## Letter

# Estimation Based Adaptive Constraint Control for a Class of Coupled String Systems

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Dear editor,

In this letter, the boundary tension constraint-based adaptive boundary control problem is investigated for a class of flexible string systems with unknown boundary disturbances and unknown parameters, with the purpose of suppressing transverse and longitudinal vibrations. Firstly, on the basis of the Hamiltonian principle, two partial differential equations for a transverse-longitudinal coupled string with spatiotemporally varying tension are derived. Secondly, the transverse and longitudinal adaptive boundary controllers are designed by constructing Lyapunov functions. A barrier Lyapunov function is adopted to confine the boundary tension within the constraint and disturbance observers are designed to estimate the unknown boundary disturbances. Finally, stability analysis of the closed-loop system is performed. The simulation results further illustrate the effectiveness of the proposed method.

String structures are widely existed in modern industry, such as telephone lines, cableways, transmission belts, crane suspensions, crane cables for subsea installations [1]. However, in practical applications, string systems are subject to external environmental disturbances such as aerodynamic forces, wind or currents, which cause the vibration of systems [2], [3]. Undesirable vibrations degrade system performance, reduce efficiency, and even create premature fatigue problems [4]. In addition, the actual tension on the string is often not a constant from the internal structural analysis and it varies with the vibration displacement and time [5]. Excessive tension on the string structure causes it to develop large vibrations [6]. Consequently, during production work, the tension value needs to be constrained to a restricted space to avoid serious hazards. At the same time, effective control mechanisms need to be utilized to reduce string vibrations, thereby improving the operational performance of the system.

The string system can be described by partial differential equations (PDEs). PDE systems possess infinite dimensional properties, which makes system dynamics difficult to control [7]. Recently, boundary control has been regarded by many scholars as an economical and easy-to-implement approach for controlling infinite dimensional systems [8]. It requires only actuators and sensors at the boundary of system and does not affect the dynamics model of system [9]. Given these advantages, boundary control has been applied in many research areas [10], [11]. In addition, adaptive control approach is an effective method to deal with parameter uncertainty [12], which can actively respond to the dynamic characteristics of system, thus improving the control. Researchers have done a lot of research work on this topic [13]−[15]. He *et al*. [16] considered the transverse and longitudinal displacements of string system. Krstic and Smyshlyaev [17] studied the adaptive control problem of parabolic PDEs based on the PDE-backstepping method. In spite of the great progress that has been made in adaptive boundary control, there are still some challenging difficulties. In practical engineering, since physical constraints, environ- mental limitations and safety needs, certain states in the system are usually subject to certain restrictions.

The barrier Lyapunov function (BLF) is an efficient method to tackle the constraint problem [18]. Therefore, more and more scholars are using BLF to address the constrained control problems of nonlinear systems [19], [20]. He and Ge [21] proposed a cooperative control strategy for a portal crane system with tension constraint. A study on vibration suppression of the flexible riser with top tension constraint was presented in [22]. However, the research results on the control of string systems with the boundary tension constraint are limited. In practical engineering, excessive tension increases the wear of string with respect to the tip load. Therefore, it is essential and meaningful to study the boundary tension constraint issue to suppress the vibration of the string system.

In this paper, the coupling effect arising from the transverse and longitudinal displacements of the string system are considered. We study the vibration suppression problem of the string with unknown parameters, unknown time-varying boundary disturbances and the boundary tension constraint. Using the Lyapunov direct method, we apply adaptive boundary control to ensure effective restraint of string vibration. The main contributions of this paper can be found in Section II of the Supplementary Material.

**System description and preliminaries:** We consider a class of coupled string systems with spatiotemporally varying tension [16], where the governing equations are

$$\begin{aligned}&\rho x_{tt}(s,t)-T[s,x_s(s,t)]x_{ss}(s,t)-\kappa_s(s)x_s^3(s,t)\\&\quad -EA[x_{ss}(s,t)y_s(s,t)+x_s(s,t)y_{ss}(s,t)]\\&\quad -T_s[s,x_s(s,t),x_{ss}(s,t)]x_s(s,t)-[3\kappa(s)\\&\quad +3EA/2]x_s^2(s,t)x_{ss}(s,t)=f_x(s,t)\end{aligned} \tag{1}$$

$$\rho y_{tt}(s,t)-EA[x_s(s,t)x_{ss}(s,t)+y_{ss}(s,t)]=f_y(s,t) \tag{2}$$

where $(s,t)\in(0,\ell)\times[0,\infty)$, and the boundary conditions as

$$x(0,t)=y(0,t)=0 \tag{3}$$

$$\begin{aligned}&m x_{tt}(\ell,t)+T[\ell,x_s(\ell,t)]x_s(\ell,t)+EAx_s(\ell,t)y_s(\ell,t)\\&\quad +[\kappa(\ell)+EA/2]x_s^3(\ell,t)=U_x(t)+d_x(t)\end{aligned} \tag{4}$$

$$m y_{tt}(\ell,t)+EAy_s(\ell,t)+EA/2\cdot x_s^2(\ell,t)=U_y(t)+d_y(t) \tag{5}$$

where $t\in[0,\infty)$, $s$ and $t$ denote the spatial and time variables, $\ell$ indicates the length of the string. The transverse and longitudinal vibrational displacements of the string are denoted by $x(s,t)$ and $y(s,t)$. $U_x(t)$ and $U_y(t)$ are the control forces imposed at the boundary of the string. $d_x(t)$ and $d_y(t)$ are unknown time-varying boundary disturbances on the end-point load. $f_x(s,t)$ and $f_y(s,t)$ are unknown distributed disturbances. Moreover, $EA$ stands for the axial stiffness, $m$ means the mass of the end-point load, $\rho$ is the uniform mass per unit length of the string and $T[s,x_s(s,t)]$ stands for the tension on the string, which is expressed as [23]

$$T[s,x_s(s,t)]=T_0(s)+\kappa(s)x_s^2(s,t) \tag{6}$$

with $T_0(s)>0$ is the initial tension, and $\kappa(s)\ge 0$ is the nonlinear elastic modulus. The boundary tension $T(\ell,t)=T[\ell,x_s(\ell,t)]$ is requested to satisfy $|T(\ell,t)|<T_M$, where $T_M$ is the constraint of the boundary tension. The dynamics analysis can be found in Section III of the Supplementary Material.

**Control objective:** The control objective is to deal with the coupling effect between transverse and longitudinal displacements under time-varying disturbances by designing transverse and longitudinal controllers at the boundary of the string such that: 1) The boundary tension on the string does not violate the constraint; 2) The transverse and longitudinal vibrations of the string are effectively suppressed and remain near the equilibrium position under time-varying disturbances; 3) The closed-loop system is stable and all signals are bounded.

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Assumption 1: The boundary disturbances $d_x(t)$, $d_y(t)$ and distributed disturbances $f_x(s,t)$, $f_y(s,t)$ are bounded, i.e., $|f_x(s,t)| \le f_1$, $|f_y(s,t)| \le f_2$, $\forall(s,t) \in [0,\ell] \times [0,\infty)$ and $|d_x(t)| \le d_1$, $|d_y(t)| \le d_2$, $|d_{xt}(t)| \le d_3$, $|d_{yt}(t)| \le d_4$, $\forall t \in [0,\infty)$, where $f_1$, $f_2$, $d_1$, $d_2$, $d_3$ and $d_4$ are positive constants.

Assumption 2: The functions $\kappa(s)$ and $T_0(s)$ are assumed to be bounded, i.e., $\underline{\kappa} \le \kappa(s) \le \bar{\kappa}$ and $\underline{T_0} \le T_0(s) \le \overline{T_0}$, where $\underline{\kappa}$, $\bar{\kappa}$, $\underline{T_0}$, and $\overline{T_0}$ are positive constants.

Property 1 [24]: If the kinetic energy and potential energy of the system described by (1)−(5) are bounded, then $x_t(s,t)$, $x_s(s,t)$, $x_{st}(s,t)$, $x_{ss}(s,t)$, $x_{sst}(s,t)$, $y_t(s,t)$, $y_s(s,t)$, $y_{st}(s,t)$, $y_{ss}(s,t)$, $y_{sst}(s,t)$ are bounded on $(s,t) \in [0,\ell] \times [0,\infty)$.

For simplicity of presentation, the notion $(\Delta) = (\Delta)(s,t)$ is defined, the following text will be developed using abbreviated notation.

Adaptive boundary controller design and stability analysis: To address the coupling effect between transverse and longitudinal displacements under time-varying disturbances, adaptive controllers $U_x(t)$ and $U_y(t)$ at the boundary of the string are designed combining the Lyapunov direct method with BLF and the stability of the system is analyzed. Here, the case of string with the boundary tension constraint, unknown system parameters and unknown boundary disturbances are considered, the proposed adaptive boundary controllers ensure that the boundary tension does not violate the constraint and compensate for the uncertainty of the system.

1) Adaptive boundary controller design: To facilitate controllers design and compensate for the uncertainty caused by unknown parameters and boundary disturbances, the following auxiliary signals are introduced:

$$\zeta(t) = x_t(\ell,t) + x_s(\ell,t) \tag{7}$$

$$\eta(t) = y_t(\ell,t) + y_s(\ell,t). \tag{8}$$

For the coupled string system described by (1)−(5), the following two adaptive boundary controllers are designed:

$$\begin{aligned} U_x(t) =& -k_1\zeta(t) + \widehat{T}_0(\ell,t)x_s(\ell,t) + \widehat{EA}(t)x_s(\ell,t)y_s(\ell,t) \\ & - \widehat{m}(t)x_{st}(\ell,t) - \widehat{d}_x(t) - \Big\{k_2x_t(\ell,t) + \widehat{EA}(t)x_s(\ell,t)y_s(\ell,t) \\ & + \frac{\widehat{m}(t)\zeta(t)x_s(\ell,t)x_{st}(\ell,t)}{b^2 - x_s^2(\ell,t)} + \widehat{T}_0(\ell,t)x_s(\ell,t) \\ & + \left[\frac{\widehat{EA}(t)}{2} - 2\kappa(\ell)\right]x_s^3(\ell,t)\Big\}\Big/\ln\frac{2b^2}{b^2 - x_s^2(\ell,t)} \\ & + \left[2\kappa(\ell) + \widehat{EA}(t)/2\right]x_s^3(\ell,t) \end{aligned} \tag{9}$$

$$U_y(t) = -\widehat{m}(t)y_{st}(\ell,t) - k_3\eta(t) - k_4y_t(\ell,t) - \widehat{d}_y(t) \tag{10}$$

where $b > 0$ is a constant which satisfies $|x_s(\ell,t)| < b$, $k_1 - k_4$ are positive control gains, $\widehat{d}_x(t)$, $\widehat{d}_y(t)$, $\widehat{T}_0(\ell,t)$, $\widehat{m}(t)$, and $\widehat{EA}(t)$ are the estimation of $d_x(t)$, $d_y(t)$, $T_0(\ell)$, $m$, and $EA$. Further, define the disturbance estimation errors as $\widetilde{d}_x(t) = d_x(t) - \widehat{d}_x(t)$ and $\widetilde{d}_y(t) = d_y(t) - \widehat{d}_y(t)$. Let parameter estimation errors be $\widetilde{T}_0(\ell,t) = T_0(\ell) - \widehat{T}_0(\ell,t)$, $\widetilde{m}(t) = m - \widehat{m}(t)$ and $\widetilde{EA}(t) = EA - \widehat{EA}(t)$. In this paper, the disturbance problem for $d_x(t)$ and $d_y(t)$ are solved by designing disturbance observers, which are shown as follows:

$$\widehat{d_{xt}}(t) = \delta_1\zeta(t)\ln\frac{2b^2}{b^2 - x_s^2(\ell,t)} - \delta_1\gamma_1\widehat{d}_x(t) \tag{11}$$

$$\widehat{d_{yt}}(t) = \delta_2\eta(t) - \delta_2\gamma_2\widehat{d}_y(t) \tag{12}$$

with $\delta_1$, $\delta_2$, $\gamma_1$ and $\gamma_2$ being positive parameters.

For unknown system parameters $T_0(\ell)$, $m$, and $EA$, the following adaptative laws are designed:

$$\begin{aligned} \widehat{T_{0t}}(\ell,t) =& -\frac{a}{\gamma_3}x_s(\ell,t)\zeta(t)\ln\frac{2b^2}{b^2 - x_s^2(\ell,t)} \\ & + \frac{a}{\gamma_3}x_s(\ell,t)\zeta(t) - \frac{\phi_1}{\gamma_3}\widehat{T}_0(\ell,t) \end{aligned} \tag{13}$$

$$\begin{aligned} \widehat{m}_t(t) =& -\frac{a}{\gamma_4}x_{st}(\ell,t)\zeta(t)\ln\frac{2b^2}{b^2 - x_s^2(\ell,t)} + \frac{a}{\gamma_4}y_{st}(\ell,t)\eta(t) \\ & + \frac{a}{\gamma_4}\frac{x_s(\ell,t)x_{st}(\ell,t)}{b^2 - x_s^2(\ell,t)}\zeta^2(t) - \frac{\phi_2}{\gamma_4}\widehat{m}(t) \end{aligned} \tag{14}$$

$$\begin{aligned} \widehat{EA}_t(t) =& -\frac{a}{\gamma_5}\left[\frac{x_s^3(\ell,t)}{2} + x_s(\ell,t)y_s(\ell,t)\right]\zeta(t)\ln\frac{2b^2}{b^2 - x_s^2(\ell,t)} \\ & + \frac{a}{\gamma_5}\left[\frac{x_s^3(\ell,t)}{2} + x_s(\ell,t)y_s(\ell,t)\right]\zeta(t) - \frac{\phi_3}{\gamma_5}\widehat{EA}(t) \end{aligned} \tag{15}$$

where $a$, $\gamma_3 - \gamma_5$ and $\phi_1 - \phi_3$ are positive constants.

2) Stability analysis: This section will testify the stability of the system (1)−(5) with the controllers (9) and (10). We firstly define the Lyapunov function as

$$\Gamma(t) = \Gamma_1(t) + \Gamma_2(t) + \Gamma_3(t) + \Gamma_4(t) \tag{16}$$

where

$$\begin{aligned} \Gamma_1(t) =& \frac{a\rho}{2}\int_0^\ell \left(x_t^2 + y_t^2\right)ds + \frac{a}{2}\int_0^\ell T_0(s)x_s^2 ds \\ & + \frac{a}{2}\int_0^\ell \kappa(s)x_s^4 ds + \frac{aEA}{2}\int_0^\ell \left(y_s + \frac{1}{2}x_s^2\right)^2 ds \end{aligned} \tag{17}$$

$$\begin{aligned} \Gamma_2(t) =& \frac{am}{2}\zeta^2(t)\ln\frac{2b^2}{b^2 - x_s^2(\ell,t)} + \frac{am}{2}\eta^2(t) \\ & + \frac{a}{2\delta_1}\widetilde{d}_x^{\,2}(t) + \frac{a}{2\delta_2}\widetilde{d}_y^{\,2}(t) \end{aligned} \tag{18}$$

$$\Gamma_3(t) = \lambda\rho\int_0^\ell sx_tx_s ds + \lambda\rho\int_0^\ell sy_ty_s ds \tag{19}$$

$$\Gamma_4(t) = \frac{\gamma_3}{2}\widetilde{T}_0^{\,2}(\ell,t) + \frac{\gamma_4}{2}\widetilde{m}^2(t) + \frac{\gamma_5}{2}\widetilde{EA}^{\,2}(t) \tag{20}$$

where $\lambda > 0$ is a parameter.

Next, we will present some lemmas and the theorem of this work, whose proofs are in Sections IV–VI of the Supplementary Material.

Lemma 1: The upper and lower bounds of the Lyapunov function given by (16) are

$$\begin{aligned} 0 \le& \alpha_1\left[\Pi(t) + \Gamma_2(t) + \widetilde{T}_0^{\,2}(\ell,t) + \widetilde{m}^2(t) + \widetilde{EA}^{\,2}(t)\right] \\ \le& \Gamma(t) \le \alpha_2\left[\Pi(t) + \Gamma_2(t) + \widetilde{T}_0^{\,2}(\ell,t) + \widetilde{m}^2(t) + \widetilde{EA}^{\,2}(t)\right] \end{aligned} \tag{21}$$

where $\alpha_1, \alpha_2 > 0$ and $\Pi(t) = \int_0^\ell \left(x_t^2 + y_t^2 + x_s^2 + y_s^2 + x_s^4\right)ds$.

Lemma 2: The time derivative of (16) is upper bounded, i.e.,

$$\Gamma_t(t) \le -\alpha\Gamma(t) + \varepsilon \tag{22}$$

where $\alpha, \varepsilon > 0$.

Theorem 1: For coupled strings represented by (1), (2) and boundary conditions (3)−(5), under Assumption 1 and adaptive control protocols (9), (10) with disturbance observers (11), (12) and adaptive laws (13)−(15), we can conclude that all signals of the system are bounded, the boundary tension of the string satisfies the constraint $|T(\ell,t)| < T_M$, $\forall t \in [0,\infty)$, the boundary disturbances are effectively restrained and the closed-loop system is uniformly bounded stable.

Remark 1: The main tool utilized in this paper is the estimation-based adaptive constraint control method. Details are given Section VII of the Supplementary Material.

Remark 2: $\phi_1$, $\phi_2$, $\phi_3$ in (13)−(15) are modification terms applied to improve the robustness of the closed-loop system. Details are given in Section VII of the Supplementary Material.

Remark 3: The influence of control gains $k_1$ and $k_3$ on the system stability is described here, which means that the design parameters should be carefully adapted. Details are given in Section VII of the Supplementary Material.

Remark 4: All signals of the adaptive boundary controllers (9) and (10) are available, which shows that controllers are implementable. Details are given in Section VII of the Supplementary Material.

Simulation: In this section, numerical simulation is performed for a string with 10 m described in (1)–(5). In order to verify the effectiveness of the proposed adaptive boundary control, parameters of the string system are given as $M = 20$ kg, $\rho = 17$ kg/m, $EA = 250$ N, $T_0(s) = 17\times(s+20)$ N, $\kappa(s) = 0.05(s+10)$, $T_M = 5.2\times10^2$ N.

The string is subjected to boundary disturbances of

$$d_x(t) = 0.1\sin(2t) + 0.1\sin(3t) + 0.9\sin(5t) \tag{23}$$

$$d_y(t) = 0.1\sin(2t) + 0.1\sin(3t) + 0.9\sin(5t) \tag{24}$$

and the distributed disturbances to the string are

$$f_x(s,t) = 0.1\,[3+\sin(\pi st)+\sin(2\pi st)+\sin(3\pi st)]\,s \tag{25}$$

$$f_y(s,t) = 0.1\,[1+\sin(\pi st)+\sin(2\pi st)+\sin(3\pi st)]\,s. \tag{26}$$

Figs. 1−4 shows the simulation experiment schematic. Figs. 1(a) and 1(b) show the transverse and longitudinal displacements of the string when without control. It is easy to observe that there is a noticeable vibration of the string without control. Then the adaptive boundary controllers (9) and (10) are employed in which the related parameters are selected as $b = 0.1$, $k_1 = k_3 = 5$, $k_2 = k_4 = 50$, $\delta_1 = 1000$, $\delta_2 = 2000$, $\gamma_1 = 0.007$, $\gamma_2 = 0.02$, $\gamma_3 = \gamma_4 = \gamma_5 = 0.2$, $\phi_1 = \phi_2 = \phi_3 = 5$. Then it obtains the transverse and longitudinal displacements of the string under the boundary tension constraint as shown in Figs. 1 (c) and 1(d). It is obviously seen that adaptive boundary controllers effectively suppress vibrations of the string and stabilize them near their equilibrium position.

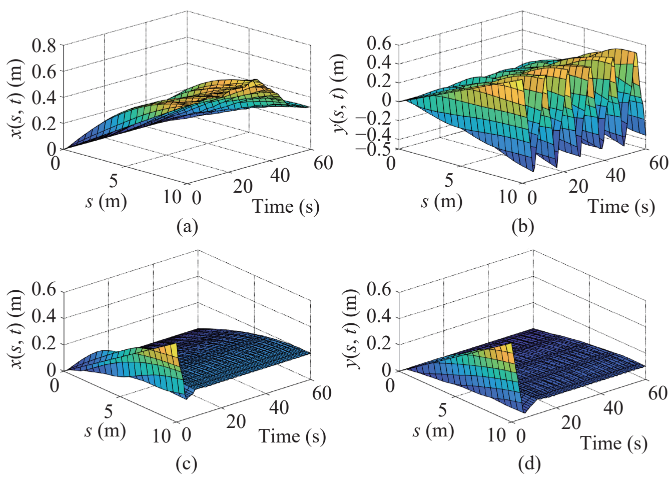


Fig. 1. The displacement of the string. (a) $x(s,t)$ without control; (b) $y(s,t)$ without control; (c) $x(s,t)$ with the adaptive boundary control; (d) $y(s,t)$ with the adaptive boundary control.

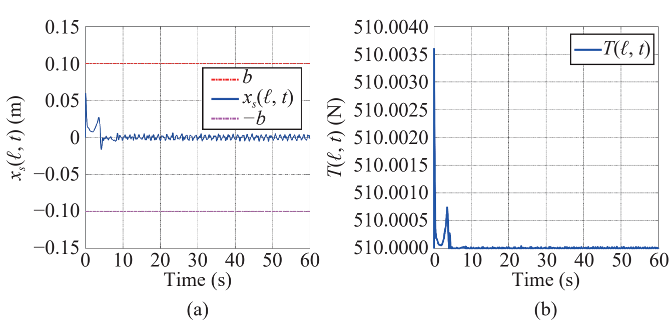


Fig. 2. Adaptive controllers applied to the coupled string system. (a) The trajectory of $x_s(\ell,t)$; (b) The trajectory of the boundary tension $T(\ell,t)$.

Figs. 2(a) and 2(b) depict the trajectories of the state $x_s(\ell,t)$ and the boundary tension $T(\ell,t)$ under the action of adaptive control, respectively. According to (6), the boundary tension is $T(\ell,t) = T_0(\ell)+\kappa(\ell)x_s^2(\ell,t)$, thus we can achieve the constraint on the boundary tension $T(\ell,t)$ by constraining $x_s(\ell,t)$. From Fig. 2, it can be clearly seen that $|x_s(\ell,t)| < b = 0.1$ and the boundary tension

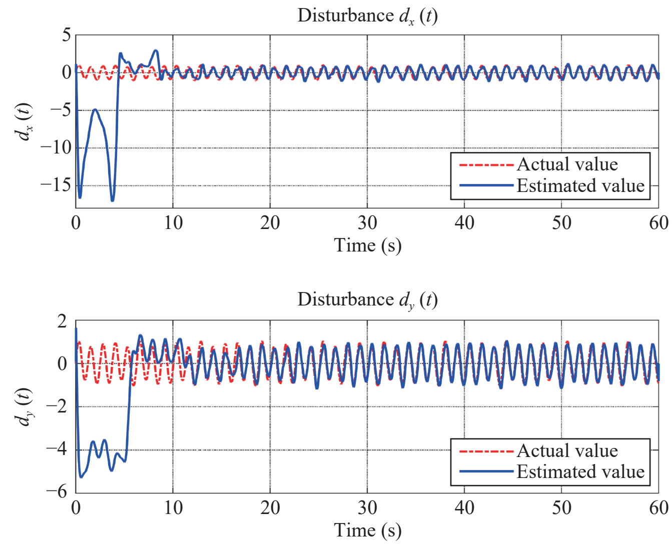


Fig. 3. Disturbance $d_x(t)$ and $d_y(t)$ tracking: Actual value (dotted line) and estimated value (solid line).

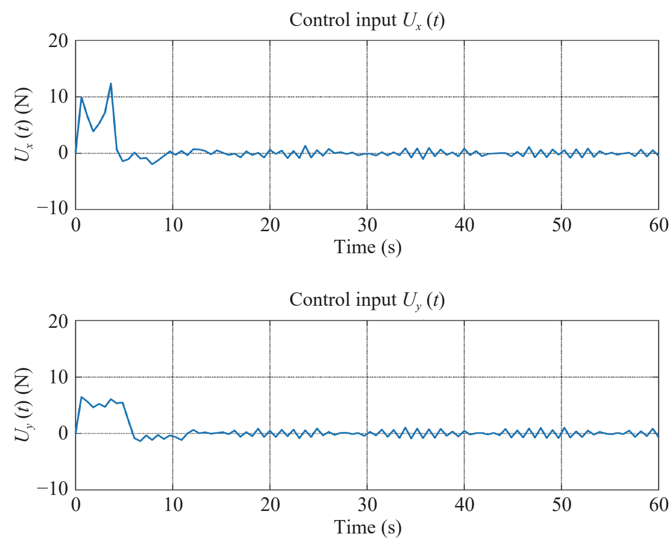


Fig. 4. The trajectory of control inputs.

satisfies its constraint condition $|T(\ell,t)| < T_M = 5.2\times10^2$ N.

Moreover, the boundary disturbances tracking under adaptive boundary control are presented in Fig. 3 . The graphs show the trajectories of the estimated and actual states. It can be seen that the estimated disturbances can better track their actual values after at 13 s. Finally, the trajectories of control inputs $U_x(t)$ and $U_y(t)$ are displayed in Fig. 4.

In summary, in the case of unknown parameters and unknown boundary disturbances in the string system, the developed adaptive boundary control scheme can restrict the boundary tension to the bounded range and suppress the string vibrations. Meanwhile, the designed disturbance observers have good estimation performance.

**Conclusions:** For a class of coupled string systems, this paper presents the adaptive boundary control scheme to suppress vibrations. The control scheme consists of two control forces, applied transversely and longitudinally at the boundary of the string. With the use of logarithmic BLF, the boundary tension is guaranteed not to violate the constraint. Disturbance observers and adaptive laws are designed to compensate for the uncertainty of the system. Finally, the effectiveness of the presented control scheme is verified by numerical simulation. It would be important and interesting to consider finite-time convergence, event-triggered vibration control and PDE cooperative control. All these issues will be our future work.

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