicted terminal state will converge to the desired state.

A. Terminal Equality Constraint

Define the terminal state error as follows:

$$E_{\Phi_i}(s_{N_p}, t+1) = \Phi_0(s_{N_p}, t+1) - \Phi_i^*(s_{N_p}, t+1) - L_{0,i} \tag{8}$$

Based on the distributed MPC-based algorithm, we have

$$\begin{aligned}\Phi_i^p(s_{N_p}, t+1) &= \frac{1}{\|I_i\|}\Big(\sum_{j\in\mathcal{I}_i, j>i}(\hat{\Phi}_j(s_{N_p}, t+1) + L_{i,j}) \\ &\quad + \sum_{j\in\mathcal{I}_i, j<i}(\hat{\Phi}_j(s_{N_p}, t+1) - L_{j,i})\Big) \\ &= \frac{1}{\|I_i\|}\Big(\sum_{j\in\mathcal{I}_i, j>i}(A(\Phi_j^*(s_{N_p}, t)) + L_{i,j}) \\ &\quad + \sum_{j\in\mathcal{I}_i, j<i}(A(\Phi_j^*(s_{N_p}, t)) - L_{j,i})\Big) \\ &= \frac{1}{\|I_i\|}\Big(\sum_{j\in\mathcal{I}_i, j>i}(\mathcal{Z}\Phi_j^*(s_{N_p}, t) + L_{i,j}) \\ &\quad + \sum_{j\in\mathcal{I}_i, j<i}(\mathcal{Z}\Phi_j^*(s_{N_p}, t) - L_{j,i})\Big) \\ &= \mathcal{Z}\Phi_i^*(s_{N_p}, t),\end{aligned} \tag{9}$$

where $\mathcal{Z}$ is given as follows

$$\mathcal{Z} = \begin{bmatrix} 1 & \tau & 0 \\ 0 & 1 & \tau \\ 0 & 0 & 1 - \frac{1}{\Delta s}\tau \end{bmatrix}. \tag{10}$$

Then, we have $\Phi_i^p(s_{N_p}, t+1) = \Phi_i^*(s_{N_p}, t+1)$, and one can know that the predicted state trajectory will converge to the optimal state trajectory.

Substituting (9) into (8), it yields

$$\begin{aligned}&E_{\Phi_i}(s_{N_p}, t+1) \\ &= \Phi_0(s_{N_p}, t+1) - \frac{1}{\|I_i\|}\Big(\sum_{j\in\mathcal{I}_i, j>i}(\hat{\Phi}_j(s_{N_p}, t+1) + L_{i,j}) \\ &\quad + \sum_{j\in\mathcal{I}_i, j<i}(\hat{\Phi}_j(s_{N_p}, t+1) - L_{j,i})\Big) - L_{0,i} \\ &= \frac{1}{\|I_i\|}\sum_{j\in\mathcal{I}_i}(\Phi_0(s_{N_p}, t+1) - \hat{\Phi}_j(s_{N_p}, t+1) - L_{0,j}) \\ &= \frac{1}{\|I_i\|}\mathcal{Z}\sum_{j\in\mathcal{I}_i}(E_{\Phi_j}(s_{N_p}, t))\end{aligned} \tag{11}$$

Rewriting (11) as an matrix form as

$$E_\Phi(s_{N_p}, t+1) = (\mathcal{D}+\mathcal{P})^{-1}\mathcal{A}\otimes\mathcal{Z}E_\Phi(s_{N_p}, t). \tag{12}$$

According to the definition of $\mathcal{Z}$, we have $\lambda_\mathcal{Z} = 1$ and $\lambda_\mathcal{Z} = 1 - \frac{1}{\Delta s}\tau$. Then, we can get that $\max(\lambda_\mathcal{Z}) = 1$. Based on the Assumption1 and Lemma1, the self-loop does not exist in the $\mathcal{G}_N$ for vehicle platoon systems and the eigenvalues of $(\mathcal{D}+\mathcal{P})^{-1}\mathcal{A}$ are all located within a unit circle. Then, we can know that $\lambda_{(\mathcal{D}+\mathcal{P})^{-1}\mathcal{A}\otimes\mathcal{Z})} < 1$, thus the terminal state error are converged to zero at $t = t + N_p$.

B. Stability Analysis for Closed-Loop Stability

The main results are given as follows.

Theorem 1: Under Assumption 1 and the proposed distributed MPC controller, the closed-loop vehicle platoon system

$$\begin{aligned}g_i(\Phi_i^p, u_i^p, \hat{\Phi}_i, \hat{\Phi}_j) &= \left\|K_i(\Phi_i^p(s,t) - \Phi_i^*(s,t))\right\| + \left\|R_i u_i^p(s,t)\right\| \\ &\quad + \left\|F_i(\Phi_i^p(s,t) - \hat{\Phi}_i(s,t)\right\| \\ &\quad + \sum_{j\in\mathcal{I}_i, j>i}\left\|\begin{matrix} G_i(\Phi_i^p(s,t) - \hat{\Phi}_j \\ (s,t) - L_{i,j}) \end{matrix}\right\| \\ &\quad + \sum_{j\in\mathcal{I}_i, j<i}\left\| G_i(\Phi_i^p(s,t) - \hat{\Phi}_j(s,t) + L_{j,i}))\right\|\end{aligned} \tag{7}$$

Algorithm 1 The distributed MPC-based method for vehicle platoon system

1. Initialization: Set vehicle $i = 1$ as the leader and broadcast the desired velocity to other vehicles. We assumed that the leader vehicle i will solve the optimal problem $\Gamma_i(t)$ by setting $F_i = G_i = 0$ before $t = 0$, and that the other vehicles in the platoon system will solve the optimal problem $\Gamma_i(t)$ by setting $F_i = 0$ before $t = 0$, implying that the assumed trajectories are unnecessary in the initialization.
Initialize $\Phi_i^p(s, 0)$ and $u_i^p(s, 0)$ as:

$$\Phi_i^p(s,0) = \begin{cases} \Phi_i(0), s = 0 \\ A(\Phi_i^p(s-1,0)) + Bu_i^p(s-1,0), others \end{cases}$$
$$u_i^p(s,0) = 0$$

2. Iteration: For any $0 \leq t < t + N_p$, implementing the following process:
(1) Transmit the assumed state trajectories $\hat{\Phi}_i(s,t)$, which is derived by solving the optimal problem $\Gamma_i(t)$, to the neighbor vehicle $j \in \mathcal{I}_i$;
(2) Solve the optimal problem $\Gamma_i(t)$ and obtain the optimal control $u_i^*(t)$ and

$$\Phi_i^*(t+1) = A_i(\Phi_i^*(t)) + B_i u_i^*(t);$$

(3) Compute $\hat{\Phi}_i(s, t+1)$ and $\hat{u}_i(s, t+1)$ as:

$$\hat{\Phi}_i(s,t+1) = \begin{cases} \Phi_i^*(s+1,t),\ s \leq Np-1 \\ A_i(\Phi_i^*(s_{N_p},t)),\ others \end{cases}$$

where s_{N_p} means $s = N_p$.

$$\hat{u}_i(s,t+1) \begin{cases} u_i^*(s+1,t),\ s \leq Np-1 \\ 0,\ others \end{cases}$$

(4) Transmit $\hat{\Phi}_i(s, t+1)$ to its neighbour vehicles $i+1$ and $i-1$. Also the vehicle i receive $\hat{\Phi}_{i+1}(s, t+1)$ and $\hat{\Phi}_{i-1}(s, t+1)$ from its neighbours, respectively;
(5) Solve the optimal problem $\Gamma_i(t)$ for $u_i^*(s, t+1)$ and set $s = s+1$ and go back to step (1).

is stable under if the following conditions are satisfied, i.e.

$$u_i^*(s_0, t) = 0, \tag{13}$$
$$\Phi_i^*(s_0, t) - \hat{\Phi}_i(s_0, t) = 0, \tag{14}$$
$$(\Phi_i^*(s_0, t) - \hat{\Phi}_j(s_0, t) - L_{i,j})_{j\in I_i, j>i} = 0, \tag{15}$$
$$(\Phi_i^*(s_0, t) - \hat{\Phi}_j(s_0, t) + L_{j,i})_{j\in I_i, j<i} = 0. \tag{16}$$

Remark 3: The above conditions are the constraints on the assumed states of the system at $s = 0$, which means the initial assumed states of the system. It requires that the assumed states of the system equal to the desired states of the system at the initial moment, this is consistent with the previous definition of the expected value and predicted value of the system.

Proof: Consider the sum of the local cost function as an candidate Lyapunov function, then the difference function is

$$\Delta V = \sum_{i=1}^{N} (J_i^*(t+1) - J_i^*(t)), \tag{17}$$

where $J_i^*(t) = \sum_{s=0}^{N_p-1} g_i(\Phi_i^*(s,t), u_i^*(s,t), \hat{\Phi}_i(s,t), \hat{\Phi}_j(s,t))$. Then rewriting (17) as

$$\begin{aligned}\Delta V = \sum_{i=1}^{N}(&\sum_{s=0}^{N_p-1} g_i(\Phi_i^*(s,t+1), u_i^*(s,t+1), \hat{\Phi}_i(s,t+1), \\ &\hat{\Phi}_j(s,t+1)) \\ &- \sum_{s=0}^{N_p-1} g_i(\Phi_i^*(s,t), u_i^*(s,t), \hat{\Phi}_i(s,t), \hat{\Phi}_j(s,t))).\end{aligned} \tag{18}$$

Next, we will prove that the difference function ΔV is strictly monotonically decreasing such that the asymptotic stability can be ensured.

According to the local optimal theory, we have

$$\begin{aligned}\Delta V \leq& \sum_{i=1}^{N}(\sum_{s=0}^{N_p-1} g_i(\hat{\Phi}_i(s,t+1), \hat{u}_i(s,t+1), \hat{\Phi}_i(s,t+1), \\ &\hat{\Phi}_j(s,t+1)) \\ &- \sum_{s=0}^{N_p-1} g_i(\Phi_i^*(s,t), u_i^*(s,t), \hat{\Phi}_i(s,t), \hat{\Phi}_j(s,t))) \\ \leq& \sum_{i=1}^{N}(-g_i(\Phi_i^*(s_0,t), u_i^*(s_0,t), \hat{\Phi}_i(s_0,t), \hat{\Phi}_j(s_0,t)) \\ &+ \sum_{s=1}^{N_p-1}(g_i(\Phi_i^*(s,t), u_i^*(s,t), \Phi_i^*(s,t), \Phi_j^*(s,t)) \\ &- \sum_{s=1}^{N_p-1} g_i(\Phi_i^*(s,t), u_i^*(s,t), \hat{\Phi}_i(s,t), \hat{\Phi}_j(s,t)))).\end{aligned} \tag{19}$$

where the definitions of $\Phi_i^*(s,t), u_i^*(s,t), \hat{\Phi}_i(s,t), \hat{\Phi}_j(s,t)$ are given previously. Combining (7) and (19), it yields

$$\begin{aligned}\Delta V \leq& \sum_{i=1}^{N}(-g_i(\Phi_i^*(s_0,t), u_i^*(s_0,t), \hat{\Phi}_i(s_0,t), \hat{\Phi}_j(s_0,t)) \\ &+ \sum_{s=1}^{N_p-1}((\left\|K_i(\Phi_i^*(s,t) - \Phi_i^*(s,t))\right\| \\ &+ \left\|R_i(u_i^*(s,t))\right\| + \left\|F_i(\Phi_i^*(s,t) - \Phi_i^*(s,t))\right\| \\ &+ \sum_{j\in I_i, j>i} \left\|G_i(\Phi_i^*(s,t) - \Phi_j^*(s,t) - L_{i,j})\right\| \\ &+ \sum_{j\in I_i, j<i} \left\|G_i(\Phi_i^*(s,t) - \Phi_j^*(s,t) + L_{j,i})\right\|) \\ &-(\left\|K_i(\Phi_i^*(s,t) - \Phi_i^*(s,t))\right\| + \left\|R_i(u_i^*(s,t))\right\| \\ &+ \left\|F_i(\Phi_i^*(s,t) - \hat{\Phi}_i(s,t))\right\| \\ &+ \sum_{j\in I_i, j>i} \left\|G_i(\Phi_i^*(s,t) - \hat{\Phi}_j(s,t) - L_{i,j})\right\| \\ &+ \sum_{j\in I_i, j<i} \left\|G_i(\Phi_i^*(s,t) - \hat{\Phi}_j(s,t) + L_{j,i})\right\|)))\end{aligned} \tag{20}$$

Furthermore, according to the properties of the matrix norm, we have

$$\Delta V \le \sum_{i=1}^{N}(-g_i(\Phi_i^*(s_0,t),u_i^*(s_0,t),\hat{\Phi}_i(s_0,t),\hat{\Phi}_j(s_0,t)) + \sum_{s=1}^{N_p-1}(\sum_{j\in I_i}\left\|G_i(\Phi_j^*(s,t)-\hat{\Phi}_j(s,t))\right\| - \left\|F_i(\Phi_i^*(s,t)-\hat{\Phi}_i(s,t))\right\|)) \quad (21)$$

Since $\sum_{i\in I_i} G_i - F_i \le 0$, it derives that $(\Phi_i^*(s,t) - \hat{\Phi}_i(s,t))^T(\sum_{j\in I_i} G_i - F_i)(\Phi_i^*(s,t) - \hat{\Phi}_i(s,t)) \le 0$. In addition,

$$\sum_{j\in I_i}\left\|G_i(\Phi_i^*(s,t)-\hat{\Phi}_i(s,t))\right\| - \left\|F_i(\Phi_i^*(s,t)-\hat{\Phi}_i(s,t))\right\| \le 0 \quad (22)$$

Then substituting (22) into (21), we have

$$\begin{aligned}\Delta V &\le \sum_{i=1}^{N}(-g_i(\Phi_i^*(s_0,t),u_i^*(s_0,t),\hat{\Phi}_i(s_0,t),\hat{\Phi}_j(s_0,t)))\\ &\le -\sum_{i=1}^{N}(\left\|K_i(\Phi_i^*(s_0,t)-\Phi_i^*(s_0,t))\right\| + \left\|R_i u_i^*(s_0,t)\right\|\\ &\quad + \left\|F_i(\Phi_i^*(s_0,t)-\hat{\Phi}_i(s_0,t)\right\|\\ &\quad + \sum_{j\in I_i, j>i}\|G_i(\Phi_i^*(s_0,t)-\hat{\Phi}_j(s_0,t)-L_{i,j})\|\\ &\quad + \sum_{j\in I_i, j<i}\|G_i(\Phi_i^*(s_0,t)-\hat{\Phi}_j(s_0,t)+L_{j,i})\|). \end{aligned} \quad (23)$$

According to the conditions in (13)-(16), one has

$$\begin{aligned}\Delta V &\le -\sum_{i=1}^{N}(\left\|K_i(\Phi_i^*(s_0,t)-\Phi_i^*(s_0,t))\right\| + \left\|R_i u_i^*(s_0,t)\right\|\\ &\quad + \left\|F_i(\Phi_i^*(s_0,t)-\hat{\Phi}_i(s_0,t)\right\|\\ &\quad + \sum_{j\in I_i, j>i}\|G_i(\Phi_i^*(s_0,t)-\hat{\Phi}_j(s_0,t)-L_{i,j})\|\\ &\quad + \sum_{j\in I_i, j<i}\|G_i(\Phi_i^*(s_0,t)-\hat{\Phi}_j(s_0,t)+L_{j,i})\|)\\ &\le 0 \end{aligned} \quad (24)$$

Then asymptotical stability of the closed-loop platoon system is guaranteed. This completes the proof. □

Remark 4: In this paper, the last-step shifting method is used to constructed the assumed states in the proposed distributed MPC algorithm. The similar strategies are also used in [35], [36]. In contrast to these results, asymptotical stability of the closed-loop platoon system can be guaranteed under both PF and BD information flow structures. In addition, another potential benefit is that the proposed algorithm removes the requirement on all states of the platoon system be known such as the leader's one, which is different from [34].

C. String Stability Analysis

The string stability is suggested and thoroughly investigated to ensure that the disturbance between each vehicle does not amplify along the string when the number of vehicles is increasing in the system [26], [27], [28]. The string stability of the vehicle platoon systems is analyzed in this section.

Theorem 2: Under Assumption 1 and Theorem 1, the proposed distributed MPC-based algorithm ensures the string stability of the vehicle platoon system if the following conditions are satisfied:

1. The optimal problem Γ_i is solved by using the proposed distributed MPC algorithm for each vehicle $i \ge 2$ under the following conditions:

$$\left|p_i^p(s_0,t)-p_0(s_0,t)-l_{0,i}\right| \le \alpha_i\left|p_{i-1}^p(s_0,t)-p_0(s_0,t)-l_{0,i-1}\right| \quad (25)$$

where $0 < \alpha_i \le 1$.

2. Problem $\Gamma_i(t)$ will be solved by the proposed distributed MPC algorithm with the following constraints:

For the leader vehicle:

$$\left|\begin{array}{c} p_1^p(s,t+1)-p_0(s,t+1)-l_{0,1}\\ -(\hat{p}_1(s,t+1)-p_0(s,t+1)-l_{0,1})\end{array}\right| \le k_{(1,s)}\left|p_1^p(s,t+1)-p_0(s,t+1)-l_{0,1}\right| \quad (26)$$

where $k_{(1,s)} > 0$;

For the other vehicles:

$$\left|\begin{array}{c} p_i^p(s,t+1)-p_0(s,t+1)-l_{0,i}\\ -(\hat{p}_i(s,t+1)-p_0(s,t+1)-l_{0,i})\end{array}\right| \le k_{(i,s)}\max\left|p_i^p(s,t+1)-p_0(s,t+1)-l_{0,i}\right| \quad (27)$$

where $k_{(i,s)} > 0$.

3. In addition, the following condition is satisfied:

$$k_{i,s}(k_{i-1,s}+1)+\alpha_i\prod_{j=1}^{s}(k_{i-1,j}+1) + \sum_{j=1}^{s-1}(k_{i,j}\prod_{\beta=1}^{s}(k_{i-1,\beta}+1)) < 1,\ \textit{for } i \ge 2. \quad (28)$$

Proof: Based on the definition of the string stability, if the following condition is satisfied:

$$\max_{t\ge 0}\left|p_i-p_0-l_{0,i}\right| \le k_s\max_{t\ge 0}\left|p_{i-1}-p_0-l_{0,i-1}\right|, \quad (29)$$

where $k_s \le 1$, then the vehicle platoon system is string stable. To verify this condition, the mathematical induction method is used.

For $s = 0$: We will firstly prove that the condition (29) is satisfied for the case that $s = 0$. Since $F_i = G_i = 0$ at initialization for the leader vehicle $i = 1$, $p_i^p(s_0,t) - p_0(s_0,t) - l_{0,i} = p_i^*(s_0,t) - p_0(s_0,t) - l_{0,i}$ at initialization, based on (25), we have

$$\left|p_i^*(s_s,t)-p_0(s_0,t)-l_{0,i}\right| \le \alpha_i\left|p_{i-1}^*(s_0,t)-p_0(s_0,t)-l_{0,i}\right|. \quad (30)$$

Then, we have

$$\max\left|p_i^*(s_0,t)-p_0(s_0,t)-l_{0,i}\right| \le \alpha_i\max\left|p_{i-1}^*(s_0,t)-p_0(s_0,t)-l_{0,i}\right| \quad (31)$$

Thus the condition (29) is satisfied for the case that $s = 0$.

For $s = 1$**:** Next, we will prove that the condition (29) is also satisfied for the case that $s = 1$. By using the triangle inequality, we have

$$\begin{aligned}&\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\le\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}-(\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i})\right|\\&\quad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|.\end{aligned}\tag{32}$$

Since p_i^p is the predicted position trajectory and p_i^* is the optimal predicted position trajectory, we have

$$\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\le\left|p_i^p(s_1,t)-p_0(s_1,t)-l_{0,i}\right|.\tag{33}$$

Substituting (33) into (32), we can obtain

$$\begin{aligned}&\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right.\\&\qquad\left.-(\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i})\right|\\&\qquad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\qquad\le\left|\begin{matrix}p_i^p(s_1,t)-p_0(s_1,t)-l_{0,i}\\-(\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i})\end{matrix}\right|\\&\qquad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|.\end{aligned}\tag{34}$$

Then substituting (27) into (34), one can get

$$\begin{aligned}&\left|p_i^p(s_1,t)-p_0(s_1,t)-l_{0,i}\right.\\&\qquad\left.-(\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i})\right|\\&\qquad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\qquad\le k_{i,1}\max\left|\hat{p}_{i-1}(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&\qquad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\end{aligned}\tag{35}$$

Furthermore, substituting (35) into (32), we can obtain

$$\begin{aligned}&\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\qquad\le\left|p_i^p(s_1,t)-p_0(s_1,t)-l_{0,i}\right.\\&\qquad\left.-(\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i})\right|\\&\qquad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\qquad\le k_{i,1}\max\left|\hat{p}_{i-1}(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&\qquad+\left|\hat{p}_i(s_1,t)-p_0(s_1,t)-l_{0,i}\right|.\end{aligned}\tag{36}$$

From the results with $s = 0$, it yields

$$\begin{aligned}&\left|p_i^*(s_0,t)-(p_0(s_0,t))-l_{0,i}\right|\\&\qquad\le\alpha_i\left|p_{i-1}^*(s_0,t)-(p_0(s_0,t))-l_{0,i}\right|\end{aligned}\tag{37}$$

Then, one can get

$$\begin{aligned}&\left|\hat{p}_i(s_1,t+1)-p_0(s_1,t+1)-l_{0,i}\right|\\&\qquad\le\alpha_i\left|\hat{p}_{i-1}(s_1,t+1)-p_0(s_1,t+1)-l_{0,i}\right|.\end{aligned}\tag{38}$$

By combining (36), (37), and (38), the following result is derived

$$\begin{aligned}&\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\qquad\le k_{i,1}\max\left|\hat{p}_{i-1}(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&\qquad+\alpha_i\left|\hat{p}_{i-1}(s_1,t)-p_0(s_1,t)-\hat{l}_{0,i}\right|.\end{aligned}\tag{39}$$

Similarly, the following inequality can be yielded by using the triangle inequality,

$$\begin{aligned}&\left|\hat{p}_i(s,t+1)-p_0(s,t+1)-l_{0,i}\right|\\&\quad-\left|p_i^*(s,t+1)-p_0(s,t+1)-l_{0,i}\right|\\&\quad\le\left|\begin{matrix}p_i^p(s,t+1)-p_0(s,t+1)-l_{0,i}\\-(\hat{p}_i(s,t+1)-p_0(s,t+1)-l_{0,i})\end{matrix}\right|.\end{aligned}\tag{40}$$

Then according to (27) and (40), the following result is derived by defining $i = i - 1$,

$$\begin{aligned}&\left|\hat{p}_{i-1}(s,t+1)-p_0(s,t+1)-l_{0,i-1}\right|\\&\quad\le k_{i-1,s}\max\left|p_{i-1}^*(s,t+1)-p_0(s,t+1)-l_{0,i-1}\right|\\&\quad+\left|p_{i-1}^*(s,t+1)-p_0(s,t+1)-l_{0,i-1}\right|.\end{aligned}\tag{41}$$

Substituting $s = 1$ into (41), we have

$$\begin{aligned}&\left|\hat{p}_{i-1}(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&\quad\le k_{i-1,1}\max\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&\quad+\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\end{aligned}\tag{42}$$

Substituting (42) into (39), one can get

$$\begin{aligned}&\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\le k_{i,1}\max(k_{i-1,1}\max\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&+\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|)\\&+\alpha_i\cdot(k_{i-1,s}\max\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|\\&+\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|)\\&\le(k_{i,1}(k_{i-1,1}+1)+\alpha_i k_{i-1,1}+\alpha_i)\\&\max\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|.\end{aligned}\tag{43}$$

Thus the following result can be derived,

$$\begin{aligned}&\max\left|p_i^*(s_1,t)-p_0(s_1,t)-l_{0,i}\right|\\&\quad\le(k_{i,1}(k_{i-1,1}+1)+\alpha_i k_{i-1,1}+\alpha_i)\\&\quad\cdot\max\left|p_{i-1}^*(s_1,t)-p_0(s_1,t)-l_{0,i-1}\right|.\end{aligned}\tag{44}$$

Then the condition in (29) is proved.

For $s = 2$: Similarly, by following the process for the case $s = 1$, we will prove that the condition (29) is also satisfied for the case that $s = 2$. By using the triangle inequality theory, we have

$$\begin{aligned}&\left|p_i^*(s_2,t)-p_0(s_2,t)-l_{0,i}\right|\\&\quad\le\left|p_i^*(s_2,t)-p_0(s_2,t)-l_{0,i}\right.\\&\quad\left.-(\hat{p}_i(s_2,t)-p_0(s_2,t)-l_{0,i})\right|\\&\quad+\left|\hat{p}_i(s_2,t)-p_0(s_2,t)-l_{0,i}\right|\end{aligned}\tag{45}$$

Similarly, by following the process of (33)-(35) and substituting it into (45), we have

$$\begin{aligned}&\left|p_i^*(s_2,t)-p_0(s_2,t)-l_{0,i}\right|\\&\quad\le k_{i,2}\max\left|\hat{p}_{i-1}(s_2,t)-p_0(s_2,t)-l_{0,i}\right|\\&\quad+\left|\hat{p}_i(s_2,t)-p_0(s_2,t)-l_{0,i}\right|\end{aligned}\tag{46}$$

By using the triangle inequality theory, we have

$$\begin{aligned}&\left|\hat{p}_i(s,t+1)-p_0(s,t+1)-l_{0,i}\right|\\&\quad-\left|p_i^*(s,t+1)-p_0(s,t+1)-l_{0,i}\right|\\&\quad\le k_{i,s}\max\left|p_i^p(s,t+1)-p_0(s,t+1)-l_{0,i}\right|\end{aligned}\tag{47}$$

Then, the following result will be derived by defining $i = i-1$ and $s = 2$,

$$\begin{aligned}&\left|\hat{p}_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i}\right| \\ &\quad\le k_{i-1,2}\max\left|p^p_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \\ &\quad+\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \\ &\quad= k_{i-1,2}\max\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \\ &\quad+\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \end{aligned} \tag{48}$$

Substituting (48) into (46), we have

$$\begin{aligned}&\left|p^*_i(s_2,t) - p_0(s_2,t) - l_{0,i}\right| \\ &\le k_{i,2}(k_{i-1,2}+1)\max\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \\ &+\left|\hat{p}_i(s_2,t) - p_0(s_2,t) - l_{0,i}\right| \end{aligned} \tag{49}$$

From the results with $s = 1$, it yields

$$\begin{aligned}&\left|p^*_i(s_1,t) - p_0(s_1,t) - l_{0,i}\right| \\ &\quad\le (k_{i,1}(k_{i-1,1}+1) + \alpha_i k_{i-1,1} + \alpha_i) \\ &\quad\max\left|p^*_{i-1}(s_1,t) - p_0(s_1,t) - l_{0,i-1}\right| \end{aligned} \tag{50}$$

Then, one can obtain

$$\begin{aligned}&\left|\hat{p}_i(s_2,t+1) - (p_0(s_2,t+1)) - l_{0,i}\right| \\ &\quad\le (k_{i,1}(k_{i-1,1}+1) + \alpha_i k_{i-1,1} + \alpha_i) \\ &\quad\max\left|\hat{p}_{i-1}(s_2,t+1) - (p_0(s_2,t+1)) - l_{0,i-1}\right| \end{aligned} \tag{51}$$

By combining (48)-(51), the following result is derived,

$$\begin{aligned}&\left|p^*_i(s_2,t) - p_0(s_2,t) - l_{0,i}\right| \\ &\le k_{i,2}(k_{i-1,2}+1)\max\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \\ &+(k_{i,1}(k_{i-1,1}+1) + \alpha_i k_{i-1,1} + \alpha_i) \\ &\max(k_{i-1,2}\max\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right| \\ &+\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right|) \end{aligned} \tag{52}$$

The following result can be derived,

$$\begin{aligned}&\max\left|p^*_i(s_2,t) - p_0(s_2,t) - l_{0,i}\right| \\ &\le (\alpha_i(k_{i-1,1}+1)(k_{i-1,2}+1) + k_{i,1}(k_{i-1,1}+1)(k_{i-1,2}+1) \\ &+k_{i,2}(k_{i-1,2}+1)) \\ &\cdot\max\left|p^*_{i-1}(s_2,t) - p_0(s_2,t) - l_{0,i-1}\right|. \end{aligned} \tag{53}$$

Next, suppose that the following condition is hold for the case $s = n$.

$$\begin{aligned}&\left|p^*_i(s_n,t) - p_0(s_n,t) - l_{0,i}\right| \\ &\quad\le (k_{i,n}(k_{i-1,n}+1) + \alpha_i\prod_{j=1}^{n}(k_{i-1,j}+1) \\ &\quad+\sum_{j=1}^{n-1}(k_{i,j}\prod_{\beta=1}^{n}(k_{i-1,\beta}+1))) \\ &\quad\cdot\max\left|p^*_{i-1}(s_n,t) - p_0(s_n,t) - l_{0,i-1}\right|. \end{aligned} \tag{54}$$

Based on this condition, we will prove that the condition (29) is still satisfied under the proposed distributed MPC-based algorithm.

For $s = n+1$**:** By using the triangle inequality, we have

$$\begin{aligned}&\left|p^*_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\ &\le \left|\begin{matrix} p^*_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i} \\ -(\hat{p}_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i})\end{matrix}\right| \\ &+\left|\hat{p}_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\ &\le k_{i,n+1}\max\left|\hat{p}_{i-1}(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\ &+\left|\hat{p}_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right|. \end{aligned} \tag{55}$$

Following the previous process, it yields

$$\begin{aligned}&\left|\hat{p}_i(s,t+1) - p_0(s,t+1) - l_{0,i}\right| \\ &-\left|p^*_i(s,t+1) - p_0(s,t+1) - l_{0,i}\right| \\ &\le k_{i,s}\max\left|p^p_i(s,t+1) - p_0(s,t+1) - l_{0,i}\right|. \end{aligned} \tag{56}$$

Defining $i = i-1$ and substituting it into (56), one can get

$$\begin{aligned}&\left|\hat{p}_{i-1}(s_{n+1},t+1) - p_0(s_{n+1},t+1) - l_{0,i-1}\right| \\ &\quad\le k_{i-1,n+1}\max\left|\begin{matrix} p^p_{i-1}(s_{n+1},t+1) \\ +p_0(s_{n+1},t+1) - l_{0,i-1}\end{matrix}\right| \\ &\quad+\left|p^*_{i-1}(s_{n+1},t+1) - p_0(s_{n+1},t+1) - l_{0,i}\right| \\ &\quad\le k_{i-1,n+1}\max\left|\begin{matrix} p^*_{i-1}(s_{n+1},t+1) \\ +p_0(s_{n+1},t+1) - l_{0,i-1}\end{matrix}\right| \\ &\quad+\left|p^*_{i-1}(s_{n+1},t+1) - p_0(s_{n+1},t+1) - l_{0,i}\right|. \end{aligned} \tag{57}$$

Substituting (57) into (55), it yields

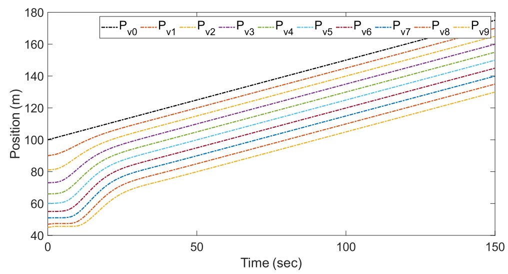
$$\begin{aligned}&\left|p^*_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\ &\le k_{i,n+1}\max(k_{i-1,n+1}\max\left|\begin{matrix} p^*_{i-1}(s_{n+1},t+1) \\ +p_0(s_{n+1},t+1) - l_{0,i-1}\end{matrix}\right| \\ &+\left|p^*_{i-1}(s_{n+1},t+1) - p_0(s_{n+1},t+1) - l_{0,i}\right|) \\ &+\left|\hat{p}_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\ &\le k_{i,n+1}(k_{i-1,n+1}+1)\max\left|\begin{matrix} p^*_{i-1}(s_{n+1},t+1) \\ +p_0(s_{n+1},t+1) - l_{0,i-1}\end{matrix}\right| \\ &+\left|\hat{p}_i(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right|. \end{aligned} \tag{58}$$

Based on the condition (54), we have

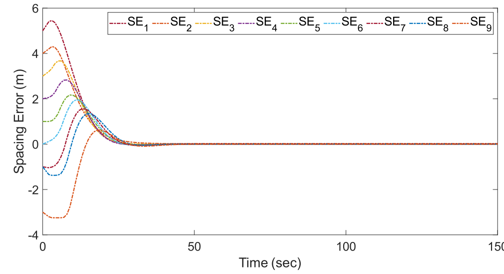
$$\begin{aligned}&\left|p^*_i(s_n,t) - p_0(s_n,t) - l_{0,i}\right| \\ &\quad\le (k_{i,n}(k_{i-1,n}+1) + \alpha_i\prod_{j=1}^{n}(k_{i-1,j}+1) \\ &\quad+\sum_{j=1}^{n-1}(k_{i,j}\prod_{\beta=1}^{n}(k_{i-1,\beta}+1))) \\ &\quad\cdot\max\left|p^*_{i-1}(s_n,t) - p_0(s_n,t) - l_{0,i-1}\right|. \end{aligned}$$

Then, we can obtain

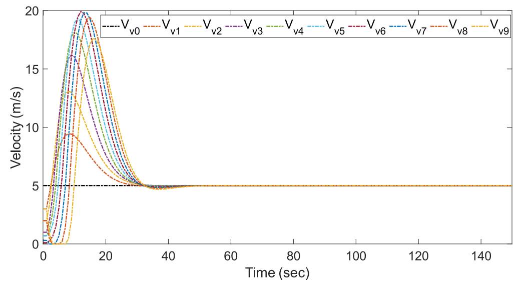
$$\begin{aligned}&\left|\hat{p}_i(s_{n+1},t+1) - p_0(s_{n+1},t+1) - l_{0,i}\right| \\ &\le (k_{i,n}(k_{i-1,n}+1) + \alpha_i\prod_{j=1}^{n}(k_{i-1,j}+1) \\ &+\sum_{j=1}^{n-1}(k_{i,j}\prod_{\beta=1}^{n}(k_{i-1,\beta}+1))) \\ &\max\left|\hat{p}_{i-1}(s_{n+1},t+1) - p_0(s_{n+1},t+1) - l_{0,i-1}\right|. \end{aligned} \tag{59}$$



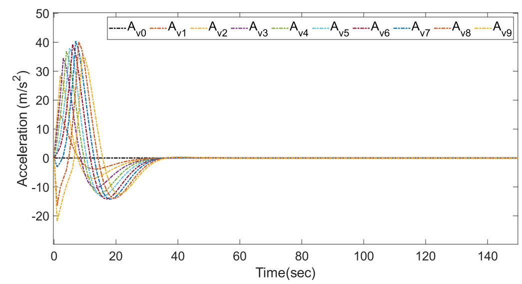
(a) The initial position and the end positions of each vehicle, where P_{Vi} represents the position of vehicle i.



(b) The spacing errors of each vehicle, where SE_i represents $p_{i-1} - p_i - l_{i-1,i}$.



(c) The velocities of each vehicle, where V_{Vi} represents the velocity of vehicle i.



(d) The accelerations of each vehicle, where A_{Vi} represents the acceleration of vehicle i.

Fig. 1. The results for vehicle platoon system under BD structure.

Then substituting (59) into (58), one can get

$$\begin{aligned}
&\left|p_i^*(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\
&\quad \le k_{i,n+1}(k_{i-1,n+1}+1) \\
&\quad \max\left|\, p_{i-1}^*(s_{n+1},t+1) + p_0(s_{n+1},t+1) - l_{0,i-1}\,\right| \\
&\quad +((k_{i,n}(k_{i-1,n}+1) + \alpha_i \prod_{j=1}^{n}(k_{i-1,j}+1) \\
&\quad + \sum_{j=1}^{n-1}(k_{i,j}\prod_{\beta=1}^{n}(k_{i-1,\beta}+1)))(k_{i-1,n+1}+1)) \\
&\quad \cdot \max\left|p_{i-1}^*(s_{n+1},t+1) + p_0(s_{n+1},t+1) - l_{0,i-1}\right| \\
&= (k_{i,n+1}(k_{i-1,n+1}+1) + \alpha_i \prod_{j=1}^{n+1}(k_{i-1,j}+1) \\
&\quad + \sum_{j=1}^{n}(k_{i,j}\prod_{\beta=1}^{n+1}(k_{i-1,\beta}+1))) \\
&\quad \cdot \max\left|p_{i-1}^*(s_{n+1},t+1) + p_0(s_{n+1},t+1) - l_{0,i-1}\right|.
\end{aligned} \tag{60}$$

From (60), the following condition can be derived.

$$\begin{aligned}
&\max\left|p_i^*(s_{n+1},t) - p_0(s_{n+1},t) - l_{0,i}\right| \\
&\quad \le (k_{i,n+1}(k_{i-1,n+1}+1) + \alpha_i \prod_{j=1}^{n+1}(k_{i-1,j}+1) \\
&\quad + \sum_{j=1}^{n}(k_{i,j}\prod_{\beta=1}^{n+1}(k_{i-1,\beta}+1))) \\
&\quad \max\left|p_{i-1}^*(s_{n+1},t+1) + p_0(s_{n+1},t+1) - l_{0,i-1}\right|.
\end{aligned} \tag{61}$$

Then according to the available condition (54), it yields

$$\begin{aligned}
&\max\left|p_i^*(s,t) - p_0(s,t) - l_{0,i}^*\right| \\
&\qquad \le k_s \max\left|p_{i-1}^*(s,t) - p_0(s,t) - l_{0,i}\right|.
\end{aligned} \tag{62}$$

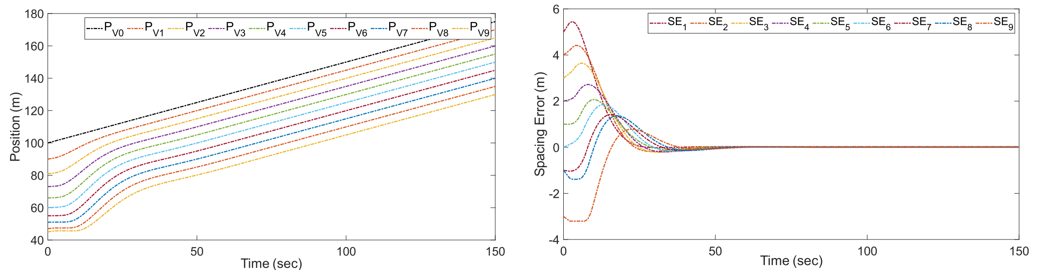
where $k_s = (k_{i,s}(k_{i-1,s}+1) + \alpha_i \prod_{j=1}^{s}(k_{i-1,j}+1) + \sum_{j=1}^{s-1}(k_{i,j}\prod_{\beta=1}^{s}(k_{i-1,\beta}+1)))$. Since $k_{i,s}(k_{i-1,s}+1) + \alpha_i \prod_{j=1}^{s}(k_{i-1,j}+1) + \sum_{j=1}^{s-1}(k_{i,j}\prod_{\beta=1}^{s}(k_{i-1,\beta}+1)) \le 1$ and the condition in (29) is hold, one can get that $k_s \le 1$. Then, the string stability of the platoon system is guaranteed. □

Remark 5: Under the proposed distributed MPC algorithm, the string stability of the discrete-time platoon system is always guaranteed under the both BD and PF structures. This is different from the results in [35] and [36], where the string stability is not considered. The string stability can be proved by the Laplacian transformation of position errors for linear continuous system. However, the system's position error cannot be directly Laplace transformed for discrete-time systems. Compared with the string stability analysis of linear continuous system in [43], the string stability analysis of discrete system is more challenging. The string stability analysis for nonlinear discrete system is also one of our future works.

V. SIMULATION

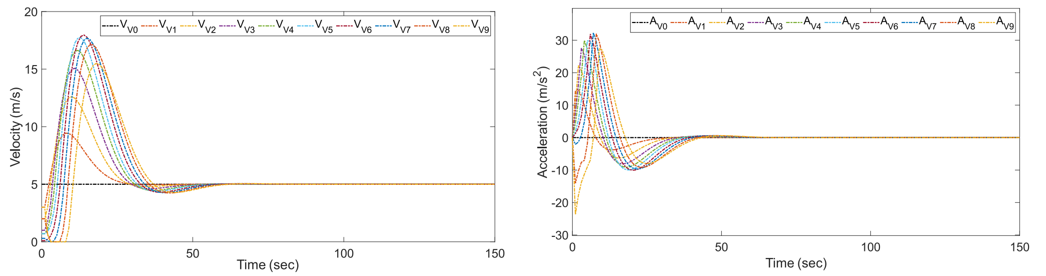
A. Simulation Parameters Settings

In this subsection, a vehicle platoon system with 10 vehicles modeled by the discrete model (1) is considered. The first vehicle is considered as a leader while the others are considered as followers. The initial position of the virtual leader is $X_0 = 100$ and the virtual leader is moving as a constant velocity of $v_0 = 5m/s$. The desired distance between each vehicle is considered as $l_{i,i-1} = 5m$. In this paper, the horizon



(a) The initial position and the end positions of each vehicle, where P_{Vi} represents the position of vehicle i.

(b) The spacing errors of each vehicle, where SE_i represents $p_{i-1} - p_i - l_{i-1,i}$.



(c) The velocities of each vehicle, where V_{Vi} represents the velocity of vehicle i.

(d) The accelerations of each vehicle, where A_{Vi} represents the acceleration of vehicle i.

Fig. 2. The results for vehicle platoon system under PF structure.

TABLE I
THE MASS AND INITIAL STATES OF EACH VEHICLE

The Mass and initial states of each vehicle in platoon system				
No./State	Mass (kg)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
Vehicle_0	5	100	5	0
Vehicle_1	5.6557	90	3	0
Vehicle_2	5.0357	81	2	0
Vehicle_3	5.8791	73	1	0
Vehicle_4	5.9340	66	0.7	0
Vehicle_5	5.6787	60	0.7	0
Vehicle_6	5.7577	55	0.1	0
Vehicle_7	5.7431	51	0.3	0
Vehicle_8	5.3922	47	2	0
Vehicle_9	5.6555	45	3	0

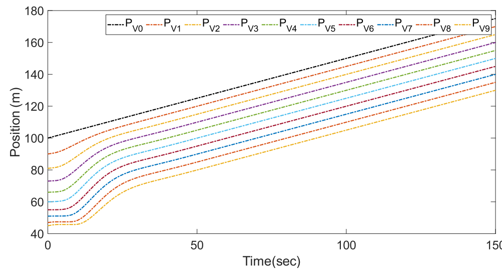
is set as $T = 10s$, the predictive horizon is set as $N_p = 15$, and the update period is set as $\tau = 0.1s$, respectively. The model parameters are $\Delta s = 0.5s$, $m_a = 1$, $c_a = 0.98m^2$, $c_g = 0.01$, and $f_{cg} = 0.5N$. All of the vehicles in the platoon system moving in the positive direction of X-axis. The initial states of each vehicle are given in Table I. The simulation results are given as follows to verify the effectiveness of the proposed algorithm.

B. The Simulation Results Under BD Structure

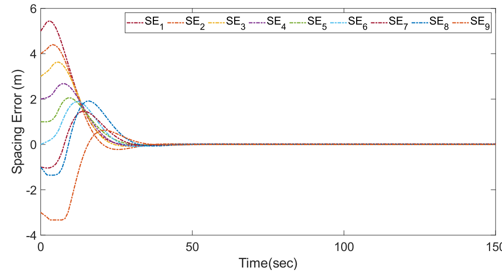
The platoon control using the proposed distributed MPC-based method under the BD structure is presented in this subsection. The initial conditions and control setting are given in previous subsection. The main results are shown in Fig.1. Under the BD structure, both the front vehicle $(i-1)_{\text{th}}$ and the following vehicle $(i+1)_{\text{th}}$ can be affect by the vehicle i_{th} in the platoon system. Fig.1(a) shows the position of each vehicle. Since a constant velocity is defined to the leader vehicle's, the final slopes of each vehicle's position is the same as that of the virtual leader. Fig.1(b) shows the spacing errors of each vehicle in the platoon system, which satisfy the system's string stability. Fig.1(c) and Fig.1(d) show the velocity and acceleration, respectively. It can be seen that the spacing errors of each vehicle are all converged to zero, the velocity and the acceleration of each vehicle are converged to the leader's velocity and acceleration, respectively. It can be seen that under the proposed distributed MPC-based algorithm, both the asymptotical stability and the string stability are guaranteed.

C. The Simulation Results Under PF Structure

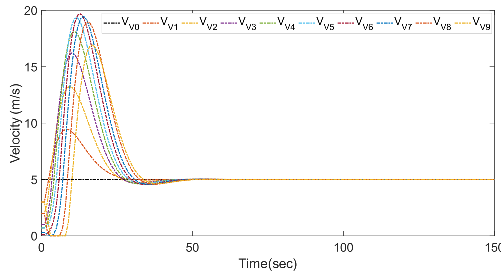
Fig.2 shows the results by using the proposed distributed MPC-based method under a PF platoon structure, which means the vehicle i_{th} only contact to the front vehicle $(i-1)_{th}$ in the system. The simulation setting is same to the ones in Subsection A. According to the PF structure, the local cost function will change as $J_i(t) = \sum_{s=0}^{N_p-1}(\|K_i(\Phi_i^p(s,t) - \Phi_i^*(s,t))\| + \|R_i u_i^p(s,t)\| + \|F_i(\Phi_i^p(s,t) - \hat{\Phi}_i(s,t)\| + \sum_{i\in I_i}\|G_i(\frac{1}{2}(\Phi_i^p(s,t) - \hat{\Phi}_{i-1}(s,t) + L_{i,i-1}))\|)$. Fig.2(a) shows the position of each vehicle. Since the virtual leader runs with a constant velocity, the final slopes of each vehicle's position is the same as it. The spacing errors are shown in in Fig.2(b), which can be seen that the string stability of the platoon system is guaranteed. The velocities and accelerations are shown in Fig.2(c) and Fig.2(d), respectively. Fig.4 shows the cost of each vehicle under the BD structure and the PF structure. From Fig.1 and Fig.2, we can know that the proposed distributed MPC-based method can guarantee the close-loop stability of the system both under the BD structure and the PF



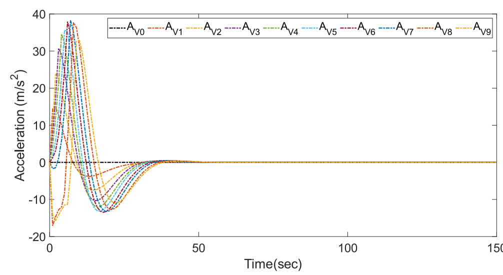
(a) The initial position and the end positions of each vehicle, where P_{Vi} represents the position of vehicle i.



(b) The spacing errors of each vehicle, where SE_i represents $p_{i-1} - p_i - l_{i-1,i}$.

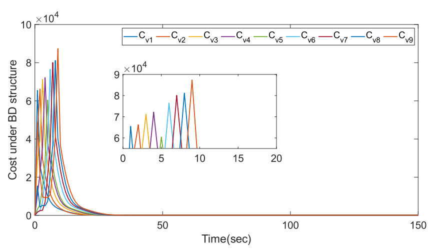


(c) The velocities of each vehicle, where V_{Vi} represents the velocity of vehicle i.

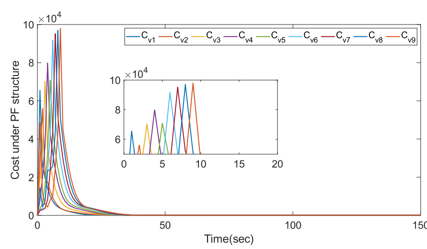


(d) The accelerations of each vehicle, where A_{Vi} represents the acceleration of vehicle i.

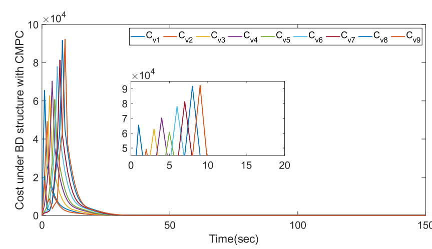
Fig. 3. The results for vehicle platoon system under BD structure with CMPC.



(a) The cost of the platoon system under BD structure.



(b) The cost of the platoon system under PF structure.



(c) The cost of the platoon system under BD structure with CMPC.

Fig. 4. The cost of the platoon system under BD structure and PF structure.

structure. The string stability of the platoon system also can be guarantee under the proposed method. The cost functions converge to zero under the distributed MPC algorithm, which is shown in Fig.4.

D. Comparison Results Under BD Structure With Centralized MPC

In this subsection, comparison results with the centralized MPC algorithm proposed in [34] are presented. Based on [34], the cost function and constraints in $\Gamma_i(t)$ are aggregated and replace $\hat{\Phi}_j(s,t)$ with $\Phi_j^p(s,t)$ in the MPC method. Since global information in the platoon system can be used in the centralized MPC method, the real-time predicted trajectory $\Phi_j^p(s,t)$ is used to avoid the local optimum instead of the assumed trajectory $\hat{\Phi}_j(s,t)$ in it. From Fig.3, it can be seen that even though the system can be converged, the string stability cannot be guaranteed. Fig.4 shows the cost of each vehicle under the BD structure and the PF structure.

VI. Conclusion

The distributed MPC algorithm is proposed for a discrete-time platoon system in this paper. The method can be applied not only to the PF-based vehicle platoon structures but also to the BD-based one. By considering the sum of the local cost function as the candidate Lyapunov function, the asymptotical stability is guaranteed. In addition, rigorous proof is also presented for the string stability issue of the platoon system. Finally, simulation results as well as comparison results are presented to show the effectiveness of the proposed control scheme. How to ensure the effectiveness of the proposed control method for some complicated cases, such as the nonlinear discrete vehicle platoon system under communication failure/delays, traffic oscillations and emergency braking, will be considered in future works.

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