Enigmas, etc.

Solution to Last Month's Quiz

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The circuit diagram from last month's problem is shown in Figure 1, where the three voltages are related as

$$v_s(t) + v_D(t) + v_L(t) = 48 \tag{1}$$

to meet Kirchhoff's voltage law. The time-domain analysis starts at $t = 0$. The source voltage $v_s(t)$ linearly increases with t up until it reaches the peak, as shown in Figure 2(a). This is simply formulated as

$$v_s(t) = 100t. \tag{2}$$

The very moment $v_s(t)$ crosses 48 V, the diode D turns on. As long as diode D stays on, $v_D(t)$ is kept at zero. Therefore, (1) leads the inductor voltage to

$$v_L(t) = 48 - 100t \quad \text{for} \quad 0.48 < t < 1. \tag{3}$$

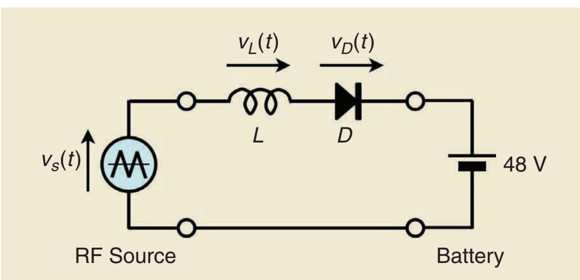

Figure 1. *The battery-charging circuit in question.*

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After passing the peak, $v_s(t)$ decreases as

$$v_s(t) = 200 - 100t. \tag{4}$$

Because diode D still remains on, (1) leads to

$$v_L(t) = 100t - 152 \quad \text{for} \quad 1 < t < \tau \tag{5}$$

where τ denotes the turn-off moment.

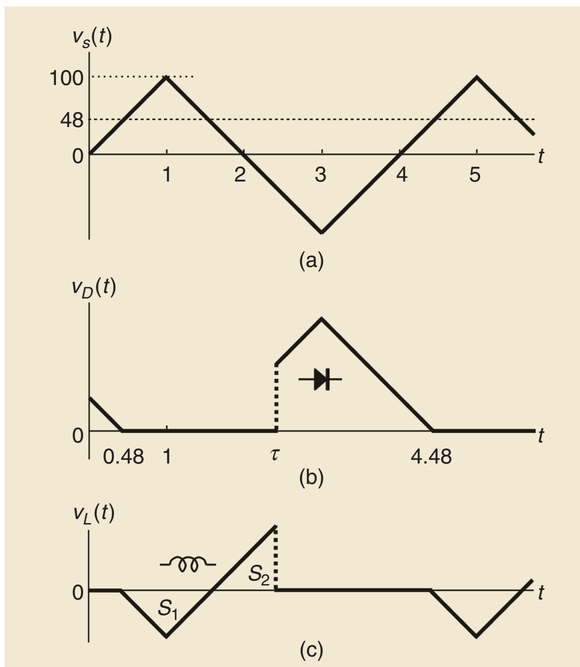

Figure 2. *The voltage waveforms of (a) the RF source, (b) the diode, and (c) the inductor.*

Looking at the downhill slope of (4), one might have thought that the moment $v_s(t)$ falls across 48 V, it makes diode D turn off. However, that is not true. Because of the inductor's inherent property, it is hard for the current to change quickly. To find where τ is exactly located, we use another inherent property of lossless inductors: the voltage across L contains no dc component. This is formulated as

$$\oint v_L(t)dt = 0 \tag{6}$$

where the circular symbol means an integral for one cycle.

Applying (6) to (3) and (5) gives

$$\int_{0.48}^{1}(48-100t)dt+\int_{1}^{\tau}(100t-152)dt=0. \tag{7}$$

Note that the integration is performed only when diode D stays on because $v_L(t)$ is kept at zero when diode D stays off. Solving (7), we find $\tau \approx 2.255$, which is located before the peak. As a result, the pyramid of $v_D(t)$ is truncated, as shown in Figure 2(b). Therefore, the correct answer to last month's quiz is (b). The zero-mean waveform of $v_L(t)$ is also worth viewing from graphics. Focusing on the two triangles indicated by S_1 and S_2 in Figure 2(c), we notice that (7) is translated into the area balance as $S_1 = S_2$.

Paralleling the waveforms of $v_D(t)$ and $v_L(t)$, a simultaneous discontinuity is observed with vertical broken lines. That is to say, the linearly increasing $v_L(t)$ suddenly transfers to $v_D(t)$ at $t=\tau$. By contrast, $v_D(t)$ holds continuity back at $t = 0.48$, which is called *zero-voltage switching* (ZVS); namely, $v_D(0.48) = 0$. It is a general feature of $L + D$ in series that D turns on in ZVS, whereas it turns off in non-ZVS. This law applies not just to the simple battery charger but also to any other diode-oriented RF power electronics, such as rectennas for wireless power transfer.

Educator's Corner *(continued from page 91)*

the 3-dB bandwidth, our simulations showed that, for higher values of k, the frequency response shape was more of a highpass filter, with an overunity gain in the left side, just like overshoot in the step response. Therefore, except for lower values of k, the upper 3-dB point is not meaningful.

Conclusions

It has been shown that a UDRC network is able to achieve a maximum gain of 1.148 for high values of k. This has not been known in the existing literature, and it is hoped that teachers and students of electrical engineering will take notice of this fact. Distributed *RC* modeling of interconnects is essential in microwave circuits as the energy transmission through them is not instantaneous, and the transmission delays play an important part in the characteristics of the designed circuit.

Acknowledgment

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