Received 4 November 2016; revised 4 February 2017; accepted 20 March 2017.
Date of publication 27 March 2017; date of current version 22 June 2017.

Digital Object Identifier 10.1109/JPETS.2017.2688020

Voltage Stability-Constrained Optimal Simultaneous Placement of PMUs and Channels Enhancing Measurement Reliability and Redundancy

MASOUD ESMAILI (Senior Member, IEEE) AND MOHAMMAD GHAMSARI-YAZDEL (Student Member, IEEE)

Department of Electrical Engineering, West Tehran Branch, Islamic Azad University, Tehran 1949663311, Iran
CORRESPONDING AUTHOR: M. ESMAILI (msdesmaili@gmail.com)

ABSTRACT In this paper, a channel-oriented method is proposed for optimal placement of phasor measurement units (PMUs) with the objective function of explicit cost of PMUs and their channels. PMU measurement channels are treated as optimization binary variables, and a PMU installed at a bus assigns channels to observe its adjacent buses only if it is economically justified. Since power system substations have different reliability levels, in order to enhance reliability of the measurement system, PMUs and their channels are encouraged to be employed at more reliable buses and branches. In addition, in order to monitor fragile areas of power systems for prevention of voltage collapse, PMUs and their channels are assigned to observe buses with vulnerable voltage stability status. Furthermore, in order for a more economical and practical solution, the most probable contingencies are identified using the Monte Carlo simulation to be incorporated in the problem. Also, PMU failures and branch outages are modeled with a technique resulting in a less cost than existing methods. Channel failure is also modeled as a new type of contingency. The efficiency of the proposed method is evaluated by testing it on standard and practical large-scale test systems.

INDEX TERMS Phasor measurement unit, measurement channel, measurement reliability, Monte Carlo simulation, voltage stability, measurement redundancy.

I. INTRODUCTION

One of practical limitations of Phosor Measurement Units (PMUs) is their channel constraint. As long as there are enough channels available in a PMU, it makes observable its host bus (by measuring its voltage phasor) and all of its connected buses (by measuring their branch current phasor and calculating voltage). Extensive research is done for Optimal PMU Placement (OPP) in literature and most works considered no limitation in PMU channels; that is, there are enough voltage and current channels as many as needed. In such a case, the OPP is simplified to minimizing the cost-weighted number of PMUs. Some other works, such as [1] and [2] addressed PMU channel limitation in OPP with the same channel limit for all PMUs. Nevertheless, each bus has different number of connected branches, and then, there is a different channel requirement for PMUs at different buses. In [3], PMU measuring channel capacities are taken into account in numerical observability analysis using a semidefinite programming where a combination of channel assignments are formulated when the number of channels of a PMU is less than branches connected to a bus. In [4], channel limitation is used to determine the optimal channel assignment of PMUs, but it is not formulated in the cost objective function.

In order to employ PMUs and their channels more efficiently, this paper proposes an OPP in which PMU channels are treated as optimization variables like the location of PMUs. The explicit cost of PMUs and channels is included in the cost objective function; where the optimal location of PMUs and their channels is decided by the optimization problem. Under such a formulation, PMUs have a base cost that covers their devices and installation works, where PMU channels as I/O cards have also their own cost. In the proposed OPP, total explicit cost of PMUs and their channels are minimized instead of number of PMUs and channels.

In order to improve the reliability of Wide Area Measurement System (WAMS), measurement reliability can be enhanced through OPP. Some papers used extra PMUs with additional cost to improve measurement reliability such as [5]. On the other hand, some papers have tried to locate PMUs at more reliable buses to improve reliability such as [6] and [7], where PMUs are located at more reliable buses to decrease the probability of losing observability. In [8], PMU availability is considered in OPP to enhance the availability of WAMS considering short-circuit faults and PMU communication failure. In [9], Optimal locations of PMUs and Phasor Data Concentrators (PDCs) are obtained to maximize reliability of the communication network.

The other point about the WAMS is that it should pay attention to less stable areas of power systems in order to minimize the probability of unobservability of such areas. Vulnerable areas of power systems are always a good candidate to start instability issues [10]. Power system whole stability status is improved if its vulnerable areas are detected and kept under control. This topic is addressed in a few papers in literature; Kumar and Thukaram [11] ranked vulnerable buses from transient stability point of view using transient energy margin in order to be observed using PMUs. However, in the current paper, we locate PMUs and their channels so that vulnerable buses from voltage stability point of view have more chance to be observed.

Another feature that should be included in OPP is Measurement Redundancy (MR). Different methods are proposed in literature to consider measurement redundancy in OPP [12]–[14]. In [12], MR is maximized in OPP along the cost objective function without extra PMUs. Mazhari *et al.* [13] proposed an OPP with MR where worth of contingencies are used to decide whether a contingency is included in OPP. Huang *et al.* [14] presented an OPP with MR formulated as integer linear programming ensuring power system observability after contingencies under controlled islanding conditions. In [15], possible solutions for optimal locations of PMUs are identified in the 1st step and then, measurement redundancy is maximized in 2nd step with addressing normal and controlled islanding operation conditions. In [16], Simulated Annealing is used to maximize measurement redundancy out of possible PMU locations considering branch and bus contingencies. In [17], measurement redundancy is maximized in OPP enhancing system stability based on a Lyapunov exponent approach. It is worth noting that providing MR needs additional PMU channels and consequently, it should be managed wisely to have a cost-effective solution. In the current paper, vulnerable buses from voltage stability point of view are determined and then, they are redundantly observed for closely monitoring the fragile areas.

In addition, a WAMS is expected to keep the power system observable not only in its base case (normal operation) but also in post-contingency states. Contingencies that are usually considered in OPP include branch outages and PMU failure. Since contingencies have different likelihoods, it is more economical to consider only highly probable contingencies because the incorporation of a longer list of contingencies leads to a higher cost of PMUs. As a result, it will be economically efficient if only probable contingencies are identified using probabilistic methods and included in OPP. A few papers in literature addressed identifying probable contingencies using different techniques. For instance, Aminifar *et al.* in [18] considered random power system outages in OPP using an iterative algorithm to obtain probability of observability at buses. In [13], the number of contingencies to be included in OPP is limited by defining a monetary value for contingencies and their likelihood. Contingencies are selected in [6] with their probability to be included in the OPP. In [7], Monte Carlo Simulation (MCS) is used to identify probable contingencies for OPP. Most literature works in this regard consider each contingency group (branch outage, PMU failure, etc.) separately. However, some of them such as [7] are based on *scenarios*, which enable us to model all types of contingencies in scenarios. Depending on the likelihood of contingencies, a scenario may include one or more types of contingencies. In addition, under a scenario framework, there is no need to discriminate $n-1$ and $n-k$ contingencies; a scenario can include $n-k$ contingencies if their probability is enough high. As a result, we use a scenario-based method in the current paper to determine the most probable contingencies to be included in the OPP.

Taking into account the above-mentioned features, the main contributions of the current paper are to propose a channel-based OPP where the locations of PMUs and their channels are treated as optimization variables and their optimal binary values are determined with minimizing an explicit cost objective function. Using the channel-based concept, a PMU may not observe some of its adjacent buses if the solution is not economically justified. In other words, in this way, PMU channels are utilized more efficiently to achieve a solution with a better cost. In addition, by considering reliability of substations, PMUs and their channels are encouraged to be placed at more reliable buses in order to enhance the reliability of whole measuring system. Also, special attention is paid to areas with vulnerable voltage stability status; in the proposed OPP, such areas have more chance to be observed by PMUs in order to closely monitor weak parts of the network. Measurement redundancy is also provided to observe vulnerable buses. Also, in the proposed method, probable scenarios of the power system are identified using MCS to be included in the OPP.

II. OPTIMAL PLACEMENT OF PMUs AND MEASUREMENT CHANNELS

A. VOLTAGE STABILITY VULNERABILITY OF BUSES

In order to calculate vulnerability index of bus i (called here ξ_i) for incorporation into the proposed method, the loadability of buses (λ_i) are determined using the CPF (Continuation Power Flow) analysis [10]. Then, the weakest and the strongest bus are determined having the loadability of λ^{min} and λ^{max}, respectively. In order to illustrate this concept, P-V curve for the IEEE 14-bus test system is plotted in Fig. 1.

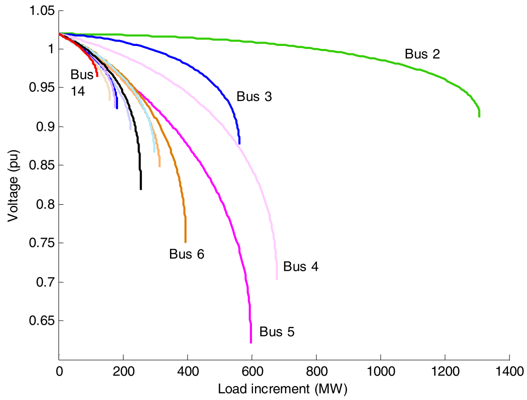

FIGURE 1. Voltage stability ranking of buses in the IEEE 14-bus test system.

As seen, buses 2 and 14 are the strongest and weakest buses from voltage stability point of view. Afterwards, the vulnerability of all buses are calculated by linear fuzzification of (1), where the weakest and strongest bus are assigned a vulnerability of 1 and 0, respectively. The vulnerability of other buses lies between 0 and 1 depending on their loadability (λ). Vulnerability index of buses is later included in the cost objective function

$$\xi_i = \frac{\lambda^{max} - \lambda_i}{\lambda^{max} - \lambda^{min}}. \tag{1}$$

B. AVAILABILITY OF COMPONENTS

The availability of a component in a given time horizon represents how much the component is reliable and it is the probability of performing its function without failure. The probability of observability of bus i due to installing a PMU at bus j is a parameter in the optimization and is defined as [18]

$$A_{ij} = a_{ij} A_j^{Bus} A_j^{VM} A_j^{PMU} A_j^{LINK} A_{ij}^{CM} A_{ij}^{BR} \tag{2}$$

where A_j^{Bus} is the availability of bus j; A_j^{VM} is the availability of voltage measuring system at bus j; A_j^{PMU} and A_j^{LINK} are the probability of successful operation of PMU and its communication link at bus j; A_{ij}^{CM} is the availability of PMU current measurement at branch ij; A_{ij}^{BR} represents the availability of branch ij; a_{ij} is a binary parameter equal to 1 if there is a branch between bus i and j or if $i = j$.

In (2), to calculate the probability of observability of bus i due to installing a PMU at the same bus (A_{ii}), we set $A_{ii}^{CM} = A_{ii}^{BR} = 1$. As a result, (2) gives the probability of observability of buses and branches. Evidently, we have different A_{ij} values across the network even if the same PMUs are employed because the availability of buses and branches varies in different locations of the power system. Considering A_{ij}, we so locate PMUs and their channels that a higher probability of observability is established in the whole network.

C. OBJECTIVE FUNCTION AND CHANNEL-BASED CONCEPT

The objective function in the proposed method is to minimize total cost of the base unit of PMUs and their channels

$$MinF = C_{PMU} \sum_{i \in SB} x_i (1 - A_{ii})(1 - \xi_i) + C_{ch} \sum_{i,j \in SB} ch_{ij}(1 - A_{ij})(1 - \xi_j) \tag{3}$$

where x_i is a binary variable equal to 1 if a PMU is to be placed at bus i; C_{PMU} is the unit cost of a base PMU (without channels) including its related costs (procurement, installation, and commissioning); SB represents set of buses; ch_{ij} is a binary variable equal to 1 if PMU at bus i assigns a voltage channel to observe its host bus ($ch_{ii} = 1$) or a current channel to observe bus j across branch ij ($ch_{ij} = 1$); C_{ch} is the unit cost of a PMU channel and includes instrumentation, wiring, and labor costs for current or voltage measurement.

The first and second term in (3) corresponds to the cost of base PMUs and channels, respectively. Focusing on the first term, since it is multiplied by $(1-A_{ii})(1-\xi_i)$, the method tries to place PMUs at more reliable buses (with higher A_{ii}) and more vulnerable buses (with higher ξ_i) in order to minimize the first term. In the second term of (3), ch_{ij} is the observability of bus j due to installing a PMU at bus i. Therefore, in order to minimize the second term, PMU channel ch_{ij} is assigned to observe bus j through a more reliable branch ij (with higher A_{ij}) or to observe more vulnerable bus j (with higher ξ_i). The ultimate result of including availability and vulnerability indices in (3) is that PMUs and their channels are assigned to more reliable locations and also to monitor more vulnerable buses.

In the proposed channel-based method, a PMU located at a bus can assign current channels to observe its adjacent buses if it is economically acceptable; this is a decision made by optimizing the cost objective function. This decision-making process is substantially different from previous works, where a PMU placed at a bus has to assign channels to all of its connected buses even if it is neither necessary nor economical. The constraint to assign channels is formulated as

$$ch_{ij} \leq a_{ij} x_i \quad \forall i, j \in SB. \tag{4}$$

Equation (4) indicates that if a PMU is placed at bus i (i.e., $x_i = 1$), the right hand is equal to 1 ($a_{ij} = 1$ if there is a branch between bus i and j). Since ch_{ij} is a binary variable, it *can* be zero (i.e., PMU at bus i does not assign a channel to observe bus j) or it *can* be one (i.e., PMU at bus i assigns a channel to observe bus j). Therefore, PMUs *can* assign current channels to observe their adjacent buses. The optimal assignment of PMU channels is determined through minimizing the objective function. This type of allocating channels to branches makes possible exploit current channels more efficiently and does not waste channels. In this case, unwanted measurement redundancy is kept low; however, we later provide measurement redundancy at locations where they are needed.

In order to illustrate the channel-based feature of the proposed method, its results are shown for the IEEE 14-bus test system on the one-line diagram of Fig. 2.

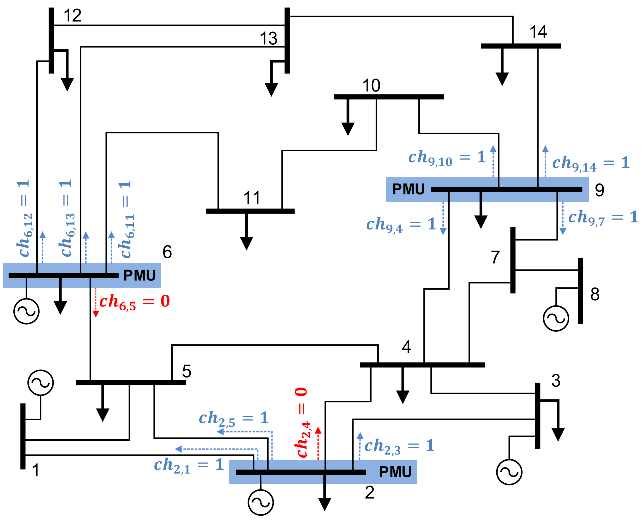

FIGURE 2. Optimal channel assignments of the channel-based method in the IEEE 14-bus test system.

This test system will be studied later in Section IV in detail. In order to focus on the channel-based concept, we ignore reliability and vulnerability terms, for now, in the objective function. As shown on Fig. 2, the proposed method places 3 PMUs at buses 2, 6, and 9 and uses 13 channels including 3 voltage channels (at PMU buses) and 10 current channels (shown with dashed blue arrows in the figure). With this configuration, the whole system is observable and there is no need to assign current channels to branches 2-4 and 6-5 by PMUs at buses 2 and 6, respectively, (these channels are shown by red arrows in the figure) because they are already observed through other routes. On the other hand, previous works locate 3 PMUs at the same buses with assigning channels to every branch connected to PMU buses (15 channels in total including blue and red routes). This means that the channel-based OPP saves 2 channels (shown by red arrows) and it results in a more optimal solution. The number of saved channels in larger test system will be considerable. The problem that is open to debate is that there is no measurement redundancy after saving the two channels. In this way, MR is omitted from unnecessary locations and it is assigned later in a controlled way to buses of interest. It is noted that in this test system, bus 8 is made observable using Zero Injection Bus (ZIB) property of bus 7.

D. OBSERVABILITY CONSTRAINTS

The total cost in (3) as the objective function is minimized subject to some constraints. The observability function for each bus is given [12] as

$$f_i = \sum_{j \in SB} ch_{ji} + \sum_{j \in SB} z_j a_{ij} u_{ij} \quad \forall i \in SB \tag{5}$$

where z_j is a binary parameter equal to 1 if bus j is a ZIB that is a bus with neither generation nor load (In a ZIB, if all branch flows but one of them are known, it is possible to apply the KCL and determine the unknown flow; this is known as the ZIB property); u_{ij} is an auxiliary binary variable equal to 1 if bus i is observable due to the ZIB property of bus j; f_i is the number of times that bus i is observed through PMU channels and ZIB properties. In order to make all buses observable, the observability function should satisfy

$$f_i \geq 1 \quad \forall i \in SB. \tag{6}$$

It is worth noting that the technique used here to model ZIBs is based on the formulation used in [19] and [20], where the auxiliary variable u_{ij} of (5) is constrained by

$$z_j = \sum_{i \in SB} a_{ij} u_{ij} \quad \forall j \in SB. \tag{7}$$

As mentioned previously, it is necessary to provide measurement redundancy for some critical buses to preserve enough reliability. For such buses, the augmented observability function of (6) should be redundant or more as

$$f_i \geq n_i s_i \quad \forall i \in SB \tag{8}$$

where n_i is the required observability for bus i ($n_i = 2$ for redundant observation); s_i is a parameter equal to 1 if bus i is critical. Here, we assume that $s_i = 1$ if the vulnerability of bus (ξ_i) is greater than a threshold vulnerability (ξ_{tresh}). In other words, those buses, which have a vulnerability greater than the threshold vulnerability, are considered as critical and they should be redundantly observed.

III. STOCHASTIC MODELING OF CONTINGENCIES IN OPP

Contingencies that are usually considered in OPP include branch outage and PMU failure. Additionally, channel failure is also considered here as a new type of contingency because the probability of failure in channels is higher than a PMU [21].

The flowchart of the proposed MCS is plotted in Fig. 3. Using the MCS, scenarios are generated and ranked according to their probability of occurrence [7]. Some scenarios can have multiple component outages (including branch outage, PMU failure, or channel failure) depending on availability of components. Those components, which appear in the set of highly probable scenarios, are more likely to fail and only this set of equipment are embedded here in the OPP. We here denote the less reliable PMU set with S_{PMU}^{out}, the less reliable branch set with S_{BR}^{out}, and the less reliable channel set with S_{CH}^{out}. Including such a limited number of components in the OPP results in a more economical and realistic solution because a longer list of contingencies increases the cost of OPP.

A. INCLUSION OF PROBABLE PMU FAILURES

To cover PMU failure, in some papers such as [19], new PMUs are needed. Nevertheless, in the method used here to cover PMU failure, a secondary (or backup) PMU with the same channels as the primary PMU is installed only at less reliable buses. In view of the fact that the base unit cost of

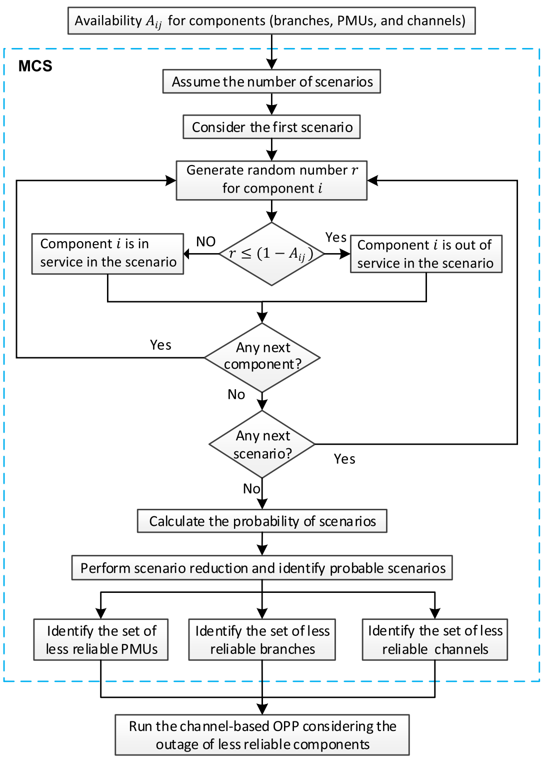

FIGURE 3. **Flowchart of the proposed Monte-Carlo simulation.**

the second PMU is about one half of the first one [1], this trend results in a more economical solution than previous techniques which place new PMUs at new buses with 100% base cost. As another advantage, since the proposed technique locates duplicate PMUs at buses, the system retains full observability even if one of two PMUs at every bus is concurrently lost, a case that cannot be covered with previous techniques.

To mathematically model the proposed PMU failure, if the existence of secondary PMUs and their channels is denoted with x_i' and ch_{ij}', respectively, the placement of secondary PMUs and their channels next to less reliable PMUs is accomplished by

$$x_i' = x_i, \quad ch_{ij}' = ch_{ij} \quad \forall i \in S_{PMU}^{out}, \quad \forall j \in SB. \qquad (9)$$

The condition $\forall i \in S_{PMU}^{out}$ in (9) ensures that the secondary PMUs and channels be installed only next to less reliable primary PMUs.

B. INCLUSION OF PROBABLE BRANCH OUTAGES

For a cost-effective solution, only branches with high probability of outage (S_{BR}^{out}) are included in the proposed OPP. The formulation employed here to incorporate selected branch outages in the OPP is based on [19]. Similar to the base case in Section II, the post-contingency observability function is formulated as

$$f_i^k = \sum_{j \in SB} ch_{ji}^k + \sum_{j \in SB} z_j a_{ij}^k u_{ij}^k \quad \forall i \in SB, \quad \forall k \in S_{BR}^{out} \quad (10)$$

where superscript k denotes branch k ($k \in S_{BR}^{out}$ means that only branches with higher outage probability are considered); a_{ij}^k is the ij entry of the connectivity matrix after removing branch k; u_{ij}^k is an auxiliry binary variable equal to 1 if bus i is observable due to the ZIB property of bus j after removing branch k; ch_{ji}^k is a binary variable equal to 1 if bus i is observable through the PMU channel of bus j after removing branch k; f_i^k is the post-contingency observability function of bus i.

In order to have the system observable after the outage of less reliable branches, the post-contingency observability function should satisfy

$$f_i^k \ge 1 \quad \forall i \in SB, \quad \forall k \in S_{BR}^{out}. \qquad (11)$$

In fact, the condition $\forall k \in S_{BR}^{out}$ in (10) and (11) makes possible that only the outage of less reliable branches, which are determined by the MCS, not all branches, is included in the OPP; this limits the number of branches and reduces required cost. Similar to (7), the ZIB properties in the post-contingency states are constrained by

$$z_j = \sum_{i \in SB} a_{ij}^k u_{ij}^k \quad \forall j \in SB, \quad \forall k \in S_{BR}^{out}. \qquad (12)$$

Also, similar to (4), the channel assignment of PMUs in post-contingency states is constrained by

$$ch_{ij}^k \le a_{ij}^k x_i \quad \forall i, j \in SB, \quad \forall k \in S_{BR}^{out}. \qquad (13)$$

Since PMU channels (ch_{ij}) are included in the cost objective function (3), in order to relate ch_{ij}^k with other variables of the optimization problem, the following equation needs to be satisfied:

$$ch_{ij}^k \le ch_{ij} \quad \forall i, \quad j \in SB, \quad \forall k \in S_{BR}^{out}. \qquad (14)$$

C. INCLUSION OF PROBABLE CHANNEL FAILURES

In order to keep the network observable in case of probable channel failures, we need a backup channel to cover the failure of those primary channels which have high probability of failure. This concept is shown in Fig. 4, where bus j is observed through the primary channel ch_{ij} of a PMU at bus i.

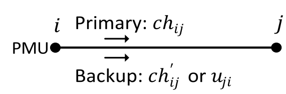

FIGURE 4. **Backup observation to cover channel failure.**

The backup observation can be provided either from channels of secondary PMUs of ch_{ij}' or ZIB property of u_{ji} (if bus i is a ZIB and bus j is using its ZIB property [12]). In other words, it is not necessary to assign a backup channel to a branch if its backup is provided through ZIB. These concepts are mathematically modeled as

$$x_i' \le x_i \quad \forall i \in S_{CH}^{out} \qquad (15)$$

$$ch_{ij}' \le a_{ij} x_i' \forall i \in S_{CH}^{out}, \quad \forall j \in S_{CH}^{out} \qquad (16)$$

$$x_i' \le \sum_{j \in SB} ch_{ij}' \quad \forall i \in S_{CH}^{out}. \qquad (17)$$

The set S_{CH}^{out} in (15)-(17) ensures to locate backup PMUs and channels in routes with high probability of channel failure. Constraint (15) implies that a backup PMU ($x_i^{'}$) *can* be placed next to the primary PMU ($x_i^{'}$ can be 1 if $x_i = 1$, but it is not always 1). Similar to primary channel constraint of (4) constraint (16) assigns backup channels to observe buses over branch *ij*. Equation (17) implies that if total backup channels from bus *i* is zero, it should not have a backup PMU.

Sum of channels provided by primary and backup observations in Fig. 4 is $ch_{ij} + ch_{ij}^{'} + u_{ji}$. In order to ensure a backup observation (through channels or ZIBs) after failure of a primary channel, the following constraint should be satisfied:

$$ch_{ij} + ch_{ij}^{'} + u_{ji} \geq 2ch_{ij} \quad \forall i \in S_{CH}^{out}, \quad \forall j \in S_{CH}^{out}. \quad (18)$$

In (18), if no channel is assigned to observe bus *j* through bus *i* (i.e., $ch_{ij} = 0$), the right-hand side becomes zero and then, the constraint has no effect on the optimization problem. However, if there is a primary channel over branch *ij* (i.e., $ch_{ij} = 1$), sum of bus *j* possible observations over branch *ij* (primary and backup channels and ZIB property as seen in Fig. 4) should be greater than 2, which means to provide a backup for the primary channel.

IV. CASE STUDIES AND NUMERICAL RESULTS

Different features of the proposed stochastic OPP are evaluated in this section by testing on standard and practical test systems. We use cost for each PMU and channel as in [1]: unit cost of each primary base PMU, secondary base PMU, and PMU channel are assumed as $20000, $10000, and $3000, respectively. Availability data for voltage and current measuring systems (A_{ij}^{VM} and A_{ij}^{CM}) are taken from [22] as 0.9985424 and 0.9995845, respectively. The availability of PMUs and their communication links is also assumed as 0.9954977 and 0.9990000 according to [23]. Also, availability of buses and branches are generated using a uniform random number generator in appropriate ranges.

A. OPTIMAL PLACEMENT OF PMUs AND CHANNELS WITHOUT MEASUREMENT REDUNDANCY

The main purpose here is to evaluate the channel-based feature of the proposed method in two cases of with and without reliability in the objective function. Results are shown in Table 1. It should be noted that the number of channels and

TABLE 1. Results of the channel-based method without considering voltage stability critical buses and reliability.

Test system	# PMUs	# Channels	Cost ($)
IEEE 14-bus	3	13	99,000
IEEE 39-bus	8	27	241,000
IEEE 57-bus	11	42	346,000
IEEE 118-bus	28	108	884,000
IEEE 300-bus	68	235	2,065,000
Polish 2383-bus	553	1831	16,553,000
Polish 2746-bus	625	2041	18,623,000

PMUs in this table are treated as optimization variables and their optimal values, as listed in the table, are obtained after solving the problem in one stage. Results from a few recent works are also reported in Table 2 for comparison; they

TABLE 2. Results from previous methods.

Test system	Method	# PMUs	# Channels	Cost ($)
IEEE 14-bus	[24], [13], [25], [19], [2]	3	15	105,000
IEEE 57-bus	[24], [13], [19], [2]	11	48	364,000
	[25]	12	53	399,000
IEEE 118-bus	[24]	29	141	1,003,000
	[13]	29	148	1,024,000
	[25]	28	137	971,000
	[19]	28	147	1,001,000
	[2]	26	131	913,000

encompass classical and some evolutionary methods including Immunity Genetic Algorithm (IGA), Cellular Learning Automata (CLA), and Adaptive Step Multidimensional Fruit-fly Optimization Algorithm (ASM-FOA). Although these literature methods do not have an explicit cost function to be compared with the proposed method, we here calculate their total observation cost from their number of PMUs and channels.

The result is that the channel-based method has made observable networks with a less cost; for instance, in case of the IEEE 14-bus test system in Table 1, the proposed method leads to $99,000 against $105,000 of previous works as shown in Table 2 (two channels are saved in the proposed method costing each channel $3000). This saving happens also in other test systems. Although in some cases, for example in IEEE 118-bus, the number of PMUs increases in the proposed method (28 against 26), it leads to a better cost due to more optimal usage of channels: the proposed method solution in Table 1 in the IEEE 118-bus test system results in $884000, which is better than the others in Table 2. In fact, the cost, not the number of PMUs, should be compared. Accordingly, these results confirm the efficiency of the channel-based method in getting a more cost-effective solution with a better usage of PMU channels.

In order to generally assess the reliability of different OPP solutions, an index is defined here as the Average Probability of Observability (APO). Each solution that results in a higher APO provides a better OPP solution from the measurement reliability point of view. APO is defined as

$$APO = \frac{\sum_{i,j \in SB} A_{ij} ch_{ij}}{\sum_{i,j \in SB} ch_{ij}} \quad (19)$$

where A_{ij} is the probability of observability of a PMU voltage or current channel already defined in (2). In (19), the numerator represents total probability of observabilities of channels, while the denominator is the total number of PMU channels.

In Table 3, results of the proposed method are presented where reliability is included but bus vulnerability is not yet included. If this table is compared with Table 1, it is seen that in IEEE 14-bus test system, the same number of PMUs and channels are reported in both tables and then, the two methods (without and with considering reliability) have resulted in the same costs ($99,000). However, the reliability-based method in other larger test systems may need a few more PMUs or channels; this is a small fee that should be paid to have a more reliable measurement system. By comparing APO of Table 3

TABLE 3. **Results of reliability-based OPP for the base case.**

Test system	# PMUs	# Channels	Cost ($)	APO	APO of Table I
IEEE 14-bus	3	13	99,000	0.9841030	0.9836750
IEEE 39-bus	9	27	261,000	0.9833363	0.9822720
IEEE 57-bus	12	42	366,000	0.9850629	0.9831022
IEEE 118-bus	29	108	904,000	0.9835153	0.9824092
IEEE 300-bus	69	235	2,085,000	0.9837100	0.9821103
Polish 2383-bus	571	1831	16,913,000	0.9838585	0.9822724
Polish 2746-bus	646	2041	19,043,000	0.9838236	0.9819828

with Table 1 solutions, it can be seen that the reliability-based method has resulted in a more reliable system.

B. VOLTAGE STABILITY-CONSTRAINED OPTIMAL PLACEMENT OF PMUs AND CHANNELS

In simulations, threshold vulnerability of voltage stability is considered as $\xi_{tresh} = 0.95$ meaning that only top 5% critical buses require measurement redundancy. In order to judge the quality of OPP solutions from voltage stability viewpoint, two indices are defined here as the Average Vulnerability of PMU locations (AVP) and the Average Vulnerability of Channel locations (AVC). A higher AVP or AVC implies that PMUs and channels are so installed that they observe fragile areas of the power system. AVP and AVC are defined as

$$AVP = \frac{\sum_{i \in SB} V_i x_i}{\sum_{i \in SB} x_i} \quad AVC = \frac{\sum_{i,j \in SB} V_j ch_{ij}}{\sum_{i,j \in SB} ch_{ij}} \tag{20}$$

where, AVP is total vulnerability of location of PMUs divided by total number of PMUs; AVC is total vulnerability of location of channels divided by total number of PMU channels.

Results of voltage stability-constrained OPP is shown in Table 4 without considering measurement redundancy for vulnerable buses; just the vulnerability index of buses is used in (3) to encourage PMUs and channels to be installed at more fragile areas. Normalized Total Observability (NTO) in this table is defined as total observability normalized by number of buses. By adding measurement redundancy for top vulnerable buses, the cost increases as shown in Table 5.

TABLE 4. **Results of the proposed method without measurement redundancy for voltage stability vulnerable buses.**

Test system	# PMUs	# Channels	Cost (k$)	AVP	AVC	NTO
IEEE 14-bus	4	13	119	0.48571	0.22490	1.0000
IEEE 39-bus	9	28	264	0.32761	0.14102	1.0256
IEEE 57-bus	12	42	366	0.08333	0.02381	1.0000
IEEE 118-bus	30	108	924	0.06534	0.01815	1.0000
IEEE 300-bus	70	235	2105	0.01429	0.00842	1.0000

TABLE 5. **Results of the proposed method with measurement redundancy for voltage stability vulnerable buses.**

Test system	# PMUs	# Channels	Cost (k$)	AVP	AVC	ET
IEEE 14-bus	6	16	168	0.48744	0.36546	0.107
IEEE 39-bus	9	30	270	0.32761	0.16463	0.214
IEEE 57-bus	12	43	369	0.08333	0.04651	0.372
IEEE 118-bus	30	110	930	0.06534	0.02691	0.612
IEEE 300-bus	71	237	2131	0.02787	0.01670	2.248

ET: Elapsed Time in seconds

For instance, in the IEEE 14-bus test system, buses 10, 12, and 14 are detected as the top 5% vulnerable ones that need a redundant observation ($f_i = 2$), whereas other buses have $f_i = 1$. However, without considering measurement redundancy for critical buses (Table 1, Table 3, and Table 4), all buses have the minimum observability of $f_i = 1$ to keep the cost at minimum. As a result, the overall cost increases. In some test systems, such as IEEE 118-bus, redundancy is provided without adding PMUs and from their additional channels. Of course, the percentage of increase due to considering voltage stability is smaller in larger test systems when Table 5 is compared with Table 4. In fact, results of Table 5 present a higher value of AVP and AVC meaning that PMUs and channels are employed at more vulnerable areas. In some test systems, such as IEEE 118-bus, AVP is the same in the two tables (0.06534); this means that they have the same number of PMUs at the same buses. However, the assignment of channels is different with a higher value of AVC is Table 5. The elapsed time is also reported in Table 5. This time, which is reported by the GAMS software package, is in orders of seconds implying a fast solution.

C. STOCHASTIC VOLTAGE STABILITY-CONSTRAINED OPTIMAL PLACEMENT OF PMUs

At first, the MCS is run to determine less reliable components (branches, PMUs, and channels) as explained in Fig. 3 and results are listed in Table 6. As seen in this table, only alimited number of contingencies is included in the problem and this eases the feasibility of solutions and final cost. As seen in this table, the percentage of selected components is lower as the system size increases. This means that the efficiency of the stochastic method increases in larger test systems.

TABLE 6. **Number of less reliable components determined by MCS.**

Test system	Selected branches	Selected PMUs	Selected channels
IEEE 14-bus	9 (out of 20)	3 (out of 4)	6 (out of 13)
IEEE 39-bus	18 (out of 46)	8 (out of 9)	15 (out of 28)
IEEE 57-bus	18 (out of 78)	7 (out of 12)	22 (out of 42)
IEEE 118-bus	33 (out of 179)	17 (out of 30)	24 (out of 108)

Results of stochastic OPP considering the probable contingencies are reported in Table 7. It is evident that by incorporating contingencies, cost increases with respect to the non-contingent cases. However, since the number of contingencies is limited to only probable ones, the cost increment is under control. In case of branch outage in Table 7, the method optimally places PMUs and channels to keep the system observable in base case and post-contingency states. In case of PMU failure in the table, the method places secondary PMUs in order to cover loss of primary PMUs (only primary PMUs that are more likely to fail). Lastly in case of channel failure, the method places secondary channels to cover failure of primary channels. In all cases, a larger NTO compared with Table 4 implies a better observability in the contingency-constraint cases.

TABLE 7. Stochastic solution of the proposed method considering probable contingencies.

Case	Parameter	IEEE 14	IEEE 39	IEEE 57	IEEE 118
Branch outage	# Primary PMUs	7	12	15	37
	# Channels, NTO	15, 1.1429	30, 1.0769	44, 1.0351	111, 1.0254
	Cost (k$)	185	330	432	1073
	AVP	0.41767	0.24571	0.06667	0.05298
	AVC	0.32825	0.13162	0.04546	0.01766
PMU failure	# PMUs (Pri. + Sec.)	4 + 3	9 + 8	12 + 7	30 + 17
	# Channels, NTO	23, 1.7143	52, 1.6410	67, 1.4386	172, 1.5424
	Cost (k$)	179	416	511	1286
	AVP	0.55510	0.34688	0.10526	0.04171
	AVC	0.21159	0.15187	0.02985	0.01140
Channel failure	# PMUs (Pri. + Sec.)	4 + 3	9 + 9	12 + 10	30 + 18
	# Channels (Pri. + Sec.), NTO	13 + 6, 1.4286	28 + 15, 1.4103	42 + 22, 1.3860	108 + 24, 1.2034
	Cost (k$)	167	399	532	1176
	AVP	0.41224	0.32761	0.09091	0.06085
	AVC	0.20350	0.07020	0.01563	0.01485

V. CONCLUSION

In this paper, a voltage stability-constrained method is proposed for optimal placement of PMUs and their channels. Also, they are located at more reliable places to increase measurement reliability and PMU channels are more economically employed. To assign measurement redundancy for more vulnerable buses from voltage stability point of view, the weakest ones are prioritized in PMU and channel placement. Compared with previous methods, the proposed model provides a better cost and also a better reliability: for example, on the IEEE 57-bus test system, it offers a solution with $346,000, while the best solution from literature leads to $364,000 (about 5% saving in the cost even with a better reliability). In addition, the probable contingencies are identified using MCS for a more cost-effective solution. Also, some new concepts are proposed to model PMU failure, channel failure, and branch outages.

REFERENCES

[1] F. Aminifar, M. Fotuhi-Firuzabad, A. Safdarian, and M. Shahidehpour, "Observability of hybrid AC/DC power systems with variable-cost PMUs," *IEEE Trans. Power Del.*, vol. 29, no. 1, pp. 345–352, Feb. 2014.

[2] E. Abiri, F. Rashidi, T. Niknam, and M. R. Salehi, "Optimal PMU placement method for complete topological observability of power system under various contingencies," *Int. J. Elect. Power Energy Syst.*, vol. 61, pp. 585–593, Oct. 2014.

[3] N. Manousakis and G. Korres, "Optimal allocation of PMUs in the presence of conventional measurements considering contingencies," *IEEE Trans. Power Del.*, to be published.

[4] S. Azizi, G. B. Gharehpetian, and A. S. Dobakhshari, "Optimal integration of phasor measurement units in power systems considering conventional measurements," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 1113–1121, Jun. 2013.

[5] Y. Wang, C. Wang, W. Li, J. Li, and F. Lin, "Reliability-based incremental PMU placement," *IEEE Trans. Power Syst.*, vol. 29, no. 6, pp. 2744–2752, Nov. 2014.

[6] O. Gómez, C. Portilla, and M. A. Ríos, "Reliability analysis of substation monitoring systems based on branch PMUs," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 962–969, Mar. 2015.

[7] M. Ghamsari-Yazdel and M. Esmaili, "Reliability-based probabilistic optimal joint placement of PMUs and flow measurements," *Int. J. Elect. Power Energy Syst.*, vol. 78, pp. 857–863, Jun. 2016.

[8] M. Sarailoo and N. E. Wu, "A new PMU placement algorithm to meet a specified synchrophasor availability," in *Proc. IEEE Power Energy Soc. Innov. Smart Grid Technol. Conf. (ISGT)*, Sep. 2016, pp. 1–5.

[9] F. H. Fesharaki, R.-A. Hooshmand, and A. Khodabakhshian, "Simultaneous optimal design of measurement and communication infrastructures in hierarchical structured WAMS," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 312–319, Jan. 2014.

[10] N. Amjady and M. Esmaili, "Improving voltage security assessment and ranking vulnerable buses with consideration of power system limits," *Int. J. Elect. Power Energy Syst.*, vol. 25, no. 9, pp. 705–715, Nov. 2003.

[11] V. S. S. Kumar and D. Thukaram, "Approach for multistage placement of phasor measurement units based on stability criteria," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2714–2725, Jul. 2016.

[12] M. Esmaili, K. Gharani, and H. A. Shayanfar, "Redundant observability PMU placement in the presence of flow measurements considering contingencies," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3765–3773, Nov. 2013.

[13] S. M. Mazhari, H. Monsef, H. Lesani, and A. Fereidunian, "A multi-objective PMU placement method considering measurement redundancy and observability value under contingencies," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2136–2146, Aug. 2013.

[14] L. Huang, Y. Sun, J. Xu, W. Gao, J. Zhang, and Z. Wu, "Optimal PMU placement considering controlled islanding of power system," *IEEE Trans. Power Syst.*, vol. 29, no. 2, pp. 742–755, Mar. 2014.

[15] S. Akhlaghi, N. Zhou, and N. E. Wu, "PMU placement for state estimation considering measurement redundancy and controlled islanding," in *Proc. IEEE Power Energy Soc. General Meeting (PESGM)*, Jul. 2016, pp. 1–5.

[16] S. Akhlaghi, "Optimal PMU placement considering contingency-constraints for power system observability and measurement redundancy," in *Proc. IEEE Power Energy Conf. Illinois (PECI)*, Feb. 2016, pp. 1–7.

[17] M. Rashidi and E. Farjah, "Lyapunov exponent-based optimal PMU placement approach with application to transient stability assessment," *IET Sci., Meas. Technol.*, vol. 10, no. 5, pp. 492–497, 2016.

[18] F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour, and A. Khodaei, "Probabilistic multistage PMU placement in electric power systems," *IEEE Trans. Power Del.*, vol. 26, no. 2, pp. 841–849, Apr. 2011.

[19] F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Contingency-constrained PMU placement in power networks," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 516–523, Feb. 2010.

[20] M. Esmaili, "Inclusive multi-objective PMU placement in power systems considering conventional measurements and contingencies," *Int. Trans. Elect. Energy Syst.*, vol. 26, no. 3, pp. 609–626, Mar. 2016.

[21] R. J. Albuquerque and V. L. Paucar, "Evaluation of the PMUs measurement channels availability for observability analysis," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2536–2544, Aug. 2013.

[22] M. J. Rice and G. T. Heydt, "The measurement outage table and state estimation," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 353–360, May 2008.

[23] F. Aminifar, S. Bagheri-Shouraki, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Reliability modeling of PMUs using fuzzy sets," *IEEE Trans. Power Del.*, vol. 25, no. 4, pp. 2384–2391, Oct. 2010.

[24] F. Aminifar, C. Lucas, A. Khodaei, and M. Fotuhi-Firuzabad, "Optimal placement of phasor measurement units using immunity genetic algorithm," *IEEE Trans. Power Del.*, vol. 24, no. 3, pp. 1014–1020, Jul. 2009.

[25] B. Ramachandran and G. T. Bellarmine, "Improving observability using optimal placement of phasor measurement units," *Int. J. Elect. Power Energy Syst.*, vol. 56, pp. 55–63, Mar. 2014.

MASOUD ESMAILI, photograph and biography not available at the time of publication.

MOHAMMAD GHAMSARI-YAZDEL, photograph and biography not available at the time of publication.