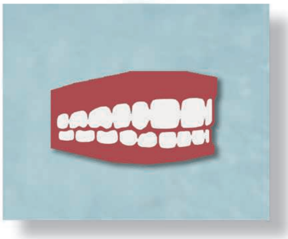
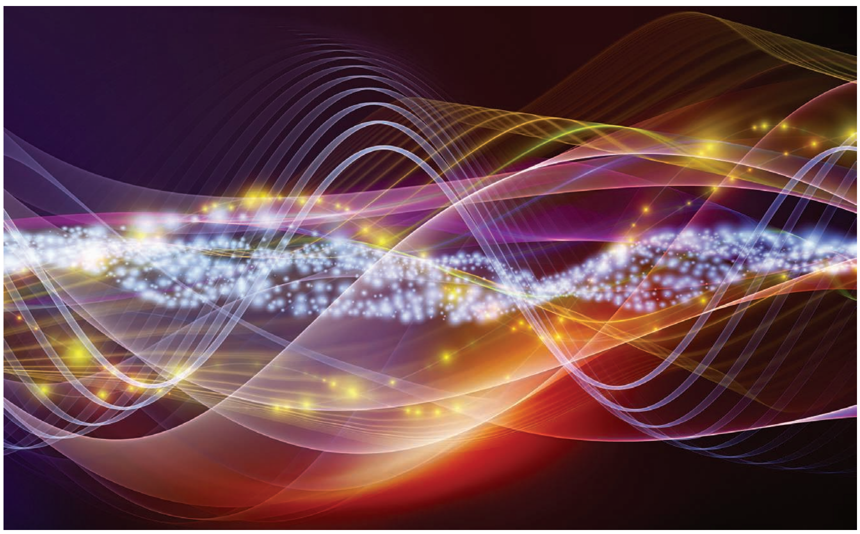


Microwave Bytes

Clipper Classes

■ **Steve C. Cripps**



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It was, admittedly, rather early in my own business traveling career that the then most prominent U.S. airline, Pan Am, introduced the "Clipper Class" cabin, which had a bit more room, no movies, and a more continuous supply of booze. This of course evolved into the now familiar, and much coveted, business class, the exorbitant cost of which I never cease to be amazed that companies are still willing to pay. I once suggested the above title to my publisher, for a book in which I would throw a few wrenches into conventional power amplifier (PA) waveform theory. It did not receive a very positive response, (the title itself that is, rather than the subject matter), mainly because they were all too young to remember "Clipper Class." However, my statutory *double entendre* still stands, and I will discuss some issues that relate to, and to a significant extent modify, the "textbook" concepts of high-efficiency PA "Classes," by taking account of the clipping process that occurs when the device voltage dips into the "knee" region of the device.

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This issue has been something of a "bee in my bonnet," to use a most archaic English expression, for as many years as I have been working on RF-PAs, and yet I have run into a surprising amount of disagreement as to how much impact it has on conventional thinking on the subject. For starters, let's look at the two fundamental pieces of conventional wisdom that cause the conflict.

Figure 1(a) shows a typical set of current-voltage characteristic curve (I-V) characteristics for a microwave power field-effect transistor (FET); given the variety of technologies available these days it is convenient to normalize the voltage to the dc level, and the current to the available peak value. Above a certain minimum value of drain voltage, the device behaves as a current sink, whose magnitude is controlled by the gate voltage, and there is a cutoff point, known variously as the threshold or pinchoff point, where the channel is completely shut off. But below that point, the current shows a monotonic increase from zero up to the

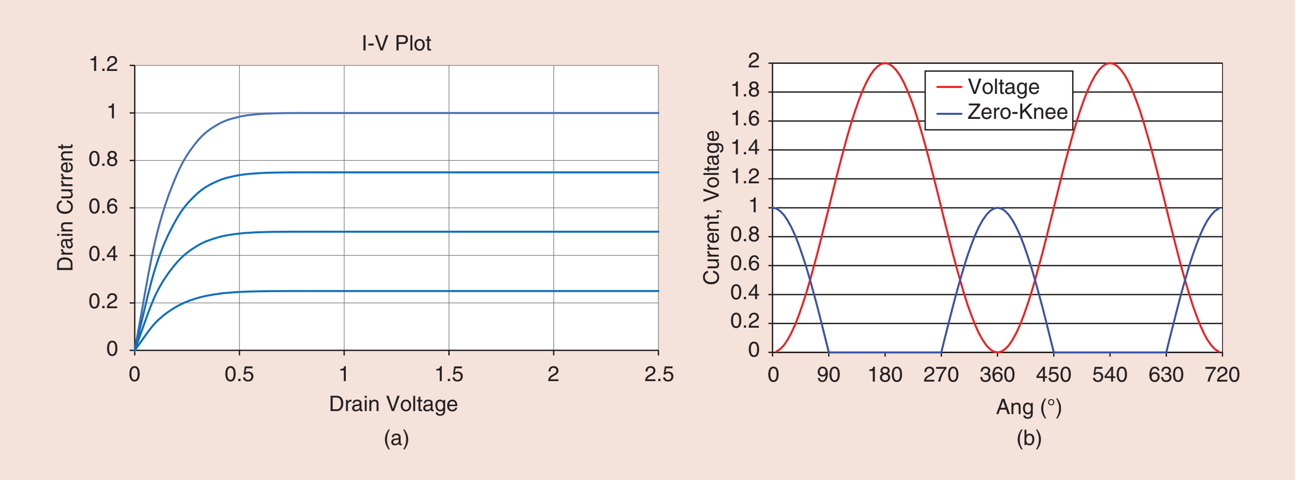

Figure 1. *(a) FET I-V characteristics, (b) Class B PA voltage (red) and current (blue). (Note: see text for normalization of currents and voltages.)*

constant, or "saturated" value. So, "zero volts equals zero current." Meanwhile, Figure 1(b) shows the waveforms of a Class B amplifier, as shown in countless books, papers, and articles for a century or more. The current has the form of a half-wave rectified sine wave, while the voltage swings sinusoidally about its dc bias point, and just "grazes" zero at a point in time coinciding with the current peaks.

Easy enough; but these two plots are, simply and fundamentally, incompatible. The I-V plots show that if the voltage grazes zero then the current must do so as well, as indicated in Figure 2. The exact shape, and extent, of the current "glitch" depends on the more detailed properties of the I-V characteristics, but it will clearly have a significant impact on the familiar power and efficiency formulations that are typically trotted out.

I have always been a strong advocate of the "KIS" ("Keep it Simple") principle; but the danger is to accept simplicity as the final solution, or even worse, as axiomatic; personally, I do find this progression occurs quite often in engineering problems (I will leave the topic of conjugate match for another time 😉). The "zero-knee" assumption, so widely used, is a very useful simplification and leads to nice simple expressions for power and efficiency, the famous 78.5% ($\pi/4$) result in this case for the efficiency of a Class B amplifier. But including the effect of the I-V knee, and thus allowing for current clipping effects, makes the analysis more difficult, and as a minimum involves determining a suitable formulation for the I-V characteristic.

Looking at a typical set of curves, as in Figure 1, the obvious reaction is to think in terms of an exponential function, so that the I-V curves resemble the charging of an *RC* network. This has, with a few nips and tucks, been widely used in nonlinear device models used in circuit simulation software programs but is less suited to deriving closed-form expressions for power and efficiency for classical PA modes. This opens up the more general question of whether closed-form analytical formulation of an engineering problem is necessary, or even relevant, to the designer who is content to use the CAD approach; can one not "crunch the numbers" just as quickly, and maybe more conveniently, than deriving and then evaluating long and complicated formulae? It is an interesting and wide-ranging issue, which leads into further questions about how engineering is taught and the content of undergraduate courses. However, in scanning the pages of *IEEE Transactions*, it certainly appears that the need for analytical verification of a new technique is still very much regarded as fundamental in the eyes of reviewers, editors, and editorial committees. In most cases, the analytical formulation can give deeper insight into the key parameters of a system, and how they interact with

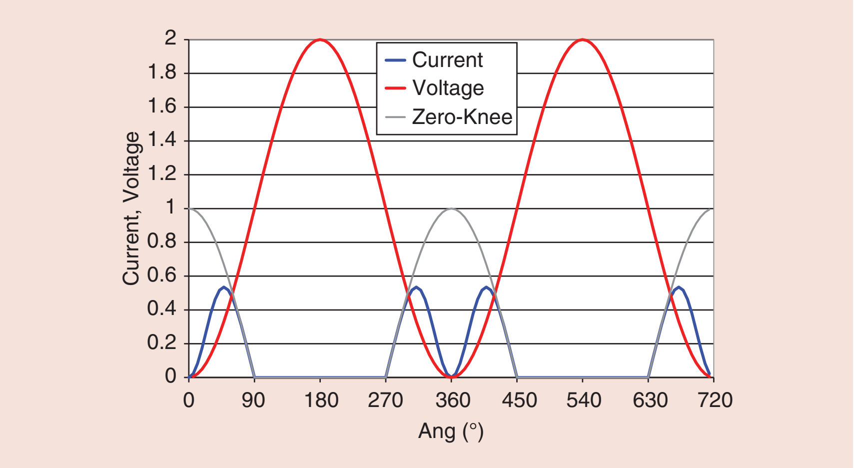

Figure 2. *Class B waveforms incorporating the I-V characteristic in Figure 1.*

each other, although that is not to say that this applies in every case.

A model for the I-V knee characteristic has been proposed [1], which has a simple polynomial form, and which is more suitable for PA wavefom analysis. For an output voltage V_d, and a "zero-knee," or unclipped, current waveform I_z, the clipped current can be represented as

$$I_d = \{1-(1-V_d)^N\}.I_z, \qquad (1)$$

where I_z represents the unclipped, or zero-knee, current, and N is an even integer whose value can be selected to characterize the sharpness, or otherwise, of the I-V characteristic.

Fitting such a model to a specific device raises a further question of how the I-V is measured. The question of whether an I-V characteristic can be assumed quasi static depends on various issues of device physics, most notably thermal and memory effects, and more controversially, transit time effects at higher frequencies. As such, it is highly desirable to use a dynamic measurement of the I-V characteristic at or near to the frequency of use rather than the "slow sweep" used by commercial device curve tracers. This is quite possible using an active load-pull system, so that the device plane current and voltages can be swept for a set of resistive terminations, giving the "fan" plot shown in Figure 3. Obviously, some interpretation is required due to the presence of reactive parasitic elements in the device, but Figure 3 shows how the various sweeps are "enveloped" by the dynamic I-V characteristic for two cases of N. This dynamic measurement usually shows a softer knee characteristic than obtained using conventional curve tracers; at higher frequencies phenomena such as "knee walkout" are usually observed, although as is often the case with device physics, giving an observed effect a fancy name does not actually tell us anything about its cause 😉.

The clipping model can be engaged for the two cases of $N = 4$, $N = 6$, and is shown in Figure 3. Higher values of N will cause a sharper clipping action, but the impact on power and efficiency is not substantially different between the two N values:

N	V_d (amplitude normalized to dc)	Power (relative dB)	Efficiency (%)
"zero-knee"	1	0	78.5
4	1	−4.3	63.2
6	1	−3.5	65.6

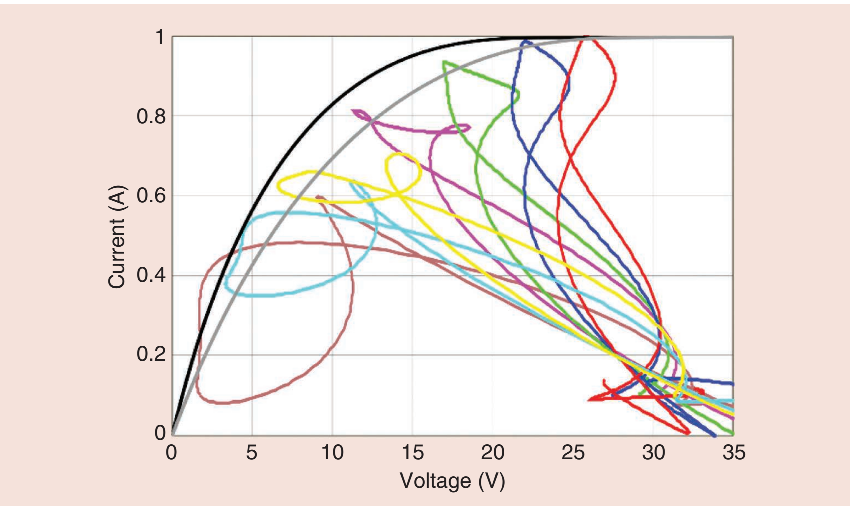

Figure 3. *The dynamic measurement of I-V characteristics.*

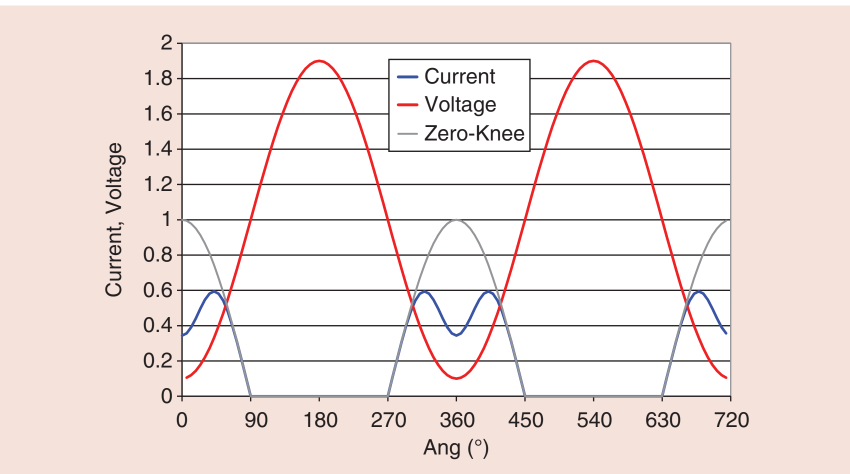

Figure 4. *Class B waveforms with backed-off voltage swing.*

Compared to the zero-knee values, it is the power that takes a major hit; efficiency shows a somewhat milder reduction due to the fact that the clipped current has reduced dc, as well as RF, components. Both gallium nitride (GaN) and laterally-diffused metal-oxide semiconductor (LDMOS) power FETs do appear to show such "soft" knee characteristics, although as already noted, a dynamic measurement can also appear to stretch the knee somewhat from a dc slow-sweep result.

Under the heading of "the oldest rule in the PA deign handbook," stretching well back into the vacuum tube era, an obvious way of reducing the clipping effect is to reduce the voltage swing by reducing the load resistance so that at full drive the voltage only dips briefly into the knee region. A typical case is shown in Figure 4; clearly, the depth of the current glitch is reduced but the reduced voltage swing will impact both power and efficiency, as shown by the following computed values:

N	**V_d**	**Power (rel. dB)**	**Efficiency (%)**
4	0.9	−2.8	64.3
6	0.9	−2	66.3

Further variations on the output voltage will generate a family of power-efficiency tradeoffs, as indeed is observed in practice and is the main function of load-pull systems. Computation of individual cases is one way to get a feel for this design tradeoff, but in the hope of getting a deeper feel for the impact of clipping in PAs it is instructive to consider a full mathematical analysis of the clipped waveforms.

Some Math; Don't Panic …

Let us, with some justified reservation, apply the "N" model to a Class B set of waveforms, where the voltage is assumed to remain sinusoidal as its amplitude varies. As already defined, the voltage will be normalized such that the dc supply, V_{dc}, is unity and the peak current is also unity. So, the device output voltage can be represented as

$$V_d = (1 - V\cos\theta),$$

and the Class B clipped current, using the N model,

$$I_d = \{1 - (V\cos\theta)^N\}\cos\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2},$$
$$= 0,\ (\text{else}).$$

The dc and RF harmonic components can be determined using discrete Fourier integration; for example, the in-phase fundamental component of I_d, I_1, is given by

$$I_1 = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}(1 - V^N\cos^N\theta)\cos^2\theta d\theta,$$

which may look somewhat formidable, but in fact for $N = 4$ can be shown to evaluate to

$$I_1 = \frac{1}{2} - \frac{5}{16}V^4, \quad (2)$$

a somewhat remarkable result that demonstrates the value of the N model in symbolic analysis of clipped waveforms. (It is worth also commenting, that without access to a symbolic math software package such as *Mathematica*, I am not sure such a compact result would ever have emerged!) As the voltage amplitude V decreases from the "zero-grazing" value of unity, the clipping term rapidly diminishes, and the fundamental current approaches the ideal unclipped value of 1/2. But at the unity value the fundamental component is reduced by a factor of 3/8, or −4.3 dB (as earlier), relative to the ideal unclipped case. The dc component can be determined in a similar manner,

$$I_{dc} = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}(1 - V^N\cos^N\theta)\cos\theta\, d\theta,$$

which for the $N = 4$ case evaluates to

$$I_{dc} = \frac{1}{\pi}\left(1 - \frac{8}{15}V^4\right). \quad (3)$$

So, just as the fundamental RF component reduces by a factor of 3/8 at zero-grazing, so the dc component reduces by a factor of 7/15, so the efficiency reduction factor of 45/56 indicates a much lower impact of clipping on efficiency than on power.

It is of considerable interest to pursue this further and evaluate the second harmonic component of current. It is a well-established design issue that the relatively high second harmonic component in a Class B waveform requires a short circuit termination to remove any second harmonic component in the output voltage. This is not just simply an issue of meeting spectral emission specifications; the in-phase second harmonic component, I_2, with $N = 4$, is given by

$$\frac{2}{\pi}\left(\frac{1}{3} - \frac{8}{21}V^4\right). \quad (4)$$

Equation (4) is very significant, as it shows that another effect of clipping is to reduce substantially the relative level of second harmonic current component; indeed, for the zero-grazing case where $V = 1$, (4) shows it almost at the point of crossing zero, which in fact occurs when $V = \sqrt[4]{7/8}$, or about 0.97. Does this explain why in practice we find that a harmonic short never seems to have quite the major impact that the textbooks aver?

The results in (2)–(4) have used the output voltage amplitude as the independent variable. In practice, this voltage is not independent but will be a function primarily of the drive level and the output load. Equation (3) directly implies a value for the load resistance RL,

$$I_1 = \frac{1}{2} - \frac{5}{16}V^4 = \frac{V}{R_L}, \quad (5)$$

so that (5) can be solved for V, given a value for RL. This enables these results on power and efficiency to be plotted as a function of RL, as shown in Figure 5.

This plot reveals a pair well-known home truths for RFPA designers, but is novel inasmuch as the physical origin of these truths is now largely quantified.

1) Maximum power and maximum efficiency occur at substantially different load values.
2) the efficiency plot is quite flat as the load value is increased beyond the shallow optimum but power falls quite sharply.

The behavior of the second harmonic component has some possibly less well-known implications for RFPA design; as already commented, the observation that it crosses zero in the vicinity of the power and efficiency maximum points has implications for harmonic termination design. But the regime where it reverses sign opens up the possibility of using a second harmonic voltage component to reduce the minimum point of the voltage waveform. This in effect takes us into the "Class J sector," where an in-phase second harmonic enables a higher fundamental component. This will be considered next.

Clipped Class J Modes

Figure 6 shows the voltage and current waveforms for an ideal zero-knee

Class J amplifier [2]. Basically, adding an in-phase second harmonic component to a raised cosine wave increases the peaks but raises the minima above zero. As such the entire waveform can be scaled up to reach the zero-grazing condition again. The optimum situation occurs [2] when the second harmonic component has a normalized amplitude of $1/(2\sqrt{2})$; at this point the waveform can be scaled up by a factor of $\sqrt{2}$. But if the voltage and current waveforms are in the standard antiphase condition, the second harmonic impedance will have a substantial negative sign, and hence unrealizable with a passive matching network. Various attempts have been made to inject second harmonic power [2], [3] to implement such a condition, but a more prosaic approach is to introduce a phase shift between current and voltage to move the second harmonic impedance to a purely reactive value. In the ideal case of zero-knee, this phase shift will be 45° referred to the fundamental, so there will be a power factor of $1/\sqrt{2}$ to be applied to the current/voltage product when determining the fundamental power. In the optimum case, this actually reduces the power and efficiency back to their Class B values, but the key point is that these values can be obtained without a short circuited second harmonic, a condition that is difficult in practice to maintain over an extended bandwidth.

The effects of knee clipping on Class J, and the associated continuum of modes, has not to my knowledge ever been fully analyzed. It is, in truth, a complicated problem and something that I have filled more than one notebook attempting to do. As such what follows could be characterized as some extracts from these notes, which I have not before attempted to publish; it seems that basic but useful theoretical results no longer satisfy the appetites of reviewers and editors of the IEEE Microwave Theory and Techniques Society (MTT-S) journals, but that's another discussion (☺).

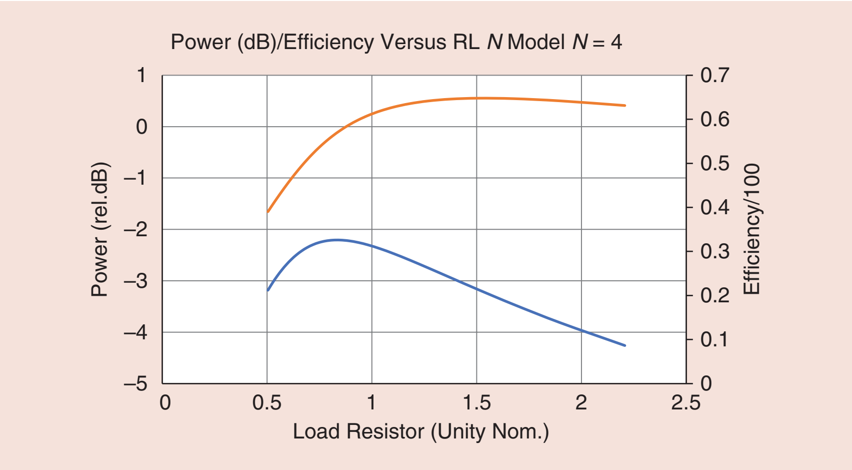

Figure 5. *The power and efficiency variation versus load resistance, case of N = 4.*

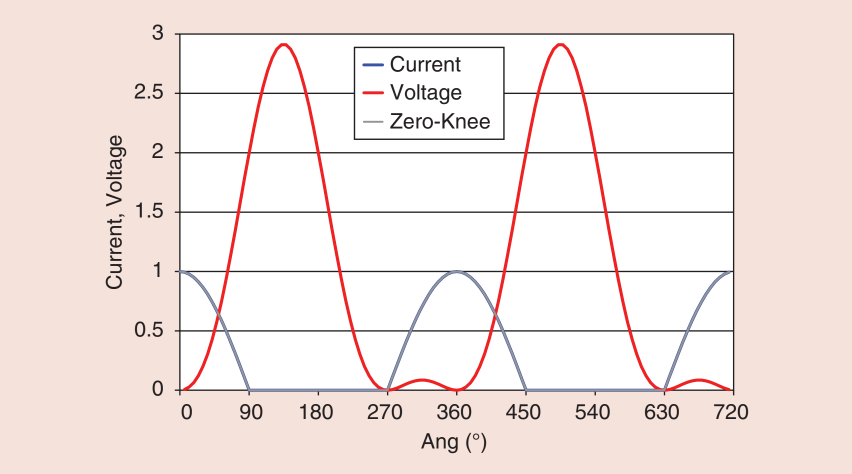

Figure 6. *The class J current and voltage (zero-knee assumption).*

There are some very interesting and practically useful results that emerge using the N model on continuous, or "Class BJ" modes, which center on the reduction of the 45° phase offset between voltage and current, which becomes possible due to the reduction of the second harmonic component of the clipped current waveforms, as noted for the Class B mode in the previous section. Applying the N model to the Class J voltage waveform does, admittedly, result in considerably greater algebraic complexity, but with the help of a symbolic math program is still a worthwhile exercise and gives some deeper insight than direct numerical computation. (An example of the latter approach was demonstrated in [5]).

The voltage waveform for a Class BJ amplifier can be defined as

$$v = 1 - V_1 \cos\theta + V_2 \cos 2\theta.$$

So, using the N clipping model in (1), the clipped current for Class B bias will be

$$I_d = \{1 - [V_1 \cos(\theta + \emptyset) - V_2 \cos(2\theta + 2\emptyset)]^4\}\cos\theta,$$
$$-\frac{\pi}{2} < 0 < \frac{\pi}{2},$$

where $\emptyset$ represents the offset phase between the voltage and current waveforms.

So, we are particularly interested in the in-phase second harmonic component, which in the unclipped case, and with the phase offset ϕ set at zero, will show a negative resistive impedance, hence the need to offset the voltage by 45° to move the second harmonic to a pure reactance. The clipping of the current waveform will enable this offset angle to be reduced considerably, resulting in improved power and efficiency. So, with some trepidation, we note that this component, I_{2r}, will be given by the following integral,

$$I_{2r} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}\{1-[V_1\cos(\theta+\emptyset) - V_2\cos(2\theta+2\emptyset)]^4\} \cos\theta \,.\, \cos(2\theta+2\emptyset)\,.\,d\theta,$$

which can be set to zero to establish possible pairings of the voltage amplitude components V_1 and V_2, for selected values of the phase offset $\emptyset$, that result in a purely reactive second harmonic current component, thus maintaining alignment with the original provision of the Class J mode of a reactive 2H termination. Once again, although it would take me most of the rest of my days to evaluate this manually, *Mathematica* trots out the result in a matter of seconds,

$$I_{2r} = \frac{2}{3\pi}\cos 2\emptyset - F_0(\emptyset)V_1^4 + F_1(\emptyset)V_1^3V_2 + F_2(\emptyset)V_1^2V_2^2 + F_3(\emptyset)V_1V_2^3 + F_4V_2^4, \quad (6)$$

where the $F(\emptyset)$ functions are straightforward, albeit somewhat lengthy, summations of various amplitudes of cosines of even multiples of $\emptyset$, which can be easily evaluated for a given value of the offset angle $\emptyset$.

Thus, by stipulating values for the offset angle and a value for the fundamental component v_1, we end up with a fourth-degree equation that may, or may not, yield a value for v_2 that gives the required condition of a reactive 2H termination. These days with the power of a math solver we no longer need to worry about numerically solving fourth-degree polynomial equations, although in this case it would be a reasonable approximation to drop terms in v_2^4 and v_2^3, given that v_2 will be a "small" value, less than 0.25 for most useful solutions. These solutions are plotted in Figure 7, and reveal something rather interesting; for a given value of offset angle the solutions map an apparently linear relationship between v_1 and v_2; essentially it appears that within the ranges of interest the clipped Class J solutions follow a relationship that can be expressed as

$$v_1 - v_2 = F_j(\emptyset),$$

which in turn says that the voltage minimum remains nearly constant for each selected value of the offset angle $\emptyset$. What this appears to tell us is that the required Class J solutions occur for different values of voltage minima.

Of probably greater interest is the power and efficiency values that correspond to these solutions for V_1 and V_2. Needless to say, *Mathematica* delivers expressions for the fundamental and dc components of current in short order, and they have the same general form as (6). I will refrain from printing the "full nine yards" of these expressions, but it is instructive to examine the dominant terms in each, essentially ignoring the V_2^2 and V_2^3 components. The in-phase fundamental component of current is

$$I_{1r} = \left(\frac{1}{2} - \frac{5}{16}v_1^4\right)\cos\emptyset + \frac{1}{\pi}\left(2 + \frac{7}{6}\cos 2\emptyset + \frac{2}{15}\cos 4\emptyset - \frac{1}{70}\cos 6\emptyset\right)v_1^3 v_2 + (\text{etc.}). \quad (7)$$

The first term in the first bracket represents the value for the fundamental current component with no clipping; the second term in the first bracket represents the "shortfall" caused by the voltage clipping with no second harmonic ($V_2 = 0$). The second bracket will clearly be a positive, or "payback" component due to the presence of the second harmonic component, for "lower values" of the offset angle $\emptyset$. Similar observations can be made for the dc component,

$$I_{\text{dc}} = \frac{1}{\pi} - \frac{1}{\pi}v_1^4\left(\frac{3}{8} + \frac{1}{6}\cos 2\emptyset - \frac{1}{60}\cos 4\emptyset\right) + v_1^3 v_2\cos\emptyset.\,(\text{etc.}). \quad (8)$$

Putting this all together (finally 😉) gives the series of curves in Figure 8, which plots power versus efficiency for several values of $\emptyset$. Also shown (dashed) in Figure 8 is the corresponding plot for clipped Class B, as derived in the previous section.

So, the use of a second harmonic component in the voltage waveform can be seen to offer some significant efficiency benefits, albeit with some compromise on RF power. But the key benefit of the Class BJ modes is the more flexible requirement on the second harmonic termination, which is generally reactive, rather than a short circuit. It is of interest to look at a typical set of

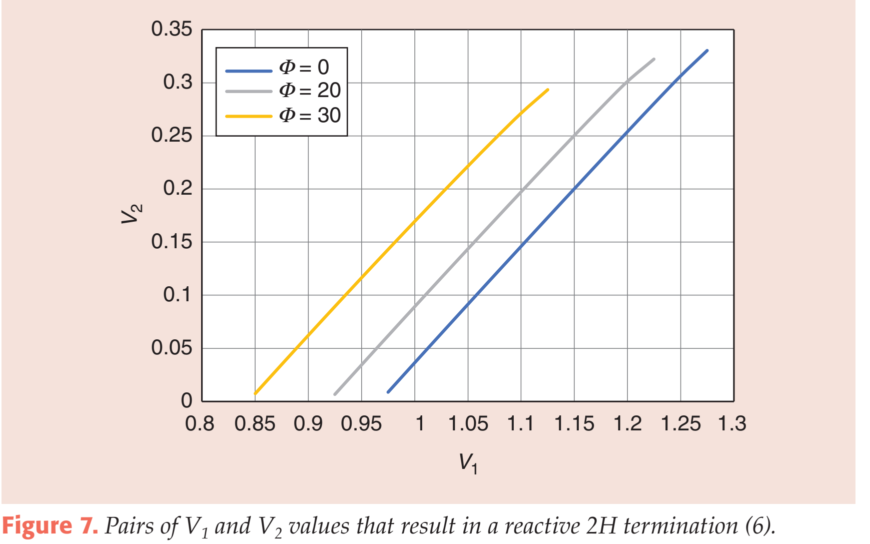

Figure 7. *Pairs of V_1 and V_2 values that result in a reactive 2H termination (6).*

waveforms, these are shown in Figure 9 for a particular case in Figure 8.

This reminds me of some interesting results I obtained some years ago when I was trying to set some exam questions for an M.Sc. course on RFPAs. Intended initially as a purely academic exercise to see whether the incumbents had actually learned anything, I set a question about the current waveform shown in Figure 10. It is essentially a "half-wave sawtooth," and the initial requirement was to determine the fundamental and dc components. Unlike the ubiquitous half-wave cosine, this waveform is neither even nor odd and has both cosine and sine components. Their evaluation is a nice little exercise in "integration by parts," the details of which I will not repeat, but the results are that the cosine and sine components, I_{1c} and I_{1s}, are given by

$$I_{1c} = \frac{1}{\pi}, \quad I_{1s} = \frac{2}{\pi^2},$$

and the dc component, $I_{dc} = 1/4$.

The next part of the question was to determine the power and efficiency, assuming a raised cosine voltage function, as shown in Figure 10; this will of course not capture all of the available power due to the presence of a quadrature current component, but that's the following part of the question, so the fundamental rf power, P_{rf}, is given by

$$P_{rf} = \frac{1}{2} . 1 . \frac{1}{\pi},$$

which is a factor of $\pi/2$, or very nearly 2 dB lower than the Class B case. The efficiency, η, is given by

$$\eta = \frac{2}{\pi},$$

again, an unexceptional 64% in comparison to the zero-knee Class B efficiency of 78.5%.

So, next we are asked to determine the optimum phasing of the voltage, and the resulting power and efficiency. So, the magnitude of the fundamental component is given by

$$I_1 = \sqrt{\left(\frac{1}{\pi}\right)^2 + \left(\frac{2}{\pi^2}\right)^2},$$

which evaluates to about 0.38, still 1.2 dB lower than Class B.

As such, my "academic" example may appear to be just that; a useful learning exercise but little practical use. But if we probe deeper, and this was not part of the exam question(!), it gets more interesting. If we evaluate the second harmonic components, I_{2r}, I_{2q}:

$$I_{2r} = 0; \quad I_{2q} = \frac{1}{2\pi}.$$

The zero in-phase second harmonic component opens up the possibility of adding an in-phase second harmonic voltage component, as in Class J, but without the need for a phase offset, resulting in the waveforms shown in Figure 11. The fundamental voltage component can now increase by up to a factor of $\sqrt{2}$, which results in the eye-catching numbers for power and efficiency,

$$P_{rf} = +0.3\ \text{dB}\,(\text{rel} \rightarrow \text{Class B}),$$
$$\text{effcy} = 90\%.$$

Very nice, but how do we persuade our transistors to generate a sawtooth wave? Well, just in case you didn't see this one coming, take a look back at Figure 9; it would appear that the knee clipping effect can be put to some positive use after all, by "shaping" the current into something like the "academic" sawtooth, in Figure 10. Meanwhile, I hear some distant voices from the wilderness that are saying "… *isn't this starting to look like Class E?*," not to

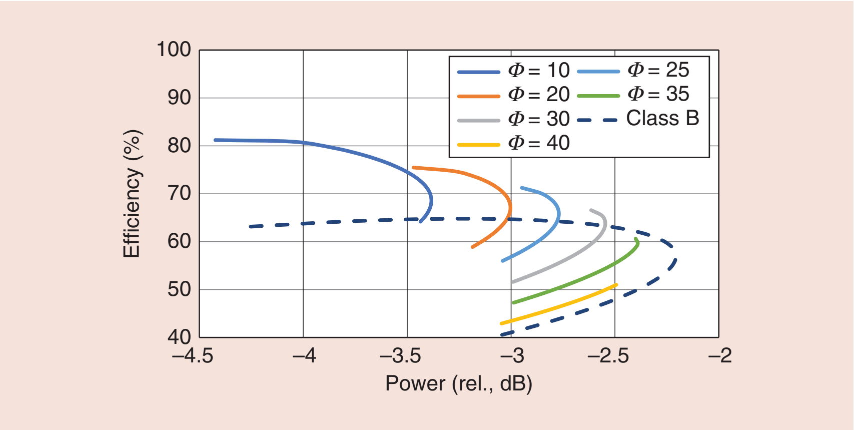

Figure 8. *The clipped Class J family of PA modes, plotted as a function of the offset angle between current and voltage (clipped Class B values shown dotted).*

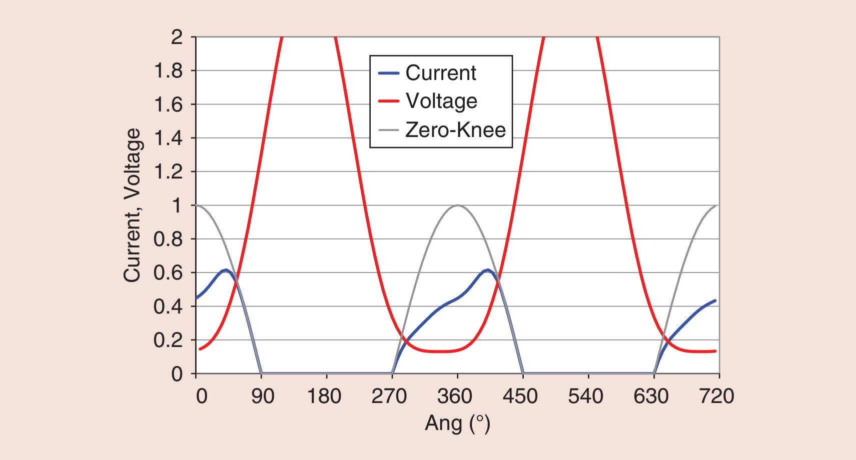

Figure 9. *The clipped Class J mode;* $\emptyset = 25^\circ$, $v_1 = 1.15$, $v_2 = 0.28$, *Power (rel) = –2.9 db, efficiency = 70.1%.*

mention another voice from the same direction saying "… *Aha, the switch mode skeptic rediscovers Class E!*"

I will avoid the temptation to get into an argument with myself about switch modes, other than to suggest that maybe the progression from Figure 10 to Figure 11 represents, perhaps, some kind of a long-sought reconciliation between "switched" and "analog" modes. You pays your money, you takes your choice; but it's time to roll another letter in the alphabet.

Clipped Class F

The basic idea behind the Class F mode is largely demonstrated in Figure 11. If, by some means or another (and that's the tricky bit to which I return shortly), some antiphase third harmonic is added to the original Class B voltage cosine wave, the peak-to-peak value is reduced, and in particular the minima no longer graze zero. So, the entire waveform can be scaled up such that zero-grazing is restored. It can be shown [6] that this scaling factor passes through a maximum value when the normalized amplitude of the third harmonic component is 1/6, and has a value of $2/\sqrt{3}$, or about 1.155, resulting in a power increase of 0.6 dB and an efficiency of 90.7%. This is a much celebrated result but of course assumes the zero-knee idealization. It also poses some awkward questions from a design viewpoint; how do we engineer the third harmonic component of voltage, especially given that the perfect half-wave rectified cosine wave itself has no third harmonic component? The usual answer is to terminate the nonexistent third harmonic current component with an open circuit, so that

$$0 \times \infty = 0.16666\ldots.(!),$$

which is obviously unsound in principle but, rather insidiously, does usually seem to work quite well in practice.

This little "Class F Mystery," as I have called it in the past, has had numerous attempts over the decades at being resolved, which in general recognize that certain problems in physics become intractable if they are idealized too much. A particular favorite of mine, which I recall being used very early in my undergrad engineering courses as an example of

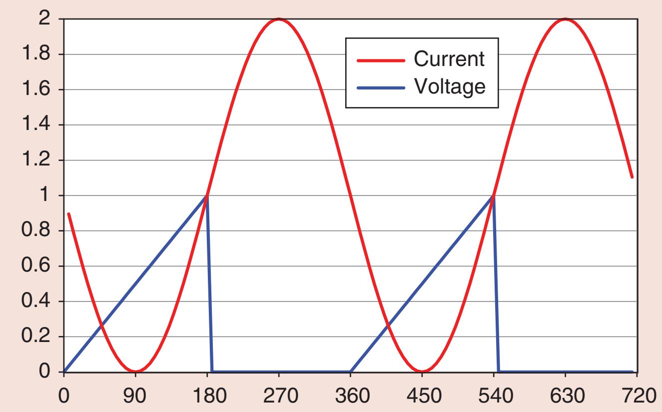

Figure 10. *The "academic" PA mode with half-wave sawtooth current waveform.*

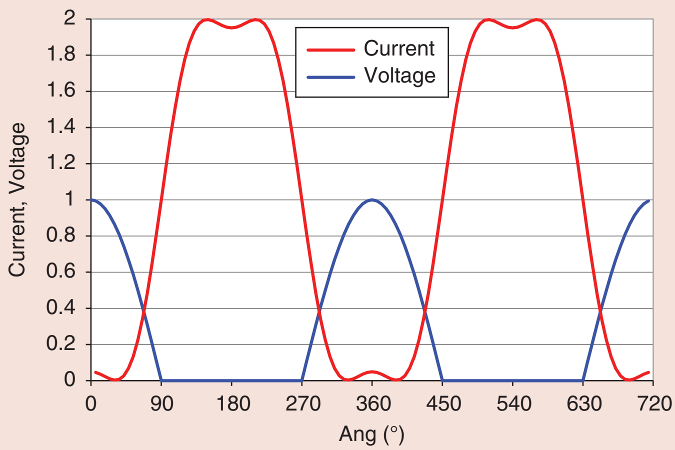

Figure 11. *Ideal Class F waveforms.*

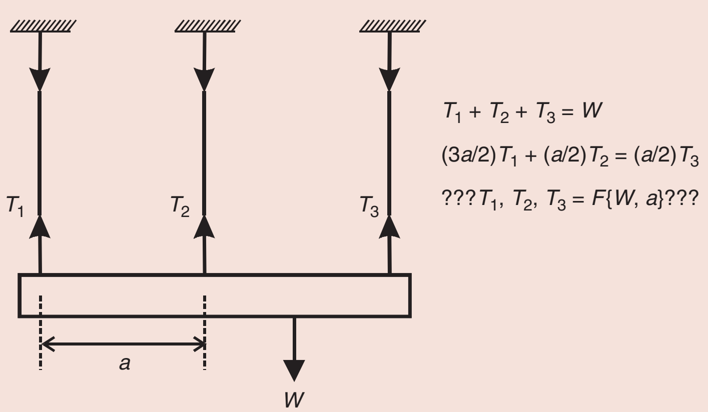

Figure 12. *A "statically indeterminate" structure.*

where classical mechanics gives way to engineering mechanics, is shown in Figure 12. A rigid beam of weight W is supported from a horizontal ceiling with by three pieces of inelastic strings of equal length. Applying the usual laws of statics to determine the three tensions in the strings, we only get two equations relating the three unknowns. Such a structure is termed "statically indeterminate." In actual practice, the strings will be just slightly elastic, and the beam slightly nonrigid, and if these factors are included then the problem can be resolved, albeit with a good deal more complexity.

In the case of a Class F PA, we have to start including some "real world" departures from the ideal zero-knee current source model of the transistor to get some more meaningful relations between the harmonic terminations and the RF performance.

Applying the "N model" is one fairly straightforward way of releasing the Class F mode from its ideality straitjacket, and to obtain some more meaningful design guidelines, particularly on how the power and efficiency depend on nonopen circuit third harmonic terminations. In fact, this is some analysis that I have already presented in the public domain [7] and as such I will reprise the story somewhat more briefly. If we start off with an unclipped Class B current waveform and define a Class F voltage waveform to have the form

$$v = 1 - (v_1 \cos\theta - v_3 \cos 3\theta),$$

thus, initially assuming a short circuit second harmonic termination and resistive terminations at fundamental and third harmonics, the N model can be applied to determine the clipped current, giving

$$i = [1 - \{v_1 \cos\theta + v_3 \cos 3\theta\}^N] . \cos\theta, \quad \text{for } -\frac{\pi}{2} < \theta < \pi/2. \tag{9}$$

It is appropriate at this point to reconsider the value of N. I have shown earlier that a dynamic I-V characteristic of a typical GaN device lies between N model curves for $N = 4$ and $N = 6$. For mathematical convenience I am using $N = 4$, although in fact using a higher N value does not escalate the mathematics all that much, given that the higher powers of v_2 can be ignored. It is perhaps fortuitous that the current device technologies of widespread use in microwave PAs, GaN and LDMOS, do both display softer knee characteristics that enable a more concise analysis.

Going through a similar procedure as before for the Class BJ waveforms, the clipped fundamental component with $N = 4$ comes out to be

$$I_1 = \left\{\frac{1}{2} - \frac{5}{16}v_1^4 + \frac{5}{8}v_1^3 v_3 - \frac{9}{8}v_1^2 v_3^2 + \frac{3}{8}v_1 v_3^3 - \frac{3}{16}v_3^4\right\}, \tag{10}$$

and as before we expect "useful" v_3 values to be less than 0.2 so higher powers can be ignored. Indeed, most of the "action" here can be perceived from the first three terms in (9); the first represents the unclipped value, the second shows the very substantial reduction due to the clipping process, but the "payback" from having a third harmonic voltage component comes from the third term. Higher degree terms have a much smaller effect, both positive and negative.

The same applies to the other components of interest; the dc,

$$I_{dc} = \frac{1}{\pi}\left\{1 - \frac{8}{15}v_1^4 + \frac{32}{35}v_1^3 v_2 - \frac{208}{105}v_1^2 v_2^2 + \frac{1376}{3465}v_1 v_2^3\right\}, \tag{11}$$

and the third harmonic,

$$I_{3r} = \frac{1}{32}\{-5v_1^4 + 24v_1^3 v_3 - 18v_1^2 v_3^2 + 24 v_1 v_3^3\}. \tag{12}$$

The third harmonic component of current, if we comply with the folk lore of PA design, should be zero to support an open circuit termination. So, equating (12) to zero gives a range of combinations of fundamental and third harmonic voltage components that result from an open circuit 3H termination. Solving cubics happens to have been a nerdy interest of mine from an early age, and it's still a fascinating story, but in fact we only need to perform the task once in this case to determine the range of v_1 and v_3 pairings, since dividing through by v_1^4 gives a single solution for the normalized value of v_3,

$$24\left(\frac{v_3}{v_1}\right)^3 - 18\left(\frac{v_3}{v_1}\right)^2 + 24\left(\frac{v_3}{v_1}\right) - 5 = 0, \tag{13}$$

which has the single real solution of $(v_3/v_1) \cong 0.237$. This represents a voltage waveform that is significantly more double-peaked than the classical unclipped value of 1/6, or 0.1666.

One of the main reasons for doing this analysis was to explore how well the Class F mode holds up as the 3H termination moves away from an open circuit value, which for bandwidths any greater than a few percent will inevitably happen. Before embarking on that quest, it is, however, relevant to evaluate the power and efficiency for the open circuit condition over a useful range of v_1 using (12) and (13), which is shown in Figure 13.

Clearly, as the design amplitude of fundamental voltage V_1 increases the clipping is more severe and the relative power drops quite precipitously while the efficiency still rises. But it seems appropriate to select the median value of $V_1 = 1$ to pursue further investigations as this represents a reasonable compromise between power and efficiency.

To explore the effects of third harmonic terminations on a Class F amplifier it is necessary to include a third harmonic phase term in the voltage expression, so that

$$I_d = 1 - v_1 \cos\theta - v_3 \cos(3\theta + \emptyset),$$

and once again, with now increased trepidation, we can see how this modifies the expressions for the various current components. Once again, with the necessary help from *Mathematica*, the resulting expressions are surprisingly manageable, and I will not list all of them, as they can be found in [7]. Essentially, all of the key current components come out as polynomial expressions in v_1, v_3, and $\emptyset$. As such it is a somewhat challenging multi-dimensional problem to present in graphical form, but it has already been noted, from Figure 13 that the value of v1 is likely to be close to unity and so we can proceed to use this value to evaluate the effect of the 3H termination. So, for example, the expression for efficiency, ignoring higher powers of v_3, becomes

$$\eta = \frac{\pi}{2} \cdot \frac{\frac{3}{16} - \frac{9}{8}v_3^2 - \frac{5}{8}v_3 \cos\emptyset}{\frac{7}{15} - 2v_3^2 + \frac{32}{35}v_3 \cos\emptyset},$$

which with a certain amount of extra mathematical manipulation enables contours of constant efficiency to be plotted, and is shown in Figure 14.

Obviously, these results are entirely theoretical, use an approximate model for the knee characteristic, assume a perfect short at all other harmonics (most notably the second), and ignore reactive device parasitics, especially (and conveniently) those that cause instability. It does, however, to my way of thinking, bring some more solid rationale into Class F design, replacing what has hitherto verged on folklore (see "real" design example in [7]). In particular, significant efficiency improvement can be obtained with a third harmonic termination that can be substantially displaced from an open circuit, as shown in Figure 14. That said, as with most folklore, there is a grain of truth inasmuch as power and efficiency will indeed be maximum when the open circuit is realized.

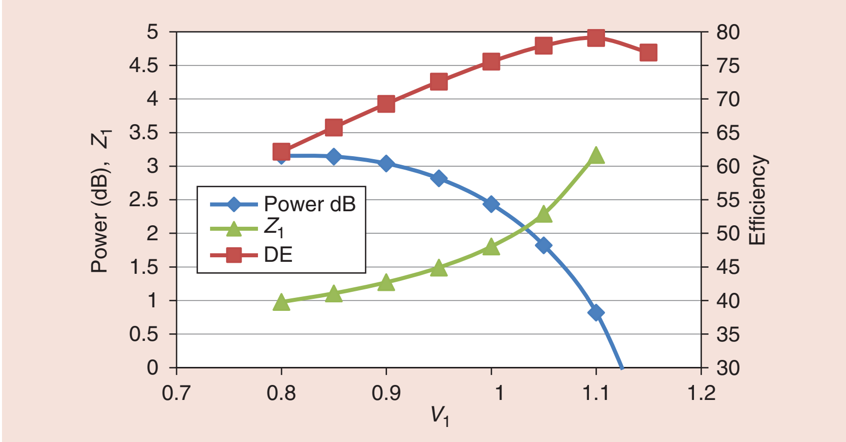

Figure 13. *The relative power (dB, left), efficiency (%, right), and normalized fundamental load (left), for Class F mode using a range of fundamental voltage components.*

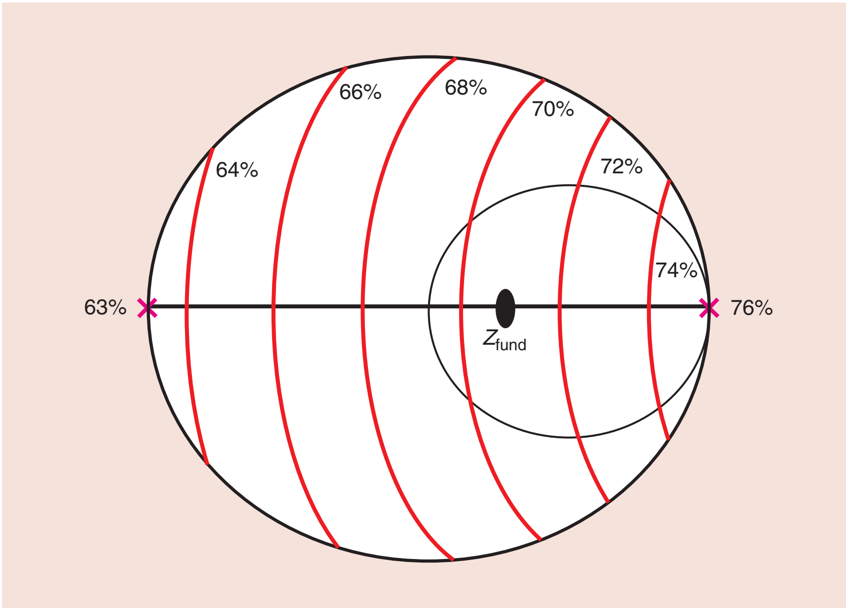

Figure 14. *The power and efficiency variation as a function of third harmonic termination.*

Wrapping Up

I think what comes out of this is the notion that the I-V knee can be put to good use, rather than being regarded as a "spoiler" of classical PA mode theory. Once the voltage drops into the knee region, the current waveform can be reshaped by quite small variations in voltage. I remember a conversation I had with Nathan Sokal, the longtime advocate of the Class E switch mode, on how it is in effect this sensitive knee-shaping function that really gets switch modes "out of jail" (my phrase, not his ☺) when applied to much higher microwave frequencies. Transistors, fundamentally, cannot "switch" fast enough using the input gate voltage but can appear to do so if the current is tailored with an appropriate voltage plunging into the knee region. The same can be said for ongoing attempts to implement the Chireix outphasing PA, in which transistors have to be operated well into the clipping region to "look" like voltage sources. Much like the Class F "folklore," the voltage-source assumption isn't by any means totally wrong, but the underlying action is more accurately modeled by using the I-V characteristic [8].

In conclusion, I pick up on my introductory comments concerning the value of symbolic level mathematical analysis in tackling engineering problems. Make no mistake, this analysis is approximate, using models that are behavioral rather than physics-based. But engineering problems typically have a number of independent variables, and the task of the number-crunching

(continued on page 29)

Circuits Society, IEEE Circuits and Systems Society, IEEE Electron Devices Society, IEEE Signal Processing Society, and IEEE Engineering in Medicine and Biology Society. The RWW organizers are grateful to have these experts who volunteer their time to review the more than 170 papers submitted to the conference. Papers submitted from 24 countries touch on many technical areas related to state-of-the-art radio and wireless technologies. Some representative topics among the submissions include

- antenna technologies, including multiple input/multiple output and multiantenna communications
- passive components and packaging
- device materials, modeling, and characterization
- advanced circuit design and topologies
- high-speed and broadband wireless technologies
- emerging wireless technologies
- wireless system architecture and propagation channel modeling
- wireless digital signal processing and artificial intelligence (AI)
- millimeter-wave and terahertz technologies
- wireless sensors and sensor networks for radar, positioning, and the IoT
- systems, hardware, mission concepts, and operations for space applications
- biomedical, environmental, and imaging applications.

Throughout the week, technical papers will be presented at either oral sessions or interactive poster sessions. A number of high-quality workshops, special sessions, and panels will complement the regular technical sessions to explore current R&D trends and promote technical and professional interaction. We will also continue the tradition of the Demo Track, which offers a unique opportunity for in-person, hands-on experience on new wireless technologies. The Demo Track will engage all attendees, including students, faculty, and industrial practitioners—both experts and novices—in an intimate setting based on real-world wireless setups. The workshops will cover trending topics on AI-based radar technologies, solid-state power amplifiers, and machine learning techniques for RF circuits and systems.

As a tradition, RWW 2022 will include a session of technical talks given by the MTT-S Tatsuo Itoh Distinguished Microwave Lecturers class of 2022–2024. In addition, the RWW 2022 program will integrate several focused sessions in response to recent industrial developments. This will create additional opportunities to network with the leading experts in radio and wireless technologies.

Besides a program full of established researchers and engineers, RWW also fosters a platform for the technical and professional development of young professionals and students. The submissions include more than 60 student papers from around the globe, which go through the same rigorous review process as the professional papers do. Based on initial evaluation by the TPC, a group of finalists are selected to participate in the Student Paper Competition, which will feature both elevator-pitch presentations and interactive forum discussions. Don't miss out on the opportunity to meet these bright young researchers! The best student paper will be recognized during the plenary session. Throughout the technical program, there will be multiple coffee breaks, interactive forums, and social events, allowing young professionals to network with peers, mentors, and industrial representatives.

There are so many things to explore in the conference venue during RWW 2022. We appreciate your participation and look forward to seeing you at RWW 2022.

Microwave Bytes *(continued from page 25)*

engineering simulator, both designer and machine, can be seriously stretched when attempting to analyze all possible combinations. Obviously, as computing horsepower has advanced, the timescale has reduced, but even presenting the results of a multidimensional problem poses difficulties. I have to be honest and admit that presentations consisting of a long sequence of computer-generated graphs, showing the effect of varying different parameters, usually challenges my attention span. Unfortunately, the same applies to presentations that display lengthy mathematical expressions, but on the other hand, these usually do expose some relationships between the various parameters that may well be missed in a simulation approach.

Whether this applies to clipped power amplifier modes, I leave it to the reader to decide.

References

[1] R. J. Quaglia, D. J. Shepphard, and S. C. Cripps, "A reappraisal of optimum output matching conditions in microwave power transistors," *IEEE Trans. Microw. Theory Techn.*, vol. 65, no. 3, pp. 838–845, 2017.

[2] A. Dani, M. Roberg, and Z. Popovic, "PA efficiency and linearity improvement using external harmonic injection," *IEEE Trans. Microw. Theory Techn.*, vol. 60, no. 12, pp. 4097–4106, 2012.

[3] A. Almuhaisen, P. Wright, J. Lees, P. J. Tasker, S. C. Cripps, and J. Benedikt, "Novel wideband high efficiency active harmonic injection power amplifier concept," in *Proc. IEEE Int. Microw. Symp.*, 2010, pp. 664–667.

[4] F. You and J. Benedikt, "An optimum load-impedance calculation and mining method based on I-V curves," *IEEE Trans. Indust. Electron.*, vol. 66, no. 7, pp. 5254–5263, 2019.

[5] S. C. Cripps, *RF Power Amplifiers for Wireless Communications*. Norwood, MA: Artech House, 2007.

[6] S. C. Cripps, "A new analysis of the class F mode," in *Proc. INMMIC*, 2020, pp. 1–3.

[7] T. W. Barton et al., "Inphasing signal component separation for an X-band outphasing power amplifier," *IEEE Trans. Microw. Theory Tech*, vol. 69, no. 3, pp. 1661–1674, 2021.