Letter

Dynamic Event-triggered State Estimation for Nonlinear Coupled Output Complex Networks Subject to Innovation Constraints

Jun Hu, Chaoqing Jia, Hui Yu, and Hongjian Liu

Dear editor,

This letter investigates the recursive state estimation (RSE) problem for a class of coupled output complex networks via the dynamic event-triggered communication mechanism (DETCM) and innovation constraints (ICs). Firstly, a DETCM is employed to regulate the transmission sequences. Then, in order to improve the reliability of network communication, a saturation function is introduced to constrain the measurement outliers. A new RSE method is provided such that, for all output coupling, DETCM and ICs, an upper bound of state estimation error covariance (SEEC) is presented in a recursive form, whose trace can be minimized via parameterizing the state estimator gain matrix (SEGM). Moreover, the theoretical analysis is given to guarantee that the error dynamic is uniformly bounded. Finally, a simulation example is illustrated to show the effectiveness of the proposed RSE method.

Related work: Over the past few decades, complex networks (CNs) have been applied to plentiful domains such as biological networks and power grids. In recent years, the dynamical analyses regarding the CNs have attracted increasing research attention from many researchers, particularly the state estimation issue as mentioned in [1]−[3]. It should be noted that most of the published results have been devoted to proposing the state estimation algorithms for state coupled CNs. As is known to all, the CNs can be divided into state coupled CNs and coupled output CNs (COCNs) on the basis of the type of connection among different network units. As a matter of fact, the CNs could be coupled with the observation outputs due to the unknown system states, limited network resources or channel occupations. Up to now, some results on the dynamical analyses of COCNs have been presented, see e.g., [4]. Nevertheless, very few authors have proposed the state estimation strategies for COCNs [5], especially the RSE algorithm. Hence, it is essential to further develop novel RSE method to shorten such a gap.

Compared with the traditional systems, the networked systems have some potential advantages in simple installation, high flexibility and convenient maintenance [6], [7]. However, in the networked environment, a large number of data have rights to share an identical network channel, which results in the occupation of network resources. Nowadays, the research effort has been dedicated to employing the protocol-based scheduling strategies to reduce the transmission frequency thereby improving the communication quality [8]. The well-investigated protocol schedules mainly include event-triggered protocol, stochastic protocol, round-robin protocol, etc. Among them, the event-triggered protocol is a prevalent one to govern the transmission sequences in the process of developing the state estimation strategy [9], [10]. According to the triggered conditions, the event-triggered scheme can be categorized into the static event-triggered communication mechanism and the DETCM [11], [12]. As an extended version of static event-triggered situation, the DETCM has been widely applied to deal with the design issue of state estimation strategy for CNs. For example, [13] has developed a DETCM-based H_∞ approach to handle the state estimation problem for CNs, where an auxiliary variable has been adopted to describe the scheduling criterion and the corresponding analysis has been provided to reveal the impact from communication constraint onto the estimation performance.

On another research frontier, the observation output may be contaminated due to sudden environmental changes or malicious cyber attacks. It should be pointed out that plenty of results have been reported on the design of state estimation strategy subject to malicious attacks [14], [15]. On the contrary, only a few references focus on the problem of state estimation with outliers. With the purpose of enhancing the reliability of the observation outputs, it is very essential to develop an outlier-resistant state estimation method for dynamical systems. Up to now, a saturation function has generally been employed to constrain the measurement outliers [16]. For instance, [17] and [18] have presented the Kalman-type state estimation algorithms for networked systems with ICs, where the certain saturation level and the adaptive saturation threshold have been respectively adopted to attenuate the impacts of outliers. It is worth mentioning that the relevant research on the outlier-resistant RSE strategy in the context of CNs has not gained adequate attention, which is one of the current research motivations.

The main difficulty and challenge encountered in the design of RSE strategy can be listed as: 1) How to attenuate the effect caused by the measurement outliers properly in the framework of RSE algorithm; 2) How to handle the DETCM and ICs simultaneously in the design of RSE algorithm for a class of COCNs; and 3) How to provide a sufficient condition to ensure that the estimation error is bounded uniformly. The main contributions can be summarized as follows. a) We make the first attempt to develop an optimized RSE scheme for COCNs subject to DETCM and ICs. b) A minimized upper bound of SEEC is provided by parameterizing the SEGM. c) A sufficient condition is given to ensure that the error dynamic is uniformly bounded.

Problem statement: In this letter, we consider the following class of COCNs with N nodes:

$$\begin{aligned} x_{i,s+1} &= h(x_{i,s}) + \sum_{l=1}^{N} g_{il}\Gamma y_{l,s} + B_{i,s}\varrho_{i,s} \\ y_{i,s} &= C_{i,s}x_{i,s} + \upsilon_{i,s} \end{aligned} \tag{1}$$

where the subscripts i and s indicate the network node and sampling instant respectively. $x_{i,s} \in \mathbb{R}^{n_x}$ stands for the state vector to be estimated and $x_{i,0}$ with expectation $\bar{x}_{i,0}$ signifies the initial value of $x_{i,s}$, $y_{i,s} \in \mathbb{R}^{n_y}$ denotes the system output measured by sensors. $\varrho_{i,s} \in \mathbb{R}^{n_\varrho}$ and $\upsilon_{i,s} \in \mathbb{R}^{n_y}$ are employed to represent the zero-mean noise signals with variances $Q_{i,s} > 0$ and $\mathcal{R}_{i,s} > 0$, respectively. $B_{i,s}$ and $C_{i,s}$ are known matrices with proper dimensions. $\mathcal{G} = [g_{il}]_{N\times N}$ and $\Gamma \in \mathbb{R}^{n_x \times n_y}$ denote the outer coupled matrix and inner coupled matrix, respectively. The nonlinear function $h(x_{i,s})$ is assumed to be continuously differentiable. In this letter, we suppose that $x_{i,0}$, $\varrho_{i,s}$ and $\upsilon_{i,s}$ are mutually independent.

For the purpose of enhancing communication efficiency, a DETCM is employed to govern the transmission sequences. The triggered instant is denoted by $0 \le \tau_0^i < \tau_1^i < \cdots < \tau_r^i < \tau_{r+1}^i < \cdots$. The event-triggered generator is described by

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$$f(\theta_i,\eta_{i,s},\sigma_i,\varepsilon_{i,s},s)=\frac{\eta_{i,s}}{\theta_i}+\sigma_i-\|\varepsilon_{i,s}\| \tag{2}$$

where $\varepsilon_{i,s}=y_{i,s}-y_{i,\tau_r^i}$ with y_{i,τ_r^i} denoting the latest transmitted output. θ_i and σ_i are known positive parameters and $\eta_{i,s}$ denotes a dynamic variable evolved by $\eta_{i,s+1}=\lambda_i\eta_{i,s}+\sigma_i-\|\varepsilon_{i,s}\|$, $\eta_{i,0}\geq 0$ with λ_i being a scalar-value parameter and $\eta_{i,0}$ being the initial value of $\eta_{i,s}$. If $f(\theta_i,\eta_{i,s},\sigma_i,\varepsilon_{i,s},s)\leq 0$, then the remote estimator has access to the current output. Otherwise, such a triggered mechanism is executed by the zero-order holders.

Note that the transmitted data governed by DETCM usually suffer from the sudden environmental changes due to the network vulnerability. Hence, an ICs strategy is carried out to attenuate the outliers. Here, define a vector $\vartheta_{i,s}=\mathrm{col}_{n_y}\{\vartheta_{i\pi,s}\}(\pi=1,2,\ldots,n_y)$ and the function $\mathrm{sat}(*)$ satisfies $\mathrm{sat}(\vartheta_{i,s})=\mathrm{col}_{n_y}\{\mathrm{sat}_{\vartheta_{i\pi,s}}(\vartheta_{i\pi,s})\}$ with $\mathrm{sat}_{\vartheta_{i\pi,s}}(\vartheta_{i\pi,s})=\max\{-\sigma_{i\pi,\max},\min\{\sigma_{i\pi,\max},\vartheta_{i\pi,s}\}\}$ and $\sigma_{i\pi,\max}>0$ being the π-th saturation threshold.

In this letter, the following state estimator is constructed

$$\hat{x}_{i,s+1|s}=h(\hat{x}_{i,s|s})+\sum_{l=1}^{N}g_{il}\Gamma\hat{y}_{l,s|s}$$
$$\hat{x}_{i,s+1|s+1}=\hat{x}_{i,s+1|s}+K_{i,s+1}\mathrm{sat}(y_{i,\tau_{r+1}^i}-C_{i,s+1}\hat{x}_{i,s+1|s}) \tag{3}$$

where $\hat{x}_{i,s+1|s}$ and $\hat{x}_{i,s|s}$ respectively denote the prediction and estimation. $\hat{y}_{i,s|s}=C_{i,s}\hat{x}_{i,s|s}$ is the output observation. $K_{i,s+1}\in\mathbb{R}^{n_x\times n_y}$ is the SEGM to be designed later.

In what follows, we introduce the following lemmas.

Lemma 1 [19]: Give matrices H, M, N and O with $OO^T\leq I$. If matrix $G>0$ and scalar $\gamma>0$ satisfy condition $\gamma^{-1}I-NGN^T>0$, then one has

$$(H+MON)G(H+MON)^T\leq H(G^{-1}-\gamma N^TN)^{-1}H^T+\gamma^{-1}MM^T.$$

Lemma 2 [20]: Taking (2) into account, let $\varphi_1>0$, and $\varphi_3>0$ be real-value scalars. A formulation of the upper bound of $\Theta_{i,s+1}=\mathbb{E}\{\eta_{i,s}^2\}$ is expressed by

$$\vec{\Theta}_{i,s+1}=[(1+\varphi_1)(1+\varphi_2^{-1})/\theta_i^2+(1+\varphi_2)(1+\varphi_3)\lambda_i^2]\vec{\Theta}_{i,s}$$
$$+[(1+\varphi_1^{-1})(1+\varphi_2^{-1})+(1+\varphi_2)(1+\varphi_3^{-1})]\sigma_i^2.$$

Main results: The SEEC is calculated firstly based on the related definition. Subsequently, the upper bound of SEEC is derived and minimized via designing the SEGM properly.

First of all, the error dynamic of prediction is calculated by

$$e_{i,s+1|s}=x_{i,s+1}-\hat{x}_{i,s+1|s}=h(x_{i,s})-h(\hat{x}_{i,s|s})$$
$$+\sum_{l=1}^{N}g_{il}\Gamma C_{l,s}e_{l,s|s}+\sum_{l=1}^{N}g_{il}\Gamma\upsilon_{l,s}+B_{i,s}\varrho_{i,s}.$$

The nonlinear function $h(x_{i,s})$ is linearized around $\hat{x}_{i,s|s}$ leading to $h(x_{i,s})=h(\hat{x}_{i,s|s})+\mathcal{H}_{i,s}e_{i,s|s}+o(|e_{i,s|s}|)$ with $\mathcal{H}_{i,s}$ being the Jacobian matrix and $o(|e_{i,s|s}|)$ being the high-order term which is approximately estimated by $o(|e_{i,s|s}|)=F_{i,s}\aleph_{i,s}E_{i,s}e_{i,s|s}$. Here, $F_{i,s}$ and $E_{i,s}$ are known matrices and $\aleph_{i,s}$ is employed to describe the linearization error satisfying $\aleph_{i,s}\aleph_{i,s}^T\leq I$. It is obvious that

$$e_{i,s+1|s}=O_{i1,s}+O_{i2,s}+O_{i3,s}e_{i,s|s}+B_{i,s}\varrho_{i,s} \tag{4}$$

where $O_{i1,s}=\sum_{l=1}^{N}g_{il}\Gamma C_{l,s}e_{l,s|s}$, $O_{i2,s}=\sum_{l=1}^{N}g_{il}\Gamma\upsilon_{l,s}$ and $O_{i3,s}=\mathcal{H}_{i,s}+F_{i,s}\aleph_{i,s}E_{i,s}$.

Next, the state estimation error will be given. For simplicity and convenience, define a function $\xi(*,*)$ satisfying equation $\xi(\tilde{p},\tilde{q})=0$ if $\tilde{p}\leq\tilde{q}$ holds. Otherwise, $\xi(\tilde{p},\tilde{q})=1$. Set $\Omega_{i,s+1}:=y_{i,\tau_{r+1}^i}-C_{i,s+1}\hat{x}_{i,s+1|s}$. For the π-th component in $\Omega_{i,s+1}$, we have

$$\mathrm{sat}_{\Omega_{i\pi,s+1}}(\Omega_{i\pi,s+1})=[1-\xi(|\Omega_{i\pi,s+1}|,\sigma_{i\pi,\max})]\Omega_{i\pi,s+1}$$
$$+\xi(|\Omega_{i\pi,s+1}|,\sigma_{i\pi,\max})\mathrm{sign}(\Omega_{i\pi,s+1})\sigma_{i\pi,\max}$$

with $\mathrm{sign}(*)$ being the sign function. Subsequently, the function $\mathrm{sat}(\Omega_{i,s+1})$ can be written as

$$\mathrm{sat}(\Omega_{i,s+1})=[I-\Xi(\Omega_{i,s+1})]\Omega_{i,s+1}+\Xi(\Omega_{i,s+1})S(\Omega_{i,s+1})$$

where

$$\Xi(\Omega_{i,s+1})=\mathrm{diag}_{n_y}\{\xi(|\Omega_{i\pi,s+1}|,\sigma_{i\pi,\max})\}$$
$$S(\Omega_{i,s+1})=\mathrm{col}_{n_y}\{\mathrm{sign}(\Omega_{i\pi,s+1})\sigma_{i\pi,\max}\}.$$

Based on $e_{i,s+1|s+1}=x_{i,s+1}-\hat{x}_{i,s+1|s+1}$, we can obtain

$$e_{i,s+1|s+1}=\mathcal{A}_{i1,s+1}e_{i,s+1|s}+\mathcal{A}_{i2,s+1}\varepsilon_{i,s+1}-\mathcal{A}_{i2,s+1}\upsilon_{i,s+1}-\mathcal{A}_{i3,s+1} \tag{5}$$

where $\mathcal{A}_{i1,s+1}=I-K_{i,s+1}[I-\Xi(\Omega_{i,s+1})]C_{i,s+1}$, $\mathcal{A}_{i2,s+1}=K_{i,s+1}[I-\Xi(\Omega_{i,s+1})]$ and $\mathcal{A}_{i3,s+1}=K_{i,s+1}\Xi(\Omega_{i,s+1})S(\Omega_{i,s+1})$.

Consequently, the prediction error covariance (PEC) and SEEC are provided in explicit forms.

Theorem 1: The PEC and SEEC satisfy the recursions:

$$X_{i,s+1|s}=\mathbb{E}\{O_{i3,s}e_{i,s|s}e_{i,s|s}^TO_{i3,s}^T+O_{i1,s}O_{i1,s}^T+O_{i2,s}O_{i2,s}^T\}$$
$$+B_{i,s}Q_{i,s}B_{i,s}^T+\Phi_{i1,s}+\Phi_{i1,s}^T \tag{6}$$

$$X_{i,s+1|s+1}=\mathbb{E}\{\mathcal{A}_{i1,s+1}X_{i,s+1|s}\mathcal{A}_{i1,s+1}^T+\mathcal{A}_{i2,s+1}\varepsilon_{i,s+1}\varepsilon_{i,s+1}^T\mathcal{A}_{i2,s+1}^T$$
$$+\mathcal{A}_{i2,s+1}\mathcal{R}_{i,s+1}\mathcal{A}_{i2,s+1}^T+\mathcal{A}_{i3,s+1}\mathcal{A}_{i3,s+1}^T\}$$
$$+\sum_{\kappa=1}^{4}(\Upsilon_{i\kappa,s+1}+\Upsilon_{i\kappa,s+1}^T) \tag{7}$$

where

$$\Phi_{i1,s}=\mathbb{E}\left\{O_{i3,s}e_{i,s|s}O_{i1,s}^T\right\}$$
$$\Upsilon_{i1,s+1}=\mathbb{E}\left\{\mathcal{A}_{i1,s+1}e_{i,s+1|s}\epsilon_{i,s+1}^T\mathcal{A}_{i2,s+1}^T\right\}$$
$$\Upsilon_{i2,s+1}=-\mathbb{E}\left\{\mathcal{A}_{i1,s+1}e_{i,s+1|s}\mathcal{A}_{i3,s+1}^T\right\}$$
$$\Upsilon_{i3,s+1}=-\mathbb{E}\left\{\mathcal{A}_{i2,s+1}\epsilon_{i,s+1}\upsilon_{i,s+1}^T\mathcal{A}_{i2,s+1}^T\right\}$$
$$\Upsilon_{i4,s+1}=-\mathbb{E}\left\{\mathcal{A}_{i2,s+1}\epsilon_{i,s+1}\mathcal{A}_{i3,s+1}^T\right\}.$$

Proof: The proof of this theorem can be readily obtained and it is omitted here for brevity. ■

Theorem 2: Give some positive scalars φ_t $(t=1,2,\ldots,9)$ and γ_1. If the following two matrix equations:

$$\mathcal{X}_{i,s+1|s}=(1+\varphi_4)[\mathcal{H}_{i,s}(\mathcal{X}_{i,s|s}^{-1}-\gamma_1E_{i,s}^TE_{i,s})^{-1}\mathcal{H}_{i,s}^T+\gamma_1^{-1}F_{i,s}F_{i,s}^T]$$
$$+B_{i,s}Q_{i,s}B_{i,s}^T+(1+\varphi_4^{-1})\sum_{j=1}^{N}g_{ij}\sum_{l=1}^{N}g_{il}\Gamma C_{l,s}\mathcal{X}_{l,s|s}C_{l,s}^T\Gamma^T$$
$$+\sum_{l=1}^{N}g_{il}^2\Gamma\mathcal{R}_{l,s}\Gamma^T \tag{8}$$

$$\mathcal{X}_{i,s+1|s+1}=(1+\varphi_5+\varphi_6)(1+\varphi_9)(I-K_{i,s+1}C_{i,s+1})\mathcal{X}_{i,s+1|s}$$
$$\times(I-K_{i,s+1}C_{i,s+1})^T+K_{i,s+1}\mathcal{M}_{i,s+1}K_{i,s+1}^T \tag{9}$$

with

$$\mathcal{M}_{i,s+1}=(1+\varphi_5+\varphi_6)(1+\varphi_9^{-1})\mathrm{tr}(C_{i,s+1}\mathcal{X}_{i,s+1|s}C_{i,s+1}^T)I$$
$$+(1+\varphi_5^{-1}+\varphi_7+\varphi_8)\Big[(1+\varphi_1)\frac{\vec{\Theta}_{i,s+1}}{\theta_i^2}+(1+\varphi_1^{-1})\sigma_i^2\Big]I$$
$$+(1+\varphi_7^{-1})\mathrm{tr}(\mathcal{R}_{i,s+1})I+(1+\varphi_6^{-1}+\varphi_8^{-1})\sum_{\pi=1}^{n_y}\sigma_{i\pi,\max}^2I$$

have positive definite solutions $\mathcal{X}_{i,s+1|s}$ and $\mathcal{X}_{i,s+1|s+1}$ under constraint $E_{i,s}\mathcal{X}_{i,s|s}E_{i,s}^T<\gamma_1^{-1}I$, then $\mathcal{X}_{i,s+1|s+1}$ is an upper bound of

$\mathcal{X}_{i,s+1|s+1}$. In addition, if the SEGM is selected as

$$K_{i,s+1} = (1+\varphi_5+\varphi_6)(1+\varphi_9)\mathcal{X}_{i,s+1|s}C_{i,s+1}^T\Delta_{i,s+1}^{-1} \tag{10}$$

with

$$\Delta_{i,s+1} = (1+\varphi_5+\varphi_6)(1+\varphi_9)C_{i,s+1}\mathcal{X}_{i,s+1|s}C_{i,s+1}^T + \mathcal{M}_{i,s+1}$$

then $\mathrm{tr}(\mathcal{X}_{i,s+1|s+1})$ is minimized.

Proof: By resorting to the induction, based on the condition $X_{i,0|0} = \mathcal{X}_{i,0|0} > 0$, assume that $\mathcal{X}_{i,s|s} \geq X_{i,s|s}$. Next, we will prove that $\mathcal{X}_{i,s+1|s+1} \geq X_{i,s+1|s+1}$.

First of all, based on *Lemma 1* and assumption $\mathcal{X}_{i,s|s} \geq X_{i,s|s}$, we can get

$$\mathbb{E}\{O_{i3,s}e_{i,s|s}e_{i,s|s}^TO_{i3,s}^T\} \leq \mathcal{H}_{i,s}(\mathcal{X}_{i,s|s}^{-1}-\gamma_1E_{i,s}^TE_{i,s})^{-1}\mathcal{H}_{i,s}^T+\gamma_1^{-1}F_{i,s}F_{i,s}^T$$

where $\gamma_1 > 0$ is a scalar. Furthermore, it is easy to know that $\mathbb{E}\{O_{i1,s}O_{i1,s}^T\} \leq \sum_{j=1}^N g_{ij}\sum_{l=1}^N g_{il}\Gamma C_{l,s}\mathcal{X}_{l,s|s}C_{l,s}^T\Gamma^T$ as well as $\mathbb{E}\{O_{i2,s}O_{i2,s}^T\} = \sum_{l=1}^N g_{il}^2\Gamma\mathcal{R}_{l,s}\Gamma^T$. According to the inequality $\alpha\beta^T + \beta\alpha^T \leq \varphi\alpha\alpha^T + \varphi^{-1}\beta\beta^T$ with $\varphi > 0$ being a scalar, one has $X_{i,s+1|s} \leq \mathcal{X}_{i,s+1|s}$.

In what follows, we handle the uncertain terms in (7). Notice that $S(\Omega_{i,s+1})S^T(\Omega_{i,s+1}) \leq \sum_{\pi=1}^{n_y}\sigma_{i\pi,\max}^2 I$ and $\Xi(\Omega_{i,s+1})\Xi^T(\Omega_{i,s+1}) \leq I$. Hence, we get $\mathbb{E}\{\mathcal{A}_{i3,s+1}\mathcal{A}_{i3,s+1}^T\} \leq \sum_{\pi=1}^{n_y}\sigma_{i\pi,\max}^2K_{i,s+1}K_{i,s+1}^T$ and $\mathbb{E}\{\mathcal{A}_{i2,s+1}\mathcal{R}_{i,s+1}\mathcal{A}_{i2,s+1}^T\} \leq \mathrm{tr}(\mathcal{R}_{i,s+1})K_{i,s+1}K_{i,s+1}^T$. Considering Lemma 2, we can further get

$$\mathbb{E}\{\varepsilon_{i,s+1}\varepsilon_{i,s+1}^T\} \leq \left[\frac{(1+\varphi_1)\vec{\Theta}_{i,s+1}}{\theta_i^2} + (1+\varphi_1^{-1})\sigma_i^2\right]I.$$

Recall the fundamental inequality leading to $X_{i,s+1|s+1} \leq \mathcal{X}_{i,s+1|s+1}$.

Finally, the SEGM will be parameterized. Let $\partial\mathrm{tr}(\mathcal{X}_{i,s+1|s+1})/\partial K_{i,s+1} = 0$ resulting in

$$K_{i,s+1} = (1+\varphi_5+\varphi_6)(1+\varphi_9)\mathcal{X}_{i,s+1|s}C_{i,s+1}^T\Delta_{i,s+1}^{-1}.$$

Consequently, the proof of this theorem is complete. ■

Boundedness analysis: The uniform boundedness of the error dynamic is verified and new sufficient criterion is given accordingly.

Assumption 1: There exist some positive scalars $\bar{b}$, $\bar{\gamma}$, $\bar{c}$, $\bar{g}$, $\bar{h}$, $\underline{\sigma}$, $\bar{\sigma}$, $\bar{q}$, $\underline{r}$, $\bar{r}$, $\bar{f}$, $\bar{e}$ and $\bar{\mu}$ satisfying

$$B_{i,s}B_{i,s}^T \leq \bar{b}I,\quad \Gamma\Gamma^T \leq \bar{\gamma}I,\quad C_{i,s}C_{i,s}^T \leq \bar{c}I,\ g_{il} \leq \bar{g}$$

$$\mathcal{H}_{i,s}\mathcal{H}_{i,s}^T \leq \bar{h}I,\quad \underline{\sigma} \leq \sum_{\pi=1}^{n_y}\sigma_{i\pi,\max}^2 \leq \bar{\sigma},\ Q_{i,s} \leq \bar{q}I$$

$$\underline{r}I \leq \mathcal{R}_{i,s} \leq \bar{r}I,\quad F_{i,s}F_{i,s}^T \leq \bar{f}I,\quad E_{i,s}E_{i,s}^T \leq \bar{e}I \tag{11}$$

and $\vec{\Theta}_{i,s} \leq \bar{\mu}$ with the constraint condition

$$(1+\varphi_1)(1+\varphi_2^{-1})/\theta_i^2 + (1+\varphi_2)(1+\varphi_3)\lambda_i^2 < 1$$

for each $i, l \in \{1,2,\ldots,N\}$.

For convenience, some notations are defined as follows:

$$\bar{\tau} = (1+\varphi_4)[(\bar{x}^{-1}-\gamma_1\bar{e})^{-1}\bar{h}+\gamma_1^{-1}\bar{f}] + \bar{q}\bar{b} + (1+\varphi_4^{-1})\bar{c}\bar{x}\bar{\gamma}\bar{g}^2N^2 + \bar{g}^2N\bar{\gamma}\bar{r}$$

$$\bar{\rho} = 1/[(1+\varphi_6^{-1}+\varphi_8^{-1})\underline{\sigma} + (1+\varphi_7^{-1})\underline{r}n_y]$$

$$\bar{\kappa} = [\bar{\rho}\bar{\tau}(1+\varphi_5+\varphi_6)(1+\varphi_9)]^2\bar{c}$$

$$\begin{aligned}\tilde{x} = {} & 2(1+\varphi_5+\varphi_6)(1+\varphi_9)\bar{\tau} + \{2(1+\varphi_5+\varphi_6)(1+\varphi_9)\bar{\tau}\bar{c} \\ & + (1+\varphi_5+\varphi_6)(1+\varphi_9^{-1})n_y\bar{\tau}\bar{c} + (1+\varphi_5^{-1}+\varphi_7+\varphi_8)[(1+\varphi_1) \\ & \times(\bar{\mu}/\theta_i^2) + (1+\varphi_4^{-1})\sigma_i^2] + (1+\varphi_7^{-1})n_y\bar{r} + (1+\varphi_6^{-1}+\varphi_8^{-1})\bar{\sigma}\}\bar{\kappa}.\end{aligned}$$

Theorem 3: If $\mathcal{X}_{i,0|0} \leq \bar{x}I$ with $\bar{x}$ being a positive scalar holds, then $\mathcal{X}_{i,s|s} \leq \bar{x}I$ is established for each $s > 0$ under $\bar{x}\,\bar{e} < \gamma_1^{-1}$ and $\tilde{x} \leq \bar{x}$.

Proof: Take $\bar{x}\,\bar{e} < \gamma_1^{-1}$ and (11) into account resulting in

$$(\mathcal{X}_{i,s|s}^{-1} - \gamma_1E_{i,s}^TE_{i,s})^{-1} \leq (\bar{x}^{-1}-\gamma_1\bar{e})^{-1}I \tag{12}$$

$$\sum_{j=1}^N g_{ij}\sum_{l=1}^N g_{il}\Gamma C_{l,s}\mathcal{X}_{l,s|s}C_{l,s}^T\Gamma^T \leq \bar{c}\bar{x}\bar{\gamma}\bar{g}^2N^2I. \tag{13}$$

It follows from Assumption 1, (12) and (13) that $\mathcal{X}_{i,s+1|s} \leq \bar{\tau}I$. Notice that

$$\Delta_{i,s+1}^{-1} \leq \frac{1}{[(1+\varphi_6^{-1}+\varphi_8^{-1})\underline{\sigma} + (1+\varphi_7^{-1})\underline{r}n_y]}I := \bar{\rho}I.$$

According to the expression in (10), one has

$$K_{i,s+1}K_{i,s+1}^T \leq [\bar{\rho}\bar{\tau}(1+\varphi_5+\varphi_6)(1+\varphi_9)]^2\bar{c}I := \bar{\kappa}I.$$

Consequently, it is easy to know that $\mathcal{X}_{i,s+1|s+1} \leq \tilde{x}I$. So far, it can be concluded that $\mathcal{X}_{i,s+1|s+1} \leq \bar{x}I$ holds based on $\tilde{x} \leq \bar{x}$. ■

An illustrative example: The usefulness of the developed optimized RSE scheme is shown. The specific system matrices are presented as follows:

$$B_{1,s} = [-0.12-0.1\sin(0.1s)\ \ -0.12]^T,\ B_{2,s} = [-0.06\ 0.08]^T$$
$$B_{3,s} = [-0.01\ 0.04]^T,\ C_{1,s} = [1.2\ 1.5]$$
$$C_{2,s} = [-5-0.1\cos(0.8s)\ \ 0.5],\ C_{3,s} = [0.4\ \ 2].$$

The coupled parameters are given by

$$\mathcal{G} = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{bmatrix},\ \Gamma = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

The system state is $x_{i,s} = [x_{i,s}^1\ x_{i,s}^2]^T$ and the state estimation is $\hat{x}_{i,s|s} = [\hat{x}_{i,s|s}^1\ \hat{x}_{i,s|s}^2]^T (i = 1,2,3)$. The initial parameters are set as

$$\hat{x}_{1,0|0} = \mathbb{E}\{x_{1,0}\} = [0.2\ 0.2]^T,\ \hat{x}_{2,0|0} = \mathbb{E}\{x_{2,0}\} = [0.3\ 0.3]^T$$
$$\hat{x}_{3,0|0} = \mathbb{E}\{x_{3,0}\} = [0.4\ 0.4]^T,\ \mathcal{X}_{i,0|0} = 0.5I_2.$$

The related parameters are chosen as $\varphi_1 = \varphi_7 = \varphi_8 = 1$, $\varphi_2 = 4.5$, $\varphi_3 = \varphi_9 = 0.1$, $\varphi_4 = 8$, $\varphi_5 = \varphi_6 = 2$ and $\gamma_1 = 0.1$. Let the saturation thresholds be $\sigma_{i1,\max} = 18$. The matrices are $F_{i,s} = 0.2I_2$ and $E_{i,s} = 0.3I_2$. The other parameters are $\eta_{i,0} = 2$, $\vec{\Theta}_{i,0} = 4$, $\sigma_i = 8$, $\theta_i = 20$, $\lambda_i = 2/\theta_i$, $Q_{i,s} = 0.2$ and $\mathcal{R}_{i,s} = 0.02$. Moreover, the nonlinear function $h(x_{i,s})$ is

$$h(x_{i,s}) = \begin{bmatrix} -1.02x_{i,s}^1 - 1.24x_{i,s}^2 - 0.01\cos(x_{i,s}^1x_{i,s}^2) \\ 0.17x_{i,s}^1 - 0.2x_{i,s}^2 + 0.01\sin(x_{i,s}^1x_{i,s}^2) \end{bmatrix}.$$

It should be pointed out that the outliers are characterized by zero-mean noises with variances 1000. The occurrence period of outliers is $T = 2$ and the initial value of outliers is equal to zero. Moreover, the logarithms of the sum of mean-square error (MSE) with $T = 2$ and $T = 20$ under 200 iterations are respectively presented in Table 1, i.e., Case 1: log(MSE) with $T = 2$ and Case 2: log(MSE) with $T = 20$.

Table 1. The Comparisons of log(MSE) With Different T

s	…	45	46	47	48	49	50	51	52	…
Case 1	…	3.8	4.8	4.8	5.0	4.9	4.9	4.7	4.1	…
Case 2	…	3.4	3.6	3.7	3.0	3.5	3.0	3.7	3.6	…

The simulation results are depicted in Fig. 1. The curves of actual state and estimation are presented in Figs. 1(a)−1(c). The sum of upper bounds under 200 iterations with and without saturation constraints are shown in Fig. 1(d). Similarly, the curves of log(MSE) under 200 iterations with and without saturation constraints are presented in Fig. 1(e). According to the comparison, it can be confirmed that the developed state estimation algorithm performs

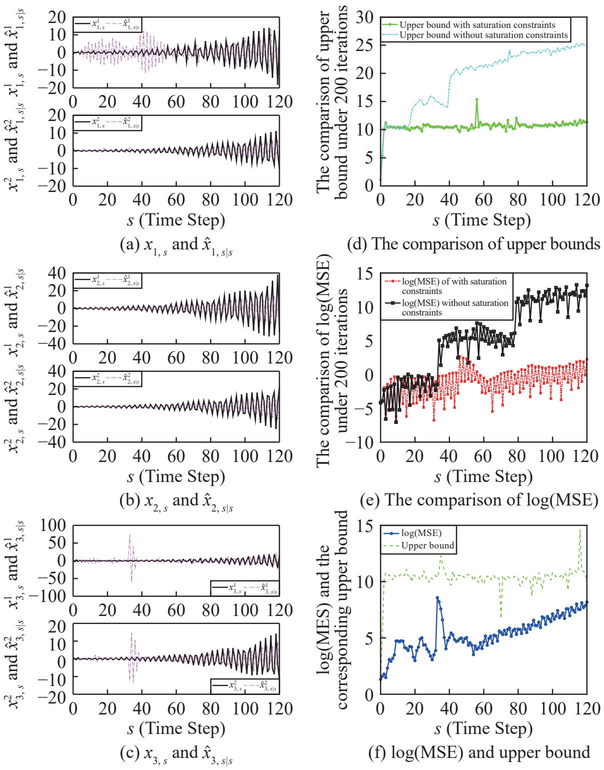

Fig. 1. The illustration of proposed RSE method.

well than the strategy without saturation constraint scheme. Moreover, Fig. 1(f) draws the log(MSE) and the upper bound.

Conclusions: This letter has investigated the DETCM-based RSE problem for a class of COCNs with ICs. Firstly, the DETCM has been adopted to adjust the mode of data transmission, thereby avoiding unnecessary energy consumption. Then, an attractive saturation constraint method has been introduced to attenuate the measurement outliers. An RSE approach to COCNs has been developed such that, for both DETCM and ICs, the trace of SEECUB is minimized via designing the SEGM properly. In addition, a theoretical result in regard to the mean-square uniform boundedness of error dynamic has been analyzed in detail.

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