An Adaptive RBF Neural Network Control Method for a Class of Nonlinear Systems

Hongjun Yang and Jinkun Liu

***Abstract*—This paper focuses on designing an adaptive radial basis function neural network (RBFNN) control method for a class of nonlinear systems with unknown parameters and bounded disturbances. The problems raised by the unknown functions and external disturbances in the nonlinear system are overcome by RBFNN, combined with the single parameter direct adaptive control method. The novel adaptive control method is designed to reduce the amount of computations effectively. The uniform ultimate boundedness of the closed-loop system is guaranteed by the proposed controller. A coupled motor drives (CMD) system, which satisfies the structure of nonlinear system, is taken for simulation to confirm the effectiveness of the method. Simulations show that the developed adaptive controller has favorable performance on tracking desired signal and verify the stability of the closed-loop system.**

***Index Terms*—Adaptive control, neural network (NN), nonlinear system, radial basis function.**

I. INTRODUCTION

ADAPTIVE control has been successfully used for designing controllers for uncertain dynamic systems. The principal theory in adaptive control utilizes output feedback in the model-free unknown system [1], [2]: direct and indirect. Direct adaptive control, which we use in this study, intends that the parameters of the controller are directly regulated to reduce the output error between the desired model and the controlled plant. A direct adaptive control scheme for tracking the end effector of a two-link flexible-joint manipulator has been developed in [3]. In [4], a direct adaptive neural control is proposed for a class of uncertain non-affine nonlinear systems with unknown non-symmetric input saturation. The apparent advantage of direct adaptive control is the high computational efficiency. With direct approach [5], [6], the controllers compel the feedback system to track as closely as possible a desired signal with unknown parameters dynamics. In addition, the approaches given in [7] and [8] introduce indirect adaptive control scheme for underwater vehicle-manipulator systems (UVMSs) and the synchronization of chaotic systems separately.

Neural networks (NNs) are well known for their ability to approximate uncertainties in model-uncertain systems with complex and unknown functions. Many NN controllers are used with adaptive control technique [9]−[11]. The main performance of the scheme is that controller does not depend on the accurate structural information and model parameters for the plants. To design a neural network controller, multivariable feedback linearization is employed in [12] to convert the nonlinear model to linear one. To solve the control problem with uncertain parameters, neural network control combined with sliding mode control [13], model predictive control [14], back stepping control [15], and iterative adaptive dynamic programming algorithm [16] have been developed for a large class of nonlinear systems. Radial basis function neural networks (RBFNNs) have been successfully used in variety of applications widely and accepted to be an effective method for solving many control problems with dynamic uncertainty [17]. A RBFNN is employed to adaptively learn an upper bound of uncertain dynamics of a battery equivalent circuit model in real time [18]. In [19], a RBFNN approach with a fusion of multiple signal candidates in precision motion control is studied. The authors in [20] have studied the robust Mars atmospheric entry guidance design based on RBFNNs and second-order sliding mode control. A new approach that combines fuzzy control with RBFNNs is proposed in [21] to improve the single neuron proportional-integral-derivative (PID) control technology.

In this paper, we focus on developing an adaptive RBFNN control method for a class of unknown single-input single-output (SISO) nonlinear systems with bounded external disturbances. In [22], an adaptive controller has been proposed for a class of unknown nonlinear systems using high-order neural networks, which avoids singularity problem and guarantees regional stability of the closed-loop system. However, the form of system in it may be a little conservative. Based on [22], we propose a novel adaptive RBFNN control whose main contributions are summarized as follows:

1) We propose a single parameter adaptive (SPA) control method to reduce the amount of NN computations. The method can make the number of online adaptive parameters drop to only one, shortening the time on operation. With this method, the adaptive controller is designed concisely.

2) The SPA control method can be applied to the system with more complex and indeterminate dynamic model, compared with [22]. Thus, the SPA method is more general.

The uniform ultimate boundedness of the closed-loop

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system is guaranteed by the proposed controller. The dynamic model of coupled motor drives (CMD) system [23], which is commonly used in industrial robots, exactly satisfies the structure of the nonlinear systems in this study. It is finally taken for simulation to confirm the performance of the proposed control scheme. The distinct simulations illustrate the effectiveness of the proposed scheme and guarantee the stability of the closed-loop system.

The paper is organized as follows. Section II presents the dynamic model of the SISO nonlinear system and control objective. Then, desired control design is proposed in Section III. In Section IV, a SPA RBFNN controller is proposed. The performance of trajectory tracking and the system's stability are all proved. Simulations for the CMD system are given in Section V to show the effectiveness of the developed scheme. Section VI concludes the paper.

II. System Description

Consider the SISO nonlinear system in the presence of external disturbance described by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(x_1, x_2, x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(x) + f_3(x)u + d(t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1\ x_2\ x_3\ x_4]^T \in \mathbb{R}^4$ and $y \in \mathbb{R}$ denote the state vector and output respectively. $u \in \mathbb{R}$ represents control input; $f_1(x_1, x_2, x_3)$, $f_2(x)$ and $f_3(x)$ are unknown nonlinear smooth functions; $d(t)$ is the external disturbance bounded by a positive constant d_0, i.e., $|d(t)| \le d_0$. Since all physical quantities are in limited region, the state x belongs to a compact subset $\Phi \in \mathbb{R}^4$.

Define the desired trajectory vector y_d, the tracking error e and an error function s as

$$y_d = \begin{bmatrix} x_d & \dot{x}_d & \ddot{x}_d & \dddot{x}_d \end{bmatrix}^T \tag{2}$$

$$\begin{aligned} e &= [e_1\ e_2\ e_3\ e_4]^T = [e_1\ \dot{e}_1\ \ddot{e}_1\ \dddot{e}_1]^T \\ &= \begin{bmatrix} x_1 - x_d & x_2 - \dot{x}_d & f_1 - \ddot{x}_d & \dot{f}_1 - \dddot{x}_d \end{bmatrix}^T \end{aligned} \tag{3}$$

$$s = c_1 e_1 + c_2 e_2 + c_3 e_3 + e_4 \tag{4}$$

where $c_i > 0$, $i = 1, 2, 3$ are appropriately chosen such that polynomial $\lambda^3 + c_3\lambda^2 + c_2\lambda + c_1$ is Hurwitz, i.e., $e \to 0$ as $s \to 0$. The desired trajectory vector $y_d \in \Phi_d \subset \mathbb{R}^4$ is assumed known and continuous. $\dot{x}_d$, $\ddot{x}_d$, $\dddot{x}_d$ represent first, second, and third order time derivative of x_d, respectively. The objective is to force x_1 to follow x_d.

From (1), (3) and (4), the time derivative of s can be written as

$$\begin{aligned} \dot{s} &= c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 + \dot{e}_4 \\ &= c_1(x_2 - \dot{x}_d) + c_2(f_1 - \ddot{x}_d) \\ &\quad + c_3\left(\frac{\partial f_1}{\partial x_1}x_2 + \frac{\partial f_1}{\partial x_2}f_1 + \frac{\partial f_1}{\partial x_3}x_4 - \dddot{x}_d\right) \\ &\quad + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_1}x_2\right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_2}f_1\right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_3}\right)x_4 \\ &\quad + \frac{\partial f_1}{\partial x_3}\left[f_2(x) + f_3(x)u + d(t)\right] - \ddddot{x}_d \\ &= v + f_4(x)u + d(t) \end{aligned} \tag{5}$$

where

$$\begin{aligned} f_1 &= f_1(x_1, x_2, x_3) \\ f_4(x) &= \frac{\partial f_1}{\partial x_3}f_3(x) \\ v &= c_1(x_2 - \dot{x}_d) + c_2(f_1 - \ddot{x}_d) \\ &\quad + c_3\left(\frac{\partial f_1}{\partial x_1}x_2 + \frac{\partial f_1}{\partial x_2}f_1 + \frac{\partial f_1}{\partial x_3}x_4 - \dddot{x}_d\right) \\ &\quad + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_1}x_2\right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_2}f_1\right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_3}\right)x_4 \\ &\quad + \frac{\partial f_1}{\partial x_3}f_2(x) - \ddddot{x}_d. \end{aligned}$$

Assumption 1: The sign of $f_4(x)$ is known and $f_4(x) \ne 0$, $\forall x \in \Phi$. Without loss of generality, we could assume $f_4(x) > 0$ because of the sign of $f_4(x)$ being known.

Assumption 2: There is a smooth function $\bar{f}_4(x)$ such that $|f_4(x)| \le \bar{f}_4(x)$, and $\bar{f}_4(x)$ is bounded as $\bar{f}_4(x) \le \bar{f}_4 \in \mathbb{R}$.

III. Desired Controller Design

In order to design the desired controller $\bar{u}$, we first assume that $f_1(x_1, x_2, x_3)$, $f_2(x)$ and $f_3(x)$ are known, and the system is ideal and has no disturbance, i.e., $d(t) = 0$.

Theorem 1: Considering (1), Assumptions 1 and 2, and $d(t) = 0$, the desired controller is designed by

$$\bar{u} = -\frac{1}{f_4(x)}\left\{v + \left[\frac{1}{\varepsilon} + \frac{1}{\varepsilon f_4(x)} - \frac{\dot{f}_4(x)}{2f_4(x)}\right]s\right\} \tag{6}$$

where ε is a positive parameter. Thus, we obtain the result of $\lim_{t\to\infty}\|e(t)\| = 0$.

Proof: Setting $u = \hat{u}$ and substituting (6) into (5), we obtain

$$\begin{aligned} \dot{s} &= -\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon f_4(x)} - \frac{\dot{f}_4(x)}{2f_4(x)}\right)s \\ &= -\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon f_4(x)}\right)s + \frac{\dot{f}_4(x)}{2f_4(x)}s. \end{aligned} \tag{7}$$

Choosing a Lyapunov function as $V = \frac{1}{2f_4(x)}s^2$, we get the time derivative of it as follows:

$$\begin{aligned} \dot{V} &= \frac{1}{f_4(x)}s\dot{s} - \frac{\dot{f}_4(x)}{2f_4^2(x)}s^2 \\ &= \frac{1}{f_4(x)}s\left[-\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon f_4(x)}\right)s + \frac{\dot{f}_4(x)}{2f_4(x)}s\right] - \frac{\dot{f}_4(x)}{2f_4^2(x)}s^2 \\ &= -\left(\frac{1}{\varepsilon f_4(x)} + \frac{1}{\varepsilon f_4^2(x)}\right)s^2 \le 0 \end{aligned} \tag{8}$$

Because of $f_4(x) > 0$ and stability theorem, the result $\dot{V} \le 0$ indicates that $\lim_{t\to\infty}|s| = 0$, then we have $\lim_{t\to\infty}\|e(t)\| = 0$. ■

According to (8), we come to the conclusion that the convergence rate of the tracking error e is closely relevant to the parameter ε. We change (6) into another form, where the desired controller $\bar{u}$ can be regarded as a function of the following variable:

$$\bar{u} = \bar{u}(\alpha)$$
$$\alpha = \begin{bmatrix} x^T & s & \frac{s}{\varepsilon} & v \end{bmatrix}^T \in \Phi_\alpha \subset \mathbb{R}^7 \tag{9}$$

and compact subset Φ_α is defined as

$$\Phi_\alpha = \left\{ \begin{pmatrix} \mathbf{x}^T & s & \frac{s}{\varepsilon} & v \end{pmatrix} | x \in \Phi \, ; \; y_d \in \Phi_d \right\}. \tag{10}$$

RBFNNs will be used to design the desired controller $\bar{u}$ in the following way. Since nonlinear smooth functions $f_1(x_1, x_2, x_3)$, $f_2(x)$ and $f_3(x)$ are unknown, the ideal controller $\bar{u}$ is not available. The reason why s/ε is also chosen in RBFNN as input when s belongs to it is that it will make s and s/ε differ largely when ε is very small.

Since $\bar{u}$ in (9) is continuous on Φ_α, there exists an ideal RBFNN weight vector $\bar{W}$ as follows:

$$\bar{u}(\alpha) = \bar{W}^T h(\alpha) + \mu_\varepsilon, \quad \alpha \in \Phi_\alpha \tag{11}$$

where μ_ε is the RBFNN approximation error such that $|\mu_\varepsilon| \leq \mu_0$, $\mu_0 > 0$, and the radial-basis function vector $h(\alpha)$ which denotes the output of hidden layer is given by

$$h = [h_1 \; h_2 \; \ldots \; h_j \; \ldots \; h_m]^T, \quad j = 1, 2, \ldots, m \tag{12}$$

$$h_j = \exp\left(-\frac{\|\alpha - A_j\|^2}{(b_j)^2} \right), \quad \alpha \in \Phi_\alpha \tag{13}$$

where $\alpha \in \mathbb{R}^7$ is the input vector, and in this study the number of input neural nets in the input layer is seven; m denotes the number of hidden neural nets in the hidden layer; h_j is Gaussian function; $A_j = [a_j, a_j, \ldots, a_j]^T \in \mathbb{R}^7$; $a \in \mathbb{R}^m$ and $b \in \mathbb{R}^m$ represent the center of the receptive field and the width of Gaussian function respectively. The ideal RBFNN weight vector $\bar{W}$ is bounded as $\|\bar{W}\|_F \leq w_{\max}$, $w_{\max} > 0$.

To improve the computational procedure, we define a positive constant as follows:

$$\phi = \|\bar{W}\|_F^2. \tag{14}$$

Since $\|\bar{W}\|_F \leq w_{\max}$, ϕ is obviously bounded. Let $\hat{\phi}$ be the estimate of ϕ, and $\tilde{\phi} = \hat{\phi} - \phi$. The norm of ideal RBFNN weight vector $\bar{W}$ will be estimated via $\hat{\phi}$ in following design, which is the main contribution of this paper.

IV. Actual Controller Design

In this section, we propose a single parameter adaptive (SPA) control method. The main contribution is that the number of online adaptive parameters is decreased to only one parameter $\hat{\phi}$, instead of a vector $\bar{W}$, thus shortening the time on operation. The adaptive controller is designed as follows:

$$u = -\frac{1}{2} s \hat{\phi} h^T h. \tag{15}$$

The adaptive law will be given later. Substituting (15) into (5), we have

$$\dot{s} = v + f_4(x) \left(-\frac{1}{2} s \hat{\phi} h^T h \right) + d(t). \tag{16}$$

Adding and subtracting $f_4(x)\bar{u}(\alpha)$ on the right-hand side of (16) and from (11), respectively, we obtain

$$\dot{s} = v + f_4(x) \left(-\frac{1}{2} s \hat{\phi} h^T h - \bar{W}^T h - \mu_\varepsilon \right) + f_4(x)\bar{u}(\alpha) + d(t). \tag{17}$$

Then, substituting (6) into (17), it yields

$$\dot{s} = f_4(x) \left(-\frac{1}{2} s \hat{\phi} h^T h - \bar{W}^T h - \mu_\varepsilon \right) - \left[\frac{1}{\varepsilon} + \frac{1}{\varepsilon f_4(x)} - \frac{\dot{f}_4(x)}{2 f_4(x)} \right] s + d(t). \tag{18}$$

Theorem 2: With the controller (15), and the adaptive law

$$\dot{\hat{\phi}} = \frac{\gamma}{2} s^2 h^T h - \kappa \gamma \hat{\phi} \tag{19}$$

where $\gamma > 0$, $\kappa > 0$, the tracking error $e(t)$ in (3) is bounded in the compact subset Φ for all time, and can be made arbitrarily small by using appropriate parameters. The closed-loop system is uniformly ultimately bounded and the state x_1 will follow the desired trajectory x_d.

Proof: A Lyapunov function could be denoted as

$$V = \frac{1}{2} \left(\frac{s^2}{f_4(x)} + \frac{1}{\gamma} \tilde{\phi}^2 \right). \tag{20}$$

Differentiating (20) with respect to time, and from (18), we have

$$\begin{aligned} \dot{V} &= \frac{s\dot{s}}{f_4(x)} - \frac{\dot{f}_4(x)}{2 f_4^2(x)} s^2 + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}} \\ &= \frac{s}{f_4(x)} \left[f_4(x) \left(-\frac{1}{2} s \hat{\phi} h^T h - \bar{W}^T h - \mu_\varepsilon \right) \right] \\ &\quad - \frac{s}{f_4(x)} \left[\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon f_4(x)} - \frac{\dot{f}_4(x)}{2 f_4(x)} \right) s + d(t) \right] \\ &\quad - \frac{\dot{f}_4(x)}{2 f_4^2(x)} s^2 + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}} \\ &= -\frac{1}{2} s^2 (\tilde{\phi} + \phi) h^T h - s \bar{W}^T h \\ &\quad - \left(\frac{1}{\varepsilon f_4(x)} + \frac{1}{\varepsilon f_4^2(x)} \right) s^2 - \frac{d(t)}{f_4(x)} s - \mu_\varepsilon s + \frac{1}{\gamma} \tilde{\phi} \dot{\hat{\phi}}. \end{aligned} \tag{21}$$

Using the facts that

$$s^2 \phi h^T h + 1 = s^2 \|\bar{W}\|^2 h^T h + 1 \geq -2 s \bar{W}^T h$$

$$\left| \frac{d(t)}{f_4(x)} s \right| \leq \frac{s^2}{\varepsilon f_4^2(x)} + \frac{\varepsilon}{4} d^2(t)$$

$$|\mu_\varepsilon s| \leq \frac{s^2}{2 \varepsilon f_4(x)} + \frac{\varepsilon}{2} \mu_\varepsilon^2 f_4(x) \leq \frac{s^2}{2 \varepsilon f_4(x)} + \frac{\varepsilon}{2} \mu_\varepsilon^2 \bar{f}_4$$

and noting that $|\mu_\varepsilon| \leq \mu_0$, $|d(t)| \leq d_0$, we obtain

$$\begin{aligned} \dot{V} &\leq \tilde{\phi} \left(-\frac{1}{2} s^2 h^T h + \frac{1}{\gamma} \dot{\hat{\phi}} \right) \\ &\quad - \left(\frac{1}{\varepsilon f_4(x)} + \frac{1}{\varepsilon f_4^2(x)} \right) s^2 - \frac{d(t)}{f_4(x)} s - \mu_\varepsilon s + \frac{1}{2} \\ &\leq \tilde{\phi} \left(-\frac{1}{2} s^2 h^T h + \frac{1}{\gamma} \dot{\hat{\phi}} \right) - \frac{s^2}{2 \varepsilon f_4(x)} \\ &\quad + \frac{\varepsilon}{2} \mu_0^2 \bar{f}_4 + \frac{\varepsilon}{4} d_0^2 + \frac{1}{2}. \end{aligned} \tag{22}$$

Considering the adaptive law (19), and from (22), we have

$$\begin{aligned}\dot{V} \le & -\kappa\tilde{\phi}\hat{\phi} - \frac{s^2}{2\varepsilon f_4(x)} + \frac{\varepsilon}{2}\mu_0^2\bar{f}_4 + \frac{\varepsilon}{4}d_0^2 + \frac{1}{2}\\ \le & -\frac{\kappa}{2}\left(\tilde{\phi}^2 - \phi^2\right) - \frac{s^2}{2\varepsilon f_4(x)} + \frac{\varepsilon}{2}\mu_0^2\bar{f}_4 + \frac{\varepsilon}{4}d_0^2 + \frac{1}{2}\\ \le & -\frac{\kappa}{2}\tilde{\phi}^2 - \frac{s^2}{2\varepsilon f_4(x)} + \frac{\varepsilon}{2}\mu_0^2\bar{f}_4 + \frac{\varepsilon}{4}d_0^2 + \left(\frac{1}{2} + \frac{\kappa}{2}\phi^2\right).\end{aligned}$$

Setting $\kappa = \eta/\gamma$, $\eta > 0$, we obtain

$$\begin{aligned}\dot{V} \le & -\frac{\eta}{2\gamma}\tilde{\phi}^2 - \frac{s^2}{2\varepsilon f_4(x)} + \frac{\varepsilon}{2}\mu_0^2\bar{f}_4 + \frac{\varepsilon}{4}d_0^2 + \frac{1}{2} + \frac{\eta}{2\gamma}\phi^2\\ \le & -\beta_0 V + \delta\end{aligned}$$

where $\beta_0 = \min\{\eta, 1/\varepsilon\}$, $\delta = \frac{\varepsilon}{2}\mu_0^2\bar{f}_4 + \frac{\varepsilon}{4}d_0^2 + \frac{1}{2} + \frac{\eta}{2\gamma}\phi^2$. Solving the above inequality using Lemma B.5 in [24], we have

$$\begin{aligned}V(t) \le & \, e^{-\beta_0 t}V(0) + \delta \times \int_0^t e^{-\beta_0(t-\tau)}d\tau\\ \le & \, e^{-\beta_0 t}\left[V(0) - \frac{\delta}{\beta_0}\right] + \frac{\delta}{\beta_0} \quad \forall t \ge 0. \qquad (23)\end{aligned}$$

By the definition of V, we have $V \ge \frac{1}{2}\frac{s^2}{f_4(x)}$. Thus, we obtain $|s| \le \sqrt{2f_4(x)V} \le \sqrt{2\bar{f}_4 V}$. From (23), and noting $\sqrt{a+b} \le \sqrt{a} + \sqrt{b}$ $(a > 0,\ b > 0)$, we obtain

$$\begin{aligned}|\mathrm{s}| \le & \sqrt{2\bar{f}_4\left[e^{-\beta_0 t}V(0) + \frac{\delta}{\beta_0}(1 - e^{-\beta_0 t})\right]}\\ \le & \sqrt{2\bar{f}_4}\left[e^{-\beta_0\frac{t}{2}}\sqrt{V(0)} + \sqrt{\frac{\delta}{\beta_0}(1 - e^{-\beta_0 t})}\right]\\ \le & \sqrt{2\bar{f}_4}\left[e^{-\beta_0\frac{t}{2}}\sqrt{V(0)} + \sqrt{\frac{\delta}{\beta_0}}\right] \quad \forall t \ge 0. \qquad (24)\end{aligned}$$

Since $V(0)$ is bounded, the inequality (24) shows that s is bounded as $\lim_{t\to 0}|\mathrm{s}| \le \sqrt{2\bar{f}_4}\cdot\sqrt{\delta/\beta_0}$, $\forall t \ge 0$. The inequality (23) also shows that $V(t)$ is bounded which implies $\tilde{\phi}$ is bounded too. Thus, the closed-loop system is proven uniformly ultimately bounded.

The inequality (24) indicates that the tracking error $e(t)$ converges to a small residual set Φ for all time, then the state x in system (1) will follow the desired trajectory x_d. The tracking error $e(t)$ can be made arbitrarily small by appropriately choosing the parameters ε, η and γ. The increases in γ and η, or decrease in ε will make the tracking performance better. ■

V. Simulation Results

In order to prove the effectiveness of the control scheme, we take a CMD system for simulation. Its schematic is shown in Fig. 1, and the linearized dynamics in the presence of external disturbance could be written as follows [23]:

$$\begin{aligned}& J_l\ddot{\theta}_2 + c_{12}\dot{\theta}_2 + k\left(\theta_2 - g_r^{-1}\theta_1\right) = 0\\ & J_d\ddot{\theta}_1 + c_{11}\dot{\theta}_1 + kg_r^{-1}\left(g_r^{-1}\theta_1 - \theta_2\right) = T_d + d(t) \qquad (25)\end{aligned}$$

where θ_1 is the drive angle position; θ_2 is the load angle position; J_l is the total inertias reflected at the load; J_d is the total inertias reflected at the drive; $g_r = (r_l r_{pl})/(r_{p2} r_d)$ is the gear ratio; T_d is control torque input; c_{11} is the drive rotary damping (modeled as viscous); c_{12} is the load rotary damping (modeled as viscous); $d(t)$ is the external disturbance, and $k = 2k_l r_l^2$ is torsional spring constant.

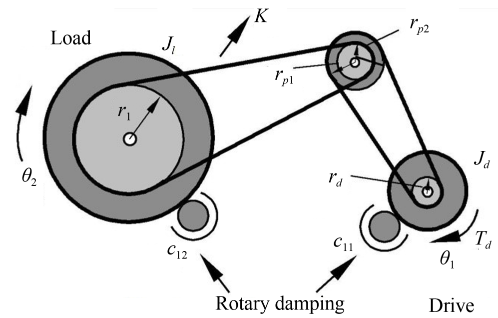

Fig. 1. Schematic of CMD system.

Let $x_1 = \theta_2$ and $x_3 = \theta_1$ be the state variables, and $u = T_d$ be the control input, then the dynamics (25) will be changed into the form as follows:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f_1(x_1, x_2, x_3)\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= f_2(x) + f_3(x)u + d(t)\end{aligned}$$

where

$$\begin{aligned}f_1(x_1, x_2, x_3) &= -\frac{c_{12}}{J_l}x_2 - \frac{k}{J_l}\left(x_1 - g_r^{-1}x_3\right)\\ f_2(x) &= -\frac{c_{11}}{J_d}x_4 - \frac{kg_r^{-1}}{J_d}\left(g_r^{-1}x_3 - x_1\right)\\ f_3(x) &= \frac{1}{J_d}.\end{aligned}$$

The parameter values in (25) for simulation are given by $J_d = 0.000425\,\mathrm{kg\cdot m^2}$, $J_l = 0.3575\,\mathrm{kg\cdot m^2}$, $d(t) = \sin(t)$, $c_{12} = 0.05\,(\mathrm{N\cdot m})\cdot\mathrm{s/rad}$, $c_{11} = 0.004\,(\mathrm{N\cdot m\cdot s})/\mathrm{rad}$, $g_r = 4$, $k = 8.45\,(\mathrm{N\cdot m})/\mathrm{rad}$.

The initial states are $x(0) = [0.5 \ \ 0 \ \ 0 \ \ 0]^T$, and the desired signal is $x_d = \sin t$. The input vector of RBFNN is $\alpha = [x_1 \ \ x_2 \ \ x_3 \ \ x_4 \ \ s \ \ s/\varepsilon \ \ v]^T$. The network structure 7-9-1 is used. The parameters of a_j and b_j in RBFNN must be chosen according to the scope of the input value. If the parameter values are chosen inappropriately, Gaussian function will not be effectively mapped, and RBF network will be invalid. In this example, according to the practical scope of x_1, x_2, x_3, x_4, s, s/ε and v, from (10), for each Gaussian function, the parameters of a and b_j are designed as $a = 0.1 \times [-2 \ \ -1.5 \ \ -1 \ \ -0.5 \ \ 0 \ \ 0.5 \ \ 1 \ \ 1.5 \ \ 2]$ and $b_j = 5$ respectively. The initial weight value is chosen as zero. In the adaptive law (20), the initial condition is $\hat{\phi}(0) = 0$. The filtered tracking gain parameters in (4) are chosen as $c_1 = 27$, $c_2 = 27$, and $c_3 = 9$. Fig. 2 shows the closed-loop RBFNN adaptive control scheme.

The simulations with different design parameters are shown in Figs. 3−6 to illustrate the tracking performance of the controller. In Figs. 3 and 4, the parameter in the ideal feedback control law (6) is taken as $\varepsilon = 0.1$ and the adaptation gain constants in the adaptive law (19) are given by $\eta = 10$ and $\gamma = 10$. In Figs. 5 and 6, the parameters are chosen as $\varepsilon = 0.01$, $\eta = 100$ and $\gamma = 100$. Figs. 3–6 show the different tracking errors and control inputs, illustrating the effectiveness of the proposed scheme. By contrast, we validate the conclusion that the smaller ε and larger η and γ may improve the tracking performance of the closed-loop system.

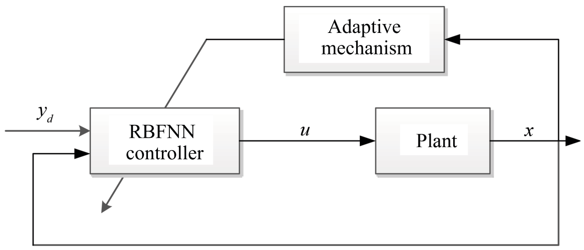

Fig. 2. Block diagram of RBF control scheme.

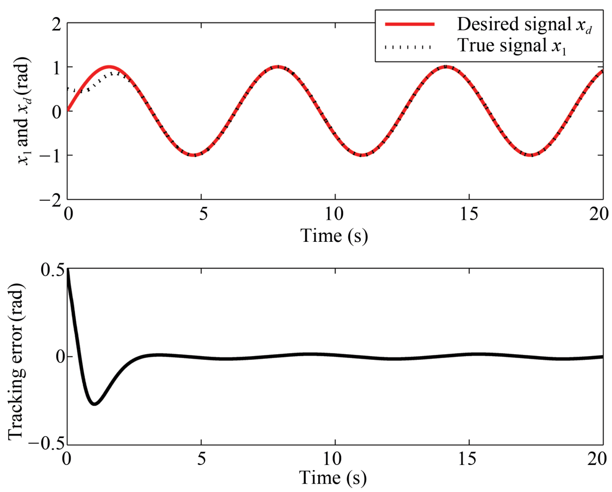

Fig. 3. Position tracking performance with the parameters $\varepsilon = 0.1$, $\eta = 10$ and $\gamma = 10$.

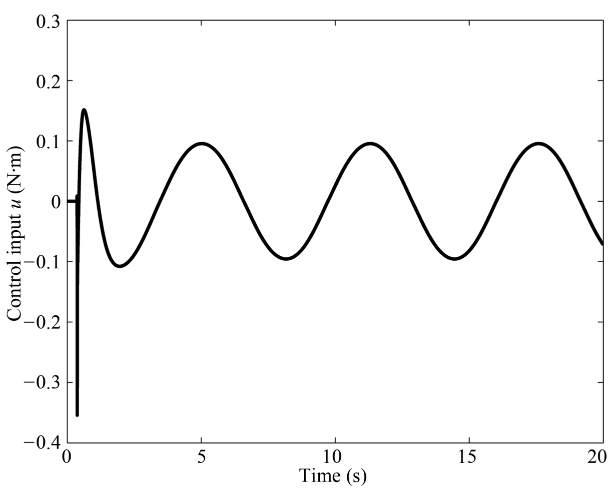

Fig. 4. Control input with the parameters $\varepsilon = 0.1$, $\eta = 10$ and $\gamma = 10$.

VI. Conclusion

An adaptive RBFNN control method has been proposed in this paper for a class of SISO nonlinear systems in the presence of bounded disturbances. By designing adaptive controller and adaptive law using a constant parameter, we prove uniform ultimate boundedness of the closed-loop system. Finally, we take the CMD system which satisfies the above mentioned for simulation to confirm the effectiveness of the method. The simulation results show the favorable performance on tracking desired signal and verify the stability of the closed-loop adaptive system through selecting the appropriate parameters.

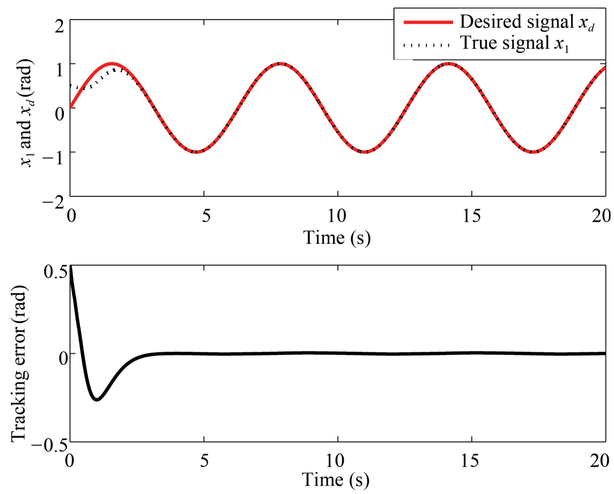

Fig. 5. Position tracking performance with the parameters $\varepsilon = 0.01$, $\eta = 100$ and $\gamma = 100$.

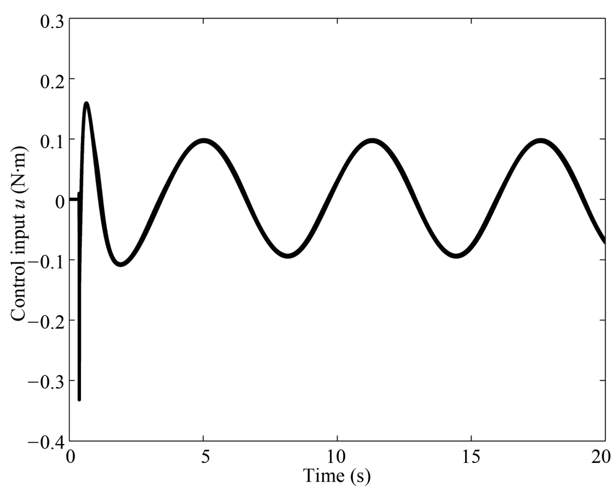

Fig. 6. Control input with the parameters $\varepsilon = 0.01$, $\eta = 100$ and $\gamma = 100$.

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