

# Ladder-Type Soil Model for Dynamic Thermal Rating of Underground Power Cables

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**ABSTRACT** This paper presents an optimal  $RC$  ladder-type equivalent circuit for the representation of the soil for dynamic thermal rating of underground cable installations. This is useful and necessary for their optimal and accurate real-time operation. The model stems from a nonuniform discretization of the soil into layers. The resistive and capacitive circuit elements are computed from the dimensions and physical parameters of the layers. The model is perfectly compatible with the International Electrotechnical Commission thermal–electric analog circuits for cables. The optimum model order is determined, for fast and slow thermal transients, from a comprehensive parametric study. It is shown that an exponential distribution of the soil layers leads to accurate results with differences of less than  $0.5\text{ }^{\circ}\text{C}$  with respect to transient finite-element simulations. An optimal model with only five layers that delivers accurate results for all practical installations and for all time scenarios is presented. The model of this paper is a simple-to-use and accurate tool to design and analyze transient operation of underground cables. It represents a relevant improvement to the available operation and monitoring tools. For illustration purposes, a step-by-step model construction example is given. The model has been validated against numerous dynamic finite-element simulations.

**INDEX TERMS** Ampacity, cable thermal rating, dynamic ratings, soil modeling, transient ratings, underground cable installations.

## NOMENCLATURE

$T_4$	External resistance of the soil.
$\theta_e$	Temperature rise of the outer surface of the cable.
$W_l$	Losses per unit length in each cable.
$\rho_T$	Thermal resistivity of the soil.
$\delta$	Soil diffusivity.
$E_i$	Exponential integral.
$D$	Diameter of the cable.
$L$	Distance from the surface of the ground to the cable axis.
$d_{pk}$	Distance from cable $k$ to the center of the hottest cable $p$ .
$d'_{pk}$	Distance from the image of the center of cable $k$ to the center of cable $p$ .
$N_c$	Number of cable in the group.
$N$	Number of discretization layers in the soil model.
$r$	Distance measured from the center of the cable.
th	Thickness of the cable layers.
$R_i$	Resistance of each layer of the soil model.

$C_i$	Capacitance of each layer of the soil model.
$b_i$	Radial position of the layer borders.
$\gamma$	Argument of the exponential distribution of layers.
$d_m$	Depth of the model.

## I. INTRODUCTION

SOIL modeling is of paramount importance for the calculation of the thermal performance of underground cables. After the conductor gauge, and perhaps the bonding technique, the soil is the major factor limiting the ampacity of an underground cable system [1], [2]. The International Electrotechnical Commission (IEC) [3], [4] and Institute of Electrical and Electronics Engineers (IEEE) [5] standards provide a methodological approach to determine the steady-state thermal rating of cables via an analog equivalent thermal–electrical circuit.

It is becoming increasingly necessary to perform accurate dynamic thermal calculations for emergency and real-time ratings [6]–[13]. The majority of these approaches follow the guidelines of the IEC standards to model transients

in underground cables. The IEC standards propose an equivalent resistance ( $T_4$ ) to represent the external resistance of the soil in steady state. The same methodology with a time-varying  $T_4$  resistance is proposed for transient (or dynamic) applications. However, for dynamic ratings, the standards only provide a formula to compute the thermal resistances and capacitances of the internal layers of the cable. The soil is modeled with the analytical solution of the diffusion equation, requiring the evaluation of exponential integrals. This solution is very precise, but it is neither convenient nor consistent with the layered (state-space compatible) modeling that is extensively used for the solution of  $RC$  dynamic systems.

Recently, a soil discretization model was proposed in [14]–[16]. The model consists of an  $RC$  ladder-type circuit suitable for the dynamic representation of the soil. The soil is discretized uniformly into layers and the model parameters are computed using standard formula. The ladder soil model is a natural extension of the existing (IEC standards)  $RC$  equivalent circuits used for cables. The model is physically sound since all its parameters are computed from the geometrical information and material properties of the soil. The model is capable of providing the temperature of the soil at any point because it is based on soil discretization. Therefore, each layer is a physical representation of the corresponding region in the soil structure. It is important to note that the dynamic  $RC$  model also computes the correct results in steady state. However, because the selected discretization of the soil was uniform, models as large as 100 sections are necessary to obtain an adequate accuracy.

In this paper, an optimal soil discretization technique is proposed from the observation of the physical diffusion of the temperature into the soil. In regions close to the cables (where temperature gradients are large), thinner soil layers are needed. At distances far from the cables (where the temperature gradients are small), thicker layers can be used without affecting the accuracy of the calculation. Thus, the model order and consequently the computational burden are greatly reduced when compared with those of [15]. As shown below, a model with only five layers can produce accurate results for all practical installations and for all realistic time scenarios. Existing techniques for the analysis of the  $RC$  circuits, e.g., parameter estimation techniques, state-space modeling, and state-space-order reduction, can be applied to the model. In particular, this model is suitable for the utilization of electrical circuit simulators (such as PSpice, EMTF, and PSCAD/EMTDC). This gives great flexibility to cable engineers to do quick, accurate, and efficient analyses with simple models without the need to solve the involved standard equations.

The final objective of this multistage research is to produce accurate transient temperature calculations to be integrated with distributed temperature sensing (DTS) systems for real-time cable ratings; therefore, the enhancements in the computation speed presented in this paper are very significant. The widespread implementation of DTS would allow for the utilization of cable systems to their maximum capabilities.

## II. SOIL MODEL AS PER IEC STANDARD 60853

The transient temperature rise of the outer surface of a cable, considering the contribution of the soil, can be evaluated by representing the cable as a line source located in a homogeneous, infinite medium with uniform initial temperature. Under these assumptions, the transient temperature rise  $\theta(t)$  at any point in the soil is governed by the following diffusion equation [2]:

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \rho_T W_I = \frac{1}{\delta} \frac{\partial \theta}{\partial t} \quad (1)$$

where  $r$  is the distance measured from the center of the cable,  $W_I$  represents the losses of the cable, and  $\delta$  is the soil diffusivity. The solution to this equation is given by

$$\theta(t) = \rho_T W_I \left[ -E_i \left( \frac{-r^2}{4\delta t} \right) \right] \quad (2)$$

where  $E_i$  is the exponential integral and  $t$  is the time span from the application of the heat source. Equations (1) and (2) are applicable to an infinite cylindrical soil and therefore do not consider the effect of the soil–air interface at the ground surface. Traditionally, an isothermal is assumed at the soil–air interface when the cables are buried at a certain depth. This effect was studied and solved in [17] and later discussed in [18] and [19]. Nonisothermal surface can be considered using the same equations with the additional wall method proposed in [20] and made practical in [21].

The IEC standards use the Kennelly hypothesis to build a model for the soil in steady-state and in transient conditions. For steady-state calculations, the IEC standards model the soil surrounding a cable with an equivalent resistance, namely  $T_4$ , calculated as [3]

$$T_4 = \frac{1}{2\pi} \rho_T \ln(u + \sqrt{u^2 - 1}). \quad (3)$$

In (3),  $\rho_T$  is the thermal resistivity of the soil (in K m/W), and  $u$  is defined as

$$u = 2L/D \quad (4)$$

where  $L$  is the distance from the surface of the ground to the cable axis and  $D$  is the diameter of the cable. The model of  $T_4$  for transient simulations is given by

$$T_4(t) = \frac{\rho_T}{4\pi} \cdot \left[ -E_i \left( \frac{-D^2}{16\delta t} \right) + E_i \left( \frac{-L^2}{\delta t} \right) \right]. \quad (5)$$

More details can be found in [2]. Nevertheless, to model the cable for transient simulations, the IEC standard does not propose a formula for the thermal capacitances of the soil, but instead it proposes two different solutions, one for long durations (normally durations greater than about 1 h) and another one for short durations (for durations of about 10 min to 1 h). In the standard, durations longer than  $RC/3$  are defined as long durations, where  $R$  is the total thermal resistance of the cable and  $C$  is the total thermal capacitance. On the other hand, short durations are considered to be shorter than  $RC/3$ . The formula for long durations are grouped in sub-section 4.2

of the IEC standard 60853 [3] and the formula for short durations are grouped in sub-section 4.3.

The IEC standards state that the transient temperature rise of the outer surface of the hottest cable,  $\theta_e(t)$ , for long durations can be computed as

$$\theta_e(t) = \frac{\rho_T W_I}{4\pi} \cdot \left[ -E_i \left( \frac{-D^2}{16\delta t} \right) + E_i \left( \frac{-L^2}{\delta t} \right) \right] + \sum_{k=1}^{N_c-1} \left[ -E_i \left( \frac{-d_{pk}^2}{4\delta t} \right) + E_i \left( \frac{-d_{pk}^{\prime 2}}{4\delta t} \right) \right] \quad (6)$$

where  $W_I$  is the total power loss per unit length of each cable in the group,  $d_{pk}$  is the distance from cable  $k$  to the center of the hottest cable  $p$ ,  $d_{pk}^{\prime}$  is the distance from the image of the center of cable  $k$  to the center of the hottest cable  $p$ , and  $N_c$  is the number of cables in the group. It is important to note that this formula holds for single-core cables and also for three-core cables. Nonetheless, for the case of single-core cables, the summation term in (6) is not needed.

For short durations, the transient temperature rise of the outer surface of the hottest cable,  $\theta_e(t)$ , can be computed as

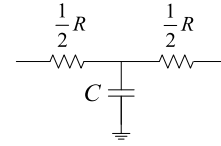
$$\theta_e(t) = \frac{\rho_T W_I}{4\pi} \cdot \left[ -E_i \left( \frac{-D^2}{16\delta t} \right) \right] + \sum_{k=1}^{N_c-1} \left[ -E_i \left( \frac{-d_{pk}^2}{4\delta t} \right) \right] \quad (7)$$

where the influence of the images has been suppressed because they are negligible for short durations. The summation term in (6) is likely to be also negligible for short period of time unless the cables are touching or are very close [3]. This nonlinear formulation involves the solution of an exponential integral function at every time step.

The standard solution is very precise, but it is not convenient and consistent with the modeling that the IEC standards themselves propose for each individual layer of the cable [3], where every cable layer is modeled with its equivalent resistance and its equivalent thermal capacitance.

### III. MULTILAYER SOIL MODEL

An alternative to the exponential equation of the IEC standards is proposed in [15] and [16] to model the soil. The underlying idea is to subdivide the soil surrounding the cables



**FIGURE 1.** Equivalent electrothermal circuit ( $T$  equivalent circuit) used for each of the individual layers modeled in this paper.

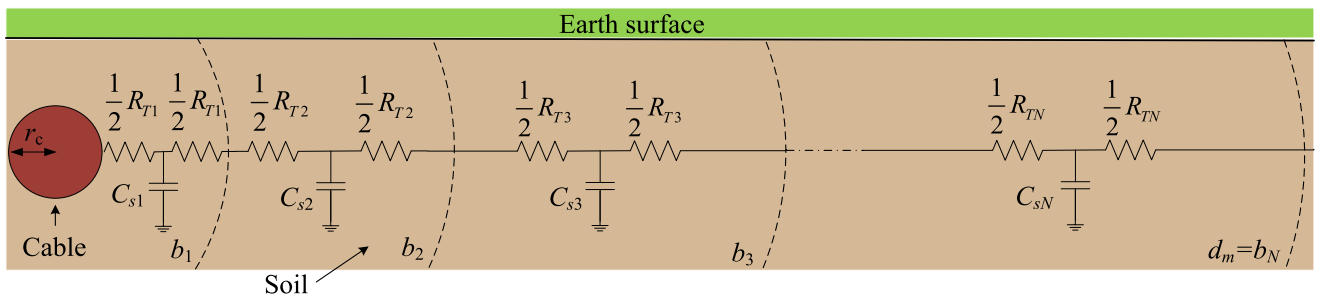
into several concentric layers. Each soil layer is represented with its  $RC$  thermal  $T$  equivalent circuit (Fig. 1) to be compatible with the IEC standards [2], [3]. The soil model parameters are computed from the thermal resistivity, the heat capacity of the soil, and the dimensions of each layer using the following formula, which are applicable to hollow cylindrical shapes [2]:

$$R = \frac{\rho}{2\pi} \log \left( 1 + \frac{th}{r_{\text{int}}} \right) \quad (8)$$

$$C = \pi (r_{\text{ext}}^2 - r_{\text{int}}^2) \cdot C_p \quad (9)$$

where  $\rho$  is the thermal resistivity of the layer under study,  $th$  is the thickness,  $r_{\text{ext}}$  and  $r_{\text{int}}$  stand for the layer external radius and internal radius, respectively, and  $C_p$  is the heat capacity of the material. This formulation is consistent with the one used in the IEC standards [3], [4], and in [15]. However, note that in this paper,  $R$  is used as the symbol for thermal resistances where the IEC standards use  $T$ . Each layer is represented with a  $T$  equivalent circuit, as shown in Fig. 1.

The physical discretization of the soil can be observed in Fig. 2, where the  $T$  equivalent circuits representing each layer are illustrated. The values of  $R$  and  $C$  in the circuit of Fig. 2 are defined in (8) and (9). Once the equivalent resistances and capacitances are calculated for each layer, the  $RC$  ladder of the soil can be constructed by concatenating all the  $T$  equivalent subcircuits into a complete ladder model, as shown in Fig. 3. In this figure, the ladder model of the soil is shown together with the ladder model of the cable, creating the complete electrothermal circuit promoted in [15] and [16] and also used in this paper. In Fig. 3, an example using a cable with four layers is represented, i.e., conductor, insulation, sheath, and jacket. Therefore,  $R_1$  and  $R_2$  represent the thermal resistance



**FIGURE 2.** Physical discretization of the soil to capture both slow and fast transients. The cable is the dark red heat source and the dashed lines represent the boundaries of the soil layers of the model. The inner layers of the cable are also represented by  $RC$  circuits (not shown).

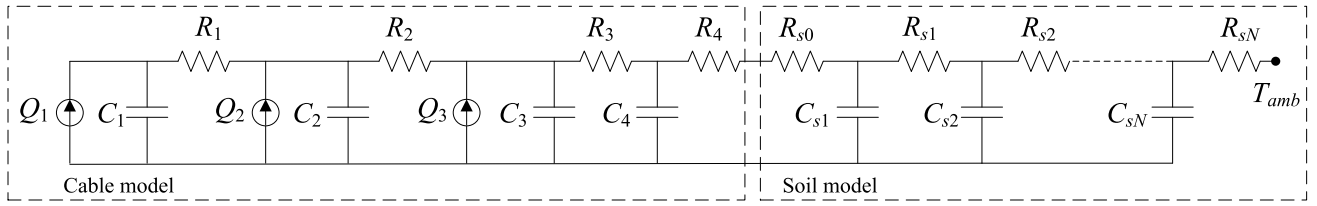


FIGURE 3. Complete ladder-type equivalent circuit for the cable and its surrounding soil.

of the insulation,  $R_3$  and  $R_4$  represent the resistance of the jacket, while  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  correspond to the thermal capacitances of the conductor, insulation, sheath, and jacket, respectively. Heat sources  $Q_1$ ,  $Q_2$ , and  $Q_3$  represent the losses in the conductor, insulation, and sheath, respectively.

The computation of the thermal resistances is given in (8). The thermal capacitances can be computed using (9). The method to compute losses is described in [4], which includes temperature, frequency, and voltage dependencies. Parameters  $R_{si}$  and  $C_{si}$  represent the thermal resistances and capacitances of the soil subdivisions and  $T_{amb}$  is the datum ambient temperature. The relationships between the resistances shown in Figs. 2 and 3 are the following:

$$R_{s0} = \frac{1}{2}R_{T1} \quad (10a)$$

$$R_{si} = \frac{1}{2}(R_{Ti} + R_{T_{i+1}}) \quad \text{for } i = 1, 2, \dots, N-1 \quad (10b)$$

$$R_{sN} = \frac{1}{2}R_{TN}. \quad (10c)$$

To obtain an accurate dynamic representation of the thermal transients, a multilayer model of the soil is needed. This is so to physically represent the diffusion of heat in the soil. When current circulates in a conductor, the temperature of the soil layers close to (or touching) the cable increases quickly. Therefore, an  $RC$  circuit with a small time constant should be used to represent the fast transients. The heat takes (much) longer to reach the soil layers that are far from the cable. Then, an  $RC$  circuit with a large time constant is adequate to represent the long time necessary for the heat to reach far soil layers. Since fast transients have smaller time constants, narrower soil subdivisions are needed for the soil close to the cable. In contrast, the soil layers that are further apart from the cable play a major role in the slow transients with larger time constants. Therefore, they can be discretized in thicker layers. This attribute is not considered in the models presented in [15] and [16]. Nonetheless, this feature is the key to obtaining a model with only a few sections with the same accuracy as a large order model, but with reduced computation time. Thus, the reduced-order model is suitable for real-time calculations. Fig. 2 shows how the size of the subdivision increases as one moves away from the cable.

#### A. DISTRIBUTION OF LAYERS

In this section, a discretization approach is proposed for the subdivision of soil into layers. As discussed above, numerous

and thin layers are preferable near the cable and thicker layers are sufficient in the far region of the soil. Since the analytical solution of the heat diffusion problem is an exponential integral (5), an exponential discretization of the soil is proposed as follows:

$$b_i = r_c + (d_m - r_c) \cdot \frac{e^{\gamma \cdot i} - 1}{e^{\gamma \cdot N} - 1} \quad (11)$$

where  $b_i$  are the radial positions of the layer borders,  $r_c$  is the radius of the cable,  $N$  is the number of layers of the discretization, and  $d_m$  is the depth of the model. Finally,  $i = 0, 1, \dots, N$  represents the index of the layer. Therefore, there are  $N + 1$  boundaries that correspond to  $N$  layers in the soil model.  $\gamma$  is the argument of the exponential distribution and it has to be a positive number. Small values of  $\gamma$  represent quasi-linear distributions, hence they imply the same number of layers in the proximity of the cable than in the far soil. This case represents the linear model presented in [15] and [16]. The fast transients may not be captured correctly unless a very large number of layers is selected (100 sections are used in [15]). On the other hand, large values of  $\gamma$  imply that a single (and thick) layer represents the effects of the far soil. This leads to a situation where the slow transients are not properly captured. Note that the exponential discretization proposed in (11) assumes that the layers of the model must have progressively increasing thicknesses (as shown in Fig. 2). This assumption is physically sound because the heat flux and temperature gradient are higher close to the cable, where the model is discretized finer [2]. The optimum value of  $\gamma$  will be computed in Section III-D.

Other two important parameters of the soil model are the depth of the model  $d_m$  (i.e., the position of the last layer that is considered) and the number of layers  $N$ . The impact of the aforementioned parameters is investigated in the following sections.

#### B. DEPTH OF THE MODEL

The depth of the model,  $d_m$ , is a parameter that depends on the burial depth of the cable. As it has been defined in (11), the value of  $d_m$  will have an effect in the final thermal resistance representing the external environment of the cable. This resistance is defined as  $T_4$  in the IEC standard [3]. For single isolated buried cables, the value of  $T_4$  is calculated using (3) and (4). These formulae were first introduced in [17], but the final formulation was proposed in [18]

and was further developed in [19]. The expressions for  $T_4$  are obtained using the method of images and consider the soil–air interface as an isotherm. These expressions give very accurate results for steady-state analysis of buried cables. Therefore, to assure that the model of this paper computes the steady state correctly, the model should have the same total external resistance as computed by (3). The same assumption is made in the models presented in [15]. Therefore, (3) should be equated to the sum of the resistances of all layers given in (8), yielding

$$T_4 = \sum_{i=0}^{N-1} \frac{\rho_T}{2\pi} \log \left( 1 + \frac{b_{i+1} - b_i}{b_i} \right). \quad (12)$$

For the case when  $N = 1$ , a simple algebraic manipulation of (12) leads to

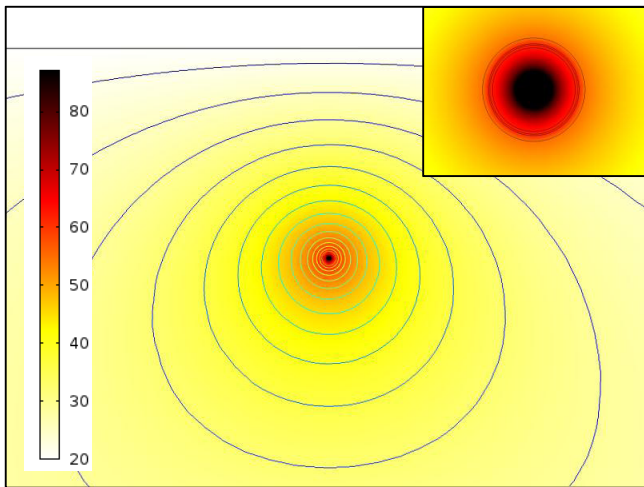
$$d_m = L + \sqrt{L^2 - r_c^2}. \quad (13)$$

Note also that due to the properties of logarithms (12) can be rewritten as

$$T_4 = \frac{\rho_T}{2\pi} \log \left( \prod_{i=0}^{N-1} \frac{b_{i+1}}{b_i} \right) = \frac{\rho_T}{2\pi} \log \left( \frac{b_N}{b_0} \right) \quad (14)$$

and the value of (14) is equivalent to a model with one single layer. Therefore, the result obtained in (13) is valid for any  $N$ .

To support the validity of the formula and to corroborate the results from the standards, the results of the models obtained from (13) are compared against finite-element method (FEM) simulations. To verify the proper depth of the model, several depths are investigated and compared to the results obtained with FEM (Fig. 4). A cable buried at 3 m carrying a current of 1000 A presents a steady-state temperature computed by FEM of 84.72 °C. This same result should be obtained with the ladder circuit for a model depth of 6 m (this is because



**FIGURE 4.** Finite-element simulation of one cable buried at 3 m with a loading of 1000 A. The top right figure shows a zoom into the temperature distribution inside the six-layer cable used for this paper.

(13) can be approximated to  $2L$  in this case). Effectively, in this situation, the result given by this soil model is a conductor temperature of 84.81 °C.

As a reference, Table 1 shows the final results (steady state) of several simulations using the model in Fig. 3 performed for different model depths, specifically 1, 3, 6, and 12 m. These results can be computed directly from the standards because in steady state, the temperature of the conductor is obtained with the value of  $T_4$  described in the standards by (3). As it can be observed, with larger or smaller depths than 6 m, large inaccuracies are introduced in steady state. This is so because the external thermal resistance of the model is underestimated (in the case of  $d_m = 1$  m) or overestimated (in the case of  $d_m = 12$  m) because they do not comply with (3) and (12) and the research reported in [18] and documented in [3].

**TABLE 1.** Steady-state conductor temperature as a function of the depth of the soil model (13) with a current of 1000 A.

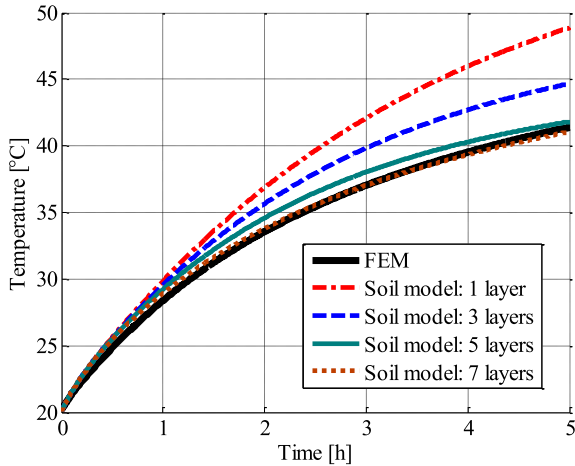
Depth of the model ( $d_m$ ) [m]	Final steady state temperature [°C]
1	63.52
3	76.25
6	84.81
12	93.81

### C. NUMBER OF LAYERS

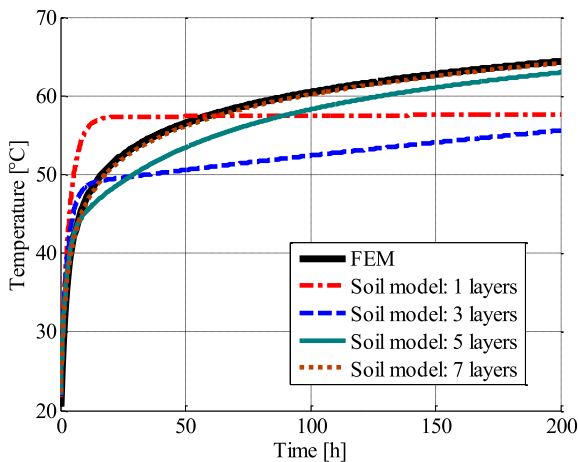
The number of layers of the model needs to be selected correctly to obtain an accurate representation of the soil. The number of layers does not affect the steady-state results because the total external thermal resistance ( $T_4$  in IEC terms) does not change. Nonetheless, the number of layers significantly affects the transient performance.

In the physical world, the temperature diffuses slowly and smoothly to a steady state because the thermal inertia is distributed homogeneously throughout the whole media. Therefore, a minimum number of layers are needed in the model to distribute correctly the thermal inertia of the soil and obtain accurate dynamic results. In the following section, the optimal model will be presented proving that a minimum amount of layers for different burial depths are required to assure a certain accuracy. In addition, it will be shown that a soil model with five layers can deliver accurate results for any practical situation of underground power cables.

Figs. 5 and 6 show the step response of soil models with one, three, five, and seven layers. The layer boundaries are computed with  $\gamma = 0.5$ . A step current of 1000 A is impressed to the conductor and its temperature as function of time is plotted. The results are compared against transient FEM simulations for durations of 5 and 200 h, respectively. As expected, because of the proper selection of the soil depth, all models eventually reach the correct steady-state temperature. It is clear, however, that lower order models (1 and 3) produce a physically incorrect representation of the soil dynamics. The initial conductor temperature rise is faster than in reality while the long-term response is slower. The reason for the erroneous behavior is the wrong distribution of the soil thermal capacitance.



**FIGURE 5.** Simulation of one step of 5 h at 1000 A through a cable situated at 3 m under the ground level. Results of the conductor temperature of the FEM simulations are shown in a dark, thick solid line. The model is tested with four different layer configurations. Best results are obtained for the model with seven layers. These results are obtained for a fixed  $\gamma = 0.5$ .



**FIGURE 6.** Simulation of one step of 200 h at 1000 A through a cable situated at 3 m under the ground level. Results of the core temperature from the FEM simulations are shown in a dark, thick solid line. The model is tested with four different layer configurations. Best results are obtained for the model with seven layers. These results are obtained for a fixed  $\gamma = 0.5$ .

The use of thick layers leads to excessively large model capacitances that create nonphysical thermal barriers (Section VII). Obviously, the model with seven layers is the most accurate. The average difference with respect to FEM is only 0.28 °C and the maximum difference is 0.51 °C, which occurs at about 30 min into the simulation. It is important to note that models with five or seven sections with linear distribution, such as the ones presented in [15], produce inaccurate results. As it is documented in [15], models with 100 sections are recommended if a linear distribution of layers is used.

#### D. OPTIMUM-ORDER MODEL

As it has been described in the previous sections, the parameters of the model that affect its performance could be opti-

mized. These parameters include the layer distribution of the model ( $\gamma$ ) and the number of layers ( $N$ ). The depth of the model ( $d_m$ ) has a direct impact on the steady-state results, therefore to have an accurate model in both steady state and transient, the depth is fixed to the value determined by (13).

The objective, or cost function ( $f$ ), is to minimize the average conductor temperature difference between the results obtained with the model and FEM simulations

$$f = \frac{1}{T} \sum_{j=1}^{T_n} |T_{\text{condFEM}}(t_j) - T_{\text{condModel}}(t_j)| \quad (15)$$

where  $t_j$  represent the different time samples of the simulations and  $T_n$  is the total number of samples.

The optimization of  $\gamma$  and  $N$  is done by means of a 2-D analysis where both parameters are swept at the same time and their results are compared with FEM simulations.  $N$  is varied from one to 13 layers and  $\gamma$  is varied from 0.001 to four. The limits of  $\gamma$  are chosen to assure that the optimization algorithm, for a particular number of layers, finds the minimum of (15) from a linear distribution to a very pronounced exponential.

The optimum model order for slow and fast transients is different. This is so because for fast transients, higher order models are required. These models would also give accurate results for slower transients, but higher order models require more computational effort. Therefore, to decrease the simulation time for real-time applications, lower order ladder-type circuits with acceptable accuracy are recommended.

Numerous sets of simulations have been conducted to cover all practical applications in underground cables. Thus, independent optimizations have been performed for different kinds of transients, e.g., step durations of 1, 24, 168 (one week), and 720 h (one month) and for different burial depths, e.g., 0.5, 1.3, 2.2, 3, 6, 10, and 15 m. In addition, different thermal resistivities (from 0.5 to 4 kW/m) of the soil have been investigated to demonstrate the robustness of the model. In this paper, the optimum number of layers is considered to be the model with the minimum number of layers that delivers an average accuracy better than 0.5 °C.

The results of the independent optimizations show that short duration transients in the order of 1 h to one day, require models with a very small number of layers (only three layers with a relatively large value of  $\gamma$ ) since the most important factor affecting these dynamics is the model of the cable itself. Longer durations, in the order of a week, require models of five to six layers. For very long durations, the model can have a reduced number of layers in the range of three to five because in these cases, the slow dynamics have more weight than the fast transients. It is well understood that the steady state can be obtained accurately with fewer layers (even with only one). The results show that for different thermal resistivities of the soil, the obtained optimal models ( $N$  and  $\gamma$ ) are very similar because the thermal characteristics of the soil are already considered by (8) and (9).

Note that, for a fixed value of  $\gamma$ , a model with more layers always gives more accurate results. However, the drawback of a higher order models is the increased computational burden (see Section VII for more details). In addition, for deeply buried cables, the amount of soil in the model is larger; hence, thinner layers close to the cable have to be obtained either by increasing  $\gamma$  or increasing  $N$ . Simulation results show that for larger burial depths, a model with the same number of layers can deliver the necessary accuracy if  $\gamma$  is increased. Nevertheless, when the burial depth increases to large values, such as 10 or 15 m, normally the optimum model requires more layers.

Extensive independent studies show that for practical installations (burial depths lower than 15 m, simulation times between 1 h and one month, and thermal resistivities ranging from 0.5 to 4 Km/W), the necessary number of layers in the model is always between three and six while guaranteeing accuracies better than 0.5 °C.

Table 2 shows the optimal models for all time scenarios described above with independent optimization for each burial depth. One can conclude that for larger burial depths, more layers are required.

**TABLE 2. Optimum number of layers and optimum value of gamma of the soil model for different burial depths.**

Burial depth [m]	Optimum number of layers of the soil model	Exponential distribution ( $\gamma$ )
0.5	3	1.38
1.3	3	1.48
2.2	4	1.22
3	5	1.12
6	5	1.33
10	5	1.48
15	6	1.22

In addition to the independent optimizations, to find a general and versatile model that delivers accurate results in many different time scenarios and for different installations, a collective optimization has been conducted. This optimization finds the model that best performs in average for all the time scenarios, burial depths, and thermal resistivities described above. The results of this optimization show that the general model has five layers with  $\gamma = 1.32$ . The general model performs with an average accuracy of 0.44 °C across all scenarios.

**IV. NUMERICAL EXAMPLE**

In this section, a numerical example is given to calculate the optimal (general) model found in the previous section. The model calculated here has five layers and  $\gamma = 1.32$ . The radius of the cable for this example is  $r_c = 0.053$  m and the burial depth is  $L = 1$  m. Then, using (13) one can calculate  $d_m = 1.999$  m. The next step is the calculation of the boundary positions of the soil discretization using (11). The results are listed in the first row of Table 3. Then, the corresponding thickness and the interior and exterior radius of every layer can be calculated, which are listed in rows

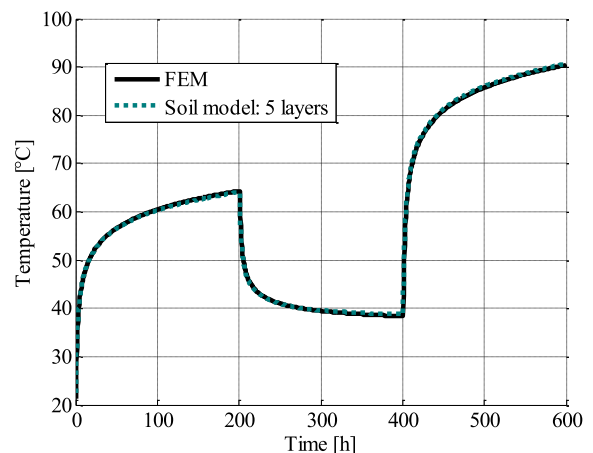
**TABLE 3. Values of thermal resistances and capacitances for the optimal soil model found in Section IV.**

$i$	0	1	2	3	4	5
$b_i$ [m]	0.053	0.06	0.087	0.189	0.571	1.999
$th_i$ [m]	-	0.007	0.027	0.102	0.382	1.428
$r_{int_i}$ [m]	-	0.053	0.06	0.087	0.189	0.571
$r_{ext_i}$ [m]	-	0.06	0.087	0.189	0.571	1.999
$R_{s_i}$ [(m <sup>2</sup> K)/W]	0.010	0.040	0.091	0.149	0.188	0.100
$C_{s_i}$ [J/K]	-	$4.00 \cdot 10^3$	$1.80 \cdot 10^4$	$1.28 \cdot 10^5$	$1.31 \cdot 10^6$	$1.66 \cdot 10^7$

two, three, and four of the same table. Next, using (8) and (9) together with a soil resistivity of  $\rho = 1$  K m/W and a  $C_p = 1.44 \cdot 10^6$  J/(K m<sup>3</sup>), the values of  $R_{Ti}$  and  $C_{si}$  corresponding to the model of Fig. 2 are calculated. Finally, with (10), the values for  $R_{si}$  are obtained. The parameter values of the model (Fig. 3) are given in rows five and six of Table 3.

**V. RESULTS**

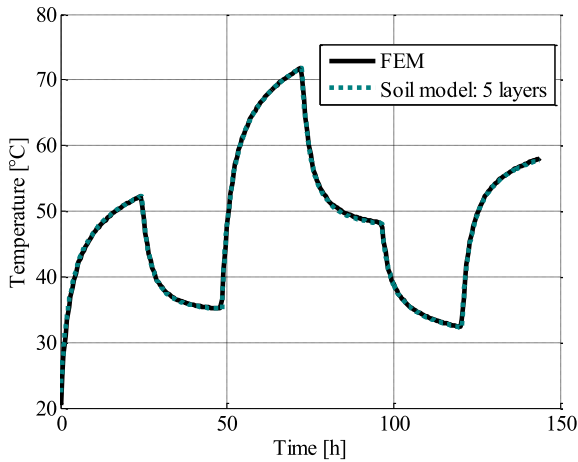
In this section, the performance of the model is assessed by comparing the results of several finite-element simulations for different loading conditions of a single cable placed at 1 m below the ground level. The cable has six layers (conductor, screen, insulation, screen, sheath, and jacket) with a cross-sectional area of 1000 mm<sup>2</sup>. In the first simulation, the current circulating through the cable has a load shape with three steps. The total simulation time is 200 h with consecutive amplitudes of 1000, 600, and 1200 A. The model used for these simulations is the model presented in Section V. The comparison of FEM and the simulations with the soil model is shown in Fig. 7. As it can be observed, the match is almost perfect and all differences are within 0.5 °C.



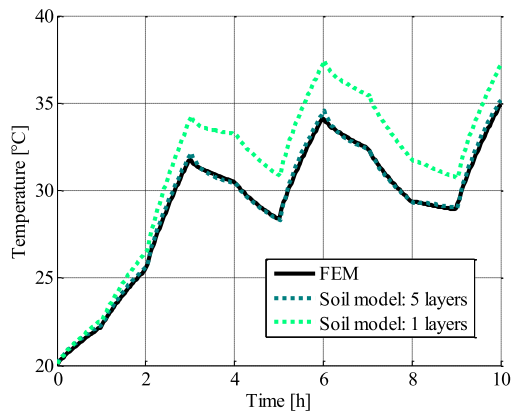
**FIGURE 7. Temperature of the conductor from a simulation of the three steps current changes to validate the model for long transient.**

A second case study is carried out with faster current variations that last for 24 h. The 24-h current steps applied to the cable are of 1000, 600, 1200, 800, 400, and 1000 A, in a successive order. Fig. 8 compares the results against FEM simulations. Once again, the mismatches between

FEM and the general model results are minimal.



**FIGURE 8.** Temperature of the conductor from a simulation of the 24-h steps current changes to validate the model in one day transients situations.



**FIGURE 9.** Temperature of the conductor from a simulation of the 1-h steps current changes to validate the model for the short transients. Note the poor performance of the model with one single layer in comparison with the general model with five layers.

To demonstrate that the performance of the developed model is also good for short durations (fast transients), one more case study is conducted. A current with 10 steps is applied. Each step is 1-h long and their amplitudes are: 500, 700, 1000, 600, 400, 1000, 600, 300, 500, and 1000 A. The results obtained from the model are compared versus FEM in Fig. 9. For the sake of comparison and illustration, the simulation results for a model with one single layer are also presented. The model with five layers perfectly matches the FEM simulations while the model with one layer shows large discrepancies. This is because the performance achieved using soil models not having enough layers is not acceptable. At the beginning of the simulation, both models behave similarly because in the first instants, the thermal evolution is mainly driven by the cable dynamics (thermal resistances and capacitances of the cable). However, after approximately 3 h, when the current in the cable starts decreasing from

1000 to 600 A, the thermal inertia of soil starts to play an important role. This clearly shows that the time constant of such thermal evolution is not correctly captured by the model and after only 6 h of simulation, differences of 5 °C are already shown. On the other hand, a soil model with five layers consists of thinner layers of soil (with lower thermal inertia) that surround the cable. This characteristic allows for the representation of faster thermal reaction of the near soil, which reproduces very closely what happens in reality.

## VI. DISCUSSION

As shown in the previous sections, if the soil is not modeled with a sufficient number of layers, the rate at which the heat flows from the cable to the far soil is not correctly represented. This is because the thermal inertia of the soil needs to be discretized correctly to avoid thermal barriers (caused by very large thermal capacitances at the wrong place).

In an extreme situation, when using a single layer to model the soil, the one-layer  $T$  equivalent circuit lumps half of the total soil resistance to the left of the thermal capacitance and the other half to the right of the thermal capacitance (Fig. 1). In practice, this is a two-node circuit; one node with no thermal inertia ( $C = 0$ ), and a second node with a very high thermal inertia (thermal capacitance equal to the capacitance of the entire soil  $C$ ). This leads to a representation of the soil that is physically incorrect. For a step of current through the cable, the first half of the soil is thermally charged very fast. This is because normally the thermal capacitance of the cable itself is very small when compared with the thermal capacitance of the soil. Therefore, the nodes that represent the cable and the first half of the soil increase their temperatures very fast. The other half of the electrothermal resistance of the soil receives heat very slowly. This is because the heat is absorbed by the large thermal capacitance and can only pass to the second half of the circuit when the capacitance has some charge. In this scenario, the one-layer thermal model shows that the cable conductor temperature increases abruptly at the beginning (but does not reach the steady-state temperature) and then drifts very slowly toward the final steady-state temperature. Note that this process occurs in two distinctive steps not emulating what happens in the physical world. This behavior can be clearly observed from Figs. 5, 6, and 9.

Noticeable, but to a lesser extent, the phenomenon just described happens as well with the three-layer model. One can observe from Figs. 5 and 6 that because the model has an insufficient number of layers, the cable temperature increases fast at the beginning followed by a relatively slow drift to steady state. This is caused by the lack of thin layers of soil close to the cable. In addition, the results show that thinner layers are a necessity near the cable surface but thicker layers are sufficient for the soil regions far from the cable, thus the correct choice of  $\gamma$  is critical. Moreover, the proper selection of  $\gamma$  provides high accuracies with lower order models. This results in a model with five layers compared with the model obtained from the linear distribution of soil sections with 100 layers presented in [15].



The lower order model is easier and simpler for engineers to implement in graphical software such as PSPICE or EMTP. It also reduces considerably the amount of time that is needed by the engine to solve the thermal problem. To assess the impact of the model size in computation speed, comparative speed tests have been conducted. Table 4 shows the computation time to solve the thermal problem with a soil model of five layers, compared with soil models of 50 and 100 layers. The results are shown for two different scenarios: 1) using a solver engine that uses efficient matrix multiplication libraries such as MATLAB or Lapack; and 2) simpler software such as standard C code. One can see that in the first scenario, the computation speed for a five-layer model is 3.5 times faster than for a 100-layer model, and in the second scenario the speed is 20 times faster. Similar results are also reported in [15], where an almost 30 times higher computation time is reported for models with 100 sections. Computation time is a significant factor in the context of real-time prediction systems, thus a model with lower computational burden leads to faster real-time algorithms.

**TABLE 4. Computation speed in milliseconds per time step on a computer with i5 CPU at 2.67 MHz with 4 GB of RAM.**

Soil model size	Computation time [ms] matrix enabled engine	Computation time [ms] not matrix enabled engine
5	0.021	0.051
50	0.031	0.337
100	0.073	0.992

## VII. CONCLUSION

This paper has introduced an optimal soil model based on a physical discretization of the soil into a few layers. The model consists of a series of lumped  $T$ -shaped  $RC$  circuits representing the thermal resistances and capacitances of the soil layers. The new optimal model provides up to 20 times faster response than the currently available approaches. The faster behavior is due to the lower order model obtained from the nonuniform spatial discretization of the soil. The model order has been optimized through a comprehensive parametric analysis of cable installation depth, thermal resistivity, and simulation time. It has been determined that a model of order five can represent all typical transients on common installations.

A numerical example illustrates how easy it is to obtain the model. In addition, all the analytical tools available for the analysis of state-space equations apply to this model. Therefore, the model is ideally suited for applications in the context of real-time operations of underground power cables. Such an accurate and fast model will represent a relevant enhancement to the actual real-time monitoring systems for power distribution and transmission cables, making them more efficient and more robust.

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