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Characteristic Parameter-Based Detuned C-Type Filter Design

GUIBIN ZHANG[1] (Member, IEEE), YANG WANG[2] (Member, IEEE), WILSUN XU[2] (Fellow, IEEE), AND ESRA SITTHER[1] (Member, IEEE)

[1]ATCO Electric, Edmonton, AB T5J 2V6, Canada

[2]Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2R3, Canada

CORRESPONDING AUTHOR: YANG WANG (yang5@ualberta.ca)

ABSTRACT The C-type filter has been widely used in industry to avoid the harmonic resonance caused by the shunt capacitors. This paper provides a deep and unique insight into the C-type filter design. It will be shown that the C-type filter can be simply characterized by two parameters–the tuning frequency and the R-ratio. The tuning frequency is predetermined based on the system need, thus the entire design process comes down to determine the R-ratio. The correlation between the R-ratio with the filter's cost and performance is then developed and an optimum value of the R-ratio is solved. Compared with the conventional design methods, the proposed method with one underdetermined variable is simpler and more straightforward. Comparative studies on actual capacitor application cases are conducted to demonstrate the usefulness of the proposed design method.

INDEX TERMS C-type filter, harmonic resonance, damping, tuning frequency, R-ratio.

I. INTRODUCTION

In recent years, the proliferation of harmonic-producing loads in power systems have led to increased incidences of shunt-capacitor related harmonic resonance [1]–[6]. To address this issue, some utilities have started to configure high voltage shunt capacitors as the detuned C-type filters. Here the term "detuned" means the filter is designed to detune potential resonances between the system and the filter bank, rather than harmonic-filtering purpose. The C-type filter configuration is shown in Fig.1 along with its frequency response. At high frequency, the filter response is dominated by the resistor R, thus exhibiting good damping characteristics. Besides, the filter has small losses as L and C_2 are tuned at the fundamental frequency, leading to a bypass of R branch.

The C-type filter consists of four elements. Therefore, four design conditions are needed to determine their parameters. So far, only two design conditions (reactive power output and fundamental frequency loss minimization) are generally agreed upon by designers and users. These two conditions are not sufficient to determine all four parameters. Additional design conditions such as voltage distortion minimization have been suggested in the literature to finalize the parameters [7]–[11]. However, most of them are established for harmonic-filtering purpose, rather than resonance-mitigation purpose.

To address the aforementioned design gap, the recent study in [12] and [13] presented the design method to guarantee the damping performance of the C-type filter. The basic idea is to design a C-type filter that exhibits sufficient damping capability even under the worst system condition. However, this design method does not take the filter cost into consideration. Besides, no guideline has been given on how to optimally select the damping capacity in practice.

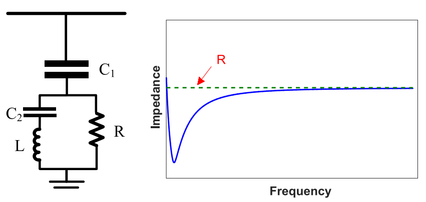

FIGURE 1. **C-type filter configuration and its frequency response.**

This paper further improves the work of [12] and presents a novel design method for the detuned C-type filter. Unlike traditional design methods that based on physical components of the filter, the proposed method simply characterizes the filter by two parameters – the tuning frequency and the R-ratio. The tuning frequency is the lowest resonance frequency to be eliminated, thus can be predetermined based on the system need. As a result, the entire design process is just to determine the R-ratio. It is further revealed that the R-ratio plays an important role to balance the filter's cost and performance. An optimum value of the R-ratio in term of this tradeoff can be obtained.

The remainder of this paper is organized as follows: Section II defines the characteristic parameters in the C-type filter design. Section III develops the design method based on the characteristic parameters. In Section IV, simplicity and effectiveness of the proposed method are demonstrated through case studies on actual capacitor application.

II. CHARACTERISTIC PARAMETERS

This section defines characteristic parameters in the C-type filter design. It will be shown in the next section that the design process becomes very simple and straightforward with these characteristic parameters.

A. PER UNIT VALUE

The C-type filter design involves four components - C_1, C_2, L and R. Without a reference, a given absolute value of an individual parameter does not have any intuitive meaning. Therefore, we suggest using the per-unit impedance value in the C-type filter design, as shown below.

- Base Voltage: nominal system voltage V_F,
- Base Frequency: nominal system frequency f_F (nominal angular frequency $\omega_F = 2\pi f_F$),
- Base MVA: the reactive power of the main capacitor, $Q_b = \omega_F C_1 V2\,F$,
- Base Impedance: the fundamental frequency impedance of the main capacitor, $Z_b = 1/(\omega_F C_1)$.

These base values can be calculated before the design process according to the system information. It is obvious that the per-unit impedance of C_1 at the fundamental frequency is always 1.0, so the design objective is to determine C_2, L and R. One inherent design condition of the C-type filter is that C_2 and L are tuned at the fundamental frequency, as shown in (1).

$$L = 1/(\omega_F^2 C_2) \tag{1}$$

Thus, C_2 will have the same per-unit impedance as L at the fundamental frequency. This means we only have two parameters L and R to be determined. Condition (1) also leads to a bypass of the R branch and, thereby, the elimination of power loss at the fundamental frequency. Accordingly, the same fundamental current flows through C_1, C_2 and L. Since the amount of the fundamental current is determined by the reactive power requirement of C_1, the larger inductance of L, the greater of the capacities/cost are needed for both components L and C_2. In other words, the per-unit impedances of L can be used as the direct cost indicator. The damping resistor R can be expressed in per-unit impedance in the same way. However, there is little physical meaning to do so, as the rating of the resistor is decided by the harmonic content, the parameter tolerances and the non-ideal operation conidiation instead of the fundamental current. It will be shown later that expressing R in term of the proposed R-ratio is more meaningful.

B. TUNING FREQUENCY

It seems to be a common industry practice that vendors are calculating the C-type filter's tuning frequency with the same concept as the single tuned filter by using the zero-damping steady state resonance frequency, i.e. by ignoring the high pass resistor R, as shown in (2).

$$\omega_{T,R=\infty} = \sqrt{\frac{C_1 + C_2}{LC_1C_2}} \tag{2}$$

For single tuned filter, the tuning frequency so defined has multiple physical meanings coinciding. At this frequency, the filter 1) presents the minimum impedance, 2) has unit power factor and 3) has the best filtering performance. However, none of the above features exist when the same concept is applied to the C-type filter. In other words, the tuning frequency defined in (2) does not make much sense in the C-type filter design. In this paper, we suggest a universal definition of the tuning frequencies for all filter topologies based on item 2) unit power factor, i.e. the steady state resonance frequency of the actual filter circuit, with the following considerations:

- Harmonic resonance is due to the reactance of the filter cancels out the reactance of the system, so it is critical to know whether the filter presents inductive or capacitive impedance at a given frequency.
- With such a definition, a filter with single tuning frequency (e.g. C-type filter) presents the capacitive impedance below the tuning frequency and the inductive impedance above the tuning frequency, which makes it easy to avoid resonance by properly positioning the tuning frequency.
- At the tuning frequency, the filter is purely resistive. Hence, it always reduces the system harmonic distortion at this frequency regardless of the system impedance value.
- A purely resistive impedance can guarantee resonance free, while the minimum filter impedance cannot.

According to Fig.1, the frequency response of the C-type filter at the tuning frequency ω_T can be calculated as,

$$Z_C(\omega_T) = R||(j\omega_T L + 1/j\omega_T C_2) + 1/j\omega_T C_1 \tag{3}$$

Since L and C_2 are tuned at the fundamental frequency, (3) can be simplified as (4) by eliminating C_2.

$$Z_C(\omega_T) = R||j(\omega_T\text{-}\omega_F^2/\omega_T)L + 1/j\omega_T C_1 \tag{4}$$

Let the imaginary part of (4) equal to zero, the tuning frequency ω_T can be derived. Mathematically, there are four solutions. The right solution is selected based on the physical concept that when R approaches the infinity, the calculation result shall match the single tuned filter. The expression of ω_T is shown in (5), as shown at the bottom of the next page.

When R is large, (2) is a good estimation of (5) for calculating the tuning frequency. However, in the design process, the filter parameters are calculated from the specified tuning frequency. While the tuning frequency may not be sensitive to certain filter parameters, the filter parameters could be very sensitive to the tuning frequency. Thus, from this point of view, an accurate mathematical relation is critical to derive the C-type filter design method. In practice, the tuning frequency of a detuned C-type filter is mainly decided by the lowest resonance point to be eliminated, such as the 3rd or 5th harmonic.

Based on (5), L can be expressed by other variables in the equation. There are two physically meaningful solutions as shown in (6) and (7).

$$L = \frac{R\omega_T(C_1 R\omega_T - \sqrt{C_1^2 R^2 \omega_T^2 - 4})}{2\omega_T^2 - 2\omega_F^2} \tag{6}$$

or

$$L = \frac{R\omega_T(C_1 R\omega_T + \sqrt{C_1^2 R^2 \omega_T^2 - 4})}{2\omega_T^2 - 2\omega_F^2} \tag{7}$$

Further defining the R-ratio ρ of the C-type filter as (8),

$$\rho = R\omega_T C_1 \tag{8}$$

Eq.(6) and (7) can be simplified as (9) and (10),

$$Z_{L,pu} = \frac{\rho(\rho - \sqrt{\rho^2 - 4})}{2(\omega_{T,pu}^2 - 1)} \tag{9}$$

or

$$Z_{L,pu} = \frac{\rho(\rho + \sqrt{\rho^2 - 4})}{2(\omega_{T,pu}^2 - 1)} \tag{10}$$

where $Z_{L,pu}$ is the per-unit impedance of L (or C_2) at the fundamental frequency and $\omega_{T,pu}$ is the per-unit tuning frequency.

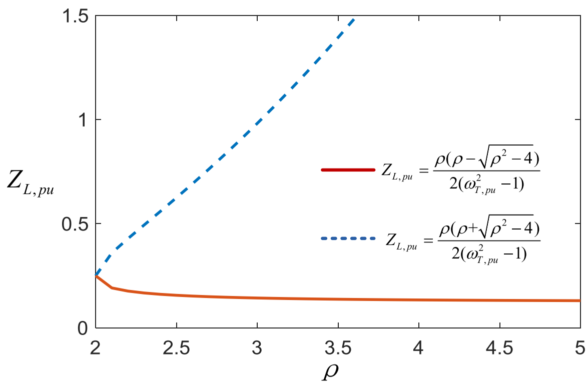

FIGURE 2. Correlation between $Z_{L,pu}$ and ρ.

Eq.(9) is the desirable solution based on the physical concept that when R approaches the infinity, the calculation shall match the single tuned filter. Besides, (10) results in larger inductance as shown the Fig.2 (tuned to the 3rd harmonic). Large inductance not only means high cost, but also means more demanding on breaker switching duty. It will be shown in Section III, the smaller inductance determined by (9) is sufficient for most practical needs. So, the solution of (10) is not further investigated in this paper.

As aforementioned in Section II.A, $Z_{L,pu}$ can be considered as the cost indicator of the C-type filter. Thus, (9) indicates the cost of a C-type filter can be characterized by its tuning frequency $\omega_{T,pu}$ and R-ratio ρ. Since $\omega_{T,pu}$ is pre-determined by the system need, the cost of a C-type filter can only be adjusted by changing R-ratio ρ. On the other hand, the R-ratio ρ also determines the damping performance of a C-type filter. In (8), C_1 is a constant value that determined by the reactive power requirement, so ρ can only be changed by the resistor R. It is well known that the larger of R, the worse of the damping performance. Therefore, the R-ratio ρ is a very useful characteristic parameter that determines both the filter's cost and damping performance.

The relationship between the R-ratio ρ and the cost indicator $Z_{L,pu}$ is shown in Fig.3 (assuming $\omega2\ T,pu = 2$). According to the figure, the larger of ρ, the lower of the cost. On the other hand, the larger of ρ, the worse of the damping performance. Therefore, there is a tradeoff between the filter's cost and damping performance.

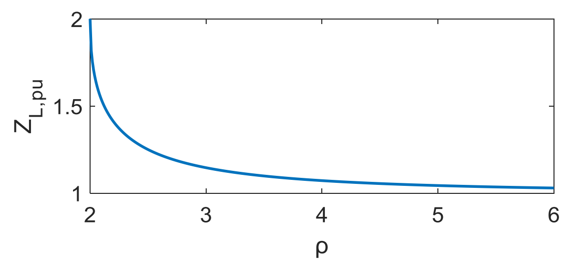

FIGURE 3. The correlation between $Z_{L,pu}$ and ρ.

III. DESIGN METHOD

With the introduction of the proposed characteristic parameters, the design process of a C-type filter is simplified with one unknown parameter – R-ratio ρ. Once ρ is determined, the entire parameter design process is complete. This section discusses the method to determine ρ, which includes two steps. First, an optimum value of ρ is defined in term of the optimum tradeoff between the performance and cost. Then ρ can be adjusted to meet the practical constraints such as the harmonic amplification ratio and the cost budget.

A. OPTIMUM VALUE OF R-RATIO

According to Fig.3, when ρ is too small, the cost ($Z_{L,pu}$) rises sharply. When ρ is too large, the damping performance would be very poor with little saving on the cost. An intuitive impression from Fig.3 is that ρ around 2.5 seems to be a good tradeoff between the performance and cost. However, such intuitive judgement is affected by how the plot is scaled. This subsection proposes a mathematical approach to achieve the optimum tradeoff between performance and cost. Since the performance and cost are in different units, they cannot be compared directly. Instead, the percentage of performance and the percentage of cost are compared.

$$\omega_T = \sqrt{-\frac{R^2 + R\sqrt{C_1^2 L^2 R^2 \omega_F^4 + 2C_1 L R^2 \omega_F^2 - 4L^2\omega_F^2 + R^2} - 2L^2\omega_F^2 + C_1 L R^2 \omega_F^2}{2L^2 - 2C_1 L R^2}} \tag{5}$$

By using the linear bijection theory, the cost indicator (9) can be normalized between 0 and 1, as shown in (11). According to (9), the minimum value of ρ for a given tuning frequency is 2, while the maximum value of ρ is infinity.

$$C_{index}(\rho) = \frac{Z_{L,pu}(\rho) - \lim_{\rho\to\infty} Z_{L,pu}(\rho)}{\lim_{\rho\to\infty} Z_{L,pu}(\rho)} = \frac{\rho^2}{2} - \frac{\rho\sqrt{\rho^2-4}}{2} - 1 \quad (11)$$

On the other hand, the damping performance can be indicated by ρ directly. The smaller of ρ, the better of the damping performance. The best performance is achieved when $\rho = 2$. Therefore, the performance index, normalized between 0 and 1, is shown in (12).

$$P_{index} = \frac{2}{\rho} \quad (12)$$

Substituting ρ in (11) by P_{index} defined in (12), the cost index can be expressed by the performance index, as shown in (13).

$$C_{index}(P_{index}) = \frac{2}{P_{index}^2} - \frac{2\sqrt{1-P_{index}^2}}{P_{index}^2} - 1 \quad (13)$$

The relationship between C_{index} and P_{index} is depicted in Fig.4. As seen from the figure, initially with the performance increases, the cost does not increase much. However, after certain point, the cost increase outweighs the performance increase. The tradeoff between the performance and the cost can be measured by the slope of the curve. For example, a slope of 1.5 means 1 per-unit increase of the performance requires 1.5 per-unit increase of the cost. As a result, it may not be very cost-effective to further improve the performance. In Fig.4, the slope of the curve always increases with P_{index}, thus the point where the slope equals to 1 is defined as the optimum tradeoff point between the performance and the cost. Beyond this point, the cost increase outweighs the performance increase.

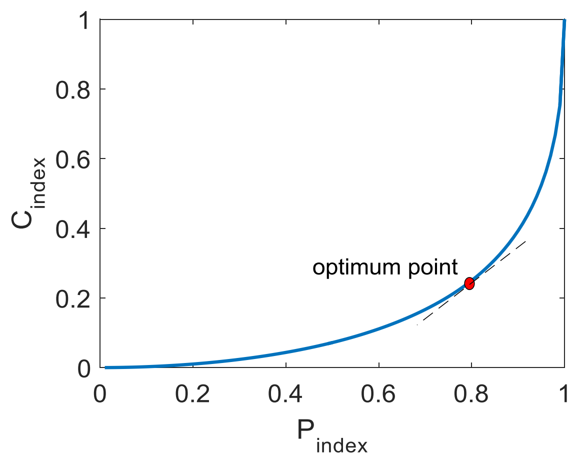

FIGURE 4. **The correlation between C_{index} and P_{index}.**

The slope of the curve in Fig.4 can be calculated as the first order derivative of (13). Accordingly, the optimum point with the slope equal to 1 can be obtained by solving (14).

$$\frac{dC_{index}}{dP_{index}} = \frac{2}{P_{index}\sqrt{1-P_{index}^2}} + \frac{4\sqrt{1-P_{index}^2}}{P_{index}^3} - \frac{4}{P_{index}^3} = 1 \quad (14)$$

The result shows the optimum P_{index} is 0.792, with the C_{index} of 0.242. This means the optimum tradeoff between the performance and the cost is 79.2% of the performance with 24.2% of the cost. Based on (12), the optimum R-ratio ρ can be determined as 2.525. As aforementioned, $\rho = 2$ reaches the maximum performance (100%) with the maximum cost (100%), while $\rho- > \infty$ reaches the minimum cost (as a single tuned filter) with no damping performance provided. The optimum ρ is 2.525, which can achieve 79.2% of the performance with 24.2% of the cost. With the optimum value of $\rho = 2.525$, it is also possible to reveal where the design is standing in term of cost and performance tradeoff for any design methods. For example, if the R-ratio of a C-type filter is 4.0, it means the design is sufficiently economical.

B. CORRELATION BETWEEN R-RATIO AND HARMONIC AMPLIFICATION RATIO

The optimum ρ is not necessary the final choice of a given design. It provides a sound start value. The value of ρ can be decreased to have the better damping performance or increased to have the cost under budget.

We have characterized the C-type filter by its tuning frequency and R-ratio. However, in practice, the utility companies usually specify a detuned C-type filter using the tuning frequency and harmonic amplification ratios. The harmonic amplification ratio (HAR) is defined as the ratio of the harmonic voltage before and after the connection of the C-type filter. According to Fig.5, HAR can be calculated based on the system impedance and the C-type filter's impedance at the harmonic frequency of interest, as shown in (15), where $R_C(\omega)$, $X_C(\omega)$ and $R_S(\omega)$, $X_S(\omega)$ are the real and imaginary part of $Z_C(\omega)$ and $Z_S(\omega)$, respectively. This is the traditional way to calculate HAR and we call it "actual" HAR (HAR_{actual}). For a given ρ and $\omega_{T,pu}$, the values of all elements C_1, C_2, L and R are readily available. The HAR_{actual} can be then calculated based on (15) with the system harmonic impedances provided by the utility. If the result does not meet the design specification, ρ should be decreased to improve the damping performance.

$$HAR_{actual}(\omega) = \left|\frac{V_C(\omega)}{V_S(\omega)}\right| = \left|\frac{Z_C(\omega)}{Z_C(\omega)+Z_S(\omega)}\right| = \sqrt{\frac{R_C^2(\omega)+X_C^2(\omega)}{(R_C(\omega)+R_S(\omega))^2+(X_C(\omega)+X_S(\omega))^2}} \quad (15)$$

The frequency response of the transmission system could be complex and unpredictable. As a result, the C-type filter

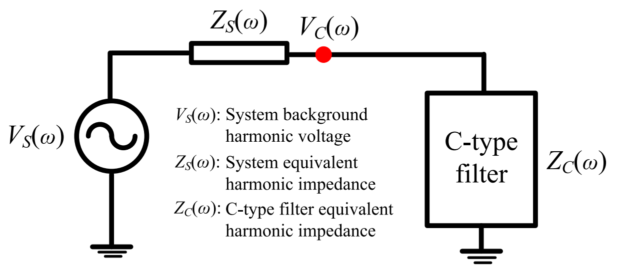

FIGURE 5. Equivalent circuit of the system and the C-type filter.

may fail to provide sufficient damping when the system impedance deviates from known values. To address this issue, a recent study [12] presented a system independent method to evaluate the damping performance of a C-type filter. It essentially assumes the worst system condition - the system impedance has no resistance and is in perfect resonance with the C-type filter. Thus, this method is more suitable to be a benchmark for generic C-type filter evaluation. Under the worst system condition $R_S = 0$ and $X_S = -X_C$, the "worst-case" HAR can be determined as (16).

$$HAR_{worst}(\omega) = \sqrt{1 + (\frac{X_C(\omega)}{R_C(\omega)})^2} \tag{16}$$

According to (16), HAR_{worst} is only related with the impedance of the C-type filter. Therefore, it can also be characterized by the proposed tuning frequency and R-ratio, as (17), as shown at the bottom of this page.

For a given tuning frequency at the 3rd harmonic, HAR_{worst} for $\rho = 2, 3, 4$ are plotted in Fig.6. Similarly, for a given R-ratio of $\rho = 3$, HAR_{worst} for $\omega_{T,pu} = 3, 4, 5$ are plotted in Fig.7. An intuitive impression of these two figures is that the maximum value of HAR_{worst} is mainly determined by the R-ratio and little changed by the tuning frequency, while the frequency that reaches the maximum HAR_{worst} is dominated by the tuning frequency and little changed with the R-ratio. In other words, the R-ratio mainly shifts the HAR_{worst} curve

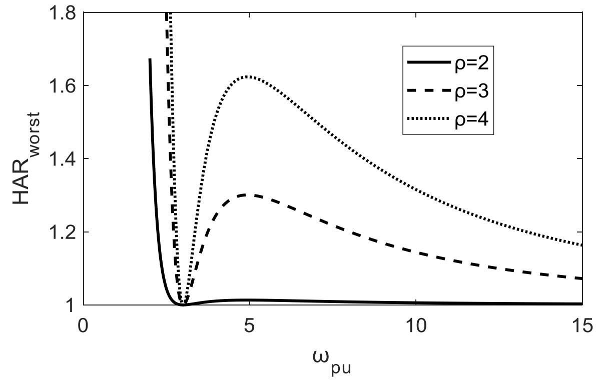

FIGURE 6. The correlation between HAR_{worst} and ρ.

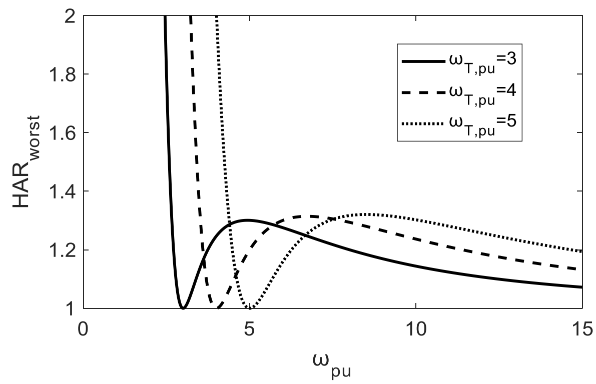

FIGURE 7. The correlation between HAR_{worst} and $\omega_{T,pu}$.

vertically, while the tuning frequency mainly shifts the curve horizontally. Their effects have very small coupling. Such a result indicates that ω_T and ρ are a pair of very effective characteristic parameters for the detuned C-type filter.

In Fig.6 and Fig.7, the focus is the maximum value of HAR_{worst}, as the damping performance can be guaranteed if the maximum HAR_{worst} is smaller than the predefined limit. The correlation between the maximum HAR_{worst} and the characteristic parameters is further studied. The frequency

$$HAR_{worst}(\rho, \omega_{T,pu}, \omega_{pu}) = \sqrt{\frac{(\omega_{pu}^2 - \omega_{T,pu}^2)^2(\rho\sqrt{\rho^2-4} - \rho^2 + \rho^2\omega_{pu}^2 - 2\omega_{pu}^2\omega_{T,pu}^2 - \rho\omega_{pu}^2\sqrt{\rho^2-4} + 2)^2}{\rho^2\omega_{pu}^2\omega_{T,pu}^2(\omega_{pu}^2-1)^4(\rho\sqrt{\rho^2-4} - \rho^2 + 2)^2} + 1} \tag{17}$$

$$\begin{aligned}\omega_{\max}(\rho, \omega_{T,pu}) = (&\sqrt[3]{(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}) + \sqrt{(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a})^2 + (\frac{c}{3a} - \frac{b^2}{9a^2})^3}} \\ &+ \sqrt[3]{(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}) - \sqrt{(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a})^2 + (\frac{c}{3a} - \frac{b^2}{9a^2})^3}} - \frac{b}{3a})^{\frac{1}{2}}\end{aligned} \tag{18}$$

where

$$\begin{cases} a = \rho^2 - \rho\sqrt{\rho^2-4} - 2\omega_{T,pu}^2 \\ b = 6\omega_{T,pu}^4 - (6 + 3\rho^2 - 3\rho\sqrt{\rho^2-4})\omega_{T,pu}^2 + 6 \\ c = \rho\sqrt{\rho^2-4} - \rho^2 + (2\omega_{T,pu}^2 + 4\rho^2 - 4\rho\sqrt{\rho^2-4} - 10)\omega_{T,pu}^2 + 2 \\ d = (2 - \rho^2 + \rho\sqrt{\rho^2-4})\omega_{T,pu}^2 \end{cases} \tag{19}$$

that reaches the maximum HAR_{worst} is the extremum of (17), which can be derived to have its first order derivative equal to zero. The result is shown in (18), as shown at the bottom of the previous page.

Thus, the maximum HAR_{worst} above the tuning frequency can be calculated as (20).

$$\max(HAR_{worst}(\rho, \omega_{T,pu})) = HAR_{worst}(\rho, \omega_{T,pu}, \omega_{\max}(\rho, \omega_{T,pu})) \quad (20)$$

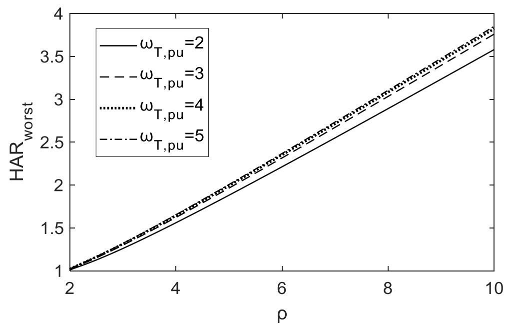

FIGURE 8. The correlation between maximum HAR_{worst} and ρ.

The correlation between the maximum HAR_{worst} and the R-ratio ρ is shown in Fig.8. It can be seen that the maximum HAR_{worst} increases almost linearly with ρ and does not change much due to the tuning frequency ω_T. This result confirms that P_{index} in (12) is a reasonable index to represent the damping performance of the filter. The correlation between the frequency that reaches the maximum HAR_{worst} and the tuning frequency ω_T is shown in Fig.9. According to the figure, ω_{max} almost increases linearly with ω_T and does not change much with the R-ratio ρ. In summary, the tuning frequency has little effect on the maximum HAR_{worst}. The R-ratio can be adjusted intuitively to meet the HAR_{worst} requirement while keeping an eye on the C-type filter cost.

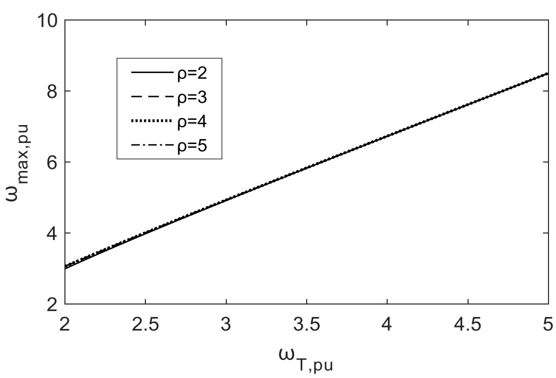

FIGURE 9. The correlation between maximum HAR_{worst} and $\omega_{T,pu}$.

The purpose of a detuned filter bank is to boost (amplify) the fundamental voltage. If the HAR of a certain harmonic is the same as the amplification ratio of the fundamental voltage, the filter bank is regarded to have no effect on the distortion of this specific harmonic. Similarly, if the HARs of all harmonics are no greater than the amplification ratio of the fundamental voltage, the filter bank is regarded to have no adverse effect on the system harmonic distortion. This means the chosen fundamental voltage amplification ratio is the tightest HAR limit for a detuned filter bank, which is typically up to 3%.

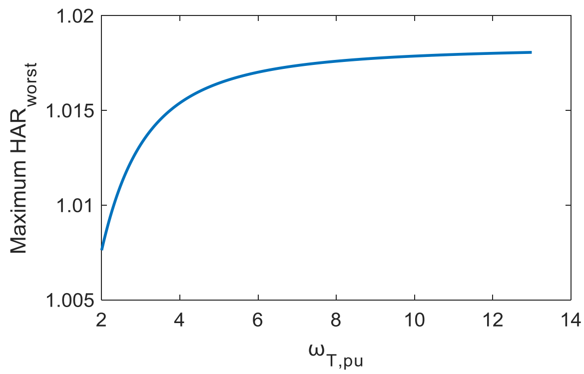

FIGURE 10. Maximum HAR_{worst} for different $\omega_{T,pu}$ with $\rho = 2$.

With (18) and (20), the best damping performance with R-ratio of 2 for the small inductance C-type filter design based on (9) is plotted in Fig.10. The result confirms that the smaller inductance given by (9) shall be able to satisfy the practical need.

In sum, this subsection discussed two methods to evaluate the damping performance of the C-type filter from practical specification perspective. One is the system dependent method that based on HAR_{actual}, and another is the system independent method that based on HAR_{worst}. The latter one places higher requirement on the damping performance of the C-type filter, while the former one may lead to certain cost saving (case-dependent). In real implementation, it is suggested to check the values of both indices. Different limits could be used for the two indices depending on the system conditions. For example, the limit of HAR_{actual} can be selected as 1.2 and the limit of HAR_{worst} can be selected as 1.5.

C. COMPLETE DESIGN PROCEDURE

In practice, the utility usually has a desire to design the C-type filter with 1) optimum performance and cost tradeoff or 2) minimum cost. The subsection describes the design procedure for these two design objectives.

1) OPTIMUM PERFORMANCE AND COST TRADEOFF

Based on the proposed design method, the optimum performance and cost tradeoff can be achieved simply by choosing the R-ratio ρ closest to $\rho = 2.525$ that meets the performance and cost requirement. The design procedure is shown in Fig.11. As seen from the flow chart, the entire design process is to adjust the R-ratio ρ, which is simple and straightforward. The C-type filter performance can be evaluated with system independent method, system dependent method or both.

2) MINIMUM COST

Based on the proposed design method, the minimum cost can be achieved by choosing the maximum R-ratio that can meet the performance requirement. Due to the limited space, the flow chart is omitted here. It should be noted that if system independent method (HAR_{worst}) is used, the desirable R-ratio can be directly solved by the numerical solution of (20). Due to the near linear relationship, the numerical solution is easy to obtain. If system dependent method (HAR_{actual}) is used, the design procedure is similar to Fig.11. The only difference is that the searching process aims to find the maximum R-ratio satisfying the performance requirement.

IV. CASE STUDY

In this section, case studies are presented to illustrate the application of the proposed characteristic parameter based design method.

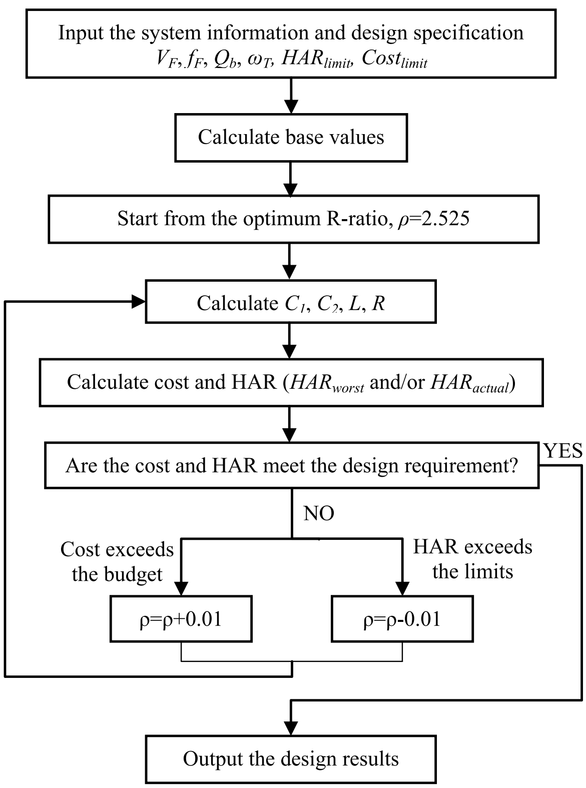

FIGURE 11. **The flowchart of optimum performance and cost tradeoff design.**

A. CASE DESCRIPTION

This case study involves one 30MVar capacitor to be connected to a 138kV substation in Alberta, Canada. The capacitor would lead to the significant harmonic amplification around the 3rd harmonic, as shown in Fig.12.

To address this issue, the utility company decides to configure the shunt capacitor as a C-type filter with the tuning frequency of 2.8th harmonic. The HAR_{limit} is selected as 1.4 to avoid the 5th harmonic voltage exceeding IEEE 519 Std. limit. Following design approaches are considered.

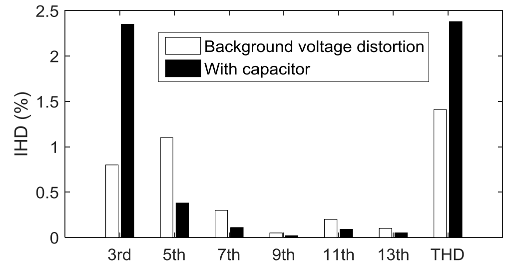

FIGURE 12. **Voltage distortion of the studied system.**

- Approach 1 - system dependent minimum cost design. Starting from the optimum R-ratio, $\rho = 2.525$, the design objective is to find the maximum ρ that satisfies $HAR_{actual} < 1.4$. No other constraints are given. The R-ratio ρ is calculated as 4.54.
- Approach 2 - system independent minimum cost design. Solving (20) with maximum HAR_{worst} equal to1.4, the R-ratio ρ can be obtained as 3.33. Since $\rho = 3.33$ is greater than the optimum value of $\rho = 2.525$, this design result is sufficiently economical. On the other hand, $\rho = 3.33$ is less than $\rho = 4.54$, so the system impedance information has a potential to further reduce the cost.
- Approach 3 – optimum performance and cost tradeoff design. Starting from $\rho = 2.525$, ρ will be adjusted as shown in Fig.11 only if the performance or cost requirement is not met. Based on (20), maximum value of HAR_{worst} with $\rho = 2.525$ and $\omega_{T,pu} = 2.8$ is 1.16. Thus, the requirement of $HAR < 1.4$ is satisfied. Since no other constraints are given, here the final value of ρ is 2.525.

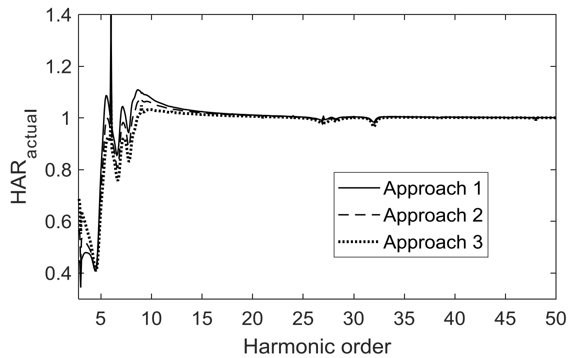

FIGURE 13. ***HAR_{actual}*** **for the three design approaches.**

B. PERFORMANCE COMPARISON

Fig.13 shows the HAR_{actual} for the above three design approaches. As seen from the figure, all results satisfy the design requirement of $HAR_{actual} < 1.4$.

Table 1 shows the parameters and damping performance of the C-type filters designed by the three approaches. It can be observed that the ratings and parameters from the three approaches are comparable, but the maximum HAR by approach 3 is significantly lower. Therefore, approach 3 yields the most cost-effective design result.

It should be mentioned that the cost and performance differences between the three design approaches would vary

TABLE 1. Design parameters and performance for three design approaches.

Design Approach		1	2	3
C_1	Value (uF)	4.17	4.17	4.17
	Q (MVar)	30.0	30.0	30.0
	Voltage (kV)	138.0	138.0	138.0
C_2	Value (uF)	27.1	25.7	23.0
	Q (MVar)	4.62	4.88	5.45
	Voltage (kV)	21.3	22.4	25.1
L	Value (mH)	259.4	273.8	305.7
	Q (MVar)	4.64	4.89	5.46
	Voltage (kV)	21.4	22.5	25.1
R	Value (Ω)	1029	753	573
	P (kW)	4.7	5.4	5.9
	Voltage (kV)	2.2	2.0	1.8
HAR (max)	Actual	1.40	1.23	1.07
	Worst-case	1.80	1.40	1.16

case by case depending on the system impedance profile, the filter bank reactive power rating, and the given HAR limits for each order of harmonics.

Different design approaches are suitable for different needs. Approach 1 better fits for the situation where reliable and accurate system impedance information is available. Approach 2 is better for the situation with no reliable system impedance information or there are too many operation scenarios to consider. Approach 3 fits in where there is incentive to exceed the minimum performance requirement with a little extra cost.

Regardless which approach to use, the optimum R-ratio is always a sound start or reference value, and the characteristic parameter based design can greatly simplify all the above design approaches.

V. CONCLUSION

This paper has proposed a novel design concept for the C-type filter. A complete and simple design method in terms of two characteristic parameters has been established. The main contributions of this paper are summarized as follows.

- This paper introduced the universal definition and explained the physical meaning of the filter tuning frequency.
- This paper introduced the definition and explained the technical significance of the R-ratio of the C-type filter.
- A new design concept based on two characteristic parameters has been proposed for the C-type filter. Since the tuning frequency is predetermined based on the system need, the entire design process is to adjust the R-ratio to meet the design specification. Compared with conventional design methods, the proposed design method with one underdetermined variable is simpler and more straightforward.
- The relationship between the R-ratio and the filter's cost and performance has been developed. A R-ratio value for the optimum tradeoff between performance and cost is calculated as the start or reference value of the design.
- Case studies illustrate the simplicity and effectiveness of the proposed design method.

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REFERENCES

[1] R. K. Varma, S. A. Rahman, T. Vanderheide, and M. D. N. Dang, "Harmonic impact of a 20-MW PV solar farm on a utility distribution network," *IEEE Power Energy Technol. Syst. J.*, vol. 3, no. 3, pp. 89–98, Sep. 2016.

[2] H. Hu, Q. Shi, Z. He, J. He, and S. Gao, "Potential harmonic resonance impacts of PV inverter filters on distribution systems," *IEEE Trans. Sustain. Energy*, vol. 6, no. 1, pp. 151–161, Jan. 2015.

[3] J. Meyer, A.-M. Blanco, M. Domagk, and P. Schegner, "Assessment of prevailing harmonic current emission in public low-voltage networks," *IEEE Trans. Power Del.*, vol. 32, no. 2, pp. 962–970, Apr. 2017.

[4] Y. Wang, J. Yong, Y. Sun, W. Xu, and D. Wong, "Characteristics of Harmonic distortions in residential distribution systems," *IEEE Trans. Power Del.*, vol. 32, no. 3, pp. 1495–1504, Jun. 2017.

[5] S. B. Gin, J. H. Sawada, and T. R. Treasure, "BC hydro harmonic resonance experience," in *Proc. IEEE Power Eng. Soc. Summer Meeting*, Jul. 2000, pp. 1088–1093.

[6] T. M. Blooming and D. J. Carnovale, "Capacitor application issues," *IEEE Trans. Ind. Appl.*, vol. 44, no. 4, pp. 1013–1026, Jul. 2008.

[7] S. H. E. Abdel Aleem, A. F. Zobaa, and M. M. Abdel Aziz, "Optimal C-type passive filter based on minimization of the voltage Harmonic distortion for nonlinear loads," in *IEEE Trans. Indus-trial Electron.*, vol. 59, no. 1, pp. 281–289, Jan. 2012.

[8] H. Hu, Z. He, and S. Gao, "Passive filter design for china high-speed railway with considering harmonic resonance and characteristic harmonics," *IEEE Trans. Power Del.*, vol. 30, no. 1, pp. 505–514, Feb. 2015.

[9] R. Horton, R. Dugan, and D. Hallmark, "Novel design methodology for C-type harmonic filter banks applied in HV and EHV networks," in *Proc. IEEE PES TD Conf. Expo.*, May 2012, pp. 1–6.

[10] Y. Xiao, J. Zhao, and S. Mao, "Theory for the design of C-type filter," in *Proc. 11th Int. Conf. Harmon. Quality Power*, 2004, pp. 11–15.

[11] B. Badrzadeh, K. S. Smith, and R. C. Wilson, "Designing passive harmonic filters for an aluminum smelting plant," *IEEE Trans. Ind. Appl.*, vol. 47, no. 2, pp. 973–983, Apr. 2011.

[12] W. Xu, T. Ding, X. Li, and H. Liang, "Resonance-free shunt capacitors-configurations, design methods and comparative analysis," *IEEE Trans. Power Del.*, vol. 31, no. 5, pp. 2287–2295, Oct. 2016.

[13] Y. Wang and W. Xu, "A shared resonance damping scheme for multiple switchable capacitors," *IEEE Trans. Power Del.*, vol. 33, no. 4, pp. 1973–1980, Aug. 2018.

GUIBIN ZHANG (M'14) received the B.S. and M.S. degrees in electrical engineering from Shandong University, Jinan, China, in 1995 and 1998, respectively, and the Ph.D. degree in electrical engineering from Zhejiang University, Hangzhou, China, in 2001. He is currently a Senior Engineer with ATCO Electric, a subsidiary of ATCO Electricity Global Business Unit. His area of focus is substation apparatus and protection.

YANG WANG (M'17) received the B.S degree in electrical engineering from Zhejiang University, Hangzhou, China, in 2012 and the Ph.D. degree in electrical and computer engineering with the University of Alberta, Edmonton, AB, Canada, in 2017. He is currently a Post-Doctoral Fellow with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada. His main research interests are power system measurement and power quality.

WILSUN XU (M'90–SM'95–F'05) received the Ph.D. degree in electrical engineering from The University of British Columbia, Vancouver, Canada, in 1989. He is currently a NSERC/iCORE Industrial Research Chair Professor with the University of Alberta, Edmonton, AB, Canada. His research interests are power quality and power disturbance analytics.

ESRA SITTHER (M'09) received the M.Eng. degree in high voltage engineering from Anna University, Chennai, India, in 1992. He is currently a Senior Engineer with ATCO Electric, a subsidiary of ATCO Electricity Global Business Unit. His technical expertise is in the domain of power system apparatus design and engineering.