Enigmas, etc.

Solution to Last Month's Quiz

■ **Takashi Ohira**

It takes tough work, in general, to analytically characterize the power conversion efficiency of nonlinear circuits. However, for this puzzle, we can exploit the two equations

$$\begin{bmatrix} V_O & I_O \\ V_P & I_P \\ V_Q & I_Q \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 50 & 1 \end{bmatrix} I_1 \tag{1}$$

$$I_1 = \pi I_2 \tag{2}$$

both already derived in this "Enigma" series. Remember that subscript O stands for dc, P for in phase, and Q for the quadrature Fourier components of the input waveform. Also remember that I_1 and I_2 denote RF source current and dc load current, respectively (see Figure 1).

First, from (1), the RF input power is calculated as

$$\begin{aligned} P_{\text{in}} &= \frac{1}{2}\begin{bmatrix} V_P & V_Q \end{bmatrix}\begin{bmatrix} I_P \\ I_Q \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} 0 & 50 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} I_1^2 \\ &= \frac{25}{4} I_1^2. \end{aligned} \tag{3}$$

Then, from (2), the dc output power is calculated as

$$\begin{aligned} P_{\text{out}} &= 50 I_2^2 \\ &= \frac{50}{\pi^2} I_1^2. \end{aligned} \tag{4}$$

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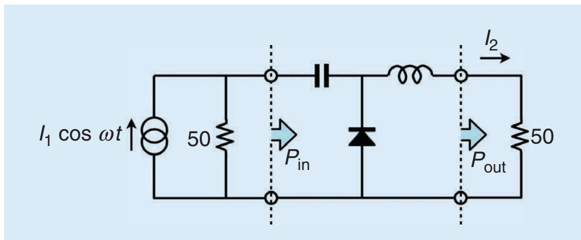

Figure 1. *The circuit diagram for analysis.*

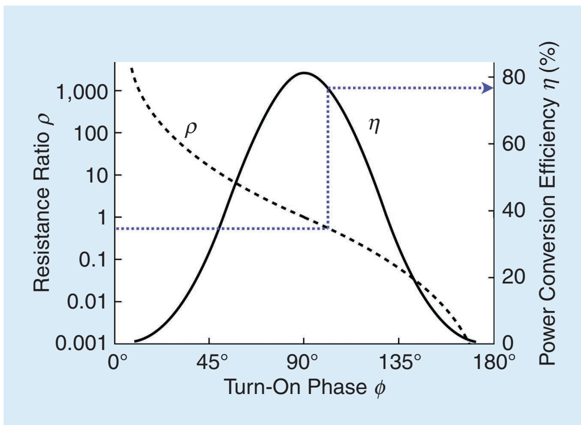

Figure 2. *The arrowed crank guides us to the power conversion efficiency.*

Finally, from the quotient of (4) to (3), we find the power conversion efficiency as

$$\begin{aligned} \eta &= \frac{P_{\text{out}}}{P_{\text{in}}} \\ &= \frac{8}{\pi^2} \\ &\approx 0.81. \end{aligned} \tag{5}$$

Therefore, the correct answer to last month's quiz is "c."

In this puzzle, the source and load resistors are both set to 50 Ω for simplicity. However, they are not always so in practical systems. To find the general solution valid for arbitrary resistances, we need more work [1]. The calculus process is too complicated to describe in this brief column. However, the final formula is amazingly elegant:

$$\eta = \frac{8}{\pi^2}\sin^3\phi. \tag{6}$$

Note that ϕ denotes the turn-on phase stemming from

$$\tan\phi - \phi = \frac{\pi}{\rho - 1} \tag{7}$$

where ρ stands for the source-to-load resistance ratio R_1/R_2.

Although (6) lets us explicitly find η from ϕ, the transcendent equation (7) cannot be solved algebraically for ϕ. We therefore take a graphical approach instead. In Figure 2, (6) and (7) are projected onto a plane with common abscissa. This chart enables us to quickly find η from a given ρ by way of ϕ. For example, we start from $\rho = 0.58$ on the left-side ordinate. Going along the cranked line, the dashed curve finds $\phi = 100^\circ$, and, finally, the solid curve finds $\eta = 77\%$. This chart also confirms (5), especially signifying that η reaches its peak, i.e., 81% when $R_1 = R_2$. The remaining 19% of the total input energy dissipates into second and higher order harmonics due to the diode's nonlinearity.

Since the efficiency is limited to 81% for this basic rectifier, one may expect even higher efficiency in future advanced RF system applications. To address this expectation, we have a viable solution exploiting a harmonic reaction technique called the *class E diode rectifier* [2], [3].

References

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[2] W. A. Nitz et al., "A new family of resonant rectifier circuits for high frequency DC-DC converter applications," in *Proc. 3rd Annu. IEEE Appl. Power Electron. Conf. Expo.*, New Orleans, LA, USA, Feb. 1988, pp.12–22, doi: 10.1109/APEC.1988.10546.

[3] T. Ohira, "Linear algebra elucidates class-E diode rectifiers," *IEEE Microw. Mag.*, vol. 23, no. 12, Dec. 2022.