

Behavior Consistency Computation for Workflow Nets With Unknown Correspondence

Mimi Wang, *Student Member, IEEE*, Guanjun Liu, *Member, IEEE*, Peihai Zhao, *Student Member, IEEE*, Chungang Yan, and Changjun Jiang

Abstract—Consistency degree calculation is established on the basis of known correspondence, but in real life, the correspondence is generally unknown, so how to calculate consistency of two models under unknown correspondence has become a problem. For this condition, we should analyze unknown correspondence due to the influence of different correspondences. In this paper we obtain the relations of transitions based on event relations using branching processes, and build a behavioral matrix of relations. Based on the permutation of behavioral matrix, we express different correspondences, and define a new formula to compute the maximal consistency degree of two workflow nets. Additionally, this paper utilizes an example to show these definitions, computation as well as the advantages.

Index Terms—Behavioral matrix, branching process, consistency degree, unknown correspondence, workflow net (WF-nets).

I. INTRODUCTION

WORKFLOW nets (WF-nets) have become one of the standard ways to model and analyze workflows [1]. In fact, WF-nets can also model many other distributed and parallel systems such as web services composition in which multiple processes interact via sending/receiving messages [2]–[4] and distributed systems [5]–[8].

To support the operation and maintenance of services systems in the open and dynamic environment, it is needed to transform from the way of building data-driven system to the construction way of behavior consciousness and from tightly-coupled architecture to loosely-coupled one. Therefore, it is necessary to model and analyze systems from the perspective of behaviors. The consistency comparison of behaviors of systems is an interesting topic [9]. For example, when a behavioral model is mined from a system's logs [10], consistency

comparison between the mined model and the real model can help one evaluate the disadvantage and advantage of a method. Consistency of two models means that their semantics match each other, and this matching relation is usually built on the basis of a mapping between the graphical data. Therefore, consistency can reflect whether the mapping is effective or not [11]. Consistency analysis has been applied in many fields such as security evaluation in cloud computing [12], and process mining [13].

In [2] consistency is defined by Weidlich *et al.* on the basis of an alignment which requires the identification of correspondences of models. Given a correspondence, the question whether two data models are equivalent is similar to the question whether a mapping between data schema is valid, which is known from the field of data integration [2]. In this area, various properties for evaluating the validity of a schema mapping have been proposed. For instance, satisfiability of a mapping of two models [9] requires the existence of such single trace that is possible in the two models after the corresponding elements have been resolved. Weidlich *et al.* studied the given correspondence of simple correspondence and complex correspondence. For the unknown correspondence, they also thought that it was a difficult job. However, in real world, the correspondence is often unknown such as Fig. 3. The consistency relation can be used to measure the two models consistency when we do not know the correspondence. And the condition of two models with unknown correspondence usually exist in the real world. For example, for a group of users behaviors patterns, we can mine their behaviors models from their logs. Obviously, the correspondence relation among these models are not given. Then under this condition we can only obtain different models from different users, and we do not know the whole profiles for these models due to their unknown correspondence relation. To manage these users and recommend more suitable or appropriate service, we should compute maximal consistency degree of these unknown influence of correspondence. To do this, we should ravel out the influence of correspondence on consistency of two WF-nets.

In addition, trace equivalence [14], [15] and bisimulation [16] are two classical notions used to determine whether two systems are of equivalent behaviors. However, these notions only ascertain whether two systems' behaviors are consistent or not. They cannot measure their degree when their behaviors are consistent. In consequence, Weidlich *et al.* proposed the notion of behavioral profile and proposed a formula to measure the degree of inconsistency of behaviors of two WF-nets [11],

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[17]–[21]. However, the results are not too accurate for some cases when we use this method. For example, there are two WF-nets, modeling the same system such that one is deadlock-free but another one is not. Obviously, their behaviors should not be equivalent. The main cause of this phenomenon is that the three behavioral relations of behavioral profiles should be refined. In addition, Weidlich *et al.* did not discuss the unknown correspondences.

What we should do is to obtain a more accurate behavioral relation and give a method to compute consistency degree of unknown correspondences. However the behavioral relation of two actions is usually difficult to obtain. Therefore we should give a method to compute the behavioral relation (see Section III-A). At the same time, for the unknown correspondence, how to compute the consistency degree is a challenge.

This paper focuses on these questions and obtains the following results:

1) By analyzing event relations of branching process, we give a method to obtain the behavioral relation of transition of WF-net.

2) Based on the relation profile of a WF-net, a behavioral matrix can be constructed. And different matrix arrangement can represent different correspondence. We propose a permutation method of behavioral matrix to obtain the maximal consistency degree.

3) We present a new formula to measure the consistency degree of behaviors of two WF-nets on the basis of behavioral matrices. And we propose to give a method to compute the maximal value, when the degree of consistency is maximal.

The paper is organized as follows: Section II introduces some basic concepts in order to understand the work of this paper easily. Section III introduces the behavioral relation and the method to obtain the relation, and a behavioral matrix. Section IV proposes a new formula to measure the consistency degree and maximal consistency degree. Section V introduces some related work. Finally, we conclude this paper in Section VI.

II. PRELIMINARIES

A. WF-net and Branching Process Unfolding

This section recalls the basic concepts and definitions used in this paper. For more details, one can refer to [1], [22]–[25] and [26].

A net is a 3-tuple $N = (P, T, F)$ where P is a finite set of places, T is a finite set of transitions, $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs, and $P \cap T = \emptyset$.

A net may be thought of as a directed graph in which a circle represents a place and a box represents a transition.

Given a net $N = (P, T, F)$ and a node $x \in P \cup T$, $\bullet x = \{y \mid (y, x) \in F\}$ is the pre-set of x , and $x^\bullet = \{y \mid (x, y) \in F\}$ is the post-set of x . If $X \subseteq P \cup T$, its pre-set and post-set are defined as follows: $\bullet X = \bigcup_{x \in X} \bullet x$ and $X^\bullet = \bigcup_{x \in X} x^\bullet$.

$(N, M_0) = (P, T, F, M_0)$ is a Petri net if

- 1) $N = (P, T, F)$ is a net;
- 2) M_0 is the initial marking;
- 3) mapping $M: P \rightarrow \mathbb{N}$ is a marking function where $\mathbb{N} = \{0, 1, 2, \dots\}$; and

4) it has the following rules:

i) a transition $t \in T$ is enabled at M , denoted by $M[t]$, if $\forall p \in \bullet t: M(p) \geq 1$;

ii) firing an enabled transition t yields a new marking M' and this is denoted by $M[t]M'$, where

$$M'(p) = \begin{cases} M(p) + 1, & \text{if } p \in t^\bullet - \bullet t \\ M(p) - 1, & \text{if } p \in \bullet t - t^\bullet \\ M(p), & \text{otherwise.} \end{cases}$$

iii) if there exist transitions t_1, t_2, \dots, t_k , and markings M_1, M_2, \dots, M_k such that $M[t_1]M_1[t_2] \dots M_{k-1}[t_k]M_k$, then M_k is reachable from M . All markings reachable from M are denoted by $R(M)$ and $M \in R(M)$.

WF-nets have become one of the standard ways to model and analyze workflows and are introduced as follows.

Definition 1 (WF-net): A net $N = (P, T, F)$ is a WF-net if it has a source place $i \in P$ with $\bullet i = \emptyset$ and a sink place $o \in P$ with $o^\bullet = \emptyset$, and $N' = (P, T \cup \{t\}, F \cup \{(o, t), (t, i)\})$ is strongly connected where $t \notin T$.

Let $\Sigma = (N, M_0) = (P, T, F, M_0)$ be a Petri net. The unfolding of Σ is the tuple $Unf(\Sigma) = (B, E, G, \rho)$, where (B, E, G) is an occurrence net, and a homomorphism $\rho: B \cup E \rightarrow P \cup T$, such that for every $e_1, e_2 \in E$, if $\bullet e_1 = \bullet e_2$ and $\rho(e_1) = \rho(e_2)$ then $e_1 = e_2$.

A branching process of a net system $\Sigma = (N, M_0)$ is a labeled occurrence net $\beta = (O, h) = (B, E, G, h)$ where the labeling function h satisfies the following properties:

1) $h(B) \subseteq S$ and $h(E) \subseteq T$ (h preserves the nature of nodes);

2) for every $e \in E$, the restriction of h to $\bullet e$ is a bijection between $\bullet e$ (in Σ) and $\bullet h(e)$ (in β), and similarly for e^\bullet and $h(e)^\bullet$ (h preserves the environments of transitions);

3) the restriction of h to $\min(O)$ is a bijection between $\min(O)$ and M_0 (β starts at M_0);

4) for every $e_1, e_2 \in E$, if $\bullet e_1 = \bullet e_2$ and $h(e_1) = h(e_2)$ then $e_1 = e_2$ (β does not duplicate the transitions of Σ).

Branching process unfolding (BPU) is the least upper bound of the set of all branching processes.

The relevant concepts and algorithms of branching process can be referred to [27], [28]. We will use *BPU* to represent branching process unfolding.

1) Two nodes x and y are in causal relation, denoted by $x \ominus y$, if the net contains a path with at least one arc leading from x to y .

2) x and y are in conflict relation, or just in conflict, denoted by $x \oplus y$, if the net contains two paths $sx1 \dots x$ and $sy1 \dots x$ starting at the same place s , and such that $x1 \neq y1$.

In words, x and y are in conflict if the net contains two paths leading to x and y which start at the same place and immediately diverge.

3) x and y are in concurrency relation, denoted by $x \oplus\oplus y$, if neither $x \ominus y$ nor $y \ominus x$ nor $x \oplus y$.

B. Behavioral Profiles and Correspondence Relation

To study the behavior relations of transitions of a WF-net, the weak order relation and behavioral profiles are defined in [11]:

Let $(N, M_0) = (P, T, F, M_0)$ be a Petri net. A pair of transitions (x, y) is in the weak order relation over T , denoted as $x \succ y$, if there exists an enabled transition sequence $t_1 t_2 \dots t_n$ such that $\exists j, k \in \{1, 2, \dots, n\}: j < k \wedge t_j = x \wedge t_k = y$.

Based on the weak order relation, the following three relations are defined in [11]: A pair of transitions (x, y) is in

- 1) the strict order relation \rightsquigarrow , if $x \succ y \wedge y \not\succeq x$;
 - 2) the exclusiveness relation $+$, if $x \not\succeq y \wedge y \not\succeq x$;
 - 3) the interleaving relation \parallel , if $x \succ y \wedge y \succ x$.
- $\mathbb{B} = \{\rightsquigarrow, +, \parallel\}$ is called the behavioral profile.

Let $(N_1, M_1) = (P_1, T_1, F_1, M_1)$ and $(N_2, M_2) = (P_2, T_2, F_2, M_2)$ be two Petri nets. Reference [2], [29] defined the following two concepts.

1) A correspondence relation $\sim \subseteq T_1 \times T_2$ associates correspondence transitions of the two systems. T_1^\sim is defined as $\{t \mid \exists t' \in T_2 : (t, t') \in \sim\}$. Similarly, we can define T_2^\sim . For Figs. 1 (a) and (b), it holds that $\sim = (A_1, A_2), (B_1, B_2), (C_1, C_2), (D_1, D_2)$.

2) Let $T_1' \subseteq T_1$ and $T_2' \subseteq T_2$ be two sets of transitions such that $T_1' \times T_2' \subseteq \sim$. If T_1' and T_2' be maximal w.r.t. \subseteq , i.e., $\forall t_1 \in (T_1 \setminus T_1') : (\{t_1\} \times T_2') \not\subseteq \sim$ and $\forall t_2 \in (T_2 \setminus T_2') : (T_1' \times \{t_2\}) \not\subseteq \sim$, then (T_1', T_2') is called a correspondence and written as $T_1' \sim T_2'$.

In fact, if the maximal value is nonexistent, then the correspondence is an unknown correspondence. Then we give the unknown correspondence concept as follows:

A unknown correspondence relation $\approx \subseteq T_1 \times T_2$ associates correspondence transitions of the two systems. T_1^\approx is defined as $\{t \mid \nexists t' \in T_2 : (t, t') \in \approx\}$ incorrect and not clear. Similarly, we can define T_2^\approx , such as Fig. 3.

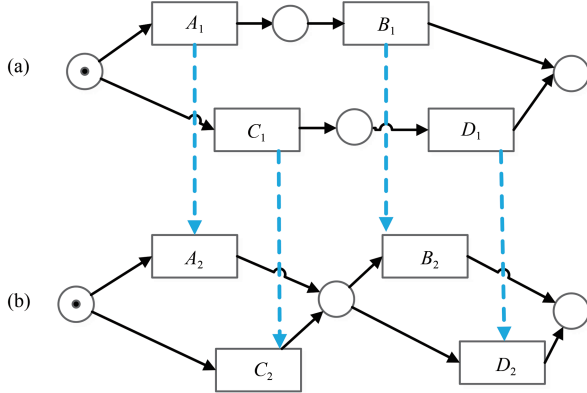


Fig. 1. Two workflow nets: (a) and (b) are in the simple correspondence relation when we give the correspondence $\{X_1\}$ and $\{X_2\}$ ($X = A, B, C, D$).

Now, we define the following complex correspondence relations.

For instance, $\{C\}$ of N_{11} and $\{C_1, C_2\}$ of N_{12} in Fig. 2 are in the complex correspondence relation. The correspondence c_1, c_2 and c_3 are all complex correspondences. Let $(N_1, M_1) = (P_1, T_1, F_1, M_1)$ and $(N_2, M_2) = (P_2, T_2, F_2, M_2)$ be two Petri nets, \mathbb{B}_1 and \mathbb{B}_2 be their behavioral profiles, and $\sim \subseteq T_1 \times T_2$ be a correspondence relation. Let $\mathbb{R}_1 \in \mathbb{B}_1 \cup \{\rightsquigarrow^{-1}\}$ and $\mathbb{R}_2 \in \mathbb{B}_2 \cup \{\rightsquigarrow^{-1}\}$. The set of behavioral profile consistent transition pairs $CT_1^\sim \subseteq (T_1^\sim \times T_1^\sim)$ for (N_1, M_1) contains all pairs (t_x, t_y) such that:

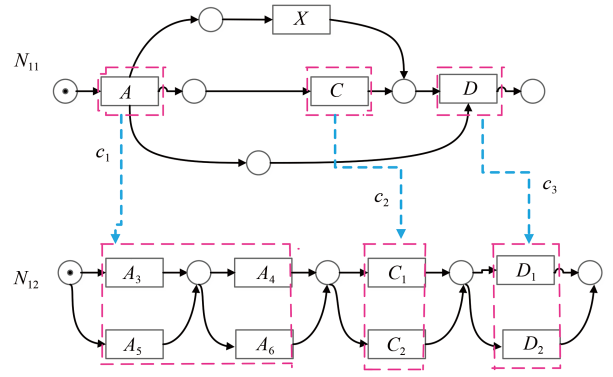


Fig. 2. Complex correspondence.

1) if $t_x = t_y$, then $\forall t_s \in T_2^\sim$ with $t_x \sim t_s : (t_x \mathbb{R}_1 t_x \wedge t_s \mathbb{R}_2 t_s) \Rightarrow \mathbb{R}_1 \simeq \mathbb{R}_2$;

2) if $t_x \neq t_y$, then $\forall t_s, t_t \in T_2^\sim$ with $t_s \neq t_t \wedge t_x \sim t_s \wedge t_y \sim t_t, (t_x, t_y) :$

i) $(t_x \mathbb{R}_1 t_y \wedge t_s \mathbb{R}_2 t_t) \Rightarrow \mathbb{R}_1 \simeq \mathbb{R}_2$; or

ii) $t_x \sim t_t \wedge t_y \sim t_s$.

The set CT_2^\sim for (N_2, M_2) is defined accordingly.

Based on these, the behavioral profiles' consistency degree [2] is defined as follows:

$$\mathbb{P}C_1^\sim = \frac{|CT_1^\sim| + |CT_2^\sim|}{|(T_1^\sim \times T_1^\sim)| + |(T_2^\sim \times T_2^\sim)|}. \quad (1)$$

C. Motivation Examples

The dining problem of philosophers is a classical example of synchronization and concurrency in computer science [30]. For simplification, we consider two philosophers whose dining model is shown in Fig. 4 (a). The dining processes are modeled by transitions b, c, d, g for philosopher 1 and by h, k, m, q for philosopher 2. As we all know Fig. 4 (a) has a deadlock. Murata and Wu used Token [31] to prevent deadlock in philosophers' dining problem. We can get Fig. 4 (b) by adding p_{12} into Fig. 4 (a) where p_{12} represents a Token. We know that Fig. 4 (b) has no deadlock. The consistency degree of behavioral profile consistency of the two WF-nets in Figs. 4 (a) and (b) is 1 by behavioral profiles consistency [2]. However, behaviors of Figs. 4 (a) and (b) should not be thought of being completely consistent from the aspect of quantification because the former has a deadlock but the latter has not. Therefore, we should obtain behavioral relations and refine them (see Section III-A).

In Fig. 3, the three WF-nets express three different systems. We do not know the correspondences of them. Therefore, we should obtain behavioral relations and refine them (see Section IV-A).

III. BEHAVIORAL SEQUENCE RELATION AND BEHAVIORAL MATRIX

To compute consistency degree, we first refine the behavioral relation of two transitions based on the event structures in BPU. Next, to formulate consistency degree conveniently, we use a matrix to represent the behavioral relations of all transitions of a WF-net.

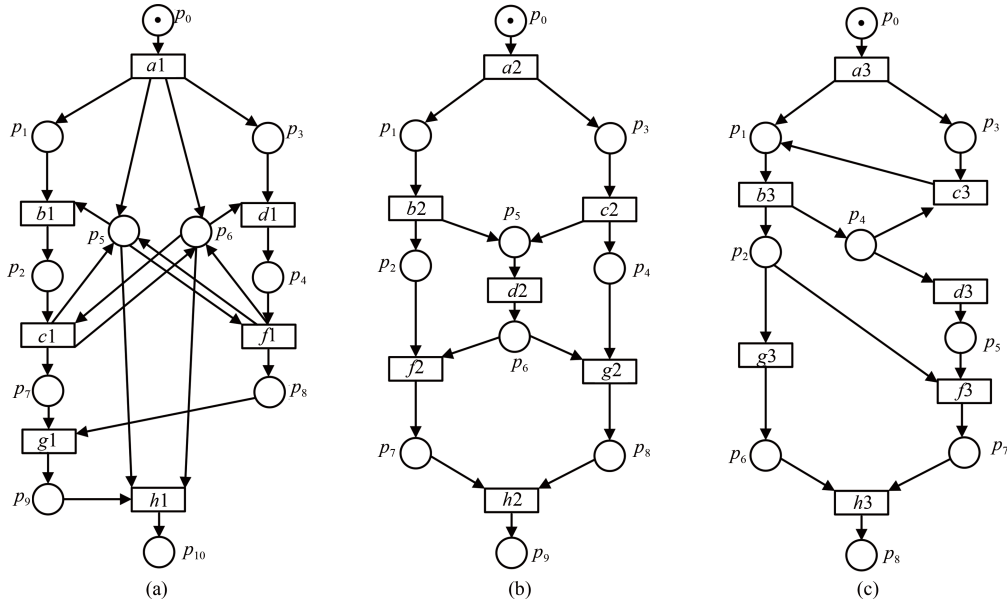


Fig. 3. Three WF-nets with unknown correspondence.

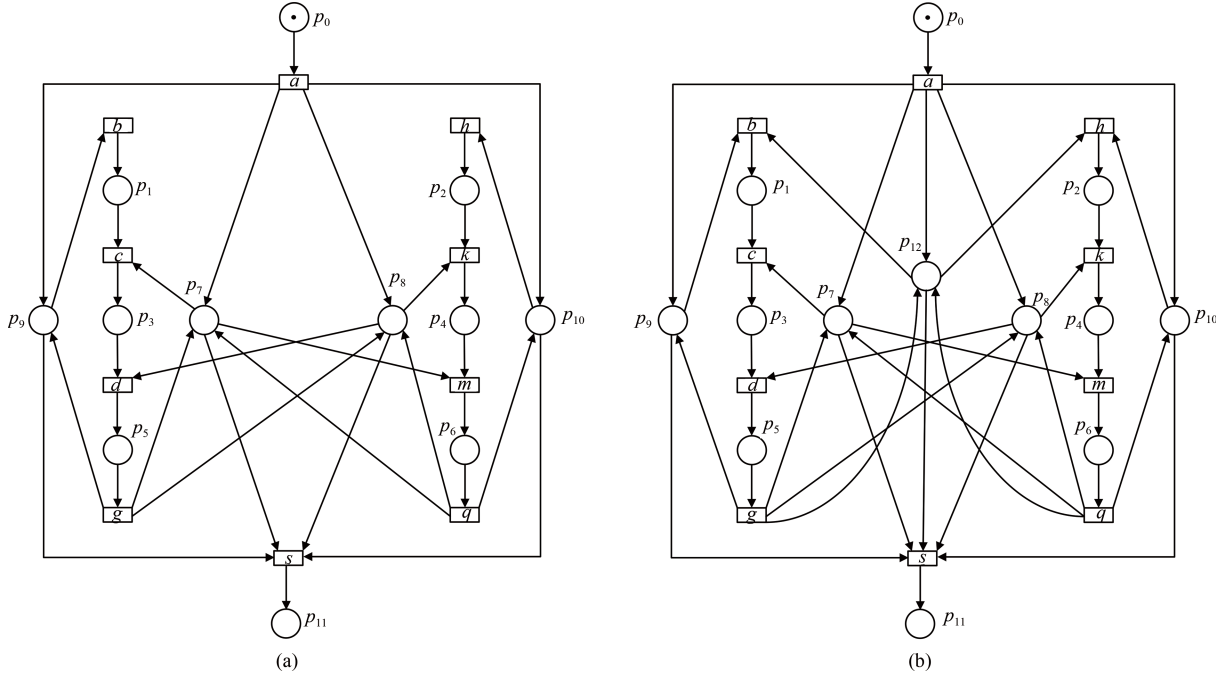


Fig. 4. Two workflow nets: (a) the WF-net of two philosophers' dining problem; and (b) the WF-net with Token.

A. Behavioral Sequence Relation

Based on the weak order relation, we define the relations between events. According to relations between events of BPU, the behavioral sequence relation can be expressed as follows.

Definition 2 (Behavioral Sequence Relation (BSR)): Let $\Sigma = (N, M_0) = (P, T, F, M_0)$ be a net system, and its unfolding of Σ is: $Unf(\Sigma) = (B, E, G, h)$, where (B, E, G) is an occurrence net, and a homomorphism $h : B \cup E \rightarrow P \cup T$. For every transition pair $(t_1, t_2) \in T \times T$ is in the behavioral sequence relations as follows.

- 1) $t_1 \Theta_1 t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
- ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 :$

$h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

- 2) $t_1 \Theta_2 t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
- ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 :$

$h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

- 3) $t_1 \Theta_3 t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
- ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 :$

$h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) =$

$$t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8.$$

5) $t_1 \Theta_5 t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

6) $t_1 \Theta_6 t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

7) $t_1 \Theta_7 t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

8) $t_1 \Theta_8 t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

9) $t_1 \Theta_9 t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

10) $t_1 \Theta_{10} t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

11) $t_1 \Theta_{11} t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

12) $t_1 \Theta_{12} t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\exists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

13) $t_1 \Theta_{13} t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\exists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

14) $t_1 \Theta_{14} t_2$, if i) $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

15) $t_1 \Theta_{15} t_2$, if i) $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$,
 ii) $\nexists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, iii) $\nexists e_5, e_6 : h(e_5) = t_1 \wedge h(e_6) = t_2 \wedge e_5 \oplus e_6$, and iv) $\nexists e_7, e_8 : h(e_7) = t_1 \wedge h(e_8) = t_2 \wedge e_7 \oplus e_8$.

$BSR = \{\Theta_1, \Theta_2, \Theta_3, \Theta_4, \Theta_5, \Theta_6, \Theta_7, \Theta_8, \Theta_9, \Theta_{10}, \Theta_{11}, \Theta_{12}, \Theta_{13}, \Theta_{14}, \Theta_{15}\}$ is the behavioral sequence relation set.

Notice that each kind of behavioral sequence relation in Definition 2 is based on the event relations between e_1 and e_2 . However, the e_1 and e_2 can be obtained by BPU. Although until now we do not have a tool to obtain this BPU, we can obtain BPU by using the algorithm of [27].

Algorithm 1. Event relation

Input: $\Sigma = (P, T, F, M_0)$, $E = \{e_{11}, e_{12}, \dots, e_{1n}\}$, BPU.

Output: Event relation ER .

Initialization

$ER = \emptyset$

For each e_{1i} and e_{1j} in BPU **Do**

If there is not any path from e_{1i} to e_{1j} , and a path from e_{1j} to e_{1i}

Then

$ER = \{\oplus\}$

End

If there is a path from e_{1i} to e_{1j} , and no a path from e_{1j} to e_{1i}

Then

$ER = \{\ominus\}$

End

If there is a path from e_{1i} to e_{1j} , and a path from e_{1j} to e_{1i}

Then

$ER = \{\oplus\}$

End

End

Return ER

The events set of the transition t_i may have only one element when the number of $h^{-1}(t_i)$ is 1. And it may have more than one element when the number of $h^{-1}(t_i)$ is greater than 1. We denote the set as E_i .

When the number of elements in E_i is greater than 1, we will compute the events relations of more than one pair.

According to Definition 2, to compute behavioral sequence relation of two transitions, we should mainly obtain the their event relations. So far, we do not know if the relation of any two transitions in a general WF-net is computable because the net system may be unbounded and its branching process may be infinite. But for bounded WF-nets, we can decide which relation any two events are in. Algorithm 1 shows the computing process.

Lemma 1: For any WF-net holds that the behavioral sequence relation of Definition 2 is mutually exclusive.

According to Definition 2, it is easy to prove the lemma.

Based on Lemma 1 and Definition 2, we know that behavioral sequence relations are complete. The formal representation is as follows:

Theorem 1: Given a WF-net N and its behavioral sequence relation BSR , then for $\forall x, y \in T$, $\exists R \in BSR$: xRy .

In fact, for any two transactions $x, y \in T$, if they are not in a kind of BSR , then there are following two cases:

1) if $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$, then they must be in one of the relations $\{\Theta_1, \Theta_3, \Theta_4, \Theta_5, \Theta_9, \Theta_{10}, \Theta_{11}, \Theta_{15}\}$;

2) if $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$, then they must be in the relation of $\{\Theta_2, \Theta_6, \Theta_7, \Theta_8, \Theta_{12}, \Theta_{13}, \Theta_{14}\}$;

if $\exists e_3, e_4 : h(e_3) = t_1 \wedge h(e_4) = t_2 \wedge e_4 \ominus e_3$, then they must be in one of the relations $\{\Theta_1, \Theta_2, \Theta_4, \Theta_5, \Theta_7, \Theta_8, \Theta_{11}, \Theta_{14}\}$;

when $\exists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \ominus e_2$ and $\nexists e_1, e_2 : h(e_1) = t_1 \wedge h(e_2) = t_2 \wedge e_1 \oplus e_2$, then they must be in one of the relations $\{\Theta_1, \Theta_4, \Theta_5, \Theta_{11}\}$.

Similarly in this way, we can see that for any two transactions $x, y \in T$, they are in one of the relation of BRS .

That is to say, behavioral sequence relations are complete, for a WF-net N and its behavioral sequence relation BRS . The proof process can be referred to the proof of completeness of behavioral profiles relations [2].

B. Behavioral Matrix

For convenience, we use behavioral matrix to describe the behavioral sequence relations of a WF-net.

Definition 3 (Behavioral Matrix): Let $\Sigma = (P, T, F, M_0)$ be a WF-net, $BRS = \{\Theta_1, \Theta_2, \Theta_3, \Theta_4, \Theta_5, \Theta_6, \Theta_7, \Theta_8, \Theta_9, \Theta_{10}, \Theta_{11}, \Theta_{12}, \Theta_{13}, \Theta_{14}, \Theta_{15}\}$ be the behavioral sequence relation over $T = \{t_1, t_2, \dots, t_n\}$. The behavioral matrix BM is an $n \times n$ matrix:

$$BM = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

such that:

$$a_{ij} = \begin{cases} \Theta_1, & t_i \Theta_1 t_j \\ \Theta_2, & t_i \Theta_2 t_j \\ \Theta_3, & t_i \Theta_3 t_j \\ \Theta_4, & t_i \Theta_4 t_j \\ \Theta_5, & t_i \Theta_5 t_j \\ \Theta_6, & t_i \Theta_6 t_j \\ \Theta_7, & t_i \Theta_7 t_j \\ \Theta_8, & t_i \Theta_8, t_j, \quad i, j = 1, 2, \dots, n. \\ \Theta_9, & t_i \Theta_9 t_j \\ \Theta_{10}, & t_i \Theta_{10} t_j \\ \Theta_{11}, & t_i \Theta_{11} t_j \\ \Theta_{12}, & t_i \Theta_{12} t_j \\ \Theta_{13}, & t_i \Theta_{13} t_j \\ \Theta_{14}, & t_i \Theta_{14} t_j \\ \Theta_{15}, & t_i \Theta_{15} t_j \end{cases}$$

Property 1: There are $n!$ kinds of correspondences for two $(n \times n)$ -order behavioral matrix BM .

Proof: In fact, for the two nets N_1, N_2 and their $(n \times n)$ -order behavioral matrices BM_1 and BM_2 , we make arrangement to n columns of BM_1, BM_2 , which can get $n \times (n-1) \times \dots \times 1 = n!$ kinds of correspondence situations. ■

For Figs.3(b) and (c), there are $7! = 5040$ kinds of correspondences.

IV. CONSISTENCY DEGREE COMPUTING

In this section, in order to find the different consistency degree on the basis of different correspondence relations, firstly we study the behavioral matrix permutation, and then look for the correspondence relation when consistency degree is the maximal value.

A. Behavioral Matrix Permutation

Different correspondences mean that their behavioral matrix has the permutation for rows and columns.

Lemma 2: The two $(n \times n)$ -order BM_1 and BM_2 , their all correspondences can be represented as $\{BM_1, BM'\}$ ($BM' = (C_1 C_2 \cdots C_m) BM_2 (C_m \cdots C_2 C_1)$, C_i ($i = 1, 2, \dots, m \leq n!$) is the permutation matrix).

Proof: $BM' = (C_1 C_2 \cdots C_m) BM_2 (C_m \cdots C_2 C_1)$ explains that it is obtained by doing the row and column transformations on BM_2 . For two matrices, one of them remains unchanged, another one does the row and column transformations, which can get all correspondences. And according to Property 1, there are $n!$ kinds of correspondence relations of two $(n \times n)$ -order BM, then the time of doing permutation is less than or equal to $n!$. ■

According to Lemma 2, the problem of maximal consistency degree of models is that the number of 0 elements in matrix $BM_1 - BM'$ is at least.

B. Computing Consistency Degree

Definition 4 (Consistency Degree): Let $\Sigma_1 = (N_1, M_1)$ and $\Sigma_2 = (N_2, M_2)$ be two net systems, BPU_1, BPU_2 be their branching process unfolding and BM_1 and BM_2 be their behavioral matrices. Consistency degree is defined as:

$$D_p = 1 - \frac{\|G\|}{\|BM_1 - BM_2\|} \quad (2)$$

where

- 1) $G = \{b_{ij} | b_{ij} \neq 0 \in (BM_1 - BM_2)\}$;
- 2) $\|BM_1 - BM_2\|$ and $\|G\|$ express the number of $BM_1 - BM_2$ and G , respectively.

Definition 4 means that the different parts from two behavioral matrices BM_1 and BM_2 are the inconsistent parts. In other words, the nonzero elements of $BM_1 - BM_2$ are non-consist parts. Therefore, the consistency degree is the maximum consistency degree if the nonzero elements of $BM_1 - BM_2$ are minimum.

Definition 5 (Maximal Consistency Degree): Let $\Sigma_1 = (N_1, M_1)$ and $\Sigma_2 = (N_2, M_2)$ be two net systems, BPU_1, BPU_2 be their branching process unfolding and BM_1 and BM_2 be their behavioral matrices. Maximal consistency degree based on BM_1 and BM_2 is defined as:

$$D_p(\max) = 1 - \frac{\min \|G'\|}{\|BM_1 - BM_2'\|} \quad (3)$$

where

- 1) $BM_2' = (C_1 C_2 \cdots C_m) BM_2 (C_m \cdots C_2 C_1)$, C_i ($i = 1, 2, \dots, m \leq n!$) is the permutation matrix;
- 2) $G' = \{b_{ij} | b_{ij} \neq 0 \in (BM_1 - BM_2')\}$;
- 3) $\|BM_1 - BM_2'\|$ and $\|G'\|$ express the number of $BM_1 - BM_2'$ and G' , respectively.

Algorithm 2 shows the computing process. According to Algorithm 2, we can see that for two behavioral matrices, we can obtain minimal $\|G'\|$, and the matrix BM_2' by permutation. And according to (3), we can obtain the maximal consistency degree.

Theorem 2: Given two WF-nets and their behavioral matrices BM_1 and BM_2 , we have that D_p computed by (2) is always less than or equal to $D_p(\max)$ computed by (3).

TABLE I
BEHAVIORAL SEQUENCE RELATION INTRODUCTION

BSR	$h^{-1}(t_1)$	$h^{-1}(t_2)$	$h^{-1}(t_1) \ h^{-1}(t_2)$	Examples
$t_1 \Theta_1 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$b1$ and $f1$ in Fig. 3 (a)
$t_1 \Theta_2 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$g1$ and $d1$ in Fig. 3 (a)
$t_1 \Theta_3 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$d1$ and $g1$ in Fig. 3 (a)
$t_1 \Theta_4 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$b1$ and $b1$ in Fig. 3 (a)
$t_1 \Theta_5 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$b1$ and $d1$ in Fig. 3 (a)
$t_1 \Theta_6 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$d2$ and $d2$ in Fig. 3 (b)
$t_1 \Theta_7 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$h2$ and $f2$ in Fig. 3 (b)
$t_1 \Theta_8 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$f2$ and $c2$ in Fig. 3 (b)
$t_1 \Theta_9 t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$f2$ and $h2$ in Fig. 3 (b)
$t_1 \Theta_{10} t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$b2$ and $d2$ in Fig. 3 (b)
$t_1 \Theta_{11} t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$b3$ and $c3$ in Fig. 3 (c)
$t_1 \Theta_{12} t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$c3$ and $c3$ in Fig. 3 (c)
$t_1 \Theta_{13} t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$c2$ and $b2$ in Fig. 3 (b)
$t_1 \Theta_{14} t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$b2$ and $a2$ in Fig. 3 (b)
$t_1 \Theta_{15} t_2$	e_1, e_3, e_5, e_7	e_2, e_4, e_6, e_8	$e_1 \ominus e_2, e_4 \ominus e_3, e_5 \oplus e_6, e_7 \oplus e_8$	$a2$ and $b2$ in Fig. 3 (b)

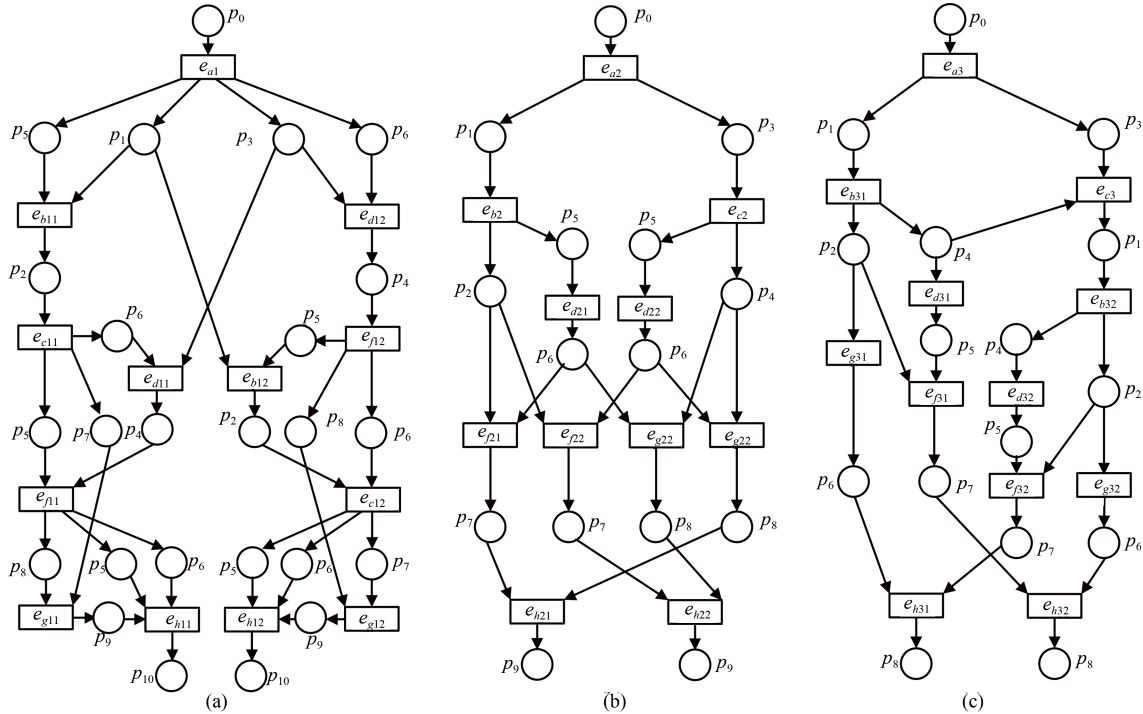


Fig. 5. BPU of Fig. 3.

Based on the above conclusion, we can see that the consistency degree based on behavioral matrix is effective to compute the consistency of two bounded WF-nets. In fact, when the WF-net is unbounded, there is no effective method to compute its branching process. We will be dedicated to do this research in the future.

C. Case Study

According to Definition 2, we can decide the behavioral sequence relation between two transitions using four kinds of relations between events. For readability, Table I shows

these relations and gives the related examples. To decide the relations between events, we must obtain the BPU. For a bounded WF-net, we can compute its BPU by the algorithms in [27] and [28]. For example, for Figs. 3 (a)–(c) we can obtain their BPUs as shown in (4) and (5).

In Figs. 3 (b) and (c), we do not know whether there is a correspondence. For these WF-nets, we easily obtain their BPU as shown in Fig. 5. For instance, the behavioral matrices of Figs. 3 (b) and (c) are shown in (4) and (5).

According to Algorithm 2, we use the permutation for matrix BM_3 . The result is shown in Fig. 6. In Fig. 6, we can see that Fig. 3 (c) does the permutations $\{1 \ 4 \ 3 \ 2 \ 6 \ 5 \ 7 \}$, i.e.,

row 4 \leftrightarrow row 2, row 5 \leftrightarrow row 6, column 4 \leftrightarrow column 2 and column 5 \leftrightarrow column 6. And according to (4) and (5), we can get the permuted matrix BM'_3 of BM_3 is shown in (6). And the $BM = BM_2 - BM'_3$ as shown in (7). Finally, according to (3), the maximal consistency degree of Figs. 3 (b) and (c) is $D_p(\max) = 1 - 17/49 \approx 0.6531$. Similarly, we can obtain all maximal consistency degrees of Fig. 3 as shown in Table II. In fact, for two 7×7 -order matrices, the time to compute their maximal consistency degree is 0.02–0.03 second.

$$BM_2 = \begin{matrix} a2 \\ b2 \\ c2 \\ d2 \\ f2 \\ g2 \\ h2 \end{matrix} \begin{pmatrix} a2 & b2 & c2 & d2 & f2 & g2 & h2 \\ \Theta_{12} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} \\ \Theta_{14} & \Theta_{12} & \Theta_{13} & \Theta_{10} & \Theta_{15} & \Theta_{10} & \Theta_{15} \\ \Theta_{14} & \Theta_{13} & \Theta_{12} & \Theta_{10} & \Theta_{10} & \Theta_{15} & \Theta_{15} \\ \Theta_{14} & \Theta_8 & \Theta_8 & \Theta_6 & \Theta_{10} & \Theta_{10} & \Theta_{15} \\ \Theta_{14} & \Theta_{14} & \Theta_8 & \Theta_8 & \Theta_{12} & \Theta_6 & \Theta_9 \\ \Theta_{14} & \Theta_8 & \Theta_{14} & \Theta_8 & \Theta_6 & \Theta_{12} & \Theta_9 \\ \Theta_{14} & \Theta_{14} & \Theta_{14} & \Theta_{14} & \Theta_7 & \Theta_7 & \Theta_{12} \end{pmatrix} \quad (4)$$

$$BM_3 = \begin{matrix} a3 \\ b3 \\ c3 \\ d3 \\ f3 \\ g3 \\ h3 \end{matrix} \begin{pmatrix} a3 & b3 & c3 & d3 & f3 & g3 & h3 \\ \Theta_{12} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} \\ \Theta_{14} & \Theta_{11} & \Theta_{11} & \Theta_9 & \Theta_9 & \Theta_{10} & \Theta_{15} \\ \Theta_{14} & \Theta_9 & \Theta_{12} & \Theta_9 & \Theta_9 & \Theta_{10} & \Theta_{15} \\ \Theta_{14} & \Theta_7 & \Theta_7 & \Theta_4 & \Theta_9 & \Theta_6 & \Theta_{15} \\ \Theta_{14} & \Theta_7 & \Theta_7 & \Theta_7 & \Theta_{12} & \Theta_6 & \Theta_9 \\ \Theta_{14} & \Theta_8 & \Theta_8 & \Theta_6 & \Theta_6 & \Theta_6 & \Theta_9 \\ \Theta_{14} & \Theta_{14} & \Theta_{14} & \Theta_{14} & \Theta_7 & \Theta_7 & \Theta_{12} \end{pmatrix} \quad (5)$$

$$BM'_3 = \begin{matrix} a3 \\ b3 \\ c3 \\ d3 \\ f3 \\ g3 \\ h3 \end{matrix} \begin{pmatrix} a3 & b3 & c3 & d3 & f3 & g3 & h3 \\ \Theta_{12} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} & \Theta_{15} \\ \Theta_{14} & \Theta_4 & \Theta_7 & \Theta_7 & \Theta_6 & \Theta_9 & \Theta_{15} \\ \Theta_{14} & \Theta_9 & \Theta_{12} & \Theta_9 & \Theta_{10} & \Theta_9 & \Theta_{15} \\ \Theta_{14} & \Theta_9 & \Theta_{11} & \Theta_{11} & \Theta_{10} & \Theta_9 & \Theta_{15} \\ \Theta_{14} & \Theta_6 & \Theta_8 & \Theta_8 & \Theta_6 & \Theta_6 & \Theta_9 \\ \Theta_{14} & \Theta_7 & \Theta_7 & \Theta_7 & \Theta_6 & \Theta_{12} & \Theta_9 \\ \Theta_{14} & \Theta_{14} & \Theta_{14} & \Theta_{14} & \Theta_7 & \Theta_7 & \Theta_{12} \end{pmatrix} \quad (6)$$

Similarly, we can obtain the minimal consistency degree if necessary. In fact, if Fig. 4 is based on an unknown correspondence, we can compute this condition using our method.

In fact, according to the Theorem 1 and the theorem of behavioral profiles relations in [2], we have shown that our method is more accurate.

V. RELATED WORK

Existing consistency analysis techniques for process models are based on three complementary aspects [3]: task labels, structure and behavior. In order to meet the complex and fickle application requirements, to compare the consistency between

Algorithm 2. Maximal consistency degree

Input: matrix BM_1 , matrix BM_2 .

Output: C , $\|G'\|$, BM'_2

Initialization

$n = \text{size}(BM_1, 1)$

$\text{all_perm} = \text{perms}(1 : n)$

$\text{num} = \text{size}(\text{all_perm}, 1)$

$\|G'\| = n^2$

$C = \text{zeros}(1, n)$

For ($i = 1 : \text{num}$) $\text{temp} = BM_2(:, \text{all_perm}(i, :))$ **Do**

$\text{temp} = \text{temp}(\text{all_perm}(i, :), :)$

$G = \text{sum}(\text{sum}((BM_1 - \text{temp}) = 0))$

If $G < \|G'\|$ $\|G'\| = G$ **Then**

$C = \text{all_perm}(i, :)$

End

End

$BM'_2 = BM_2(:, C)$

$BM'_2 = BM'_2(C, :)$

Return C , $\|G'\|$, BM'_2

models and multiple interaction components has become a new trend. Existing models behavior consistency methods are mainly divided into two aspects: one is the semantic behavior consistency [32]–[38], other one is process behavior consistency [39]–[52]. [39] gave a consistency method of using an expected value to measure a model and the expected one, but did not involve the degree of consistency. [40] proposed the concept of process-oriented protocol and proposed a standard of detect consistency automatically. [41] proposed a method of determining whether the overall model can run locally or not based on semantic consistency, and gave an algorithm of generating local models from the entire model. But it did not consider behavior satisfiability. [42] proposed a method which would determine behavior pattern and tiny deviations once determining the matching. [43], [44] explained non-conformance and consistency only for a execute sequence. [45]–[47] introduced the measure similarity method of correcting distance in semantics. [48] proposed a more standard and more open architecture to assess the service interaction. [49] used trace equivalence or bisimulation to analyze the behavioral consistency between the models. [50], [51] defined the similarity degree between two process models by using causal footprints. [52] is a family of binary relations that represented the behavior of a process in a $n \times n$ matrix.

$$BM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Theta_{12} - \Theta_4 & \Theta_{13} - \Theta_7 & \Theta_{10} - \Theta_7 & \Theta_{15} - \Theta_6 & \Theta_{10} - \Theta_9 & 0 \\ 0 & \Theta_{13} - \Theta_9 & 0 & \Theta_{10} - \Theta_9 & 0 & \Theta_{15} - \Theta_9 & 0 \\ 0 & \Theta_8 - \Theta_9 & \Theta_8 - \Theta_{11} & \Theta_6 - \Theta_{11} & 0 & \Theta_{10} - \Theta_9 & 0 \\ 0 & \Theta_{14} - \Theta_6 & 0 & 0 & \Theta_{12} - \Theta_6 & 0 & 0 \\ 0 & \Theta_8 - \Theta_7 & \Theta_{14} - \Theta_7 & \Theta_8 - \Theta_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (7)$$

TABLE II
MAXIMAL CONSISTENCY DEGREE OF FIG. 3

Two nets in Fig. 3	Permutation	$\min \ G'\ $	Maximal consistency degree	Time (s)
(a) and (b)	{1 6 5 4 3 2 7}	27	0.4490	0.026
(a) and (c)	{1 6 7 3 2 4 5}	20	0.5918	0.025
(b) and (c)	{1 4 3 2 6 5 7}	17	0.6531	0.024

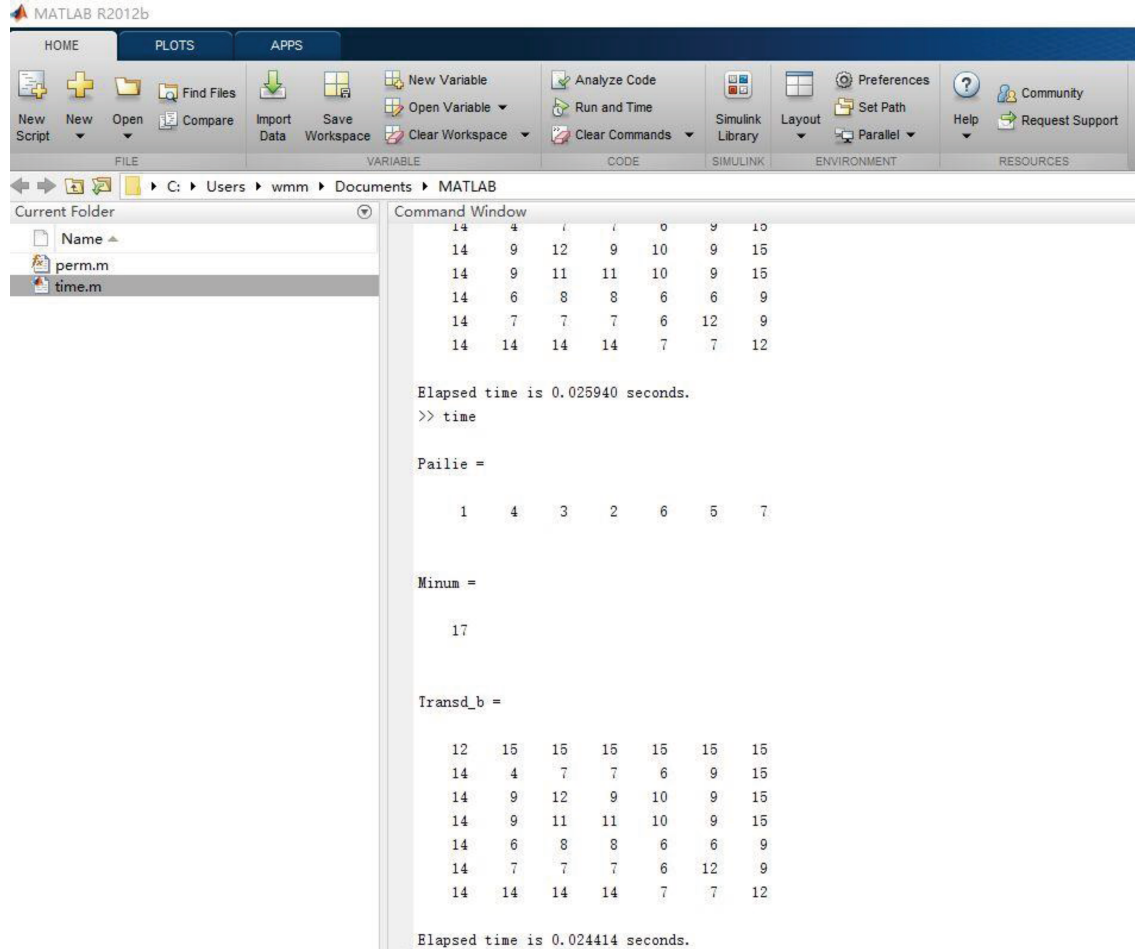


Fig. 6. Implementation result by Algorithm 2 and using MATLAB for Figs. 3 (b) and (c).

It offered a plethora of different relations and each cell in the matrix can contain more than one relation. However, 4C spectrum suffers from the same issues as the behavioral profiles because it does not guarantee any of the well-known notions of equivalence. Furthermore, the relations in this family not clear.

VI. CONCLUSIONS

In this paper, we present a method of generating behavioral sequence relations based on BPU and event relations. We refine behavioral sequence relations between two actions, and present a more accurate consistency measurement of two WF-nets with uncertain correspondence.

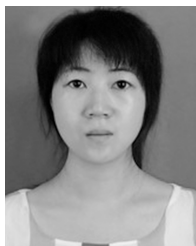
In the future, we would like to focus on the structure and behavioral characteristic influencing the consistency of WF-nets. In addition, to obtain a tool of the branching process unfolding is our future work.

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