H_∞ Tracking Control for Switched LPV Systems With an Application to Aero-Engines

Kongwei Zhu, Jun Zhao, *Senior Member, IEEE,* and Georgi M. Dimirovski, *Senior Member, IEEE*

***Abstract*—This paper focuses on the H_∞ model reference tracking control for a switched linear parameter-varying (LPV) model representing an aero-engine. The switched LPV aero-engine model is built based on a family of linearized models. Multiple parameter-dependent Lyapunov functions technique is used to design a tracking control law for the desirable H_∞ tracking performance. A control synthesis condition is formulated in terms of the solvability of a matrix optimization problem. Simulation result on the aero-engine model shows the feasibility and validity of the switching tracking control scheme.**

***Index Terms*—Aero-engine control, H_∞ tracking control, multiple parameter-dependent Lyapunov functions, switched linear parameter-varying (LPV) systems.**

I. Introduction

AERO-engines are fairly complex multi-variable nonlinear systems with a large variation in the system dynamics. Since an accurate analytical engine model is almost impossible to be obtained, the system analysis and synthesis have to be conducted using an approximate analytical model [1]. Many techniques for aero-engine control have been presented in literatures (see, [2]−[5] and the references therein). Nonlinear approaches are often hard to apply to a multi-dimensional control system. Approximate linearization techniques of nonlinear systems only ensure the performance around specific operating points, while exact input-output feedback linearization methods lack robustness. Zhao *et al.* proposed an approximate nonlinear engine model and a feedback linearization control strategy for local linear input-output performance [3]. A widely applied nonlinear control technique is the gain scheduling [6], [7], however it has no stability or performance guarantee for off-design points [8]. Linear parameter-varying (LPV) control methods can provide a systematic gain-scheduling method with guaranteed stability performance [8]. Analysis and synthesis of LPV systems for aero-engines have been studied widely [4], [9]. A model matching problem in the H_∞ and LPV framework is also investigated for an LPV turbofan engine model [9]. Polynomial LPV synthesis and fixed-order controller scheme was discussed for reduced complexity gain-scheduled control laws for a class of aircraft turbofan engines [4].

An alternative method is switching control where a family of controllers are designed at different operating points and the system performs controller switching based on the switching logic. The applications of this strategy are stimulated by the recent development of switched systems. A switched system consists of a finite number of subsystems and a switching law which usually depends on time, states, or both that determines switching between these subsystems [10]. Stability analysis and synthesis methods for switched systems have been widely studied in many literatures [11]−[16]. References [11], [13] studied the stability for linear and nonlinear switched systems in lower triangular form under an arbitrary switching law. As shown in [14], multiple Lyapunov functions technique was used to deal with a hybrid nonlinear control problem of switched systems. Besides that, the average dwell-time approach was also employed to investigate the stability and stabilization of switched systems [15]. The global robust stabilization problem for a class of uncertain switched nonlinear systems in lower triangular form was considered by [16] under any switching signal with dwell time specifications.

An LPV system is characterized as a smooth linear system with time-varying parameters. Modern aero-engines usually work in a large parameter variation range. Similar to the control of aircraft, the control of aero-engines is different in different parameter sub-regions [17], nevertheless these are more complicated than the control of aircrafts. The aircraft systems have been constructed as switched LPV models in some papers, such as [18], [19], therefore a single LPV model may not give sufficient approximation to nonlinear engine

Manuscript received June 17, 2015; accepted November 17, 2015. This work was supported by the National Natural Science Foundation of China (61304058, 61233002) and IAPI Fundamental Research Funds (2013ZCX03-01). Recommended by Associate Editor Ximing Sun. *(Corresponding author: Jun Zhao.)*

Citation: K. W. Zhu, J. Zhao, and G. M. Dimirovski, "H_∞ tracking control for switched LPV systems with an application to aero-engines," *IEEE/CAA J. of Autom. Sinica*, vol. 5, no. 3, pp. 699−705, May 2018.

K. W. Zhu is with the State Key Laboratory of Synthetical Automation for Process Industries (Northeastern University), College of Information Science and Engineering, Northeastern University, Shenyang 110819, China, and also with the School of Informatics, Linyi University, Linyi 276000, China (e-mail: zhukongwei@163.com).

J. Zhao is with the State Key Laboratory of Synthetical Automation for Process Industries (Northeastern University), College of Information Science and Engineering, Northeastern University, Shenyang 110819, China (e-mail: zhaojun@mail.neu.edu.cn).

G. M. Dimirovski is with the School of Engineering, Dogus University, and also with the School of Electrical-Electronics Engineering and Information Technologies, St. Cyril and St. Methodius University, Skopje, Macedonia, Istanbul TR-34722, R. Turkey (e-mail: gdimirovski@dogus.edu.tr).

Digital Object Identifier 10.1109/JAS.2016.7510025

dynamics over the entire operating range. A reasonable and natural idea is to adopt several LPV models and corresponding controllers, each suitable for a specific parameter sub-region, and switching among them [17], such LPV systems then become switched LPV systems. The switching LPV control approach can obtain a better approximation of the nonlinear dynamics and better performance than a single LPV control method [20]. Switched LPV systems have received considerable attention in the recent literatures [19], [21]. A complete overview of the stability results for LPV and switched LPV systems was given in [21]. Based on multiple parameter-dependent Lyapunov functions, a switching LPV control technique was presented for an F-16 aircraft via controller state reset using hysteresis and average dwell time switching law in [18]. Switching control for LPV systems in aero-engines is still an open and interesting issue.

On the other hand, due to the emergence of switching control in robotic systems and many other manufacturing processes, tracking control research on switched systems has received increasing attention. Based on the state-dependent switching method, sufficient conditions for the solvability of the state tracking control problem were given in [22]. l_1-l_∞ output tracking control problem was researched resorting to the average dwell time approach [23]. The above-mentioned results on the tracking control problem are only about general switched systems. A few results on the tracking control problem for LPV systems are also available. In [24] a model reference controller was designed using singular value decomposition to obtain the coefficient matrices. Based on poly-topic LPV models an LPV switching tracking control scheme was proposed for a flexible air-breathing hypersonic vehicle [25]. However, the switching LPV tracking control results for aero-engines are not found at present.

Motivated by the above discussions, this technical note studies the problem of switching H_∞ tracking control for an aero-engine model via multiple parameter-dependent Lyapunov functions technique. Compared with the existing results, the main contributions of our study can be summarized as follows: 1) This paper generalizes model reference H_∞ tracking problem to the switched LPV systems. We present a control design scheme to solve the H_∞ tracking control problem, and simulation result shows the effectiveness of the proposed control design method. 2) The LPV model does not fully exhibit the desired levels of reliability and flexibility with dramatic parameter variations and a large flight range. In order to overcome the aforesaid problem and to improve design accuracy, we introduce a switched LPV model for an aero-engine. The parameter region is divided into several sub-regions and LPV controllers are designed for each parameter sub-region to satisfy specified performance criterion.

The technical note is organized as follows. Section II gives the problem formulation and preliminaries. Section III gives a tracking control design technique. In Section IV we apply the designed method to a switched LPV aero-engine model. Finally, the conclusion is given in Section V.

The notations used in this paper are fairly standard. For a matrix X, X^T denotes its transpose. He$\{X\}$ is a shorthand notation of $X+X^T$. $X>0$ $(X\geq 0)$ and $X<0$ $(X\leq 0)$ denote positive definite (positive semi-definite) and negative definite (negative semi-definite), respectively. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{S}^n$ respectively denote sets of n-dimensional real vectors, $n\times m$-dimensional real matrices and $n\times n$-dimensional symmetric real matrices, $*$ denotes an abbreviated off-diagonal block in a symmetric matrix, and diag$\{X_1,\dots,X_k\}$ denotes a block-diagonal matrix composed of $X_1,\dots,X_k$.

II. Problem Formulation and Preliminaries

In this section, the problem will be formulated, and some preliminaries about the switched LPV systems will be given.

A. Problem Formulation

Consider the following switched LPV system:

$$\dot{x}(t)=A_\sigma(\rho)x(t)+B_\sigma(\rho)u(t)+\omega(t),\quad x(0)=0 \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^{n_u}$, and $\omega(t)\in\mathbb{R}^{n_\omega}$ are the state, the control input, and the disturbance input, respectively.

Suppose that the parameter ρ is in a compact set $\mathscr{P}\subset\mathbb{R}^s$ with its parameter variation rate bounded by $\underline{v}_k\leq\dot{\rho}_k\leq\bar{v}_k$, $k=1,2,\dots,s$. The set $\mathscr{P}$ is partitioned into a finite number of closed subsets $\{\mathscr{P}_i\}_{i\in\mathbb{Z}_N}$ by means of a family of switching surfaces $\boldsymbol{S}_{ij}$ $(i,j\in\mathbb{Z}_N)$, the adjacent parameter subsets are overlapped and $\mathscr{P}=\bigcup\mathscr{P}_i$, where $Z_N=\{1,2,\dots,N\}$ is the index set. The switching signal is defined as $\sigma:\mathbb{R}_+=[0,\infty)\rightarrow Z_N$ which is assumed to be a piecewise continuous (from the right) function depending on time or the parameter. The switching sequence is $\sum=\{x_;(i_0,t_0),(i_1,t_1),\dots,(i_j,t_j),\dots|i_j\in\mathbb{Z}_N,\ j=0,1,\dots\}$. $\sigma(t)=i$ means the ith subsystem is activated at time instant t. The system matrices $A_i(\rho)$, $B_i(\rho)$, $C_i(\rho)$ are of appropriate dimensions and all of the state-space data are continuous functions of the parameter ρ. The parameter ρ is exogenous variable and independent of the state x.

The reference state $x_r(t)$ is given by the reference model

$$\dot{x}_r(t)=A_rx_r(t)+r(t),\quad x_r(0)=0 \tag{2}$$

where $x_r(t)\in\mathbb{R}^{n_r}$ is the reference state, $r(t)\in\mathbb{R}^{n_r}$ is the bounded reference input, A_r is known Hurwitz matrix with compatible dimensions.

Remark 1: The reference signal can also be given by a parameter-dependent model. For this case we can replace (2) by $\dot{x}_r(t)=A(\rho)_rx_r(t)+r(t)$, the designed process for the parameter-dependent model is similar to the model (2). Without loss of generality, we consider the reference model as (2).

Combining the system (1) with the system (2), one can get the augmented system

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_r(t) \end{bmatrix} = \begin{bmatrix} A_\sigma(\rho)x(t)+B_\sigma(\rho)u(t) \\ A_r x_r(t) \end{bmatrix} + \begin{bmatrix} \omega(t) \\ r(t) \end{bmatrix}. \quad (3)$$

Give the following performance index:

$$\int_0^{t_f} e_r^T(t)e_r(t)dt < \gamma \int_0^{t_f} \bar{\omega}^T(t)\bar{\omega}(t)dt \quad (4)$$

where $e_r(t) = x(t) - x_r(t)$ denotes the state error, t_f is the control termination time, $\bar{\omega}(t) = (\omega^T(t), r^T(t))^T$, $\gamma > 0$ is the disturbance attenuation level.

The H_∞ tracking control performance of the switched system (3) can be stated as follows:

1) the system (3) is asymptotically stable, when the $\bar{\omega}(t) = 0$;

2) under the zero-initial condition, the inequality (4) holds for all $\bar{\omega}(t) \neq 0$, where γ is a constant number.

Our objective is to design both controller $u(t)$ and a switching law σ to enforce $x(t)$ of the system (1) to track the reference state $x_r(t)$ of the system (2).

B. Preliminaries

The following assumptions are adopted which are useful in our later development.

Assumption 1 [17]: The matrix function triple $(A_i(\rho), B_i(\rho), C_i(\rho))$ is parameter-dependent stabilizable and detectable for the parameter ρ.

Assumption 2 [12]: The switching signal σ has finite number of switchings occurring in any finite time interval. The subsystems and controllers are synchronously switching.

Lemma 1 [26]: Let D, E be real matrices of appropriate dimensions with $\|F\| \leq 1$. Then for any scalar $\gamma > 0$, the following inequality holds

$$DFE + E^T F^T D^T \leq \gamma^{-1} D D^T + \gamma E^T E. \quad (5)$$

III. Main Results

In this section, we will design the state feedback parameter-dependent controller and a switching law $\sigma(t)$ to solve the H_∞ tracking control problem via multiple Lyapunov functions approach.

The state feedback controller is given as

$$u(t) = K_i(\rho)e_r(t), \quad i \in \mathbb{Z}_N. \quad (6)$$

From augmented system (3) with controller (6) we have the following closed-loop system:

$$\dot{\bar{x}}(t) = \bar{A}_i(\rho)\bar{x}(t) + \bar{\omega}(t) \quad (7)$$

where

$$\bar{x}(t) = [x^T(t), x_r^T(t)]^T$$

$$\bar{A}_i(\rho) = \begin{bmatrix} A_i(\rho)+B_i(\rho)K_i(\rho) & -B_i(\rho)K_i(\rho) \\ 0 & A_r \end{bmatrix}.$$

Now we deal with the issue of parameter-dependent switching law $\sigma(t)$ to achieve H_∞ tracking performance with the help of parameter-dependent state feedback controller given in (6).

Firstly, we choose multiple parameter-dependent Lyapunov function candidate for the system (1) with the system (2) as

$$V_i(\bar{x}, \rho) = \bar{x}^T X_i(\rho)\bar{x} \quad (8)$$

where $X_i(\rho) > 0$, $i \in \mathbb{Z}_N$.

Hysteresis Switching Law: As previously mentioned, the set $\mathscr{P}$ is partitioned into number of subsets $\{\mathscr{P}_i\}_{i\in\mathbb{Z}_N}$ by $\boldsymbol{S}_{ij}$ (i, $j \in \mathbb{Z}_N$). Suppose that $\boldsymbol{S}_{ij}$ denotes the trajectory of ρ moving unidirectional from subset $\mathscr{P}_i$ to $\mathscr{P}_j$, and it is contrary to the switching surface $\boldsymbol{S}_{ji}$. The switching signal σ is described as

$$\text{When } t = 0, \quad \sigma(0) = i, \quad \text{if } \rho(0) \in \mathscr{P}_i$$

$$\text{When } t > 0, \begin{cases} \sigma(t) = i, & \text{if } \sigma(t^-) = i \text{ and } \rho(t) \in \mathscr{P}_i \\ \sigma(t) = j, & \text{if } \sigma(t^-) = i \text{ and } \rho(t) \in \mathscr{P}_j - \mathscr{P}_i. \end{cases} \quad (9)$$

For the switched closed-loop system (7), if on the switching surface $\boldsymbol{S}_{ij}$, the matrix $X_i(\rho)$ of (8) is satisfying: $X_i(\rho) \geq X_j(\rho)$, we have the Lyapunov function of (8) is non-increasing when switching from $\mathscr{P}_i$ to $\mathscr{P}_j$. For the switching surface $\boldsymbol{S}_{ji}$, we have a similar result when switching from $\mathscr{P}_j$ to $\mathscr{P}_i$, where $i, j \in \mathbb{Z}_N$.

Remark 2: The parameter subsets are partitioned by switching surfaces and the switching law is parameter-dependent in [17]. In [27], [28] the switching law is not parameter-dependent but rather is mode-dependent for dealing with the switching LPV control problem. In this paper, we studied the tracking problem for aero-engines, considering the character of practical problem, we choose the parameter-dependent switching law.

Remark 3: For the switching surface $\boldsymbol{S}_{ij}$, when the partition way for $\mathscr{P}$ is decided, the specific value ρ corresponding to $\boldsymbol{S}_{ij}$ is given such as ρ^*. The matrix inequality $X_i(\rho) \geq X_j(\rho)$ holds only on the surface $\boldsymbol{S}_{ij}$ that is $\rho = \rho^*$.

For the closed-loop switched LPV system (7), we state the synthesis condition of switching LPV control in the following theorem.

Theorem 1: Consider the augmented closed-loop system (7) with the parameter set $\mathscr{P}$ and its overlapped covering $\{\mathscr{P}_i\}_{i\in\mathbb{Z}_N}$. If there exist positive definite matrix functions $X_i(\rho) : \mathbb{R}^s \to \mathbb{S}^{2n}$, and matrix functions $K_i(\rho) : \mathbb{R}^s \to \mathbb{R}^{n\times m}$, such that for $\forall \rho \in \mathscr{P}_i$,

$$\begin{bmatrix} \mathrm{He}\{X_i(\rho)\bar{A}_i(\rho)\} + \dot{X}_i(\rho,\dot{\rho}) + Q & X_i(\rho) \\ X_i(\rho) & -\gamma I \end{bmatrix} < 0 \quad (10)$$

and

$$X_i(\rho) - X_j(\rho) \geq 0, \quad \rho \in \boldsymbol{S}_{ij} \quad (11)$$

where $Q = \begin{bmatrix} I & -I \\ -I & I \end{bmatrix}$ and $i, j \in \mathbb{Z}_N$. Then, under the switching signal satisfying (9), the feedback controller (6) solves the H_∞ tracking control problem for the system (1).

Proof: For the multiple parameter-dependent Lyapunov functions candidate (8), computing time derivative along the state variables trajectory of the system (7), we have

$$\begin{aligned}\dot{V}_i(\bar{x},\rho) = {}& \bar{x}^T(t)\left[X_i(\rho)\bar{A}_i(\rho)+\bar{A}_i^T(\rho)X_i(\rho)+\dot{X}_i(\rho,\dot{\rho})\right]\bar{x}(t)\\ &+2\bar{\omega}^T(t)X_i(\rho)\bar{x}(t)\end{aligned} \tag{12}$$

when $\bar{\omega}(t)=0$, $\dot{V}_i(\bar{x},\rho)=\bar{x}^T(t)[X_i(\rho)\bar{A}_i(\rho)+\bar{A}_i^T(\rho)X_i(\rho)+\dot{X}_i(\rho,\dot{\rho})]\bar{x}(t)$.

The matrix inequality (10) implies

$$X_i(\rho)\bar{A}_i(\rho)+\bar{A}_i^T(\rho)X_i(\rho)+\dot{X}_i(\rho,\dot{\rho})<-Q.$$

Because $Q\geq 0$, that is

$$X_i(\rho)\bar{A}_i(\rho)+\bar{A}_i^T(\rho)X_i(\rho)+\dot{X}_i(\rho,\dot{\rho})<0$$

which tell us that $\dot{V}_i(\bar{x},\rho)<0$, for any $\rho\in\mathscr{P}_i$. Moreover, due to the switching condition (11), at each switching surface $\boldsymbol{S}_{ij}$, we have $V_i(\bar{x},\rho)\geq V_j(\bar{x},\rho)$. Therefore, the augmented system (7) with $\bar{\omega}(t)=0$ is asymptotically stable under the controller (6) and the switching law (9).

Now we show the performance index (4) under the zero initial condition with $\bar{\omega}(t)\neq 0$.

From (12), applying Lemma 1, we have

$$2\bar{\omega}^T(t)X_i(\rho)\bar{x}(t)\leq\gamma^{-1}\bar{x}^T(t)X_i(\rho)X_i(\rho)\bar{x}(t)+\gamma\bar{\omega}^T(t)\bar{\omega}(t).$$

Then,

$$\begin{aligned}\dot{V}_i(\bar{x},\rho)\leq{}&\bar{x}^T(t)[\mathrm{He}\{X_i(\rho)\bar{A}_i(\rho)\}+\dot{X}_i(\rho,\dot{\rho})\\&+\gamma^{-1}X_i(\rho)X_i(\rho)]\bar{x}(t)+\gamma\bar{\omega}^T(t)\bar{\omega}(t).\end{aligned}$$

From the inequality (10) and with Schur complement, we have

$$\mathrm{He}\{X_i(\rho)\bar{A}_i(\rho)\}+\dot{X}_i(\rho,\dot{\rho})+\gamma^{-1}X_i(\rho)X_i(\rho)<-Q \tag{13}$$

that is

$$\dot{V}_i(\bar{x},\rho)<-\bar{x}^T(t)Q\bar{x}(t)+\gamma\bar{\omega}^T(t)\bar{\omega}(t) \tag{14}$$

since

$$\bar{x}^T(t)Q\bar{x}(t)=\begin{bmatrix}x(t)\\x_r(t)\end{bmatrix}^T\begin{bmatrix}I&-I\\-I&I\end{bmatrix}\begin{bmatrix}x(t)\\x_r(t)\end{bmatrix}=e_r^T(t)e_r(t)$$

it follows from (14) that

$$\dot{V}_i(\bar{x},\rho)<-e_r^T(t)e_r(t)+\gamma\bar{\omega}^T(t)\bar{\omega}(t). \tag{15}$$

Integrating both sides of (15) from zero to t_f we get

$$\begin{aligned}\int_0^{t_f}\sum_{i\in\mathbb{Z}_N}\dot{V}(\bar{x},\rho)dt&=\sum_{j=0}^{t_f}\sum_{i_j\in\mathbb{Z}_N}\int_{t_{i_j}}^{t_{i_j+1}}\dot{V}(\bar{x},\rho)dt\\&=V(\bar{x}(t_f),\rho)-V(\bar{x}(0),\rho)\\&<-\int_0^{t_f}e_r^T(t)e_r(t)dt+\gamma\int_0^{t_f}\bar{\omega}^T(t)\bar{\omega}(t)dt.\end{aligned}$$

According to the zero initial condition and $V(\bar{x}(t),\rho)>0$, it is easy to derive

$$\int_0^{t_f}e_r^T(t)e_r(t)dt<\gamma\int_0^{t_f}\bar{\omega}^T(t)\bar{\omega}(t)dt.$$

Therefore, under the switching law (9), the H_∞ tracking controller (6) solves the H_∞ tracking control problem for the switched LPV system (1). ■

Remark 4: The appearance of matrix Q in the inequality (10) is because of the $e_r(t)$. To show the relation between $e_r(t)$ and the states $x(t)$, $x_r(t)$, we just rewrite $e_r^T(t)e_r(t)$ into a compact form using matrix $Q=[I,-I;-I,I]$ and connect them together, then we introduce $e_r(t)$ into the inequality (10).

Since the matrix inequalities condition (10) of Theorem 1 are non-convex in $K_i(\rho)$ and parameter matrix variable $X_i(\rho)$, we convert these conditions into solvable linear matrix inequalities (LMIs).

Theorem 2: Consider the system (7) with the parameter set $\mathscr{P}$ and its overlapped covering $\{\mathscr{P}_i\}_{i\in\mathbb{Z}_N}$. If there exist positive definite matrix functions $Y_i(\rho):\mathbb{R}^s\to\mathbb{S}^n$, matrix functions $W_i(\rho):\mathbb{R}^s\to\mathbb{R}^{n\times m}$ and a constant $\gamma>0$, such that for any $\rho\in\mathscr{P}_i$, the following LMI holds

$$\begin{bmatrix}\Psi_{i11}&\Psi_{i12}&Y_i(\rho)&I&0\\ *&\Psi_{i22}&Y_i(\rho)&0&I\\ *&*&-I&0&0\\ *&*&*&-\gamma I&0\\ *&*&*&*&-\gamma I\end{bmatrix}<0 \tag{16}$$

and for $\rho\in\boldsymbol{S}_{ij}$

$$Y_i(\rho)-Y_j(\rho)\leq 0 \tag{17}$$

where

$$\begin{aligned}\Psi_{i11}&=\mathrm{He}\{A_i(\rho)Y_i(\rho)+B_i(\rho)W_i(\rho)\}-\sum_{k=1}^{s}\{\underline{v}_k,\bar{v}_k\}\frac{\partial Y_i(\rho)}{\partial\rho_k}\\ \Psi_{i12}&=-B_i(\rho)W_i(\rho)\\ \Psi_{i22}&=\mathrm{He}\{A_rY_i(\rho)\}-\sum_{k=1}^{s}\{\underline{v}_k,\bar{v}_k\}\frac{\partial Y_i(\rho)}{\partial\rho_k}.\end{aligned}$$

Then, under the switching law (9), the controller (6) with controller gain given by $K_i(\rho)=W_i(\rho)Y_i^{-1}(\rho)$, $i\in\mathbb{Z}_N$ solves the H_∞ tracking control problem for the system (1).

Proof: Choosing $X_i(\rho)=\mathrm{diag}\{\tilde{X}_i(\rho),\tilde{X}_i(\rho)\}$, where $\tilde{X}_i(\rho)\in\mathbb{S}^n$, and performing the congruence transformation via $\mathrm{diag}\{\tilde{X}_i^{-1}(\rho),\ \tilde{X}_i^{-1}(\rho),\ I\}$ on both sides of (10) result in

$$\begin{bmatrix}\Upsilon_{i,11}&\Upsilon_{i,12}&I&0\\ \Upsilon_{i,21}&\Upsilon_{i,22}&0&I\\ I&0&-\gamma I&0\\ 0&I&0&-\gamma I\end{bmatrix}<0 \tag{18}$$

where

$$\begin{aligned}\begin{bmatrix}\Upsilon_{i,11}&\Upsilon_{i,12}\\ \Upsilon_{i,21}&\Upsilon_{i,22}\end{bmatrix}&=\begin{bmatrix}\Xi_{i,11}&\Xi_{i,12}\\ *&\Xi_{i,22}\end{bmatrix}\\&\quad+\begin{bmatrix}\tilde{X}_i^{-1}(\rho)\tilde{X}_i^{-1}(\rho)&-\tilde{X}_i^{-1}(\rho)\tilde{X}_i^{-1}(\rho)\\ -\tilde{X}_i^{-1}(\rho)\tilde{X}_i^{-1}(\rho)&\tilde{X}_i^{-1}(\rho)\tilde{X}_i^{-1}(\rho)\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\Xi_{i,11} &= \mathrm{He}\{[A_i(\rho)\tilde{X}_i^{-1}(\rho)+B_i(\rho)K_i(\rho)\tilde{X}_i^{-1}(\rho)]\}\\ &\quad +\tilde{X}_i^{-1}(\rho)\dot{\tilde{X}}_i(\rho,\dot{\rho})\tilde{X}_i^{-1}(\rho)\\ \Xi_{i,12} &= -B_i(\rho)K_i(\rho)\tilde{X}_i^{-1}(\rho)\\ \Xi_{i,22} &= \mathrm{He}\{[A_r\tilde{X}_i^{-1}(\rho)]\}+\tilde{X}_i^{-1}(\rho)\dot{\tilde{X}}_i(\rho,\dot{\rho})\tilde{X}_i^{-1}(\rho).\end{aligned}$$

Because $\tilde{X}_i(\rho)\tilde{X}_i^{-1}(\rho)=I$, it is easy to see

$$\frac{d}{dt}[\tilde{X}_i(\rho)\tilde{X}_i^{-1}(\rho)]=0.$$

Then, we have

$$\frac{d}{dt}[\tilde{X}_i(\rho)]\tilde{X}_i^{-1}(\rho)+\tilde{X}_i(\rho)\frac{d}{\mathrm{d}t}\tilde{X}_i^{-1}(\rho)]=0.$$

Finally we can obtain

$$\tilde{X}_i^{-1}(\rho)\frac{d}{dt}[\tilde{X}_i(\rho)]\tilde{X}_i^{-1}(\rho)=-\frac{d}{dt}[\tilde{X}_i^{-1}(\rho)].$$

Defining $Y_i(\rho)=\tilde{X}_i^{-1}(\rho)$, then $\Xi_{i,11}$ and $\Xi_{i,22}$ can be formulated as

$$\begin{aligned}\Xi_{i,11} &= \mathrm{He}\{[A_i(\rho)Y_i(\rho)+B_i(\rho)K_i(\rho)Y_i(\rho)]\}\\ &\quad -\sum_{k=1}^{s}\{\underline{v}_k,\bar{v}_k\}\frac{\partial Y_i(\rho)}{\partial\rho_k}\\ \Xi_{i,12} &= -B_i(\rho)K_i(\rho)Y_i(\rho)\\ \Xi_{i,22} &= \mathrm{He}\{[A_rY_i(\rho)]\}-\sum_{k=1}^{s}\{\underline{v}_k,\bar{v}_k\}\frac{\partial Y_i(\rho)}{\partial\rho_k}.\end{aligned}$$

With Schur complement,

$$\begin{bmatrix}\Upsilon_{i,11} & \Upsilon_{i,12}\\ \Upsilon_{i,21} & \Upsilon_{i,22}\end{bmatrix}=\begin{bmatrix}\Xi_{i,11} & \Xi_{i,12} & Y_i(\rho)\\ * & \Xi_{i,22} & Y_i(\rho)\\ * & * & -I\end{bmatrix}.$$

Defining $W_i(\rho)=K_i(\rho)Y_i(\rho)$ and using Schur complement, we have solved LMIs (16). For any $\rho\in\boldsymbol{S}_{ij}$, the switching condition (11) is equivalent to

$$\begin{bmatrix}X_i(\rho) & 0\\ 0 & X_i(\rho)\end{bmatrix}-\begin{bmatrix}X_j(\rho) & 0\\ 0 & X_j(\rho)\end{bmatrix}\geq 0 \tag{19}$$

multiplying matrix $\mathrm{diag}\{\tilde{X}_i^{-1}(\rho),\tilde{X}_i^{-1}(\rho)\}$ to the right and $\mathrm{diag}\{\tilde{X}_j^{-1}(\rho),\tilde{X}_j^{-1}(\rho)\}$ to the left on both sides of inequality (19), we have the following result

$$\begin{bmatrix}\tilde{X}_j^{-1}(\rho) & 0\\ 0 & \tilde{X}_j^{-1}(\rho)\end{bmatrix}-\begin{bmatrix}\tilde{X}_i^{-1}(\rho) & 0\\ 0 & \tilde{X}_i^{-1}(\rho)\end{bmatrix}\geq 0 \tag{20}$$

consequently, the matrix inequality (17) can be obtained. ■

Remark 5: The diagonal structure of $X_i(\rho)$ may bring about some conservativeness, but based on this form we can give the solvable parameter-dependent LMIs. For this kind of LMIs, we cannot directly solve them. By forming a grid method for parameter values, we can approximately convert (16) and (17) to a finite collection of solvable LMIs. Then the continuous matrix function (6) can be formed by interpolation [8].

Remark 6: The notation $\sum_{k=1}^{s}\{\underline{v}_k,\bar{v}_k\}\frac{\partial}{\partial\rho_k}$ represents the combination of derivative terms with its variation rate which is taken as $\underline{v}_k$, or $\bar{v}_k$, $k=1,2,\ldots,s$. So it means in each inequality there are 2^s different LMIs to be checked. These approximate constraints may be conservative, but it is convenient to solve these LMIs by using MATLAB toolbox [17].

IV. Simulation Example

We will apply the designed method to a turbofan engine model to show the effectiveness of the designed scheme.

A. The Switched LPV Model of Aero-Engines

The turbofan engine model is corresponding to a large, high-bypass ratio two-spool turbofan engine similar to the GE90. It is based on the data from GE-90K engine of commercial modular aero-propulsion system simulation (CMAPSS) [29]. The input is W_F (fuel flow rate), the states are N_f (fan speed) and N_c (core speed). The altitude means the distance of the engine from the sea-level, the Mach number means the number determining the relative speed between the air flow and the aero-engine divided by the sound velocity.

A switched LPV model is established with the method of curve-fitting and small deviation linearization. A turbofan model is modeled by two different scheduling parameter sets as a switched LPV system. The turbofan engine model data is from [29]. The altitude and the fan speed are normalized by 10 000 and 3000, respectively. Then according to the curve-fitting method, based on a family of local linearized models, a switched LPV model of turbofan engine is given as

$$\dot{x}(t)=A_i(\rho)x(t)+B_i(\rho)u(t)+\omega(t)$$

where $x(t)=[\Delta N_f,\Delta N_c]^T$, $u(t)=\Delta W_F$. $\Delta N_f=N_f-N_{fe}$ is the fan speed increment, $\Delta N_c=N_c-N_{ce}$ is the core speed increment and $\Delta W_F=W_F-W_{Fe}$ is the fuel flow rate increment, respectively. Here $N_{fe}=2324/3000$ and $N_{ce}=8719/3000$. $\omega(t)$ is given by the health parameter input which can represent the disturbances or the effects of engine components aging [29].

For simplicity we only consider the Mach number as the gain scheduling parameter and let the altitude as 0. The variation range of the parameter ρ is $[0.20,0.9]$. To divide the parameter $\mathscr{P}$ into two sub-regions of $\mathscr{P}_1=[0.20,0.65]$ and $\mathscr{P}_2=[0.55,0.9]$, where $\mathscr{P}_1\bigcup\mathscr{P}_2=\mathscr{P}$ and the overlapping region is $[0.55,0.65]$. Here, we can divide the parameter set into smaller intervals and establish more subsystems for aero-engine to get better accuracy. Without loss of generality, we partition the parameter set into two subsets. $A_i(\rho)$, $B_i(\rho)$, $i=1$, 2 are parameterized in ρ by means of curve fitting, which are given as

$$A_1=\begin{bmatrix}-3.3786 & 1.3844\\ 0.7288 & -4.3411\end{bmatrix}+\rho\begin{bmatrix}-1.3835 & 0.0910\\ -1.2388 & -0.4899\end{bmatrix}$$

$$B_1=\begin{bmatrix}240.6075\\ 668.8695\end{bmatrix}+\rho\begin{bmatrix}-1\\ 105.8\end{bmatrix}$$

$$A_2=\begin{bmatrix}-1.2267 & 0.3977\\ -0.8172 & -0.6659\end{bmatrix}+\rho\begin{bmatrix}-1.3204 & 0.4585\\ 1.7429 & -2.5165\end{bmatrix}$$

$$B_2 = \begin{bmatrix} 259.4093 \\ 588.7365 \end{bmatrix} + \rho \begin{bmatrix} -23.8 \\ 186.2 \end{bmatrix}.$$

The reference model is given as

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \quad x_r(0) = 0$$

where $x_r(t) = [\Delta N_f, \Delta N_c]^T$, and the state space data of A_r is

$$A_r = \begin{bmatrix} -2.915 & 1.0362 \\ 0.7871 & -3.4432 \end{bmatrix}.$$

B. H_∞ Model Reference Tracking Control Problem of the Switched LPV Model

We conduct simulation for the turbofan engine model with a varying parameter. The time-varying Mach number trajectory is shown in Fig. 1. Choosing disturbance $\omega(t) = [e^{-2t}, e^{-2t}]^T$ and the reference input $r(t) = [0.01\sin(0.01t), 0.01\sin(0.01t)]^T$. In addition, the initial value of the switched LPV system (1) is assumed to be $x(t_0) = [\Delta N_f, \Delta N_c]^T = [0.2, 0.25]^T$.

To solve the optimization problem (16) and (17) in Theorem 2, we obtain

$$Y_1(\rho) = \begin{bmatrix} 2.2844 & -0.711 \\ -0.711 & 3.4579 \end{bmatrix} + \rho \begin{bmatrix} 0.0059 & -0.006 \\ -0.006 & 0.0143 \end{bmatrix}$$

$$Y_2(\rho) = \begin{bmatrix} 1.5191 & -0.234 \\ -0.234 & 3.1166 \end{bmatrix} + \rho \begin{bmatrix} 0.0102 & -0.008 \\ -0.008 & 0.0535 \end{bmatrix}$$

$$K_1(\rho) = [-0.0021, -0.0028] + \rho[0.0010, -0.0004]$$

$$K_2(\rho) = [-0.0026, -0.0024] + \rho[0.0005, 0.0001].$$

According to Theorem 2, we solve the H_∞ tracking control optimization problem for the switched system (1), and the H_∞ disturbance attenuation index is $\gamma = 0.4672$ over the entire parameter set. Compared with the method of [9] for single LPV model, we get $\gamma = 0.4810$. The switched LPV H_∞ tracking scheme has a smaller disturbance attenuation level and a better performance than general LPV control method.

The H_∞ tracking control problem for the switched system (1) is solved. The switching signal is shown in Fig. 2. The tracking control error is depicted in Fig. 3. The fan speed increment and fuel flow increment are shown in Figs. 4 and 5, respectively. The simulation result shows the effectiveness of the designed scheme.

V. Conclusion

In this paper, we have studied the H_∞ model reference tracking control problem for the switched LPV systems. A switched LPV turbofan engine model for different parameter regions has been constructed from a family of linearized engine models. Sufficient conditions have been developed to guarantee the H_∞ model tracking performance using the multiple parameter-dependent Lyapunov functions method. The desired tracking controller gain has been obtained by a set of LMIs. The tracking control under a hysteresis switching law was applied to the obtained switched LPV engine model, and promising simulation result is obtained.

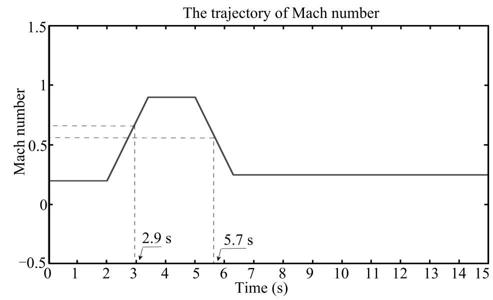

Fig. 1. The gain scheduling parameter.

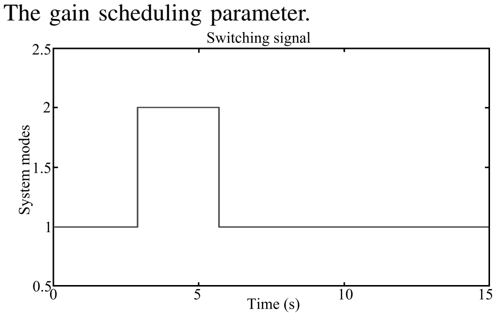

Fig. 2. The switching signal.

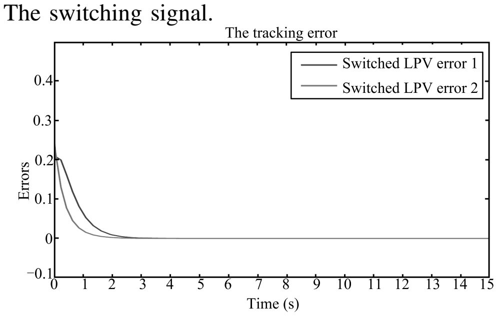

Fig. 3. The tracking control error.

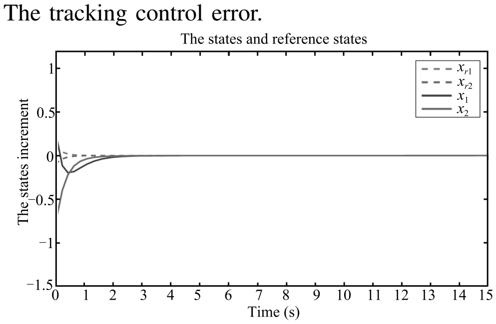

Fig. 4. The fan speed increment.

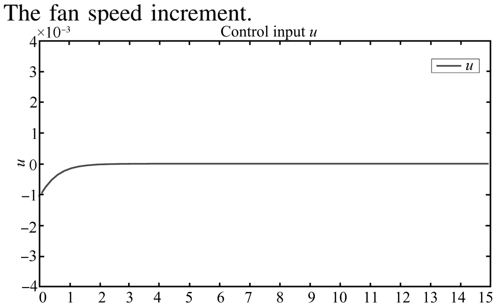

Fig. 5. The fuel flow increment.

References

[1] D. Henrion, L. Reberga, J. Bernussou, and F. Vary, "Linearization and identification of aircraft turbofan engine models," in *Proc. IFAC Symp. Automatic Control in Aerospace*, St Petersburg, Russia, 2004.

[2] D. K. Frederick, S. Garg, and S. Adibhatla, "Turbofan engine control design using robust multivariable control technologies," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 6, pp. 961−970, Nov. 2000.

[3] H. Zhao, J. F. Liu, and D. R. Yu, "Approximate nonlinear modeling and feedback linearization control for aeroengines," *J. Eng. Gas Turbines Power*, vol. 133, no. 11, pp. 111601, May 2011.

[4] W. Gilbert, D. Henrion, J. Bernussou, and D. Boyer, "Polynomial LPV synthesis applied to turbofan engines," *Control Eng. Pract.*, vol. 18, no. 9, pp. 1077−1083, Sep. 2010.

[5] J. X. Mu, D. Rees, and G. P. Liu, "Advanced controller design for aircraft gas turbine engines," *Control Eng. Pract.*, vol. 13, no. 8, pp. 1001−1015, Aug. 2005.

[6] M. Pakmehr, N. Fitzgerald, E. M. Feron, J. S. Shamma, and A. Behbahani, "Gain scheduling control of gas turbine engines: stability by computing a single quadratic Lyapunov function," in *Proc. of ASME Turbo Expo 2013: Turbine Tech. Conf. and Expo.*, San Antonio, Texas, USA, 2013.

[7] G. Wolodkin, G. J. Balas, and W. L. Garrard, "Application of parameter-dependent robust control synthesis to turbofan engines," *J. Guid. Control Dyn.*, vol. 22, no. 6, pp. 833−838, Nov. 1999.

[8] W. J. Rugh and J. S. Shamma, "Research on gain scheduling," *Automatica*, vol. 36, no. 10, pp. 1401−1425, Oct. 2000.

[9] G. J. Balas, "Linear, parameter-varying control and its application to a turbofan engine," *Int. J. Robust Nonlinear Control*, vol. 12, no. 9, pp. 763−796, Jul. 2002.

[10] H. Lin and P. J. Antsaklis, "Stability and stabilizability of switched linear systems: a survey of recent results," *IEEE Trans. Autom. Control*, vol. 54, no. 2, pp. 308−322, Feb. 2009.

[11] R. C. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no. 11, pp. 1819−1823, Nov. 2010.

[12] D. Liberzon, *Switching in Systems and Control*. Boston, USA: Birkhäuser, 2003, pp. 21−34

[13] J. Fu, R. C. Ma, and T. Y. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, pp. 360−373, Apr. 2015.

[14] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Trans. Autom. Control*, vol. 43, no. 4, pp. 475−482, Apr. 1998.

[15] X. M. Sun, J. Zhao, and D. J. Hill, "Stability and L_2-gain analysis for switched delay systems: a delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769−1774, Oct. 2006.

[16] R. C. Ma, Y. Liu, S. Z. Zhao, M. Wang, and G. D. Zong, "Nonlinear adaptive control for power integrator triangular systems by switching linear controllers," *Int. J. Robust Nonlinear Control*, vol. 25, no. 14, pp. 2443−2460, Sep. 2015.

[17] B. Lu and F. Wu, "Switching LPV control designs using multiple parameter-dependent Lyapunov functions," *Automatica*, vol. 40, no. 11, pp. 1973−1980, Nov. 2004.

[18] B. Lu, F. Wu, and S. W. Kim, "Switching LPV control of an F-16 aircraft via controller state reset," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 2, pp. 267−277, Mar. 2006.

[19] X. He and J. Zhao, "Multiple Lyapunov functions with blending for induced L_2-norm control of switched LPV systems and its application to an F-16 aircraft model," *Asian J. Control*, vol. 16, no. 1, pp. 149−161, Jan. 2014.

[20] S. Lim and J. P. How, "Modeling and H_∞ control for switched linear parameter-varying missile autopilot," *IEEE Trans. Control Syst. Technol.*, vol. 11, no. 6, pp. 830−838, Nov. 2003.

[21] F. Blanchini, S. Miani, and C. Savorgnan, "Stability results for linear parameter varying and switching systems," *Automatica*, vol. 43, no. 10, pp. 1817−1823, Oct. 2007.

[22] Q. K. Li, J. Zhao, and G. M. Dimirovski, "Robust tracking control for switched linear systems with time-varying delays," *IET Control Theory Appl.*, vol. 2, no. 6, pp. 449−457, Jun. 2008.

[23] L. L. Hou, G. D. Zong, Y. Q. Wu, and Y. C. Cao, "Exponential l_2-l_∞ output tracking control for discrete-time switched system with time-varying delay," *Int. J. Robust Nonlinear Control*, vol. 22, no. 11, pp. 1175−1194, Jul. 2012.

[24] A. Abdullah and M. Zribi, "Model reference control of LPV systems," *J. Franklin Inst.*, vol. 346, no. 9, pp. 854−871, Nov. 2009.

[25] Y. Q. Huang, C. Y. Sun, C. S. Qian, and L. Wang, "Non-fragile switching tracking control for a flexible air-breathing hypersonic vehicle based on polytopic LPV model," *Chin. J. Aeronaut.*, vol. 26, no. 4, pp. 948−959, Aug. 2013.

[26] Y. Y. Cao, Y. X. Sun, and C. W. Cheng, "Delay-dependent robust stabilization of uncertain systems with multiple state delays," *IEEE Trans. Autom. Control*, vol. 43, no. 11, pp. 1608−1612, Nov. 1998.

[27] L. X. Zhang and P. Shi, "l_2-l_∞ model reduction for switched LPV systems with average dwell time," *IEEE Trans. Autom. Control*, vol. 53, no. 10, pp. 2443−2448, Nov. 2008.

[28] Q. G. Lu, L. X. Zhang, H. R. Karimi, and Y. Yang, "H_∞ control for asynchronously switched linear parameter-varying systems with mode-dependent average dwell time," *IET Control Theory Appl.*, vol. 7, no. 5, pp. 673−683, Mar. 2013.

[29] H. Richter, *Advanced Control of Turbofan Engines*. New York, USA: Springer, 2012.

Kongwei Zhu graduated from Shandong Normal University (SNU), China, in 1997. He received the M.Sc. degree from Southeast University (SEU), Nanjing, China, in 2006. He received the Ph.D. degree from the College of Information Science and Engineering, Northeastern University (NEU), Shenyang, China, in 2018. He is also a Lecturer at the School of Informatics, Linyi University, Shandong, China. His research interests include switched LPV systems and aeroengines control.

Jun Zhao is a Professor at Northeastern University. He received the B.Sc. and M.Sc. degrees in mathematics from Liaoning University, Shenyang, China, in 1982 and 1984, respectively, and the Ph.D. degree in control theory and applications from Northeastern University, Shenyang, China, in 1991. He was a Post-Doctoral Fellow with Northeastern University from 1992 to 1993. Since 1994, he has been a Professor with the College of Information Science and Engineering, Northeastern University. He was a Senior Visiting Scholar with the Coordinated Science Laboratory, University of Illinois at Urbana Champaign, Champaign, IL, USA, from 1998 to 1999, was a Research Fellow with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong, from 2003 to 2005, and was a Fellow of the School of Engineering with Australian National University, Melbourne, ACT, Australia, from 2007 to 2010. His research interests include switched systems, nonlinear systems, and network synchronization.

Georgi M. Dimirovski is a Research Professor (lifetime) of automation and systems engineering at the School of Electrical-Electronics Engineering and Information Technologies at St. Cyril and St. Methodius University of Skopje, Macedonia, and a Guest Professor of computer science and information technologies at Dogus University of Istanbul. He is elected as a Foreign Member of the Academy of Engineering Sciences in Belgrade, Serbia, since 2004 and an Active Member of European Academy of Sciences and Arts in Salzburg, Austria since 2015. He was awarded honorary 'Pro University' Professor of Obuda University of Budapest, Hungary in 2011, and the Outstanding Associate Editor 2012 by the *International Journal of Automation and Computing* and Springer. As an Associate Editor, he serves on Editorial Boards of Journal of the *Franklin Institute, Asian Journal of Control, and International Journal of Automation and Computing.* Before, as Associate Editor he served *IEEE Transactions on Systems, Man, and Cybernetics, Proceedings of the Institution of Mechanical Engineers Part I: Systems and Control Engineering*. During 1988−1993, he served the European Science Foundation on Its Executive Council. During 1985−1991, he served three terms as President of Yugoslav Association for ETAN, then Yugoslav IFAC NMO, and of the ETAI Society, Macedonian IFAC NMO. He served the IFAC in many capacities during 1996−2011, Technical Board Member inclusive. Professor Dimirovski received the 2009 IET Premium Award for the best article of the IET-CTA journal in 2008. His research interests include fuzzy, neural, nonlinear, switched systems and control, computational intelligence, as well as artificial intelligence and society.