Stabilization of Uncertain Systems With Markovian Modes of Time Delay and Quantization Density

Jufeng Wang and Chunfeng Liu

***Abstract*—This work studies the stabilization of a class of control systems that use communication networks as signal transmission medium. The lateral motion of independently actuated four-wheel vehicle is modeled as an uncertain-linear system. Time delay and quantization density are modeled as Markov chains. The networked control systems (NCSs) with plants being lateral motion are first transformed to switched linear systems with uncertain parameters. Sufficient and necessary conditions for the stochastic stability of closed-loop networked control systems are then established. By solving the matrix inequalities, this work presents an output-feedback controller that depends on the modes of time delay and quantization density. The controller performance is illustrated via a vehicular lateral motion system.**

***Index Terms*—Networked control system (NCS), quantization, stabilization, time delay, vehicle lateral motion.**

I. Introduction

FOUR-wheel independently actuated vehicles in which each wheel is independently actuated by an in-wheel motor, have attracted increasing research efforts in recent years due to their actuation flexibility and fast speed. As is well known, stability is an important problem to be considered in system analysis and design. To ensure the stability of vehicle lateral motion, the study on lateral motion control has been actively conducted since effective lateral motion control can prevent unintended vehicle behavior.

In [1], through an analytical method, a vehicle lateral-plane motion stability control approach is presented. In [2], a control law combined with a stabilization algorithm of the yaw motion is given based on a robot dynamic model. To maintain lateral stability, there are various controllers proposed, such as a sliding mode controller [3], a state observer [4] and an output constrained controller [5]. Nevertheless, the results in them cannot be applied to the networked control systems (NCSs) whose plants, controllers and actuators are located at different places, and signals are transmitted from one place to another through communication networks.

Manuscript received April 4, 2017; accepted June 6, 2017. This work was supported by Humanities and Social Sciences Youth Foundation of the Ministry of Education (17YJC630093). Recommended by Associate Editor Mengchu Zhou. *(Corresponding author: Chunfeng Liu.)*

Citation: J. F. Wang and C. F. Liu, "Stabilization of uncertain systems with Markovian modes of time delay and quantization density" *IEEE/CAA J. of Autom. Sinica*, vol. 5, no. 2, pp. 463−470, Mar. 2018.

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Digital Object Identifier 10.1109/JAS.2017.7510823

With the very rapid advances in communication network, NCSs have gained wide applications in modern vehicles [6]−[8]. Compared to the traditional control systems, NCSs have many advantages such as system flexibility and reduced cost. Despite such advantages, the use of communication networks makes the system analysis more complicated.

Owing to bandwidth limitation, data cannot be sent with infinite precision in communication networks. To reduce network congestion, quantizers are always used in a signal transmission process. In [9]−[11], the stabilization of linear time-invariant systems with quantization is investigated. Nevertheless, time delay is not taken into account.

In practice, time delay always occurs since sampling data is transmitted through a network, and it may cause system instability. Therefore, the stabilization problem of NCSs with time delay has attracted much research [12]−[16]. In many cases, time delay is random and can be modeled as Markov chains [17]−[22]. In [17], the state-feedback controller's gain is constant. This controller is called a mode-independent controller in our paper. In [18]−[22], a feedback controller that depends on time delay is designed. We call such controller that depends on physical variables, e.g., time delay and quantization density, as a mode-dependent controller.

Compared to NCSs with only time delay or quantization, it is more difficult to analyze those with both time delay and quantization. In [23]−[28], the latter are studied, but their feedback controllers are all static and controlled plants are all deterministic systems. As is well known, the stability condition of a system with mode-dependent controller is less conservative than that of a system with a mode-independent controller. Meanwhile, due to interference, the parameters of vehicle lateral dynamics, e.g., longitudinal speed and cornering stiffness coefficients, are subject to change. Such change may destroy the stability of otherwise stable closed-loop lateral-motion systems. If only a deterministic model of a vehicle lateral dynamics is considered in system analysis, the resultant system may exhibit a high degree of vulnerability. Accordingly, in this paper, we model the vehicle lateral dynamics as an uncertain system.

It should be pointed out, the stabilization problem of NCSs with time delay and quantization has not been fully investigated, and most of the results in the existing literature are focused on sufficient conditions for the stability of NCSs. It is worth mentioning that sufficient and necessary conditions for the stability of NCSs have been studied in [18]−[20]. However, the plant studied in the literature mentioned above is a deterministic system or a Markovian jump linear system. Moreover, the controller design considers time delay only but not quantization. Therefore, the results cannot be directly applied to our case where the plant is an uncertain lateral motion system having time delay and quantization error. To the authors' best knowledge, the stabilization of vehicle lateral motion with time delay and quantization has not been fully investigated, especially the sufficient and necessary conditions for the stability of uncertain lateral motion systems having time delay and quantization density with Markovian characterization. This fact motivates the present study.

This work for the first time studies the stabilization of vehicle lateral motion subject to both quantization and time delay. The lateral motion of independently actuated four-wheel vehicle is modelled as an uncertain system. To incorporate the correlation between the current time delay (quantization density) and time delay (quantization density) in the next transmission, the quantization density and time delay are modeled as two homogeneous Markov chains. The sufficient and necessary conditions for the stochastic stability of networked vehicle lateral motion are derived under an output-feedback controller that depends on the modes of time delay and quantization density. A practical lateral motion example is presented to demonstrate the effectiveness of the proposed controller.

II. Networked Control System Model

The structure of our concerned NCS is shown in Fig. 1. Its plant is vehicle lateral dynamics shown in Fig. 2 [22]. A two-degree-of-freedom model is adopted to describe the plant. With the fact that side slip angle, steering angle and lateral acceleration are small, the tire lateral force F_{yf} (F_{yr}) is approximately linear with the tire slip angle α_f (α_r), and the state-space model of the lateral motion control can be approximately written as [4], [22]

$$\begin{aligned}\dot{x}(t) &= A_1x(t) + B_1u(t)\\ y(t) &= C_1x(t)\end{aligned} \tag{1}$$

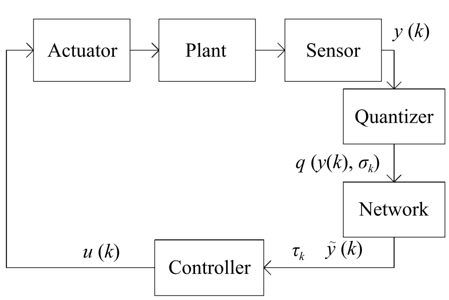

Fig. 1. The structure of NCS.

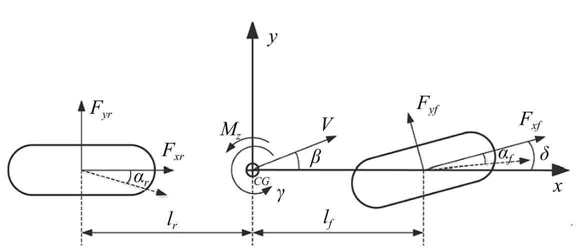

Fig. 2. Vehicle lateral dynamics.

where

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, \quad u = \begin{bmatrix} \delta \\ M_z \end{bmatrix}$$

$$A_1 = \begin{bmatrix} \frac{-2(C_f+C_r)}{mV} & \frac{-2(C_fl_f-C_rL_r)}{mV^2} - 1 \\ \frac{-2(C_fl_f-C_rl_r)}{I_z} & \frac{-2(C_fl_f^2+C_rl_r^2)}{I_zV} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{2C_f}{mV} & 0 \\ \frac{2C_fl_f}{I_z} & \frac{1}{I_z} \end{bmatrix}.$$

Here, $y(t)$ is the output of the plant, C_1 is a constant matrix of appropriate dimensions, γ is the yaw-rate, m is the vehicle mass, V is the longitudinal speed, M_z is the yaw moment, δ is the front wheel steering angle, C_f and C_r are the cornering stiffness of each front tire and rear tire, I_z is the vehicle yaw inertia, and l_f and l_r are the distances from the front and rear axles to the center of gravity.

In the NCS, the sensor, controller and actuator are all time-driven. With a sampling period T, the continuous state-space model (1) can be transformed into a discrete one as follows [29]:

$$\begin{aligned}x(k+1) &= Ax(k) + B(k)u(k)\\ y(k) &= Cx(k)\end{aligned} \tag{2}$$

where

$$A = e^{A_1T}, \quad B = \int_0^T e^{At}dt \cdot B_1, \quad C = C_1.$$

From the process of modeling, we know that model (2) well describes the actual vehicle lateral dynamics, and is affected by parametric uncertainties (e.g., the uncertainties on the longitudinal speed and cornering stiffness coefficients). In order to consider the model approximation and model parameter uncertainty, we modify the discrete-linear system (2) into a discrete-uncertain system described by

$$\begin{aligned}x(k+1) &= [A + \Delta A(k)]x(k) + [B + \Delta B(k)]u(k)\\ y(k) &= Cx(k)\end{aligned} \tag{3}$$

where $\Delta A(k)$ and $\Delta B(k)$ are unknown matrices representing the time-varying norm-bounded uncertainties that satisfy the following condition

$$[\Delta A(k)\ \Delta B(k)] = MJ(k)[Y_1\ Y_2]$$

where M, Y_1 and Y_2 are known real constant matrices with appropriate dimensions, and $J(k)$ is the unknown time-varying matrix function subject to $J(k)^TJ(k) \leq I$. In Fig. 1, τ_k represents the sensor-to-controller delay, and $q(\cdot, \sigma_k)$ stands for the quantizer with quantization density ρ_{σ_k}, $0 < \rho_{\sigma_k} < 1$, $\sigma_k \in N_\eta^+ = \{1, 2, \ldots, \eta\}$. $\eta \in N^+ = \{1, 2, \ldots\}$. Let $N = \{0, 1, 2, \ldots\}$, the set of natural numbers, and the value of σ_k corresponds to that of ρ_{σ_k}. To ease the network congestion, the quantization density is designed to be a function of the network load which is related to the network induced delay [30]. Considering the correlation between the current time delay (quantization density) and time delay (quantization density) in the next transmission, τ_k and σ_k (ρ_{σ_k}) are modeled as two homogeneous Markov chains that take values in $N_\tau = \{0, 1, \ldots, \tau\}$, $\tau \in N$ and N_η^+ $(\{\rho_1, \rho_2, \ldots, \rho_\eta\})$, respectively. Their transition probability matrices are $\Gamma = [\lambda_{lh}]$ and $\Xi = [\alpha_{ij}]$, respectively. τ_k and σ_k (ρ_{σ_k}) jump from modes l to h and from modes i (ρ_i) to j (ρ_j) with probabilities λ_{lh} and α_{ij}, respectively

$$\begin{aligned}\lambda_{lh} &= Pr(\tau_{k+1} = h|\tau_k = l)\\ \alpha_{ij} &= Pr(\sigma_{k+1} = j|\sigma_k = i)\\ &= Pr(\rho_{\sigma_{k+1}} = \rho_j|\rho_{\sigma_k} = \rho_i)\end{aligned} \tag{4}$$

where λ_{lh}, $\alpha_{ij} \geq 0$ and

$$\sum_{h=0}^{\tau} \lambda_{lh} = 1, \quad \sum_{j=1}^{\eta} \alpha_{ij} = 1. \tag{5}$$

The quantizer $q(y, j)$ is proposed as follows:

$$q(y, j) = [q_1(y_1, j),\ q_2(y_2, j), \ldots,\ q_p(y_p, j)]^T.$$

Let the quantization density ρ in [23] equal ρ_j. We can then describe the corresponding set of quantization levels of quantizer $q(y, j)$ as follows:

$$U_j = \Big\{ \pm \boldsymbol{u}_i^{(j)} : \boldsymbol{u}_i^{(j)} = \rho_j^i \boldsymbol{u}_0, \quad \boldsymbol{u}_0 > 0, \\ i = 0, \pm 1, \pm 2, \ldots \Big\} \bigcup \Big\{0\Big\}.$$

The corresponding $q_l(y_l, j)$ is defined as follows:

$$q_l(y_l, j) = \begin{cases} \boldsymbol{u}_i^{(j)}, & \text{if } \frac{1}{1+\delta_j}\boldsymbol{u}_i^{(j)} < y_l \leq \frac{1}{1-\delta_j}\boldsymbol{u}_i^{(j)} \\ 0, & \text{if } y_l = 0 \\ -q_l(-y_l, j), & \text{if } y_l < 0 \end{cases} \tag{6}$$

with $\delta_j = (1 - \rho_j)/(1 + \rho_j)$.

From (6), $q(y(k), j)$ can be rewritten as

$$q(y(k), j) = (I + H(j))y(k)$$

where $H(j)$ is an uncertain diagonal matrix satisfying

$$H^T(j)H(j) \leq \delta_j^2 I.$$

Considering the time delay, we have

$$\begin{aligned}\tilde{y}(k) &= q\left(y(k - \tau_k), \sigma_{k-\tau_k}\right)\\ &= \left(I + H(\sigma_{k-\tau_k})\right) Cx(k - \tau_k).\end{aligned} \tag{7}$$

Define

$$X(k) = \begin{bmatrix} x^T(k) & x^T(k-1) & \ldots & x^T(k-\tau) \end{bmatrix}^T \tag{8}$$

then, we have

$$\begin{aligned}X(k+1) &= \tilde{A}X(k) + \tilde{B}u(k)\\ \tilde{y}(k) &= \left(I + H(\sigma_{k-\tau_k})\right) \tilde{C}(\tau_k)X(k)\end{aligned} \tag{9}$$

where

$$\tilde{A} = \begin{bmatrix} A + \Delta A(k) & 0 & \ldots & 0 & 0 \\ I & 0 & \ldots & 0 & 0 \\ 0 & I & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & I & 0 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} B + \Delta B(k) \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$\tilde{C}(\tau_k) = \begin{bmatrix} 0 & \ldots & 0 & C & 0 & \ldots & 0 \end{bmatrix} \tag{10}$$

C is the $(1 + \tau_k)$th block of $\tilde{C}(\tau_k)$.

A two-mode-dependent output-feedback controller is designed as

$$\begin{aligned}z(k+1) &= D(\tau_k, \sigma_{k-\tau_k})z(k) + E(\tau_k, \sigma_{k-\tau_k})\tilde{y}(k)\\ u(k) &= F(\tau_k, \sigma_{k-\tau_k})z(k) + G(\tau_k, \sigma_{k-\tau_k})\tilde{y}(k)\end{aligned} \tag{11}$$

where $z(k) \in \mathbb{R}^n$, $D(\tau_k, \sigma_{k-\tau_k})$, $E(\tau_k, \sigma_{k-\tau_k})$, $F(\tau_k, \sigma_{k-\tau_k})$ and $G(\tau_k, \sigma_{k-\tau_k})$ are appropriately dimensioned matrices.

From (9), it is easy to see that

$$\begin{aligned}z(k+1) =\ & D(\tau_k, \sigma_{k-\tau_k})z(k) + E(\tau_k, \sigma_{k-\tau_k})\\ & \times \left(I + H(\sigma_{k-\tau_k})\right) \tilde{C}(\tau_k)X(k)\\ u(k) =\ & F(\tau_k, \sigma_{k-\tau_k})z(k) + G(\tau_k, \sigma_{k-\tau_k})\\ & \times \left(I + H(\sigma_{k-\tau_k})\right) \tilde{C}(\tau_k)X(k).\end{aligned} \tag{12}$$

Define

$$\xi(k) = \begin{bmatrix} X^T(k) & z^T(k) \end{bmatrix}^T. \tag{13}$$

Combining (9) and (12) leads to the following closed-loop system

$$\xi(k+1) = \left[\bar{A} + \bar{B}K(\tau_k, \sigma_{k-\tau_k})\bar{C}(\tau_k, \sigma_{k-\tau_k})\right] \xi(k) \tag{14}$$

where

$$\bar{A} = \begin{bmatrix} \tilde{A} & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} \tilde{B} & 0 \\ 0 & I \end{bmatrix}$$

$$\begin{aligned}\bar{C}(\tau_k, \sigma_{k-\tau_k}) &= \begin{bmatrix} \left(I + H(\sigma_{k-\tau_k})\right) \tilde{C}(\tau_k) & 0 \\ 0 & I \end{bmatrix}\\ &= \begin{bmatrix} \tilde{C}(\tau_k) & 0 \\ 0 & I \end{bmatrix}\\ &\quad + \begin{bmatrix} I \\ 0 \end{bmatrix} H(\sigma_{k-\tau_k}) \left(\tilde{C}(\tau_k)\ 0\right)\\ &= \hat{C}(\tau_k) + \begin{bmatrix} I \\ 0 \end{bmatrix} H(\sigma_{k-\tau_k}) \times \left(\tilde{C}(\tau_k)\ 0\right)\end{aligned}$$

$$K(\tau_k, \sigma_{k-\tau_k}) = \begin{bmatrix} G(\tau_k, \sigma_{k-\tau_k}) & F(\tau_k, \sigma_{k-\tau_k}) \\ E(\tau_k, \sigma_{k-\tau_k}) & D(\tau_k, \sigma_{k-\tau_k}) \end{bmatrix}. \tag{15}$$

Note that matrices $\bar{A}$ and $\bar{B}$ are of the following form

$$\begin{aligned}\bar{A} &= A_a + \tilde{M}J(k)\tilde{Y}_1\\ \bar{B} &= B_b + \tilde{M}J(k)\tilde{Y}_2\end{aligned}$$

where

$$A_a = \begin{bmatrix} A & 0 & \ldots & 0 & 0 & 0 \\ I & 0 & \ldots & 0 & 0 & 0 \\ 0 & I & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & 0 & 0 \\ 0 & 0 & \ldots & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_b = \begin{bmatrix} B & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}$$

$$\tilde{M}=\begin{bmatrix} M \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ \tilde{Y_1}=\begin{bmatrix} Y_1 & 0 & \ldots & 0 \end{bmatrix},\ \tilde{Y_2}=\begin{bmatrix} Y_2 & 0 \end{bmatrix}.$$

Definition 1 [19]: The closed-loop system in (14) is stochastically stable if given every initial condition $\xi_0=\xi(0)$, $\sigma_{-\tau_0}\in N_\eta^+$ and $\tau_0\in N_\tau$, there exists a symmetric and positive definite matrix W such that the following holds:

$$\mathcal{E}\left\{\sum_{k=0}^{\infty}\|\xi(k)\|^2|\xi_0,\sigma_{-\tau_0},\tau_0\right\}<\xi_0^TW\xi_0. \tag{16}$$

Lemma 1 [31]: Given matrices $Q,\ H$ and E of appropriate dimensions with Q being symmetrical, $Q+HFE+E^TF^TH^T<0$ for all F satisfying $F^TF\leq I$, if and only if there exists some scalar $\varepsilon>0$, such that

$$Q+\varepsilon HH^T+\varepsilon^{-1}E^TE<0.$$

III. Main Results

The following theorem gives sufficient and necessary conditions for the stochastic stability of closed-loop networked lateral motion system under the proposed controller (11). Its proof is motivated by the proof in [19] and is given in Appendix.

Theorem 1: Closed-loop system (14) is stochastically stable if and only if there exists a symmetric and positive definite matrix $P(l,i)$ such that

$$\begin{aligned}\mathcal{L}(l,i)=&\left\{\sum_{h=0}^{\tau}\sum_{j=1}^{\eta}\lambda_{lh}\Xi_{ij}^{1+l-h}\times[\bar{A}+\bar{B}K(l,i)\bar{C}(l,i)]^T\right.\\&\left.\times P(h,j)\times[\bar{A}+\bar{B}K(l,i)\bar{C}(l,i)]-P(l,i)\right\}\\&<0\end{aligned} \tag{17}$$

holds for all $l\in N_\tau$ and $i\in N_\eta^+$.

Based on the results in Theorem 1, the controller design techniques are given in Theorem 2.

Theorem 2: The system in (14) is stochastically stable if and only if there exist positive scalars $\epsilon_{\iota,\varsigma}(l,i)$, $\gamma_{\iota,\varsigma}(l,i)$, $\varepsilon_{\iota,\varsigma}(l,i)$ ($\iota=0,1,\ldots,\tau$, $\varsigma=1,2,\ldots,\eta$), a symmetric and positive definite matrix $P(l,i)$ and a matrix $K(l,i)$ of appropriate dimensions, such that (18) (show at the bottom of this page) holds where

$$\bar{Z}(l,i)=\begin{bmatrix}\bar{Z}_0^T & \bar{Z}_1^T & \ldots & \bar{Z}_\tau^T\end{bmatrix}^T$$
$$\bar{Z}_h=\begin{bmatrix}\bar{Z}_{h,1}^T & \bar{Z}_{h,2}^T & \ldots & \bar{Z}_{h,\eta}^T\end{bmatrix}^T$$
$$\bar{Z}_{h,j}=(\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}}\times[A_a+B_bK(l,i)\hat{C}(l)]$$
$$\hat{Z}(l,i)=\begin{bmatrix}\hat{Z}_0^T & \hat{Z}_1^T & \ldots & \hat{Z}_\tau^T\end{bmatrix}^T$$
$$\hat{Z}_h=\begin{bmatrix}\hat{Z}_{h,1}^T & \hat{Z}_{h,2}^T & \ldots & \hat{Z}_{h,\eta}^T\end{bmatrix}^T$$
$$\hat{Z}_{h,j}=(\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}}\times[\tilde{Y}_1+\tilde{Y}_2K(l,i)\hat{C}(l)]$$
$$W(l,i)=\begin{bmatrix}W_0^T & W_1^T & \ldots & W_\tau^T\end{bmatrix}^T$$
$$W_h=\begin{bmatrix}W_{h,1}^T & W_{h,2}^T & \ldots & W_{h,\eta}^T\end{bmatrix}^T$$
$$W_{h,j}=\gamma_{h,j}(l,i)\delta_i(\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}}\times[\tilde{C}(l)\ 0]$$
$$\bar{W}(l,i)=\begin{bmatrix}\bar{W}_0^T & \bar{W}_1^T & \ldots & \bar{W}_\tau^T\end{bmatrix}^T$$
$$\bar{W}_h=\begin{bmatrix}\bar{W}_{h,1}^T & \bar{W}_{h,2}^T & \ldots & \bar{W}_{h,\eta}^T\end{bmatrix}^T$$
$$\bar{W}_{h,j}=\epsilon_{h,j}(l,i)\delta_i(\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}}\times[\tilde{C}(l)\ 0]$$
$$T=\mathrm{diag}\{T_0\ \ T_1\ \ \ldots\ \ T_\tau\}$$
$$T_h=\mathrm{diag}\{T_{h,1}\ \ T_{h,2}\ \ \ldots\ \ T_{h,\eta}\}$$
$$T_{h,j}=[P(h,j)]^{-1}$$
$$\begin{aligned}\breve{M}(l,i)=\mathrm{diag}\Big\{&\varepsilon_{0,1}(l,i)\tilde{M}\tilde{M}^T\\&\varepsilon_{0,2}(l,i)\tilde{M}\tilde{M}^T\ \ \ldots\ \ \varepsilon_{\tau,\eta}(l,i)\tilde{M}\tilde{M}^T\Big\}\end{aligned}$$
$$\hat{\epsilon}(l,i)=\mathrm{diag}\{\epsilon_{0,1}(l,i)I\ \epsilon_{0,2}(l,i)I\ \ldots\ \epsilon_{\tau,\eta}(l,i)I\}$$
$$\hat{\gamma}(l,i)=\mathrm{diag}\{\gamma_{0,1}(l,i)I\ \gamma_{0,2}(l,i)I\ \ldots\ \gamma_{\tau,\eta}(l,i)I\}$$
$$\hat{\varepsilon}(l,i)=\mathrm{diag}\{\varepsilon_{0,1}(l,i)I\ \varepsilon_{0,2}(l,i)I\ \ldots\ \varepsilon_{\tau,\eta}(l,i)I\}$$

$$V^T(l,i)=\begin{bmatrix} B_bK(l,i)\begin{bmatrix} I\\0\end{bmatrix} & 0 & \ldots & 0\\ 0 & B_bK(l,i)\begin{bmatrix} I\\0\end{bmatrix} & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & B_bK(l,i)\begin{bmatrix} I\\0\end{bmatrix}\end{bmatrix}$$

$$L^T(l,i)=\begin{bmatrix} \tilde{Y_2}K(l,i)\begin{bmatrix} I\\0\end{bmatrix} & 0 & \ldots & 0\\ 0 & \tilde{Y_2}K(l,i)\begin{bmatrix} I\\0\end{bmatrix} & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & \tilde{Y_2}K(l,i)\begin{bmatrix} I\\0\end{bmatrix}\end{bmatrix} \tag{19}$$

for all $l\in N_\tau$ and $i\in N_\eta^+$.

$$\begin{bmatrix} -P(l,i) & \bar{Z}^T(l,i) & \hat{Z}^T(l,i) & W^T(l,i) & \bar{W}^T(l,i) & 0 & 0\\ \bar{Z}(l,i) & -T+\breve{M} & 0 & 0 & 0 & V^T(l,i) & 0\\ \hat{Z}(l,i) & 0 & -\hat{\varepsilon}(l,i) & 0 & 0 & 0 & L^T(l,i)\\ W(l,i) & 0 & 0 & -\hat{\gamma}(l,i) & 0 & 0 & 0\\ \bar{W}(l,i) & 0 & 0 & 0 & -\hat{\epsilon}(l,i) & 0 & 0\\ 0 & V(l,i) & 0 & 0 & 0 & -\hat{\gamma}(l,i) & 0\\ 0 & 0 & L(l,i) & 0 & 0 & 0 & -\hat{\epsilon}(l,i)\end{bmatrix}<0 \tag{18}$$

Proof: By applying the Schur complement, we obtain that (17) is equivalent to the following inequality:

$$\begin{bmatrix} -P(l,i) & S^T(l,i) \\ S(l,i) & -T \end{bmatrix} < 0 \tag{20}$$

for all $l \in N_\tau$ and $i \in N_\eta^+$, with

$$\begin{aligned} S(l,i) &= [\; S_0^T(l,i) \quad S_1^T(l,i) \quad \dots \quad S_\tau^T(l,i) \;]^T \\ S_h(l,i) &= [\; S_{h,1}^T(l,i) \quad S_{h,2}^T(l,i) \quad \dots \quad S_{h,\eta}^T(l,i) \;]^T \\ S_{h,j}(l,i) &= (\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}} \times [\bar{A} + \bar{B}K(l,i)\bar{C}(l,i)]. \end{aligned} \tag{21}$$

From Lemma 1 and the Schur complement, it is found that the inequality (20) holds if and only if there exist positive scalars $\varepsilon_{\iota,\varsigma}(l,i)$ $(\iota = 0,1,\dots,\tau,\ \varsigma = 1,2,\eta)$, such that

$$\begin{bmatrix} -P(l,i) & * & * & * \\ \bar{S}(l,i) & -T & * & * \\ \hat{S}(l,i) & 0 & -\hat{\varepsilon}(l,i) & * \\ 0 & \hat{M}(l,i) & 0 & -\hat{\varepsilon}(l,i) \end{bmatrix} < 0 \tag{22}$$

where

$$\begin{aligned} \bar{S}(l,i) &= [\; \bar{S}_0^T \quad \bar{S}_1^T \quad \dots \quad \bar{S}_\tau^T \;]^T \\ \bar{S}_h &= [\; \bar{S}_{h,1}^T \quad \bar{S}_{h,2}^T \quad \dots \quad \bar{S}_{h,\eta}^T \;]^T \\ \bar{S}_{h,j} &= (\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}} \times [A_a + B_bK(l,i)\bar{C}(l,i)] \\ \hat{S}(l,i) &= [\; \hat{S}_0^T \quad \hat{S}_1^T \quad \dots \quad \hat{S}_\tau^T \;]^T \\ \hat{S}_h &= [\; \hat{S}_{h,1}^T \quad \hat{S}_{h,2}^T \quad \dots \quad \hat{S}_{h,\eta}^T \;]^T \\ \hat{S}_{h,j} &= (\lambda_{lh}\Xi_{ij}^{1+l-h})^{\frac{1}{2}} \times [\tilde{Y}_1 + \tilde{Y}_2K(l,i)\bar{C}(l,i)] \\ \hat{M}^T(l,i) & \\ &= \begin{bmatrix} \varepsilon_{0,1}(l,i)\tilde{M} & 0 & \dots & 0 \\ 0 & \varepsilon_{0,2}(l,i)\tilde{M} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \varepsilon_{\tau,\eta}(l,i)\tilde{M} \end{bmatrix}. \end{aligned}$$

According to the Schur complement, (22) can be rewritten as

$$\begin{bmatrix} -P(l,i) & * & * \\ \bar{S}(l,i) & -T + \breve{M}(l,i) & * \\ \hat{S}(l,i) & 0 & -\hat{\varepsilon}(l,i) \end{bmatrix} < 0. \tag{23}$$

By using Lemma 1 and the Schur complement, we conclude that (23) is equivalent to (18). From Theorem 1, we complete the proof. ■

The conditions in Theorem 2 form a set of linear matrix inequalities with some inversion constraints. $K(l,i)$ can be solved by an iterative linear matrix inequality approach which is called as the cone complementarity linearization algorithm whose detail can be found in [32]. Accordingly, $D(l,i)$, $E(l,i)$, $F(l,i)$ and $G(l,i)$ can be obtained from (15). Next, we give an example to show the performance of the proposed controller.

IV. Numerical Example

Consider a vehicle lateral motion system in (3) with the following parameters [22]

$$\begin{aligned} &m = 800\,\text{kg}, \quad I_z = 728.6\,\text{kg.m}^2, \quad l_f = 0.85\,\text{m} \\ &l_r = 1.04\,\text{m}, \quad C_f = C_r = 10\,000\,\text{N/rad}, \quad V = 100\,\text{km/h}. \end{aligned}$$

The parameters of unknown matrices $\Delta A(k)$ and $\Delta B(k)$ are assumed as

$$\begin{aligned} M &= \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad Y_1 = [\; 0.2 \quad 0.1 \;] \\ Y_2 &= [\; 0.1 \quad 0.1 \;]. \end{aligned} \tag{24}$$

The sampling period of the sensor, controller and actuator is set as $T = 0.01$ s. The network time delay is supposed to be $\tau_k \in \{0, 1\}$, that means time delay in a practical vehicle control system is $0T = 0$ s and $1T = 0.01$ s, and its transition probability matrix is given as

$$\Gamma = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{bmatrix}.$$

The quantizer parameters are set as

$$\delta_1 = 0.02, \text{ and } \delta_2 = 0.04.$$

Thus we have two different quantization density values ρ_1 and ρ_2. The transition probability matrix of σ_k (ρ_{σ_k}) is

$$\Xi = \begin{bmatrix} 0.42 & 0.58 \\ 0.41 & 0.59 \end{bmatrix}.$$

The output matrix is

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The controller design in [19] considers neither quantization in the network environment, nor uncertainty of the system model parameters. Therefore, the approach of the controller design cannot be applied to our case.

By using Theorem 2, we obtain

$$\begin{aligned} G01 &= \begin{bmatrix} -0.9726 & -0.1533 \\ -0.7554 & -0.7954 \end{bmatrix} \\ G02 &= \begin{bmatrix} -1.0882 & -0.2297 \\ -0.9278 & -1.2981 \end{bmatrix} \\ G11 &= \begin{bmatrix} -0.3967 & -0.0488 \\ -0.2973 & -0.7077 \end{bmatrix} \\ G12 &= \begin{bmatrix} -0.3296 & -0.0365 \\ -0.3029 & -0.7083 \end{bmatrix} \end{aligned} \tag{25}$$

$$\begin{aligned} F(0,1) &= F(0,2) = F(1,1) = F(1,2) = E(0,1) \\ &= E(0,2) = E(1,1) = E(1,2) = D(0,1) \\ &= D(0,2) = D(1,1) = D(1,2) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Suppose that the initial conditions are $x(0) = [3,\ 2]$, $\tau_0 = 0$ and $\sigma_0 = 1$. One of the possible realizations of the random modes σ_k and τ_k is shown in Figs. 3 and 4, and the realization of the unknown time-varying matrix $J(k)$ is set to $\sin(k)$. Under them, the corresponding state trajectories of the closed-loop system are shown in Fig. 5. We can clearly see that the closed-loop system is stochastically stable.

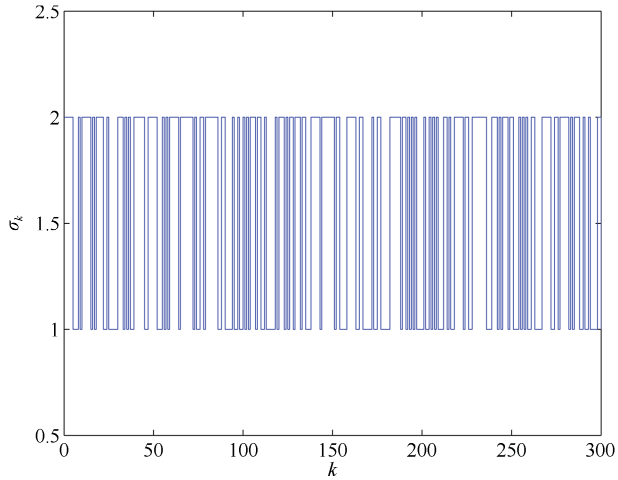

Fig. 3. Random mode σ_k.

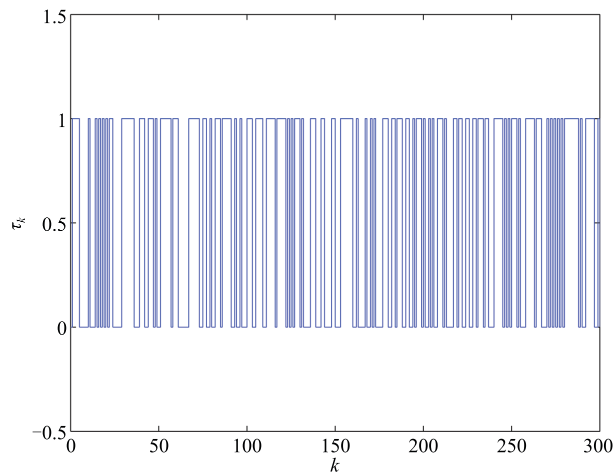

Fig. 4. Random mode τ_k.

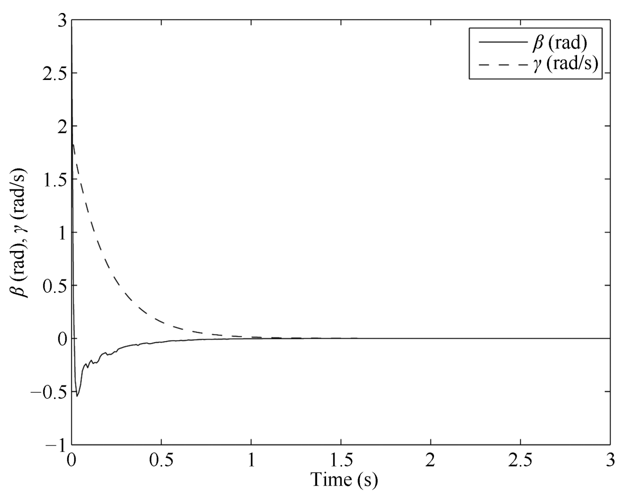

Fig. 5. State trajectories of the closed-loop system.

V. Conclusion

The stabilization problem of uncertain-linear vehicle-lateral-motion systems over networks is challenging. This work adopts Markov chains to model the stochastic changes of quantization density and time delay modes. These modes are simultaneously incorporated into the output feedback controller design. By constructing a Lyapunov function and Schur complement, this work derives sufficient and necessary conditions of stochastic stability of a given networked vehicle-lateral-motion control system in the form of a set of linear matrix inequalities with some inversion constraints. The cone complementarity linearization algorithm is employed to obtain the desired two-mode-dependent controller. The future work should address the complexity issues when a system or Markov model is of large scale.

Appendix

Proof:

Sufficiency: Construct the following Lyapunov function

$$V\left(\xi(k),k\right)=\xi^T(k)P\left(\tau_k,\sigma_{k-\tau_k}\right)\xi(k).$$

Then

$$\begin{aligned}&\mathcal{E}\left\{\triangle V(\xi(k),k)\right\}\\&\quad=\mathcal{E}\left\{V(\xi(k+1),k+1)|\xi(k),\tau_k,\sigma_{k-\tau_k}\right\}-V(\xi(k),k)\\&\quad=\mathcal{E}\left\{\xi^T(k+1)P(\tau_{k+1},\sigma_{k+1-\tau_{k+1}})\xi(k+1)|\xi(k),\tau_k,\sigma_{k-\tau_k}\right\}\\&\qquad-\left\{\xi^T(k)P(\tau_k,\sigma_{k-\tau_k})\xi(k)\right\}.\end{aligned}$$

Let

$$\tau_k=l,\quad \tau_{k+1}=h,\quad \sigma_{k-\tau_k}=i,\quad \sigma_{k+1-\tau_{k+1}}=j. \tag{26}$$

Then, the probability transition matrices are

$$\tau_k\rightarrow\tau_{k+1}:\Gamma,\quad \sigma_{k-\tau_k}\rightarrow\sigma_{k+1-\tau_{k+1}}:\Xi^{1+l-h} \tag{27}$$

and

$$\begin{aligned}&\mathcal{E}\left\{\triangle V(\xi(k),k)\right\}\\&\quad=\xi^T(k)\left\{\sum_{h=0}^{\tau}\sum_{j=1}^{\eta}\lambda_{lh}\Xi_{ij}^{1+l-h}\times[\bar{A}+\bar{B}K(l,i)\bar{C}(l,i)]^T\right.\\&\qquad\left.\times P(h,j)\times[\bar{A}+\bar{B}K(l,i)\bar{C}(l,i)]-P(l,i)\right\}\xi(k)\\&\quad=\xi^T(k)\mathcal{L}(l,i)\xi(k).\end{aligned}$$

Thus if $\mathcal{L}(l,i)<0$, then

$$\begin{aligned}\mathcal{E}\left\{\triangle V(\xi(k),k)\right\}&\le-\lambda_{\min}(-\mathcal{L}(l,i))\xi^T(k)\xi(k)\\&\le-\alpha\|\xi(k)\|^2\end{aligned} \tag{28}$$

where $\alpha=\inf\{\lambda_{\min}(-\mathcal{L}(l,i)\}>0$.

It follows from (28) that for any $n\ge1$,

$$\begin{aligned}&\mathcal{E}\left\{V(\xi(n+1),n+1)\right\}-\mathcal{E}\left\{V(\xi(0),0)\right\}\\&\qquad\le-\alpha\mathcal{E}\left(\sum_{t=0}^{n}\|\xi(t)\|^2\right).\end{aligned}$$

Furthermore, we have

$$\begin{aligned}\mathcal{E}\left(\sum_{t=0}^{n}\|\xi(t)\|^2\right)&\le\frac{1}{\alpha}\Big(\mathcal{E}\{\ V(\xi(0),0)\}\\&\qquad-\mathcal{E}\left\{V(\xi(n+1),n+1)\right\}\Big)\\&\le\frac{1}{\alpha}\mathcal{E}\{V(\xi(0),0)\}\\&\le\frac{1}{\alpha}\xi^T(0)P\left(\tau_0,\sigma_{-\tau_0}\right)\xi(0).\end{aligned} \tag{29}$$

By using Definition 1, the closed-loop system in (14) is stochastically stable.

Necessity: Assume that $Q(\tau_k, \sigma_{k-\tau_k})$ is a symmetric and positive definite matrix and $\tilde{P}(T-t, \tau_t, \sigma_{t-\tau_t})$ is a symmetric matrix. Define

$$\xi^T(t)\tilde{P}(T-t, \tau_t, \sigma_{t-\tau_t})\xi(t) = \mathcal{E}\left\{\sum_{k=t}^{T} \xi^T(k)Q(\tau_k, \sigma_{k-\tau_k})\xi(k)|\xi_t, \tau_t, \sigma_{t-\tau_t}\right\}. \quad (30)$$

Since $Q(\tau_k, \sigma_{k-\tau_k}) > 0$, as T increases, $\xi^T(t)\tilde{P}(T-t, \tau_t, \sigma_{t-\tau_t})\xi(t)$ increases. From (16), $\xi^T(t)\tilde{P}(T-t, \tau_t, \sigma_{t-\tau_t})\xi(t)$ is upper bounded. Furthermore, its limit exists and can be expressed as

$$\begin{aligned}\xi^T(t)P(l,i)\xi(t) &= \lim_{T\to\infty} \xi^T(t)\tilde{P}(T-t, \tau_t = l, \sigma_{t-\tau_t} = i)\xi(t) \\ &= \lim_{T\to\infty} \mathcal{E}\left\{\sum_{k=t}^{T}\xi^T(k)Q(\tau_k, \sigma_{k-\tau_k})\xi(k)|\xi_t, \tau_t = l\sigma_{t-\tau_t} = i\right\}\end{aligned} \quad (31)$$

where $P(l,i)$ is a symmetric matrix.

Thus, we have

$$P(l,i) = \lim_{T\to\infty} \tilde{P}(T-t, \tau_t = l, \sigma_{t-\tau_t} = i). \quad (32)$$

From (31), we obtain $P(l,i) > 0$ since $Q(\tau_k, \sigma_{k-\tau_k}) > 0$. From (30), we have

$$\begin{aligned}\mathcal{E}\Big\{&\xi^T(t)\tilde{P}(T-t, \tau_t, \sigma_{t-\tau_t})\xi(t) \\ &- \xi^T(t+1) \times \tilde{P}(T-t-1, \tau_{t+1}, \sigma_{t+1-\tau_{t+1}}) \\ &\times \xi(t+1)|\xi_t, \tau_t = l, \sigma_{t-\tau_t} = i\Big\} = \xi^T(t)Q(l,i)\xi(t).\end{aligned} \quad (33)$$

From (14), we have

$$\begin{aligned}&\mathcal{E}\Big\{\xi^T(t)\tilde{P}(T-t, \tau_t, \sigma_{t-\tau_t})\xi(t) \\ &\quad - \xi^T(t+1) \times \tilde{P}(T-t-1, \tau_{t+1}, \sigma_{t+1-\tau_{t+1}}) \\ &\quad \times \xi(t+1)|\xi_t, \tau_t = l, \sigma_{t-\tau_t} = i\Big\} \\ &= \xi^T(t)\Bigg\{\tilde{P}(T-t, l, i) - \sum_{h=0}^{\tau}\sum_{j=1}^{\eta} \lambda_{lh}\Xi_{ij}^{1+l-h} \\ &\quad \times [\bar{A} + \bar{B}K(l,i)\bar{C}(l,i)]^T \times \tilde{P}(T-t-1, h, j) \\ &\quad \times [\bar{A} + \bar{B}K(l,i)\bar{C}(l,i)]\Bigg\}\xi(t).\end{aligned} \quad (34)$$

It is easy to obtain from (33) and (34) that

$$\begin{aligned}&\xi^T(t)\Bigg\{\tilde{P}(T-t, l, i) - \sum_{h=0}^{\tau}\sum_{j=1}^{\eta} \lambda_{lh}\Xi_{ij}^{1+l-h} \\ &\quad \times [\bar{A} + \bar{B}K(l,i)\bar{C}(l,i)]^T \times \tilde{P}(T-t-1, h, j) \\ &\quad \times [\bar{A} + \bar{B}K(l,i)\bar{C}(l,i)]\Bigg\}\xi(t) \\ &= \xi^T(t)Q(l,i)\xi(t).\end{aligned} \quad (35)$$

Letting $T \to \infty$ in (35) and noticing (32), we prove that (17) holds. ■

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