Speaker's Corner

scikit-rf: An Open Source Python Package for Microwave Network Creation, Analysis, and Calibration

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With the rapid proliferation of telecommunication and RF applications, so, too, has demand grown for tools to design and characterize these devices. scikit-rf is a Python package designed to make RF/microwave engineering both robust and approachable. The package provides a modern, object-oriented library for RF network analysis, circuit building, calibration, and simulation.

Besides offering standard microwave network operations, such as reading/writing Touchstone files (.sNp files), connecting or de-embedding *N*-port networks, frequency/port slicing, concatenation, or interpolations, it

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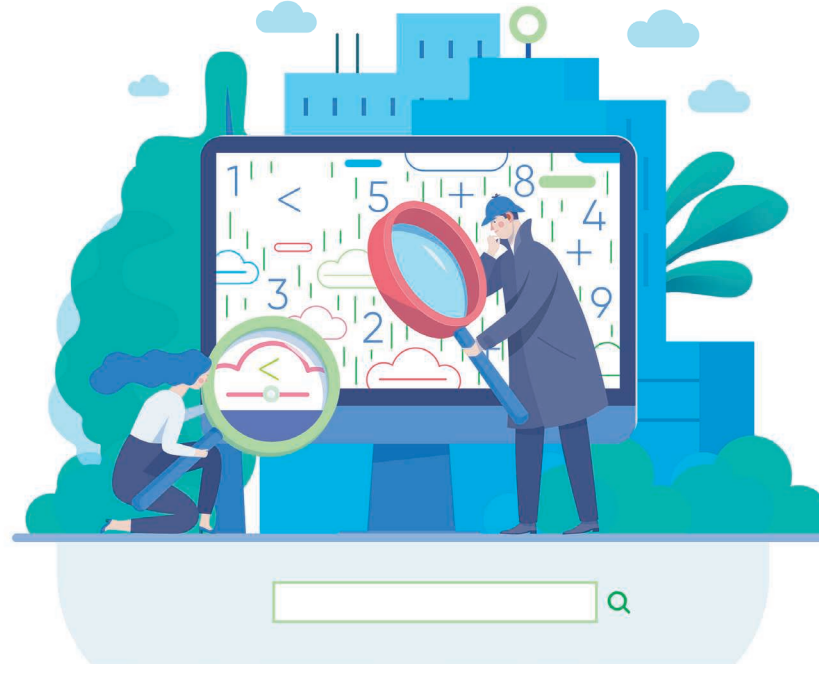

is also capable of advanced operations, such as vector network analyzer (VNA) calibrations, time gating, vector fitting, interpolating between an individual set of networks, deriving network statistical properties, and support of virtual instruments for direct communication to VNAs. The package also allows straightforward plotting of rectangular plots (decibels, magnitude, phase, group delay, and so on), Smith charts or automated uncertainty bounds.

The project was created in 2009 and has been continually developed since. The package is distributed under the Berkeley Software Distribution license and is actively developed by more than 50 volunteers on GitHub (see https://github.com/scikit-rf/scikit-rf/graphs/contributors). The project has users in several universities and research institutes around the world as well as corporate users from large companies, including Keysight, Rohde & Schwarz, National Instruments, Nvidia, and 3M. As of 2021, the package has been downloaded more than 220,000 times since its creation and has been used in more than 30 publications.

As the package is developed in Python, it is naturally compatible with the rich set of modern scientific Python libraries. Results can be shared and directly reproduced by other researchers using tools such as Binder or Google Colab. Being a Python library, it is also compatible with the robust testing and code coverage frameworks developed for that language, with reproducible modeling approaches using online virtualization services. Because it is open source, users of scikit-rf are able to see exactly what the source code is doing and, if a feature does not exist, freely contribute it to extend the library for their work and that of others. In addition, of course, it is free.

Basic Usage of scikit-rf

Detailed installation instructions can be found in the scikit-rf documentation (https://scikit-rf.readthedocs.io/en/latest/). For those familiar with Python, it is no different from installing any other package and is distributed through both pip and Conda. After installation, the package can be imported, which is assumed for all of the following examples:

```
1 import skrf
```

The following sections outline some fundamental features of scikit-rf and include code snippets to provide a starting point for new users.

Reading and Plotting Networks

The central object in the scikit-rf package is an N-port microwave `Network` object. A `Network` can be created in various ways, for example, by reading a Touchstone file named `measured_data.s2p`:

```
1 example_network = skrf.
  Network('measured_data.s2p')
```

For the reader to reproduce the given examples, a test case is provided in the package and can be imported via the following:

```
1 >>> from skrf.data import
  ring_slot
2 >>> ring_slot
3 2-Port Network: 'ring slot',
  75.0—110.0 GHz, 201 pts,
  z0=[50.+0.j 50.+0.j]
```

The basic attributes of a microwave network are provided by the following properties of the `Network` class:

- `s`: the scattering parameter matrix
- `z0`: the port impedance matrix
- `frequency`: the frequency object.

S-parameters are represented by a $(n_{bf} \times N \times N)$ NumPy array [1] (https://numpy.org/), where n_{bf} is the number of frequency points, and N is the number of ports of the network. For example, to inspect the 2×2 scattering parameters for the first frequency element, the following is used:

```
1 >>> ring_slot.s[0] # or
  ring_slot.s[0,:]
2 array([[-0.50372318 +
  0.4578448j, 0.6134571 +
  0.36678139j],
3 [0.6134571 + 0.36678139j,
  -0.19958433 + 0.6483347j]])
```

The `Network` object has numerous other properties and methods, which can be found in the documentation. If you are using IPython [2], the Jupyter Notebook [3], or any advanced Python editor, then these properties and methods can be "tabbed" out on the command line:

```
1 >>> ring_slot.s<TAB>
2 ring_slot.s ring_slot.s_
  arcl
3 ring_slot.s11 ring_slot.s_
  arcl_unwrap …
```

scikit-rf uses the *matplotlib* package [4] to generate various different types of plots for the attributes of a `Network`. For example, plotting the ring slot's scattering parameters on the Smith chart can be done in one line, with the output shown in Figure 1(a):

```
1 >>> ring_slot.plot_s_smith
  (lw = 2)
```

The frequency ranges of `Networks` can also be selected by using human-readable strings. For instance, the following line will plot the logarithmic magnitude of $S_{1,1}$ in decibels for the frequency range of 80 – 90 GHz, expressed in "natural" language. The output is shown in Figure 1(b):

```
1 >>> ring_slot['80-90ghz'].
  plot_s_db(m = 0, n = 0)
```

Other parameters are accessible, such as impedance (Z), admittance (Y), transfer (T), or chain (ABCD) parameters using `ring_slot.z`, `ring_slot.y`, `ring_slot.t`, or `ring_slot.a`, respectively. Several other features related to network processing can be found in the scikit-rf documentation. The next sections illustrate a few of them.

Network Operations

Elementwise mathematical operations on the scattering parameter matrices are accessible through overloaded operators. Hence, `Networks` can be added, subtracted, and multiplied along the frequency and port axes. For example, to plot the complex difference between a `short` and a `delayshort`, the following is used:

```
1 >>> from skrf.data import
  wr2p2_short as short
2 >>> from skrf.data import
  wr2p2_delayshort as
  delayshort
```

```
>>> difference = (short - delayshort)
>>> difference.plot_s_mag(label='Mag of difference')
```

Another common application is calculating the phase difference between two networks using the division operator (/):

```
>>> (delayshort/short).plot_s_deg(label='Detrended Phase')
```

Cascading and De-Embedding

scikit-rf supports the connection of arbitrary ports of *N*-port networks. Cascading and de-embedding two-port networks with scikit-rf can also be done though the Python power operator (**). For example, as shown in Figure 2, the cascaded connection of two individual two-port networks, `line` and `short`, can simply be calculated with

```
>>> short = skrf.data.wr2p2_short
>>> line = skrf.data.wr2p2_line
>>> delayshort = line ** short
```

De-embedding can be accomplished by cascading the inverse of a network, as shown in Figure 3. This is implemented in scikit-rf through the `Network.inv` property. To de-embed the short from `delayshort`, the following is used:

```
>>> short_2 = line.inv ** delayshort
>>> short_2 == short
True
```

As shown in the previous example, Python comparison operators, such as ==, also work with Network objects.

Statistical Analysis

The `NetworkSet` class is useful for analyzing multiple `.sNp` files initialized as `Network` objects or read from a directory. `NetworkSets` implement the same plotting routines as regular `Networks`, which is convenient to plot a group of frequency responses. In addition, the `NetworkSet` class enables statistical analyses, such as calculating and plotting the uncertainty with the method `plot_uncertainty_bounds_s_db`. This is demonstrated in the following example, with the outputs shown in Figure 4:

```
from skrf.data import ro_1, ro_2, ro_3
ntwk_list = skrf.NetworkSet([ro_1, ro_2, ro_3])
ntwk_list.plot_s_db()
ntwk_list.plot_uncertainty_bounds_s_db(label='ro mean with uncertainty, S11')
```

The standard deviations of data in a `NetworkSet` can be applied to a variety of attributes. The method `uncertainty_ntwk_triplet` can be utilized for gain output, similar to the previous example. For that case, the arguments `s_mag` and `3` are used to specify that the uncertainty bounds should be for the magnitude of the scattering parameters within the range of

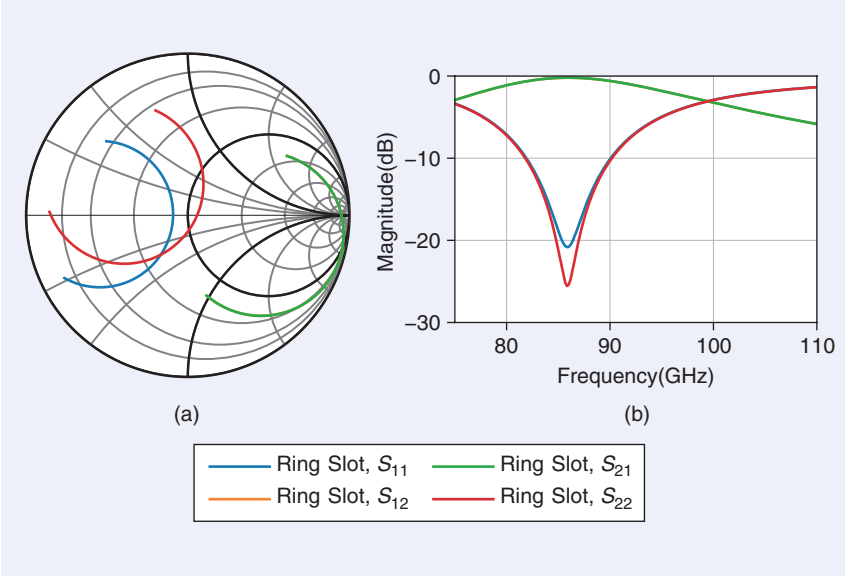

Figure 1. *The scattering parameters of (a) the* `ring_slot` *plotted on a Smith chart and (b) in logarithmic magnitude.*

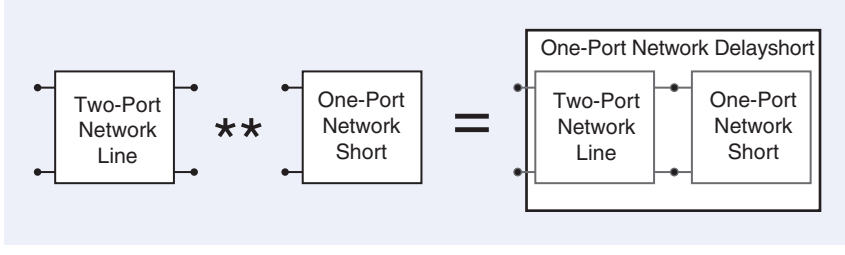

Figure 2. *The cascading of two two-port* `Networks` *in scikit-rf works with the* ** *operator.*

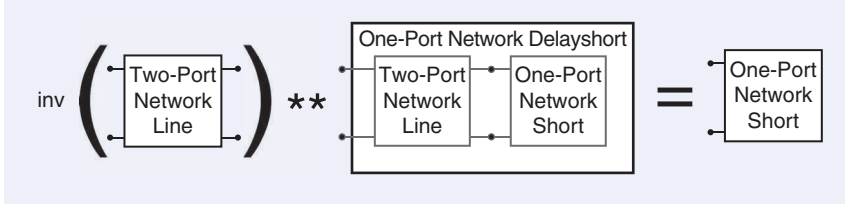

Figure 3. *De-embedding using the inverse and cascade operators in scikit-rf. inv: inverse.*

three standard deviations. The Network objects generated by this function can be saved in various formats to be reused later:

```
ntwk_mean, ntwk_lb, ntwk_ub = ntwk_list.uncertainty_ntwk_triplet('s_mag', 3)
```

Interpolation and Concatenation

A common need is to change the number of frequency points of a Network, for instance, but the previous operators and cascading functions require the networks to have matching frequencies:

```
>>> from skrf.data import wr2p2_line1 as line1
# next line fails due to different frequencies
>>> line1 + line
# next line works
>>> line.interpolate_from_f(line1.frequency) + line1
```

A related application is the need to combine networks that cover different frequency ranges. Two Networks can be concatenated (stitched) using stitch(), which concatenates the Networks along their frequency axis. For example, to combine a WR-2.2 Network with a WR-1.5 Network, the following is used:

```
>>> from skrf.data import wr2p2_line, wr1p5_line
>>> big_line = skrf.stitch(wr2p2_line, wr1p5_line)
```

Port Impedance Renormalization

Scattering parameters are defined for a given reference impedance Z_0. It can be necessary to renormalize these parameters to a different reference impedance. This example demonstrates how to use scikit-rf to renormalize the scattering parameters of a Network to different port impedances. Although trivial, this example creates a matched load in 50 Ω and then renormalizes to a 25-Ω environment, producing a reflection coefficient of 1/3. In the case of complex reference impedances, scikit-rf supports both power-wave and pseudowave scattering parameter definitions [5]:

```
>>> match_at_50 = skrf.wr10.match()
>>> match_at_50
1-Port Network: '', 75.0–110.0 GHz, 1001 pts, z0=[50.+0.j]
>>> match_at_50.s[0] # S-parameter for the first frequency point
array([[0.+0.j]])
>>> match_at_50.renormalize(25)
>>> match_at_50
1-Port Network: '', 75.0–110.0 GHz, 1001 pts, z0=[25.+0.j]
>>> match_at_50.s[0]
array([[0.33333333 + 0.j]])
```

Calibration

It is possible with scikit-rf to calibrate a device under test (DUT), assuming that an acceptable set of standards has been measured and a corresponding set of ideal responses is known. This may be referred to as *offline calibration* because it is not occurring onboard the VNA itself. One benefit of this technique is that it provides maximal flexibility for nonconventional calibrations and preserves all raw data. Self-calibration algorithms, such as thru–reflect–line (TRL) [6], do not require predefined ideal responses.

Several calibration routines are available in scikit-rf for single-, two-, or multiport networks. Traditional 12- [7] or 16-term [8] error models are available as well as eight-term models [9], short–open–load–thru (SOLT) [10], overdetermined one port [11], unknown–thru [12], short–delay–delay–load (SDDL) [13], and some special algorithms developed by our contributors. In some cases, it may be necessary or desirable to use a one-port network analyzer to determine the full set of scattering parameters of a two-port device. This technique is called *one-port, two-tier calibration* [14] and is also implemented in scikit-rf. A complete list can be found on the scikit-rf website, and only a single one-port network is given as an example in this section.

One-Port Example

A calibration in scikit-rf is generated using the Calibration object. In general, Calibration objects require two arguments: a list of measured and ideal Networks. The following example assumes that the sets of measured and ideal network standards are stored in separate directories named ideals and measured, for example, a conventional SOL calibration kit. These are used to create a one-port Calibration and subsequently correct a measured DUT:

```
from skrf.calibration import OnePort
# reads all Touchstone files located in the specified directory
my_ideals = skrf.load_all_touchstones_in_dir('ideals/')
my_measured = skrf.load_all_touchstones_in_dir('measured/')
```

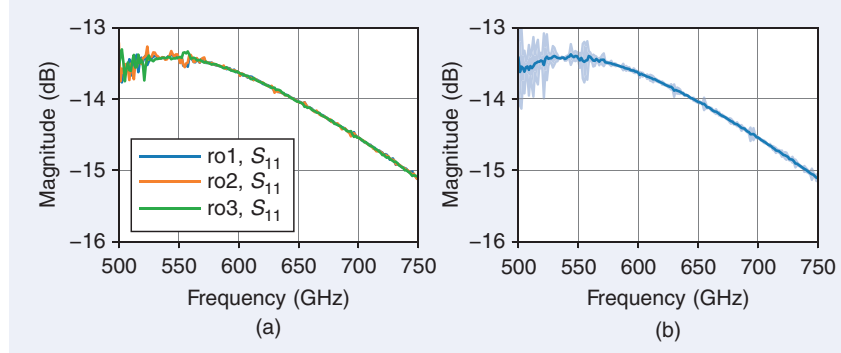

Figure 4. *The logarithmic magnitudes of S_{11} of the* ro *example networks. (a) The individual frequency responses. (b) The calculated average and uncertainty bound (3σ).*

```
# create a Calibration instance
cal = skrf.OnePort(\
    ideals = [my_ideals[k] for k in ['short', 'open','load']],
    measured = [my_measured[k] for k in ['short','open', 'load']],
)
# run calibration algorithm
cal.run()
# apply it to a DUT
dut = skrf.Network('my_dut.s1p')
dut_caled = cal.apply_cal(dut)
```

Multiline TRL Calibration

scikit-rf can also be used for wideband TRL calibration using multiple lines [15]. The necessary calibration standards in TRL calibration involve at least two transmission lines of different lengths and one or more reflective loads. The exact responses of the transmission lines and reflective loads do not need to be known and will be solved during the calibration. Ordinary SOLT or Line-Reflect-Reflect-Match [16] calibration usually moves the reference plane after the calibration to the SubMiniature version A connector or probe tip used to contact the calibration standards. TRL calibration can be used to move the reference planes to the transmission lines on the substrate being measured if every measured standard includes a similar launch to the transmission lines. This makes TRL calibration very useful when accurate short, open, load, and thru standards cannot be manufactured or when the measurement reference plane needs to be on the substrate being measured.

Time Domain and Gating

Time gating is a processing technique that is commonly used to pinpoint a response of interest in the presence of multiple reflections to isolate their effects [17], [18]. This is commonly done onboard a VNA. However, if, instead, the time gating occurs offline on a computer, the user can keep the raw measurements separate from the processed results. This is important so that the processing algorithm can be altered in the future without remeasuring the data.

In the following example, the time-gating functions of scikit-rf are used to filter out the effects of an undesired reflection. This can be done by using the method `Network.time_gate` and providing it an appropriate center and span (in nanoseconds):

```
probe_s11 = skrf.Network('./probe.s2p').s11
probe_s11.name = 'Probe'
probe_s11_gated = probe_s11.time_gate(center = 0, span = 0.2)
probe_s11_gated.name = 'Gated probe'
# time-domain plot:
s11.plot_s_db_time()
s11_gated.plot_s_db_time()
# frequency-domain plot:
s11.plot_s_db()
s11_gated.plot_s_db()
```

To see the effects of the gate, both the original and gated response are compared. As shown in Figure 5, the original response shows an interference pattern in the frequency domain due to this undesirable reflection at 0.2 ns. After gating, the response is cleaned.

Circuit Building

scikit-rf enables the building of a circuit with arbitrary topology, consisting of an arbitrary number of *N*-port `Networks` connected together. Similar to an electronic circuit simulator, the circuit must have one or more ports connected to the circuit. With the `Circuit` object, the combined responses of the *M*-port network can be calculated (and, thus, its network parameters: *S*, *Z*, and so on), where *M* is the number of ports defined. Moreover, the `Circuit` object also allows calculation of the scattering matrix *S* of the entire circuit, that is, the "internal" scattering matrices for the various intersections in the circuit. The calculation algorithm is based on [19].

Figure 6 illustrates a network with two ports, `Network` elements N_i, and intersections. To define such a circuit, the connection list needs to be defined, which describes how the networks are connected:

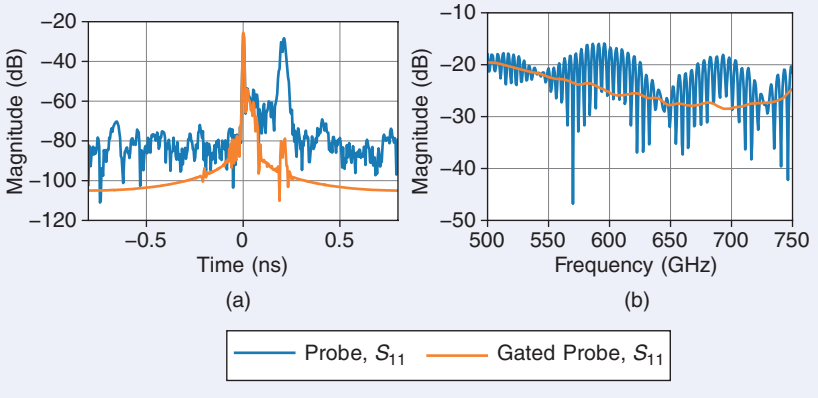

Figure 5. *A comparison of the original network response S_{11} with its gated version in the (a) time and (b) frequency domains.*

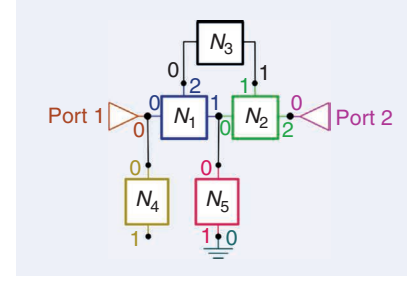

Figure 6. *An example of a* `Circuit` *made of various kinds of N-port* `Networks`*. N: an integer greater than 1.*

```
connections = [
[(network1, network1_port_nb), (network2, network2_port_nb), (network2, network2_port_nb), …],
…
]
```

The connection list to construct the `Circuit` illustrated in Figure 6 could be

```
>>> connections = [
[(port1, 0), (network1, 0), (network4, 0)],
[(network1, 1), (network2, 0), (network5, 0)],
[(network1, 2), (network3, 0)],
[(network2, 1), (network3, 1)],
[(network2, 2), (port2, 0)],
[(network5, 1), (ground1, 0)]
]
```

In this example, the `Circuit` elements `port1`, `port2`, `ground1`, and all of `network1` through `network5` are assumed to be scikit-rf `Network` objects with the same `Frequency` attribute. The individual networks can have different (real) reference impedances, and mismatches are taken into account. Note that port1 of `network4` is left open and so is not described in the connection list. Once the connection list is defined, the `Circuit` is built with

```
resulting_circuit = skrf.Circuit(connections)
```

The resulting two-port `Network` is obtained with `Circuit.network`

```
resulting_network = resulting_circuit.network
```

Vector Fitting

Microwave circuit design requires simulations in the time or frequency domain with accurate models of all involved circuit elements. For passive structures, electromagnetic field simulations or measurements can provide accurate network models in the form of sampled frequency responses, but these cannot directly be used in circuit simulators, such as SPICE. To translate the sampled frequency responses of an N-port network into a model for circuit simulations, scikit-rf provides an implementation of the well-known vector-fitting algorithm to fit the samples in the frequency domain [20]. The rational basis functions used for the fit enable the subsequent generation of linear equivalent circuits based on resistors, capacitors, inductors, and controlled current and voltage sources to be used in most types of circuit simulators [21].

The quality of the fit strongly depends on the choice of real and complex-conjugate starting poles, which should suit the number of resonances in the responses.

To summarize the vector-fitting approach, the vector $\mathbf{H}(s)$ represents the stack of fitting functions for the $N \cdot N$ individual network responses defined in the Laplace domain with $s = \sigma + j\omega$:

$$\mathbf{H}(s) = \mathbf{d} + s\mathbf{e} + \sum_{k=1}^{K} \frac{\mathbf{z}_k}{s - p_k}, \qquad (1)$$

where $\mathbf{H}(s)$ includes a series of K rational fractions with a common set of poles p_k for all of the responses of the network but with individual zero vectors $\mathbf{z}_k$ as well as individual constant and proportional vectors $\mathbf{d}$ and $\mathbf{e}$, respectively.

For example, the scattering parameters of a two-port network can be subjected to vector fitting with K poles. The objective is, therefore, to match the original network samples at all sampling frequencies $\omega_m \in [\omega_1, \omega_2, \ldots, \omega_M]$:

$$\begin{pmatrix} S_{11}(\omega_m) \\ S_{12}(\omega_m) \\ S_{21}(\omega_m) \\ S_{22}(\omega_m) \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} d_{11} + j\omega_m e_{11} + \sum_{k=1}^{K} \frac{z_{k,11}}{j\omega_m - p_k} \\ d_{12} + j\omega_m e_{12} + \sum_{k=1}^{K} \frac{z_{k,12}}{j\omega_m - p_k} \\ d_{21} + j\omega_m e_{21} + \sum_{k=1}^{K} \frac{z_{k,21}}{j\omega_m - p_k} \\ d_{22} + j\omega_m e_{22} + \sum_{k=1}^{K} \frac{z_{k,22}}{j\omega_m - p_k} \end{pmatrix}. \qquad (2)$$

The fitting process involves running an iterative least-squares algorithm [20], which is implemented in scikit-rf including the speed improvements proposed in [22] and [23].

In scikit-rf, the class `VectorFitting` is instanced with the `Network` to be fitted. A subsequent call of the `vector_fit` routine starts the fitting process with the number of real and complex-conjugate poles defined in the function arguments. Once the fitting is finished, the function `write_spice_subcircuit_s` can be called to generate a SPICE subcircuit file based on the fitted poles **p**, zeros **z**, constants **d**, and proportional coefficients **e** of the scattering parameter responses:

```
nw = skrf.Network('example.s4p')
vf = skrf.VectorFitting(nw)
vf.vector_fit(n_poles_real = 2, n_poles_cmplx = 32)
vf.write_spice_subcircuit_s('example.sp')
```

In Figure 7(a) and (b), four of the 16 vector-fitted scattering parameter responses of the example network are plotted and compared to the original network samples, showing an absolute error of less than 0.01. SPICE simulations using the exported subcircuit file in *ngspice* [24] achieve a similar accuracy. The simulated scattering parameters obtained from an ac simulation are shown in Figure 7(c).

The quality of the fit strongly depends on the choice of real and complex-conjugate starting poles, which should suit the number of resonances in the responses. Measurement noise in the sampled responses, as included in this four-port example, further contributes to the deviation of the fit with the smooth basis functions used by the vector-fitting method. The translation of the fitting parameters into the SPICE equivalent subcircuit is straightforward, and the accuracy is limited only by rounding errors. However, the accuracy of the simulation results also depends on the tolerance settings of the simulator.

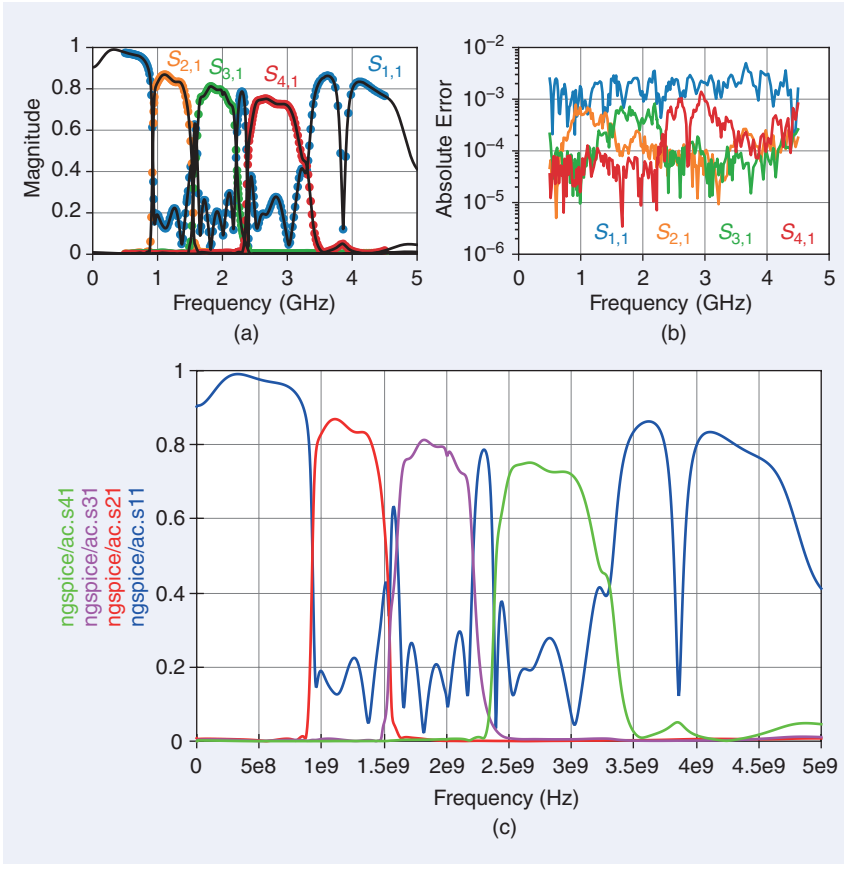

Figure 7. *Selected scattering parameter responses of a vector-fitted noisy four-port example network. (a) The magnitudes in a linear scale with markers for the samples and solid lines for the fit. (b) The absolute error magnitudes of the respective fits: absolute error* $= |S_{\text{fit},i,1} - S_{\text{sample},i,1}|$. *(c) A simulation of the scattering parameters with* ngspice *using the exported SPICE subcircuit.*

Conclusions

scikit-rf is an open source Python package produced for RF/microwave engineering. The package provides a modern, object-oriented library for RF network analysis, circuit building, and calibration. A current nonexhaustive feature list of scikit-rf (as of version 0.18) includes the following:

- microwave network operations:
  - read/write touchstone (.sNp) files
  - S/Z/Y/ABCD/T parameter conversions
  - arithmetic operations on scattering parameters
  - mixed-mode and single-ended port conversion
  - cascade/de-embed networks
  - frequency and port slicing and concatenation
  - the assembly of multiple-port networks
  - the vector fitting (S-, Z-, and Y-parameters) and export of equivalent SPICE subcircuits (based on S-parameters only)
- sets of networks:
  - the statistical properties of a set of a Network
  - the interpolation between a set of a Network (frequency or parametric)
  - methods to sort and visualize set behavior
- plotting abilities:
  - rectangular plots (decibels, magnitude, phase, and group delay)
  - Smith charts
  - automated uncertainty bounds
- calibration routines:
  - *one port*: SOL, least squares, and SDDL
  - *two ports*: TRL, multiline TRL, SOLT, unknown thru, and eight/16 terms
  - *partial*: enhanced response and one port, two paths
- virtual instruments (completeness varies by model):
  - *VNAs*: PNA, PNAX, ZVA, HP8510, and HP8720
  - *Spectrum analyzer*: HP8500
  - Other: ESP300
- transmission line physics:
  - distributed circuit, coaxial/coplanar/rectangular/circular waveguides, and free space
  - transmission line voltage, current, power, and loss calculations
  - complex characteristic impedance support.

In addition to these rich features, there are also GUIs for time gating and .sNp file viewing, with source code available at https://github.com/scikit-rf/dash-apps and functional demos hosted by Plotly at https://dash-gallery.plotly.host/Portal/.

Contributing

scikit-rf is a free and open source project. For those seeking help or reporting bugs, there is a public mailing list and GitHub issue tracker, which can be found on the scikit-rf website (http://scikit-rf.org). If you would like to participate in development, first join GitHub (https://github.com) and then take a look at the scikit-rf development pages.

Acknowledgment

scikit-rf would not be possible without the feedback, bug fixes, and new features contributed by its user community. Visit https://github.com/scikit-rf/scikit-rf/graphs/contributors to see a full list of contributors.

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Educator's Corner *(continued from page 90)*

In addition to their algebraic expressions, some of them are demonstrated in plane geometry, providing clearer vistas. We hope that the presented matrices and graphics stimulate students and young professionals to get into the wonderful world of nonlinear RF electronics.

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