An Exploration on Adaptive Iterative Learning Control for a Class of Commensurate High-order Uncertain Nonlinear Fractional Order Systems

Jianming Wei, Youan Zhang, and Hu Bao

Abstract—This paper explores the adaptive iterative learning control method in the control of fractional order systems for the first time. An adaptive iterative learning control (AILC) scheme is presented for a class of commensurate high-order uncertain nonlinear fractional order systems in the presence of disturbance. To facilitate the controller design, a sliding mode surface of tracking errors is designed by using sufficient conditions of linear fractional order systems. To relax the assumption of the identical initial condition in iterative learning control (ILC), a new boundary layer function is proposed by employing Mittag-Leffler function. The uncertainty in the system is compensated for by utilizing radial basis function neural network. Fractional order differential type updating laws and difference type learning law are designed to estimate unknown constant parameters and time-varying parameter, respectively. The hyperbolic tangent function and a convergent series sequence are used to design robust control term for neural network approximation error and bounded disturbance, simultaneously guaranteeing the learning convergence along iteration. The system output is proved to converge to a small neighborhood of the desired trajectory by constructing Lyapnov-like composite energy function (CEF) containing new integral type Lyapunov function, while keeping all the closed-loop signals bounded. Finally, a simulation example is presented to verify the effectiveness of the proposed approach.

Index Terms—Adaptive iterative learning control (AILC), boundary layer function, composite energy function (CEF), fractional order differential learning law, fractional order nonlinear systems, Mittag-Leffler function.

I. INTRODUCTION

PAST decades have witnessed tremendous research efforts aiming at the development of systematic design methods for the iterative learning control (ILC) of nonlinear systems performing control task over a finite interval repeatedly. ILC has been proven to be the most suitable and effective control scheme for such repeatable control tasks owing to its capacity

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of achieving perfect tracking by learning along iteration. Generally, according to the stability analysis tool, ILC can be categorized into two classes: traditional ILC [1]−[5] and adaptive ILC (AILC) [6]−[16]. The basic idea of traditional ILC is to use information of the previous execution to design the control signal for current operation by a learning mechanism, which allows to achieve improving performance from iteration to iteration. Furthermore, the stability conclusion of traditional ILC is usually obtained by using contraction mapping theorem and fixed point theorem, which enables traditional ILC to deal with nonlinear plants without any information of the system. Traditional ILC has been developed greatly in theory and application because of its simplicity and availability. However, the main drawback of traditional ILC lies in the requirement of the global Lipschitz continuous condition, which restricts its application to certain nonlinear systems. Besides, the usage of contraction mapping theorem rather than Lyapunov method as the key tool of stability analysis in traditional ILC makes it difficult to relax the global Lipschitz condition to local Lipschitz or even non-Lipschitz condition and cooperate with the mainstream methods of nonlinear control theory, such as adaptive control and neural control. To overcome the constraints of traditional ILC, some researchers tried to introduce the idea of adaptive control into ILC and proposed adaptive iterative learning control [6], [7]. AILC takes advantage of both adaptive control and ILC, which successfully overcomes the restriction of global Lipschitz condition, thus it enables us to use fuzzy logic systems or neural networks as approximators to deal with nonlinear uncertainties. In general, the control parameters of AILC methods are tuned along the iteration axis, and the so-called composite energy function (CEF) [8] is usually constructed to analyze the stability and convergence property of the closed-loop systems. The past decade has witnessed great progress in AILC of uncertain nonlinear systems [9]−[16].

Fractional calculus has been studied for more than 300 years. But the researches are mainly in the field of mathematical sciences [17], [18]. Until recent decade, the applications of fractional calculus develop rapidly [19], [20]. Fractional order systems allow us to describe and model a real object more accurately than the classical integer order dynamical systems. Among the investigations of fractional order systems in the past decades, control design for some fractional order systems has been a hot topic. Many different control methods have been proposed for various kinds of fractional systems [20]−[28]. Especially, the research on control and synchronization control

design for fractional order chaotic systems is very active [29]−[39].

Comparing with such a large number of results, the papers on the ILC control of fractional order systems are less. Only a few works are reported in the filed of ILC [40]−[53]. Moreover, all these literatures are from the viewpoint of traditional ILC and the stability conclusions are obtained by using contraction mapping theorem method. Therefore, as results of integer-order systems, global Lipschitz condition is required for traditional ILC schemes. As for AILC problem of fractional order systems, to the best of our knowledge, there are no results having been reported.

In this paper, we present an AILC scheme for a class of nonlinear fractional order system with both parametric and nonparametric uncertainties in the presence of disturbance. As far as we know, no works have been presented for such a problem up till now. In the proposed AILC scheme, adaptive iterative learning controller with fractional order differential type and difference type learning laws are presented and the CEF containing new integral type Lyapunov function is constructed to analyze the stability and convergence property. The main contributions of the proposed AILC scheme and stability analysis are highlighted as follows. 1) To the best of our knowledge, it is the first time, in the literature, that AILC problem of fractional order system is investigated. 2) A sliding mode surface of tracking errors is designed by using the sufficient condition for linear fractional order systems. 3) A new boundary layer function using Mittag-Leffler function is designed to deal with the initial condition problem in the ILC design of fractional order system. 4) Fractional order differential type learning laws with alignment method for unknown constant parameters is used in the AILC method for the first time and integral type Lyapunov function is used to analyze the convergence of estimation errors. 5) The hyperbolic tangent function is used to design robust control term for neural network approximation error and bounded disturbance, and a convergent series is introduced to guarantee the learning convergence along iteration index.

The rest of this paper is organized as follow. The problem formulation and preliminaries are given in Section II. The AILC design with parameter updating laws is developed in Section III. In Section IV, the CEF-based stability analysis is presented. A simulation example is presented to verify the validity of the proposed scheme in Section V, followed by conclusions in Section VI.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Preliminaries

In this subsection, some basic definitions and useful lemmas are given.

Definition 1 [18]: Fractional calculus is a generalization of integration and differentiation to noninteger-order fundamental operator $_aD_t^\alpha$, where a and t are the bounds of the operation and $\alpha \in \mathbb{R}$. The continuous integro-differential operator is defined as

$$_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \int_a^t (d\sigma)^\alpha, & \alpha < 0. \end{cases} \quad (1)$$

Definition 2 [17]: The most important function used in fractional calculus-Eulers gamma function is defined as

$$\Gamma(\alpha) = \int_0^\infty e^{-\sigma} \sigma^{\alpha-1} d\sigma. \quad (2)$$

Definition 3 [17]: Another important function in the fractional calculus named Mittag-Leffler type with two parameters is defined as

$$E_{\alpha,\beta}(z) = \sum_{j=0}^\infty \frac{z^j}{\Gamma(\alpha j + \beta)}, \quad \alpha > 0; \ \beta > 0. \quad (3)$$

Especially, when $\beta = 1$, we obtain the Mittag-Leffler function with one parameter

$$E_{\alpha,1}(z) = \sum_{j=0}^\infty \frac{z^j}{\Gamma(\alpha j + 1)} = E_\alpha(z). \quad (4)$$

For integer values of α, (4) reduces to the well-known Cauchy repeated integration formula.

The three most frequently used definitions for the general fractional differintegral are: the Grünwald-Letnikov (GL) definition, the Riemann-Liouville (RL) and Caputo definitions.

Definition 4 [17]: The Grünwald-Letnikov derivative definition of order α is described as

$$_aD_t^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^\infty (-1)^j \begin{pmatrix} \alpha \\ j \end{pmatrix} f(t - jh) \quad (5)$$

with

$$\begin{pmatrix} \alpha \\ j \end{pmatrix} = \frac{\alpha!}{j!(\alpha-j)!} = \frac{\Gamma(\alpha+1)}{\Gamma(j-1)\Gamma(\alpha-j+1)}. \quad (6)$$

Definition 5 [54]: The Riemann-Liouville fractional integral of order α of function $f(t)$ at a time instant $t \geq 0$ is defined as

$$_aI_t^\alpha f(t) = _aD_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t f(\sigma)(t-\sigma)^{\alpha-1} d\sigma. \quad (7)$$

From (7) we can write formula for the Riemann-Liouville definition of fractional derivative of order α in the following form

$$_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\sigma)}{(t-\sigma)^{\alpha-n+1}} d\sigma \quad (8)$$

for $n - 1 < \alpha < n$.

Definition 6 [17]: The Caputo fractional integral of order α of function $f(t)$ at time $t \geq 0$ is defined as

$$_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\sigma)}{(t-\sigma)^{\alpha-n+1}} d\sigma \quad (9)$$

for $n - 1 < \alpha < n$.

Remark 1: Actually, the above three definitions are equivalent under some conditions. We will use the Caputo definition in this paper. In the rest of this paper, the notation $D^\alpha(\cdot)$ indicates the Caputo derivative of order α with $a = 0$, i.e., $D^\alpha(\cdot) = _0D_t^\alpha(\cdot)$.

Lemma 1 [55], [56]: Consider the following fractional order autonomous system

$$D^\alpha x(t) = Ax(t), \quad x(0) = x_0 \tag{10}$$

where $0 < \alpha < 1$, $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$. This system is asymptotically stable if and only if $|\arg(\lambda(A))| > \alpha\pi/2$. In this situation, the components of the state vector decay toward zero like $t^{-\alpha}$. The stable region for fractional order linear systems with $0 < \alpha < 1$ is shown in Fig. 1.
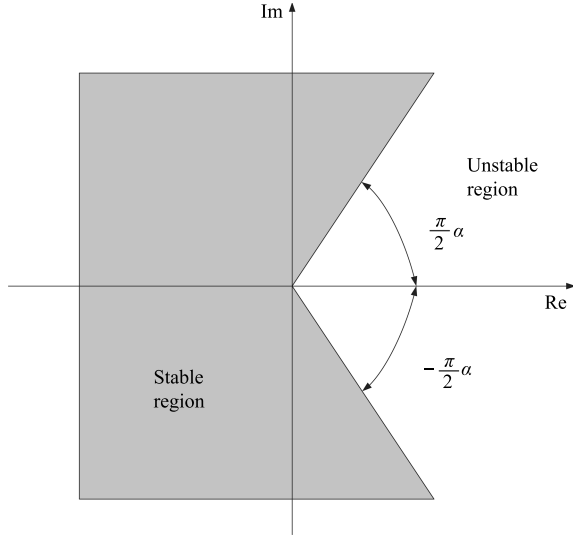

Fig. 1. Stability domain for fractional order linear systems with $0 < \alpha < 1$.

Lemma 2 [57]: The fractional system $D^\alpha y(t) = u(t)$, $0 < \alpha < 1$, is equivalent to the following continuous frequency distributed model

$$\begin{cases} \frac{\partial z(\omega,t)}{\partial t} = -\omega z(\omega,t) + u(t) \\ y(t) = \int_0^\infty \mu(\omega) z(\omega,t) d\omega \end{cases} \tag{11}$$

with weighting function $\mu(\omega) = \sin(\alpha\pi)/(\pi\omega^\alpha)$, $z(\omega,t) \in \mathbb{R}$.

B. Problem Formulation

In this paper, we consider a class of commensurate high-order uncertain nonlinear systems in the presence of disturbance which runs on a finite interval $[0, T]$ repeatedly as follows:

$$\begin{cases} D^\alpha x_{i,k}(t) = x_{i+1,k}(t), \quad i = 1, \ldots, n-1 \\ D^\alpha x_{n,k}(t) = f(x_k(t)) + \theta(t)\xi(x_k(t)) + u_k(t) + d(t) \\ y_k(t) = x_{1,k}(t) \end{cases}$$
$$\tag{12}$$

where $t \in [0, T]$ is the time, $k \in \mathbb{N}$ denotes the times of iteration; $x_{i,k}(t) \in \mathbb{R}$, $i = 1, \ldots, n$ and $y_k(t)$ are the pseudo state and output variables, respectively; $x_k(t) = [x_{1,k}(t), x_{2,k}(t), \ldots, x_{n,k}(t)]^T \in \mathbb{R}^n$ is the pseudo state vector; $f(\cdot)$ is an unknown smooth function. $\theta(t)$ is an unknown continuous time-varying parameter and $\xi(\cdot)$ is a known continuous function. $d(t)$ is unknown bounded external disturbance. $u_k(t) \in \mathbb{R}$ is the control input. The control objective of this paper is to

design the adaptive iterative learning controller to steer the output $y_k(t)$ to follow the desired reference signal $r(t)$.

Define $r_1(t) = r(t)$ and $r_{i+1}(t) = D^\alpha r_i(t)$, $i = 1, 2, \ldots, n-1$. Then we can write the desired reference vector as $x_d(t) = [r_1(t), r_2(t), \ldots, r_n(t)]^T$. Define the tracking errors as $e_{i,k}(t) = x_{i,k}(t) - r_i(t)$, $i = 1, 2, \ldots, n$. Then the tracking error vector can be given by $e_k(t) = x_k(t) - x_d(t) = [e_{1,k}(t), e_{2,k}(t), \ldots, e_{n,k}(t)]^T$. In the rest of this paper, the denotation t will be omitted when no confusion arises.

Choose the sliding surface as $e_{s,k} = [\Lambda^T \ 1] e_k$, where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{n-1}]^T$ and $\lambda_1, \ldots, \lambda_{n-1}$ are chosen suitably such that the eigenvalues of the matrix B satisfy condition of Lemma 1, where the matrix B is given by

$$B = \begin{bmatrix} 0 & & \\ \vdots & I_{n-2} & \\ 0 & & \\ -\lambda_1 & \ldots & -\lambda_{n-1} \end{bmatrix} \tag{13}$$

with I_{n-2} as unit matrix of $n-2$ dimensions. Then keeping the systems errors on this surface leads to the asymptotic stability of error systems and therefore output tracking of the desired reference signal.

To facilitate control system design, the following reasonable assumptions are made.

Assumption 1: The unknown external disturbance is bounded.

Assumption 2: The desired state trajectory $x_d(t)$ is continuous, bounded and available.

Assumption 3: The initial state errors $e_{i,k}(0)$ at each iteration are not necessarily zero, small and fixed, but assumed to be bounded.

C. RBF Neural Networks

In control engineering, two types of artificial neural networks are usually used to approximate unknown smooth functions, which specifically are linearly parameterized neural networks (LPNNs) and multilayer neural networks (MNNs). As a kind of LPNNs, the radial basis function (RBF) neural network (NN) [58] is usually used as a tool to model unknown nonlinear functions owing to its nice approximation capabilities. The RBF NN can be seen as a two-layer network in which hidden layer performs a fixed nonlinear transformation with no adjustable parameters, i.e., the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Generally, the RBF NN approximates the continuous function $Q(Z) : \mathbb{R}^q \to \mathbb{R}$ as follows

$$Q_{nn}(Z) = W^T \phi(Z)$$

where $Z \in \Omega_Z \subset \mathbb{R}^q$ is the input vector, $W = [w_1, w_2, \ldots, w_l]^T \in \mathbb{R}^l$ is the weight vector, the NN node number $l > 1$; and $\phi(Z) = [\varphi_1(Z), \ldots, \varphi_l(Z)]^T$, with $\varphi_i(Z)$ as the commonly used Gaussian functions, i.e., $\varphi_i(Z) = e^{-(Z-\mu_i)^T(Z-\mu_i)/\sigma_i^2}$, $i = 1, \ldots, l$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]$ is the center of the receptive field and σ_i is the width of the Gaussian function. It has been proven that if l is chosen sufficiently large, $W^T \phi(Z)$ can approximate any continuous function, $Q(Z)$, to any desired accuracy over a compact set

$\Omega_Z \subset \mathbb{R}^q$ in the form of $Q(Z) = W^{*T}\phi(Z) + \varepsilon(Z), \forall Z \in \Omega_Z \subset \mathbb{R}^q$ where W^* is the ideal constant weight vector, and $\varepsilon(Z)$ is the approximation error which is bounded over the compact set, i.e., $|\varepsilon(Z)| \leq \varepsilon^*, \forall Z \in \Omega_Z$, where $\varepsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an artificial quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon(Z)|$ for all $Z \in \Omega_Z \subset \mathbb{R}^q$, i.e., $W^* := \arg\min_{W \in \mathbb{R}^l}\{\sup_{Z \in \Omega_Z}|h(Z) - W^T\phi(Z)|\}$.

III. AILC SCHEME DESIGN

According to the systems dynamic equation (10) and definition of tracking errors, we can have the dynamics of tracking errors

$$\begin{cases} D^\alpha e_{i,k}(t) = e_{i+1,k}(t), & i = 1, \ldots, n-1 \\ D^\alpha e_{n,k}(t) = f(x_k(t)) + \theta(t)\xi(x_k(t)) + u_k(t) \\ \qquad + d(t) - D^\alpha r_n. \end{cases} \quad (14)$$

By taking the derivative of order α of sliding surface, one has

$$\begin{aligned} D^\alpha e_{s,k} &= D^\alpha e_{n,k} + D^\alpha\left(\sum_{i=1}^{n-1}\lambda_i e_{i,k}\right) \\ &= D^\alpha e_{n,k} + \sum_{i=1}^{n-1}\lambda_i D^\alpha e_{i,k} \\ &= f(x_k(t)) + \theta(t)\xi(x_k(t)) + u_k(t) + d(t) \\ &\quad - D^\alpha r_n + \sum_{i=1}^{n-1}\lambda_i e_{i+1,k}. \end{aligned} \quad (15)$$

According to Assumption 3, there exist known constants ε_i, such that, $|e_{i,k}(0)| \leq \varepsilon_i, i = 1, 2, \ldots, n$ for any $k \in \mathbb{N}$. In order to overcome the uncertainty from initial resetting errors, we define a novel boundary layer function by employing Mittag-Leffler function

$$\eta(t) = \varepsilon E_\alpha(-Kt), \quad K > 0 \quad (16)$$

where $\varepsilon = [\Lambda^T \; 1][\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]^T$.

Remark 2: As the boundary layer function [13]−[15] in integer order case, $\eta(t)$ has property of decreasing along time axis with initial condition $\eta(0) = \varepsilon$. Moreover, it is clear that $D^\alpha\eta(t) = \varepsilon D^\alpha E_\alpha(-Kt) = -K\varepsilon E_\alpha(-Kt) = -K\eta(t)$.

Then we can define an auxiliary error signal as

$$s_k(t) = e_{s,k}(t) - \eta(t)\text{sat}\left(\frac{e_{s,k}(t)}{\eta(t)}\right) \quad (17)$$

where $\text{sat}(\cdot)$ is the saturation function which is defined as

$$\text{sat}(\cdot) = \text{sgn}(\cdot) \times \min\{|\cdot|, 1\} \quad (18)$$

with $\text{sgn}(\cdot) = \begin{cases} 1, & \text{if } (\cdot) > 0 \\ 0, & \text{if } (\cdot) = 0 \\ -1, & \text{if } (\cdot) < 0. \end{cases}$ as the sign function.

Subsequently, it can be easily obtained that

$$\begin{aligned} |e_{sk}(0)| &= |\lambda_1 e_{1,k}(0) + \lambda_2 e_{2,k}(0) + \cdots + e_{n,k}(0)| \\ &\leq \lambda_1|e_{1,k}(0)| + \lambda_2|e_{2,k}(0)| + \cdots + |e_{n,k}(0)| \\ &\leq \lambda_1\varepsilon_1 + \lambda_2\varepsilon_2 + \cdots + \varepsilon_n = \eta(0) \end{aligned} \quad (19)$$

which implies that $s_k(0) = e_{sk}(0) - \eta(0)e_{sk}(0)/\eta(0) = 0$ is satisfied for all $k \in \mathbb{N}$. Moreover, there exists the fact that

$$s_k(t)\text{sat}\left(\frac{e_{sk}(t)}{\eta(t)}\right) = \begin{cases} 0, & \text{if } \left|\frac{e_{sk}(t)}{\eta(t)}\right| \leq 1 \\ s_k(t)\text{sgn}(e_{sk}(t)), & \text{if } \left|\frac{e_{sk}(t)}{\eta(t)}\right| > 1 \end{cases}$$
$$= s_k(t)\text{sgn}(s_k(t)) = |s_k(t)|. \quad (20)$$

To overcome the design difficulty from uncertainty $f(x_k(t))$, we employ radial basis function neural network to approximate $f(x_k(t))$ in the form of

$$f(x_k(t)) = W^{*T}\phi(x_k) + \varepsilon(x_k). \quad (21)$$

From Lemma 2, we can obtain the equivalent continuous frequency distributed model of dynamical system of s_k

$$\begin{cases} \frac{\partial z_k(\omega,t)}{\partial t} = -\omega z_k(\omega, t) + D^\alpha s_k \\ s_k(t) = \int_0^\infty \mu(\omega) z_k(\omega, t)d\omega \end{cases} \quad (22)$$

with weighting function $\mu(\omega) = \sin(\alpha\pi)/(\pi\omega^\alpha)$, $z_k(\omega, t) \in \mathbb{R}$ is the true error variable.

Define a smooth scalar positive function as

$$V_{s,k}(t) = \frac{1}{2}\int_o^\infty \mu(\omega)z_k^2(\omega, t)d\omega. \quad (23)$$

The time derivative of $V_{s,k}(t)$ can be expressed as

$$\begin{aligned} \dot{V}_{s,k}(t) &= \int_0^\infty \mu(\omega)z_k(\omega, t)\frac{\partial z_k(\omega, t)}{\partial t}d\omega \\ &= \int_0^\infty \mu(\omega)z_k(\omega, t)(-\omega z_k(\omega, t) + D^\alpha s_k)d\omega \\ &= -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega + s_k D^\alpha s_k \\ &= \begin{cases} -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega \\ \quad + s_k(D^\alpha e_{s,k} - D^\alpha\eta(t)\text{sgn}(s_k)), & |e_{s,k}| > \eta(t) \\ -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega, & |e_{s,k}| \leq \eta(t) \end{cases} \\ &= -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega \\ &\quad + s_k(D^\alpha e_{s,k} - D^\alpha\eta(t)\text{sgn}(s_k)) \\ &= -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega \\ &\quad + s_k\Big(W^{*T}\phi(x_k) + \varepsilon(x_k) \\ &\quad + \theta(t)\xi(x_k) + u_k + d(t) - D^\alpha r_n \\ &\quad + \sum_{i=1}^{n-1}\lambda_i e_{i+1,k} + K\eta(t)\text{sgn}(s_k)\Big) \\ &= -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega + s_k\Big(W^{*T}\phi(x_k) + \varepsilon(x_k) \\ &\quad + \theta(t)\xi(x_k) + d(t) - D^\alpha r_n + \sum_{i=1}^{n-1}\lambda_i e_{i+1,k} \\ &\quad - Ke_{s,k} + Ke_{s,k} + K\eta(t)\text{sgn}(s_k)\Big) \\ &= -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega \\ &\quad + s_k\Big(W^{*T}\phi(x_k) + \theta(t)\xi(x_k) + u_k \end{aligned}$$

$$+ \bar{d}(t) - D^\alpha r_n + \sum_{i=1}^{n-1} \lambda_i e_{i+1,k} + K e_{s,k} \Big) - K s_k^2 \qquad (24)$$

where $\bar{d}(t) = d(t) + \varepsilon(x_k)$ and using the equality

$$
\begin{aligned}
s_k(t)&(-K e_{sk}(t) + K\eta(t)\mathrm{sgn}\,(s_k(t))) \\
&= s_k(t)\left(-K s_k(t) - K\eta(t)\mathrm{sat}\left(\frac{e_{sk}(t)}{\eta(t)}\right)\right. \\
&\quad \left. + K\eta(t)\mathrm{sgn}\,(s_k(t))\right) \\
&= -K s_k^2(t) - K\eta(t)|s_k(t)| + K\eta(t)|s_k(t)| \\
&= -K s_k^2(t). \qquad (25)
\end{aligned}
$$

Obviously, $\bar{d}(t)$ is bounded, i.e., there exists an unknown positive constant ρ such that $|\bar{d}(t)| \le \rho$.

Then we can determine the control law as

$$
\begin{aligned}
u_k(t) = {} & D^\alpha r_n - \sum_{i=1}^{n-1} \lambda_i e_{i+1,k} - K e_{s,k} - \hat{W}_k^T \phi(x_k) \\
& - \hat{\theta}_k(t)\xi(x_k) - \hat{\rho}_k \tanh\left(\frac{s_k \hat{\rho}_k}{\Delta_k}\right) \qquad (26)
\end{aligned}
$$

where $\hat{W}_k$, $\hat{\theta}_k(t)$ and $\hat{\rho}_k$ are the estimates of W^*, $\theta(t)$ and ρ, respectively, Δ_k is a convergent series sequence which is defined as $\Delta_k = q/k^m$, l and q are constant design parameters and $q\,(\in \mathbb{R}) > 0$, $m\,(\in \mathbb{Z}_+) \ge 2$. For preceding analysis, we need the following lemmas.

Lemma 3 [59]: For any $\Delta_k > 0$ and $x \in \mathbb{R}$, the inequality $|x| - x\tanh(x/\Delta_k) \le \gamma\Delta_k$ is established, where γ is a positive constant and $\gamma = e^{-(\gamma+1)}$ or $\gamma = 0.2785$.

Lemma 4 [60]: $\lim_{k\to\infty} \sum_{j=1}^{k} \Delta_j \le 2q$.

The adaptive learning laws for unknown parameters are designed as

$$
\begin{cases}
D^\alpha \hat{W}_k(t) = \Gamma_W s_k(t)\phi(x_k) \\
\hat{W}_k(0) = \hat{W}_{k-1}(T), \quad \hat{W}_1(0) = 0
\end{cases} \qquad (27)
$$

$$
\begin{cases}
\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) + q_\theta s_k(t)\xi(x_k) \\
\hat{\theta}_0(t) = 0, \quad t \in [0, T]
\end{cases} \qquad (28)
$$

$$
\begin{cases}
D^\alpha \hat{\rho}_k(t) = q_\rho |s_k(t)| \\
\hat{\rho}_k(0) = \hat{\rho}_{k-1}(T), \quad \hat{\rho}_1(0) = 0
\end{cases} \qquad (29)
$$

where $\Gamma_W \in \mathbb{R}^{l\times l}$ is a positive square matrix and q_θ, $q_\rho > 0$ are design parameters. In the following parts, we define the estimation error of $\Theta(t)$ as $\tilde{\Theta}_k(t) = \hat{\Theta}_k(t) - \Theta(t)$ where $\Theta(t)$ denotes W^*, $\theta(t)$ and ρ.

Substituting the controller (26) back into (24) yields

$$
\begin{aligned}
\dot{V}_{s,k}(t) \le {} & -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega - s_k \tilde{W}_k^T \phi(x_k) \\
& - s_k \tilde{\theta}_k(t)\xi(x_k) + |s_k|\rho - |s_k|\hat{\rho}_k \\
& + |s_k|\hat{\rho}_k - s_k \hat{\rho}_k \tanh\left(\frac{s_k \hat{\rho}_k}{\Delta_k}\right) - K s_k^2 \\
\le {} & -\int_0^\infty \mu(\omega)\omega z_k^2(\omega, t)d\omega - s_k \tilde{W}_k^T \phi(x_k) \\
& - s_k \tilde{\theta}_k(t)\xi(x_k) - |s_k|\tilde{\rho}_k + \gamma\Delta_k - K s_k^2 \\
\le {} & -s_k \tilde{W}_k^T \phi(x_k) - s_k \tilde{\theta}_k(t)\xi(x_k) \\
& - |s_k|\tilde{\rho}_k + \gamma\Delta_k - K s_k^2. \qquad (30)
\end{aligned}
$$

From adaptive updating laws (27) and (29) it follows

$$D^\alpha \tilde{W}_k = D^\alpha \hat{W}_k - D^\alpha W^* = D^\alpha \hat{W}_k \qquad (31)$$

$$D^\alpha \tilde{\rho}_k = D^\alpha \hat{\rho}_k - D^\alpha \rho = D^\alpha \hat{\rho}_k. \qquad (32)$$

According to Lemma 2, we can obtain the distributed frequency model of (31) and (32) as follows

$$
\begin{cases}
\frac{\partial z_{W,k}(\omega,t)}{\partial t} = -\omega z_{W,k}(\omega, t) + \Gamma_W s_k(t)\phi(x_k) \\
\tilde{W}_k(t) = \int_0^\infty \mu(\omega) z_{W,k}(\omega, t)d\omega
\end{cases} \qquad (33)
$$

$$
\begin{cases}
\frac{\partial z_{\rho,k}(\omega,t)}{\partial t} = -\omega z_{\rho,k}(\omega, t) + q_\rho |s_k(t)| \\
\tilde{\rho}_k(t) = \int_0^\infty \mu(\omega) z_{\rho,k}(\omega, t)d\omega
\end{cases} \qquad (34)
$$

where $z_{W,k}(\omega, t) \in \mathbb{R}^l$ and $z_{\rho,k}(\omega, t) \in \mathbb{R}$ are the true estimation error variables.

Define a positive scalar positive function of parameter estimation errors as

$$
\begin{aligned}
V_{p,k}(t) = {} & \frac{1}{2}\int_o^\infty \mu(\omega) z_{W,k}^T(\omega, t)\Gamma_W^{-1} z_{W,k}(\omega, t)d\omega \\
& + \frac{1}{2q_\rho}\int_o^\infty \mu(\omega) z_{\rho,k}^2(\omega, t)d\omega. \qquad (35)
\end{aligned}
$$

Taking the time derivative of $V_{p,k}(t)$ results in

$$
\begin{aligned}
\dot{V}_{p,k}(t) = {} & \int_o^\infty \mu(\omega) z_{W,k}^T(\omega, t)\Gamma_W^{-1}\frac{\partial z_{W,k}(\omega, t)}{\partial t}d\omega \\
& + \frac{1}{q_\rho}\int_o^\infty \mu(\omega) z_{\rho,k}(\omega, t)\frac{\partial z_{\rho,k}(\omega, t)}{\partial t}d\omega \\
= {} & -\int_o^\infty \mu(\omega)\omega z_{W,k}^T(\omega, t)\Gamma_W^{-1} z_{W,k}(\omega, t)d\omega \\
& + \Gamma_W^{-1} D^\alpha \tilde{W}_k^T \int_o^\infty \mu(\omega) z_{W,k}(\omega, t)d\omega \\
& - \frac{1}{q_\rho}\int_o^\infty \mu(\omega)\omega z_{\rho,k}^2(\omega, t)d\omega \\
& + \frac{1}{q_\rho}D^\alpha \tilde{\rho}_k \int_o^\infty \mu(\omega) z_{\rho,k}(\omega, t)d\omega \\
\le {} & s_k \tilde{W}_k^T \phi(x_k) + |s_k|\tilde{\rho}_k. \qquad (36)
\end{aligned}
$$

Define a Lyapunov candidate as $V_k(t) = V_{s,k}(t) + V_{p,k}(t)$. Hence, we can obtain the derivative of $V_k(t)$ with respect to time by combining (30) and (36)

$$\dot{V}_k \le -K s_k^2 - s_k \tilde{\theta}_k(t)\xi(x_k) + \gamma\Delta_k. \qquad (37)$$

IV. ANALYSIS OF STABILITY AND CONVERGENCE

In this section, we will prove that the controller can guarantee the stability of the closed-loop system and the convergence of tracking errors.

The stability of the proposed AILC scheme is summarized as follows.

Theorem 1: Considering the fractional order system (12), and designing adaptive iterative learning controller (26) with parameter adaptive learning algorithms (27)−(29), the following properties can be guaranteed: 1) all the signals of the closed-loop system are bounded; 2) the pseudo error variable $s_k(t)$ converges to zero as $k \to \infty$, i.e., $\lim_{k\to\infty}\int_0^T (s_k(\sigma))^2 d\sigma \le 0$.

Proof: Define the Lyapunov-like CEF as

$$E_k(t) = V_k(t) + \frac{1}{2q_\theta} \int_0^t \tilde{\theta}_k^2(\sigma) \, d\sigma. \tag{38}$$

The proof includes four parts.
1) Difference of $E_k(t)$
Compute the difference of $E_k(t)$, which is

$$\Delta E_k(t) = E_k(t) - E_{k-1}(t)$$
$$= V_k(t) - V_{k-1}(t)$$
$$+ \frac{1}{2q_\theta} \int_0^t \left[\tilde{\theta}_k^2(\sigma) - \tilde{\theta}_{k-1}^2(\sigma) \right] d\sigma. \tag{39}$$

Considering (37), one has

$$V_k(t) \le V_k(0) + \int_0^t \left(-Ks_k^2 - s_k \tilde{\theta}_k(t) \xi(x_k) + \gamma \Delta_k \right) d\sigma$$
$$= V_{p,k}(0) - K \int_0^t s_k^2 \, d\sigma$$
$$- \int_0^t s_k \tilde{\theta}_k(\sigma) \xi(x_k) d\sigma + \gamma \Delta_k t. \tag{40}$$

Utilizing the algebraic relation $(\boldsymbol{a}-\boldsymbol{b})^T(\boldsymbol{a}-\boldsymbol{b}) - (\boldsymbol{a}-\boldsymbol{c})^T \times (\boldsymbol{a}-\boldsymbol{c}) = (\boldsymbol{c}-\boldsymbol{b})^T[2(\boldsymbol{a}-\boldsymbol{b})+(\boldsymbol{b}-\boldsymbol{c})]$ and taking the adaptive learning laws (28) into consideration, we have

$$\frac{1}{2q_\theta} \int_0^t \left[\tilde{\theta}_k^2(\sigma) - \tilde{\theta}_{k-1}^2(\sigma) \right] d\sigma$$
$$= \int_0^t s_k \tilde{\theta}_k(\sigma) \xi(x_k) d\sigma - \frac{q_\theta}{2} \int_0^t s_k^2(\sigma) \xi^2(x_k) d\sigma. \tag{41}$$

Substituting (40) and (41) back into (39), it follows that

$$\Delta E_k(t) \le V_{p,k}(0) - V_{k-1}(t) - K \int_0^t s_k^2 \, d\sigma + \gamma \Delta_k t. \tag{42}$$

Let $t = T$ in (42). From the adaptive parameter updating laws we know $V_{p,k}(0) = V_{p,k-1}(T)$. Therefore, it follows from (42) that

$$\Delta E_k(T) \le V_{p,k}(0) - V_{p,k-1}(T) - V_{s,k-1}(T)$$
$$- K \int_0^t s_k^2 \, d\sigma + \gamma \Delta_k T$$
$$\le - K \int_0^t s_k^2 \, d\sigma + \gamma \Delta_k T. \tag{43}$$

2) Finiteness of $E_k(T)$
According to (38), we know

$$E_1(t) = V_1(t) + \frac{1}{2q_\theta} \int_0^t \tilde{\theta}_1^2(\sigma) \, d\sigma. \tag{44}$$

Recalling adaptive updating law (28), we can have $\hat{\theta}_1(t) = q_\theta s_1(t) \xi(x_1)$, which leads to time derivative of $E_1(t)$ as follows

$$\dot{E}_1(t) = \dot{V}_1(t) + \frac{1}{2q_\theta} \tilde{\theta}_1^2(t)$$
$$\le - Ks_1^2 - s_1 \tilde{\theta}_1(t) \xi(x_1) + \gamma \Delta_1$$
$$+ \frac{1}{2q_\theta} \left(\tilde{\theta}_1^2(t) - 2\tilde{\theta}_1(t)\hat{\theta}_1(t) \right) + \frac{1}{q_\theta} \tilde{\theta}_1(t)\hat{\theta}_1(t)$$
$$= - Ks_1^2 - s_1 \tilde{\theta}_1(t) \xi(x_1) + \gamma \Delta_1 + s_1 \tilde{\theta}_1(t) \xi(x_1)$$

$$+ \frac{1}{2q_\theta} \left[\left(\hat{\theta}_1(t) - \theta(t) \right)^2 - 2 \left(\hat{\theta}_1(t) - \theta(t) \right) \hat{\theta}_1(t) \right]$$
$$\le - Ks_1^2 + \gamma \Delta_1 + \frac{1}{2q_\theta} \theta^2(t). \tag{45}$$

Denote $c = \max_{t\in[0,T]} \left\{ \theta^2(t)/(2q_\theta) \right\}$. Integrating the above inequality over $[0, t]$ yields

$$E_1(t) - E_1(0) \le -K \int_0^t s_1^2(\sigma) d\sigma + t \cdot c + \theta \Delta_1 t. \tag{46}$$

According to the adaptive updating laws it is clear that $E_1(0) = V_{p,1}(0)$, which is determined by W^* and ρ. Thus the boundedness of $E_1(t)$ can be ensured since

$$E_1(t) \le - K \int_0^t s_1^2(\sigma) d\sigma$$
$$+ t \cdot c + \theta \Delta_1 t + V_{p,1}(0), \quad t \in [0, T]. \tag{47}$$

Letting $t = T$ in (47), we can obtain the boundedness of $E_1(T)$

$$E_1(T) \le -K \int_0^T s_1^2(\sigma) d\sigma + T(c + \gamma \Delta_1) + V_{p,1}(0) < \infty. \tag{48}$$

Applying (43) repeatedly, we may have

$$E_k(T) = E_1(T) + \sum_{j=2}^k \Delta E_j(T)$$
$$\le - K \sum_{j=1}^k \int_0^T s_j^2(\sigma) d\sigma$$
$$+ T \cdot c_{\max} + \gamma T \sum_{j=1}^k \Delta_k + V_{p,1}(0)$$
$$\le T \cdot c_{\max} + \gamma T \sum_{j=1}^k \Delta_k + V_{p,1}(0). \tag{49}$$

Recalling Lemma 4 we have

$$\gamma T \sum_{j=1}^k \Delta_k \le \lim_{k\to\infty} \gamma T \sum_{j=1}^k \Delta_k \le 2\gamma Tq$$

which further implies the boundedness of $E_k(T)$.
3) Finiteness of $E_k(t)$
Next we will prove the boundedness of $E_k(t)$ by induction. The boundedness of $E_k(T)$ is guaranteed for all iterations. Consequently, $\forall k \in \mathbb{N}$, there exists a constant M_1 satisfying $\int_0^T \tilde{\theta}_k^2(\sigma) d\sigma \le M_1$, thus it follows

$$E_k(t) = V_k(t) + \int_0^t \tilde{\theta}_k^2(\sigma) d\sigma$$
$$\le V_k(t) + \int_0^T \tilde{\theta}_k^2(\sigma) d\sigma$$
$$\le V_k(t) + M_1. \tag{50}$$

On the other hand, from (42), we obtain

$$\Delta E_{k+1}(t) \le V_{p,k+1}(0) - V_k(t)$$

$$- K \int_0^t s_{k+1}^2 d\sigma + \gamma \Delta_{k+1} t. \tag{51}$$

Adding (51) to (50) leads to

$$
\begin{aligned}
E_{k+1}(t) &= E_k(t) + \Delta E_{k+1}(t) \\
&\leq V_k(t) + M_1 + V_{p,k+1}(0) - V_k(t) \\
&\quad - K \int_0^t s_{k+1}^2 d\sigma + \gamma \Delta_{k+1} t \\
&\leq M_1 + V_{p,k}(T) + \gamma \Delta_{k+1} t.
\end{aligned} \tag{52}
$$

As we have proven that $E_1(t)$ is bounded, therefore $E_k(t)$ is finite by induction. In the sequel, we can obtain the boundedness of $\hat{W}_k(t)$, $\hat{\theta}_k(t)$ and $\hat{\rho}_k$.

4) Learning Convergence Property

Rewrite inequality (49) as

$$
\begin{aligned}
&\sum_{j=1}^k \int_0^T s_j^2(\sigma) d\sigma \\
&\leq \frac{T \cdot c_{\max} + \gamma T \sum_{j=1}^k \Delta_k + V_{p,1}(0) - E_k(T)}{K}.
\end{aligned} \tag{53}
$$

Taking the limitation of (53), it follows that

$$
\begin{aligned}
&\lim_{k \to \infty} \sum_{j=1}^k \int_0^T s_j^2(\sigma) d\sigma \\
&\leq \lim_{k \to \infty} \frac{T \cdot c_{\max} + \gamma T \sum_{j=1}^k \Delta_k + V_{p,1}(0) - E_k(T)}{K} \\
&\leq T \cdot c_{\max} + 2\gamma q T + V_{p,1}(0).
\end{aligned} \tag{54}
$$

According to the convergence theorem of the sum of series, $\lim_{k \to \infty} \int_0^T s_k^2(\sigma) d\sigma = 0$. Since x_d is bounded, the boundedness of x_k is established. Based on the above reasoning, we can arrive at that $u_k(t)$ is bounded. ■

V. SIMULATION STUDY

In this section, a simulation study is presented to verify the effectiveness of the AILC scheme. Consider the following second-order nonlinear fractional order system:

$$
\begin{cases}
D^\alpha x_{1,k}(t) = x_{2,k}(t) \\
D^\alpha x_{2,k}(t) = f(x_k) + \theta(t)\xi(x_k) + u_k(t) + d(t) \\
y_k(t) = x_{1,k}(t)
\end{cases}
$$

where $\alpha = 0.9$, $f(x_k) = -x_{1,k}x_{2,k}\sin(x_{1,k}x_{2,k})$, $\theta(t) = 1 + 0.5\sin t$, $\xi(x_k) = \sin(x_{1,k})\cos(x_{2,k})$, $d(t) = 0.1 \times rand \times \sin(t)$ with $rand$ presenting a Gaussian white noise. The desired reference trajectory is given by $r(t) = \sin t$. The design parameters are chosen as $\varepsilon_1 = \varepsilon_2 = 1$, $\lambda = 2$, $K = 6$, $\Gamma_W = \text{diag}\{0.6\}$, $q_\theta = 2$, $q_\rho = 0.8$, $\varepsilon = \lambda\varepsilon_1 + \varepsilon_2 = 3$. It is clear that $|\lambda| > (\alpha\pi)/2$. Additionally, the boundary layer function is specified by $\eta(t) = 3E_{0.9}(-Kt)$, a graphic representation of $\eta(t)$ is shown in Fig. 2.

The parameters for neural network are chosen as $l = 30$, $\mu_j = (1/l)(2j - l)[2,3]$, $\sigma_j = 2$, $j = 1, 2, \ldots, l$. The initial condition $x_{1,k}(0)$ and $x_{2,k}(0)$ are randomly taken as $r(0) + 0.5(1$

$- 2rand)$ and $r_1(0) + 0.5(1 - 2rand)$, respectively. For ease of programming, we use the Grünwald-Letnikov definition in the simulation. The system runs on $[0, 2\pi]$ repeatedly. Parts of the simulation results are shown in Figs. 3−7.
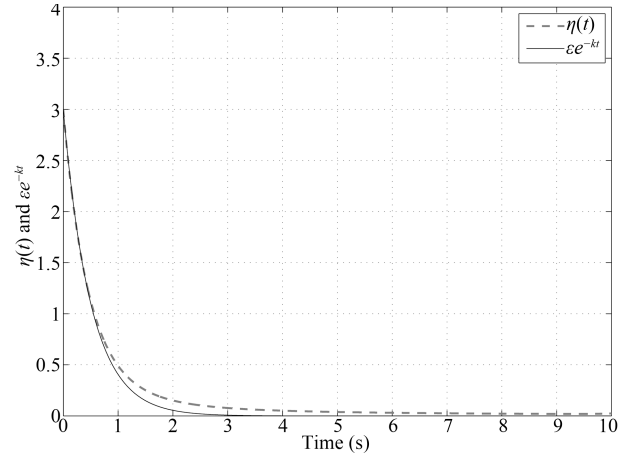

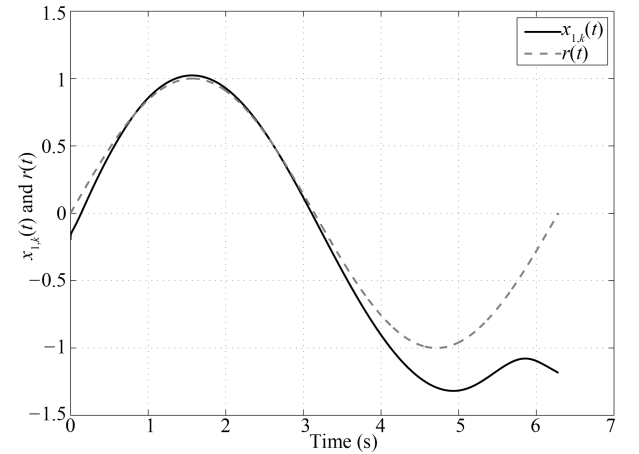Fig. 2.   Mittag-Leffler type boundary layer function $\eta(t)$.

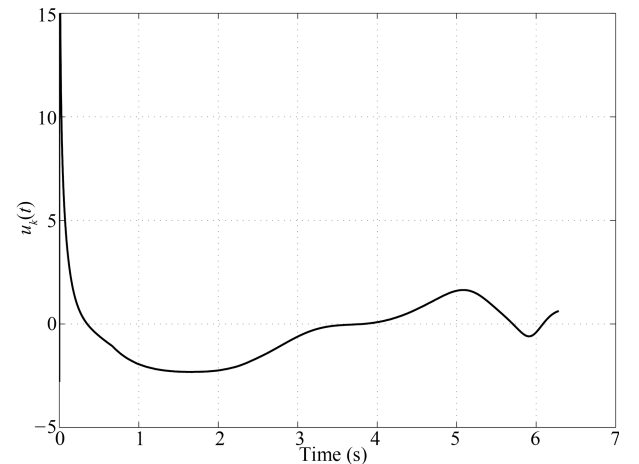Fig. 3.   System output $y_k(t)$ on $r(t)$ ($k = 1$).

Fig. 4.   Control input ($k = 1$).

Figs. 3 and 4, and Figs. 5 and 6 show the output tracking trajectory and control input of the 1st and the 30th iteration. Obviously, the signals are bounded and the tracking perfor-

mance of the 1st iteration is much worse than that of the 30th. Fig.7 gives the convergence of $\int_0^T s_k^2(t)dt$ along the iteration axis, which indicates that the proposed AILC scheme achieves perfect tracking by learning.
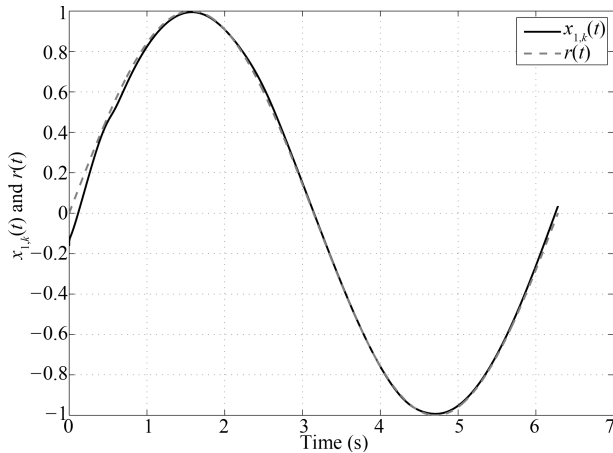

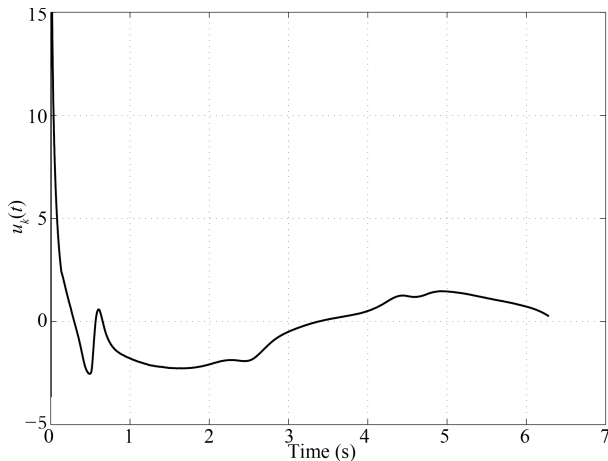Fig. 5.   System output $y_k(t)$ on $r(t)$ ($k = 30$).

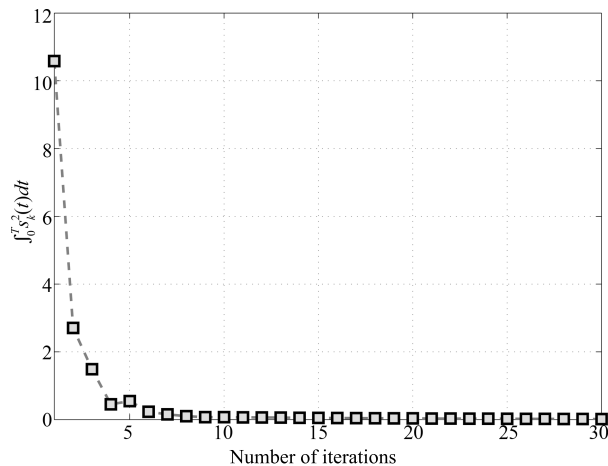Fig. 6.   Control input ($k = 30$).

Fig. 7. $\int_0^T s_k^2(t)\,dt$ versus the number of iterations.

VI. Conclusions

In this paper, an adaptive iterative learning control scheme has been presented for a class of nonlinear fractional order systems in the presence of disturbance. A new boundary layer function by introducing Mittag-Leffler function is designed to deal with the initial condition problem of ILC. RBF NN is utilized to approximate the system uncertainty while fractional order differential type updating laws are designed to estimate ideal neural weight and the upper bound of neural approximation error and disturbance. The hyperbolic tangent function with a convergent series sequence is employed to form the robust control term. Theoretical analysis by constructing Lyapunov-like CEF has been presented to show the boundedness of all signals and convergence along iteration of tracking error. Simulation results have been provided to show the validity the proposed scheme. This is the first time consideration of the AILC problem of fractional order system. Compared with traditional ILC of fractional systems, our AILC scheme relaxes the global Lipschitz condition and a new framework of stability analysis by using Lyapunov-like CEF is presented. Although we only consider the class of nonlinear fractional order systems as (12), the idea of the proposed AILC method can be applied to more kinds of fractional order systems and provide a reference for AILC design of fractional order systems.

References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. Robot. Syst.*, vol. 1, no. 2, pp. 123−140, Jun. 1984.

[2] T. Y. Kuc, K. Nam, and J. S. Lee, "An iterative learning control of robot manipulators," *IEEE Trans. Robot. Automat.*, vol. 7, no. 6, pp. 835−841, Dec. 1991.

[3] Y. Q. Chen and C. Y. Wen, *Iterative Learning Control: Convergence, Robustness and Applications*. London: Spring-Verlag, 1999.

[4] Z. Bien and J. X. Xu, *Iterative Learning Control: Analysis, Design, Integration and Applications*. Boston, USA: Kluwer Academic Publisher, 1998.

[5] J. X. Xu and Y. Tan, *Linear and Nonlinear Iterative Learning Control*. Berlin: Springer-Verlag, 2003.

[6] J. X. Xu and Z. H. Qu, "Robust iterative learning control for a class of nonlinear systems," *Automatica*, vol. 34, no. 8, pp. 983−988, Aug. 1998.

[7] M. French and E. Rogers, "Non-linear iterative learning by an adaptive Lyapunov technique," *Int. J. Control*, vol. 73, no. 10, pp. 840−850, Nov. 2000.

[8] J. X. Xu and Y. Tan, "A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainties," *IEEE Trans. Automat. Control*, vol. 47, no. 11, pp. 1940−1945, Nov. 2002.

[9] R. H. Chi, Z. S. Hou, and J. X. Xu, "Adaptive ILC for a class of discrete-time systems with iteration-varying trajectory and random initial condition," *Automatica*, vol. 44, no. 8, pp. 2207−2213, Aug. 2008.

[10] R. H. Chi, S. L. Sui, and Z. S. Hou, "A new discrete-time adaptive ILC for nonlinear systems with time-varying parametric uncertainties," *Acta Automat. Sinica*, vol. 34, no. 7, pp. 805−808, Jul. 2008.

[11] C. K. Yin, J. X. Xu, and Z. S. Hou, "An ILC scheme for a class of nonlinear continuous-time systems with time-iteration-varying parameters subject to second-order internal model," *Asian J. Control*, vol. 13, no. 1, pp. 126−135, Jan. 2011.

[12] R. K. Zhang, Z. S. Hou, R. H. Chi, and H. H. Ji, "Adaptive iterative learning control for nonlinearly parameterised systems with unknown time-varying delays and input saturations," *Int. J. Control*, vol. 88, no. 6, pp. 1133−1141, Jun. 2015.

[13] Y. C. Wang, C. J. Chien, and C. C. Teng, "Direct adaptive iterative learning control of nonlinear systems using an output-recurrent fuzzy neural network," *IEEE Trans. Syst. Man Cybern. B: Cybern.*, vol. 34, no. 3, pp. 1348−1359, Jun. 2004.

[14] C. J. Chien, "A combined adaptive law for fuzzy iterative learning control of nonlinear systems with varying control tasks," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 1, pp. 40−51, Feb. 2008.

[15] Y. C. Wang and C. J. Chien, "Decentralized adaptive fuzzy neural iterative learning control for nonaffine nonlinear interconnected systems," *Asian J. Control*, vol. 13, no. 1, pp. 94−106, Jan. 2011.

[16] C. L. Zhang and J. M. Li, "Adaptive iterative learning control for nonlinear pure-feedback systems with initial state error based on fuzzy approximation," *J. Franklin Inst.*, vol. 351, no. 3, pp. 1483−1500, Mar. 2014.

[17] I. Podlubny, *Fractional Differential Equations: an Introduction to Fractional Derivatives, Fractional Differential Equations, Some Methods of Their Solution and Some of Their Applications*. New York: Academic Press, 1999.

[18] I. Petráš, *Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation*. Berlin: Spring-Verlag, 2011.

[19] J. D. Gabano and T. Poinot, "Fractional modelling and identification of thermal systems," *Signal Process.*, vol. 91, no. 3, pp. 531−541, Mar. 2011.

[20] Y. Luo and Y. Q. Chen, "Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems," *Automatica*, vol. 48, no. 9, pp. 2159−2167, Sep. 2012.

[21] M. Ö. Efe, "Fractional fuzzy adaptive sliding-mode control of a 2-DOF direct-drive robot arm," *IEEE Trans. Syst. Man Cybern. B: Cybern.*, vol. 38, no. 6, pp. 1561−1570, Dec. 2008.

[22] H. Delavari, R. Ghaderi, A. Ranjbar, and S. Momani, "Fuzzy fractional order sliding mode controller for nonlinear systems," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 15, no. 4, pp. 963−978, Apr. 2010.

[23] J. Li, J. G. Lu, and Y. Q. Chen, "Robust decentralized control of perturbed fractional-order linear interconnected systems," *Comput. Math. Appl.*, vol. 66, no. 5, pp. 844−859, Sep. 2013.

[24] X. D. Tang, G. Tao, and S. M. Joshi, "Adaptive actuator failure compensation for parametric strict feedback systems and an aircraft application," *Automatica*, vol. 39, no. 11, pp. 1975−1982, Nov. 2003.

[25] X. S. Wang, C. Y. Su, and H. Hong, "Robust adaptive control of a class of nonlinear systems with unknown dead-zone," *Automatica*, vol. 40, no. 3, pp. 407−413, Mar. 2004.

[26] T. P. Zhang and S. S. Ge, "Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs," *Automatica*, vol. 43, no. 6, pp. 1021−1033, Jun. 2007.

[27] T. P. Zhang and S. S. Ge, "Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form," *Automatica*, vol. 44, no. 7, pp. 1895−1903, Jul. 2008.

[28] J. X. Xu, J. Xu, and T. H. Lee, "Iterative learning control for systems with input deadzone," *IEEE Trans. Automatic Control*, vol. 50, no. 9, pp. 1455−1459, Sep. 2005.

[29] A. Razminia and D. F. M. Torres, "Control of a novel chaotic fractional order system using a state feedback technique," *Mechatronics*, vol. 23, no. 7, pp. 755−763, Oct. 2013.

[30] C. L. Li, K. L. Su, J. Zhang, and D. Q. Wei, "Robust control for fractional-order four-wing hyperchaotic system using LMI," *Optik*, vol. 124, no. 22, pp. 5807−5810, Nov. 2013.

[31] C. Yin, S. Dadras, S. M. Zhong, and Y. Q. Chen, "Control of a novel class of fractional-order chaotic systems via adaptive sliding mode control approach," *Appl. Math. Modell.*, vol. 37, no. 4, pp. 2469−2483, Feb. 2013.

[32] C. C. Yang and C. J. Ou, "Adaptive terminal sliding mode control subject to input nonlinearity for synchronization of chaotic gyros," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 18, no. 3, pp. 682−691, Mar. 2013.

[33] X. Huang, Z. Wang, Y. X. Li, and J. W. Lu, "Design of fuzzy state feedback controller for robust stabilization of uncertain fractional-order chaotic systems," *J. Franklin Inst.*, vol. 351, no. 12, pp. 5480−5493, Dec. 2014.

[34] I. Pan, S. Das, and S. Das, "Multi-objective active control policy design for commensurate and incommensurate fractional order chaotic financial systems," *Appl. Math. Modell.*, vol. 39, no. 2, pp. 500−514, Jan. 2015.

[35] S. Bhalekar and V. Daftardar-Gejji, "Synchronization of different fractional order chaotic systems using active control," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 15, no. 11, pp. 3536−3546, Nov. 2010.

[36] T. Z. Li, Y. Wang, and Y. Yang, "Designing synchronization schemes for fractional-order chaotic system via a single state fractional-order controller," *Optik*, vol. 125, no. 22, pp. 6700−6705, Nov. 2014.

[37] S. K. Agrawal and S. Das, "Function projective synchronization between four dimensional chaotic systems with uncertain parameters using modified adaptive control method," *J. Process Contr.*, vol. 24, no. 5, pp. 517−530, May 2014.

[38] L. K. Gao, Z. H. Wang, K. Zhou, W. J. Zhu, Z. D. Wu, and T. D. Ma, "Modified sliding mode synchronization of typical three-dimensional fractional-order chaotic systems," *Neurocomputing*, vol. 166, pp. 53−58, Oct. 2015.

[39] Y. Gao, C. H. Liang, Q. Q. Wu, and H. Y. Yuan, "A new fractional-order hyperchaotic system and its modified projective synchronization," *Chaos Soliton. Fract.*, vol. 76, pp. 190−204, Jul. 2015.

[40] Y. Q. Chen and K. L. Moore, "On D^α-type iterative learning control," in *Proc. 40th IEEE Conference on Decision and Control*, Orlando, Florida, USA, 2001. pp. 4451−4456.

[41] H. S. Ahn, K. L. Moore, and Y. Q. Chen, "Stability analysis of discrete-time iterative learning control systems with interval uncertainty," *Automatica*, vol. 43, no. 5, pp. 892−902, May 2007.

[42] Y. Li, Y. Q. Chen, and H. S. Ahn, "Fractional order iterative learning control," in *ICROS-SICE International Joint Conference 2009*, Fukuoka, Japan, pp. 3106−3110, 2009.

[43] H. S. Li, J. C. Huang, D. Liu, J. H. Zhang, and F. L. Teng, "Design of fractional order iterative learning control on frequency domain," in *Proc. 2011 IEEE International Conference on Mechatronics and Automation*, Beijing, China, pp. 2056−2060, 2011.

[44] Y. Li, Y. Q. Chen, and H. S. Ahn, "A generalized fractional-order iterative learning control," in *Proc. 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, Orlando, USA, pp. 5356−5361, 2011.

[45] Y. Li, Y. Q. Chen, and H. S. Ahn, "On the PD^α-type iterative learning control for the fractional-order nonlinear systems," in *Proc. 2011 American Control Conference*, San Francisco, CA, USA, pp. 4320−4325, 2011.

[46] Y. H. Lan, "Iterative learning control with initial state learning for fractional order nonlinear systems," *Comput. Math. Appl.*, vol. 64, no. 10, pp. 3210−3216, Nov. 2012.

[47] Y. Li, Y. Q. Chen, and H. S. Ahn, "On P-type fractional order iterative learning identification," in *Proc. 13th International Conference on Control, Automation and Systems*, Gwangju, Korea, 2013. pp. 219−225.

[48] Y. H. Lan and Y. Zhou, "D^α type iterative learning control for fractional-order linear time-delay systems," *Asian J. Control*, vol. 15, no. 3, pp. 669 −677, 2013.

[49] Y. Li, L. Zhai, Y. Q. Chen, and H. S. Ahn, "Fractional-order iterative learning control and identification for fractional-order Hammerstein system," in *Proc. 11th World Congress on Intelligent Control and Automation*, Shenyang, China, pp. 840−845, 2014.

[50] Y. Li, Y. Q. Chen, and H. S. Ahn, "Fractional order iterative learning control for fractional order system with unknown initialization," in *Proceedings of the 2014 American Control Conference (ACC)*, Portland, Oregon, USA, pp. 5712−5717, 2014.

[51] Y. Li, Y. Q. Chen, and H. S. Ahn, "A high-gain adaptive fractional-order

iterative learning control," in *Proc. 11th IEEE International Conference on Control & Automation (ICCA)*, Taiwan, China, pp. 1150−1155, 2014.

[52] M. Lazarević and P. Mandić, "Feedback-feedforward iterative learning control for fractional order uncertain time delay system-PD α type," in *Proc. 2014 International Conference on Fractional Differentiation and Its Applications (ICFDA)*, Catania, Italy, pp. 1−6, 2014.

[53] L. Yan and J. Wei, "Fractional order nonlinear systems with delay in iterative learning control," *Appl. Math. Comput.*, vol. 257, pp. 546−552, Apr. 2015.

[54] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*. The Netherlands: Elsevier, pp. 69−83, 2006.

[55] D. Matignon, "Stability results for fractional differential equations with applications to control processing," in *Computational Engineering in Systems and Applications*, Lille, France, pp. 963−968, 1996.

[56] J. Sabatier, M. Moze, and C. Farges, "LMI stability conditions for fractional order systems," *Comput. Math. Appl.*, vol. 59, no. 5, pp. 1594−1609, Mar. 2010.

[57] J. C. Trigeassou, N. Maamri, and A. Oustaloup, "Lyapunov stability of linear fractional systems: part 1-definition of fractional energy," in *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Portland, Oregon, USA, 2013.

[58] M. M. Gupta and D. H. Rao, *Neuro-Control Systems: Theory and Applications*. New York, USA: IEEE Press, 1994.

[59] M. M. Polycarpou, "Stable adaptive neural control scheme for nonlinear systems," *IEEE Trans. Automatic Control*, vol. 41, no. 3, pp. 447−451, Mar. 1996.

[60] S. Zhu, M. X. Sun, and X. X. He, "Iterative learning control of strict-feedback nonlinear time-varying systems," *Acta Automat. Sinica*, vol. 36, no. 3, pp. 454−458, Mar. 2010.

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