Guaranteed Cost Consensus for High-dimensional Multi-agent Systems With Time-varying Delays

Zhong Wang, Ming He, Tang Zheng, Zhiliang Fan, and Guangbin Liu

***Abstract*—Guaranteed cost consensus analysis and design problems for high-dimensional multi-agent systems with time-varying delays are investigated. The idea of guaranteed cost control is introduced into consensus problems for high-dimensional multi-agent systems with time-varying delays, where a cost function is defined based on state errors among neighboring agents and control inputs of all the agents. By the state space decomposition approach and the linear matrix inequality (LMI), sufficient conditions for guaranteed cost consensus and consensualization are given. Moreover, a guaranteed cost upper bound of the cost function is determined. It should be mentioned that these LMI criteria are dependent on the change rate of time delays and the maximum time delay, the guaranteed cost upper bound is only dependent on the maximum time delay but independent of the Laplacian matrix. Finally, numerical simulations are given to demonstrate theoretical results.**

***Index Terms*—Guaranteed cost consensus, high-dimensional, multi-agent system, time-varying delay.**

I. INTRODUCTION

RECENTLY many researchers paid much attention to the distributed coordination control of multi-agent systems due to its broad practical applications including formation control of mobile agents [1], synchronization in wireless sensor networks [2], distributed automatic generation control for cyber-physical microgrid system [3], and rendezvous [4] or flocking [5] of multiple vehicles. Consensus is an essential problem for multi-agent systems and it has been extensively investigated (see the survey papers [6]−[8] and the references therein).

In the existing works, an important topic on the consensus problems is the effect of time delays which include computational delays of agents and transmission delays in the transfer of data between agents. Based on the frequency domain analysis, the Lyapunov function and the concept of delayed and hierarchical graph, many conclusions for low-dimensional multi-agent systems with time delays were obtained for single-integrator cases (e.g., [9]−[12]) and double-integrator cases (e.g., [13]−[16]). For high-dimensional cases, the dynamics of each agent can be described as high-order integrator [17]−[19]. In fact, agents of high-dimensional cases were depicted by the state space approach in most of works [20]−[23], where the analysis approach included the frequency domain analysis, the Lyapunov function approach and the property of nonnegative matrix.

With the development of the control theory, it is usually desirable to design a controller which not only stabilizes the system but also ensures an adequate level of performance. To this end, a design approach is so-called guaranteed cost control [24]−[26], where the performance and the energy consumption are considered simultaneously. In the aforementioned works on consensus problems, only the consensus regulation performance for multi-agent systems was considered. However, the energy consumption is also essential for the consensus. Thus, the idea of guaranteed cost control should be applied for the consensus problem to simultaneously consider the consensus regulation performance and the control energy consumption. To the best of our knowledge, there are very few works to investigate guaranteed cost consensus problems. In [27]−[30], the guaranteed cost consensus problems for low-dimensional multi-agent systems were investigated. For high-dimensional multi-agent systems, [31] studied the guaranteed cost leader-follower control. In [32], the problem of event based guaranteed cost consensus for high-dimensional multi-agent systems was considered based on a sampled-data event triggering mechanism. It should be pointed out that the effects of time delays for guaranteed cost consensus problems were not considered in [28]−[32].

Motivated by this, guaranteed cost consensus problems for high-dimensional multi-agent systems with time-varying delays are investigated in the current paper. Comparing with the existing works, there are two contributions. On one hand, the guaranteed cost consensus problems are introduced into high-dimensional multi-agent systems. In [27]−[30], each agent of multi-agent systems was described as low-dimensional integrator dynamics. But those analysis approaches cannot be directly used to the guaranteed cost consensus problem for high-dimensional case. On the other hand, the effects of time-varying delays are considered, while the works in [28]−[32] did not consider this issue. In [27], the guaranteed cost consensus problem for first-order multi-agent systems with time delays were studied, but the time delay is time-invariant.

The remainder of this paper is organized as follows. In Section II, the graph theory is given and the problem descrip-

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tion of guaranteed cost consensus is presented. In Section III, sufficient conditions for guaranteed cost consensus are given, an upper bound of the cost function is determined, and an approach is proposed to obtain the consensus function. In Section IV, a numerical simulation is given. Finally, concluding remarks are stated in Section V. Throughout the current paper, $\{A_1, A_2, \ldots, A_n\}$ represents a block-diagonal matrix with A_i $(i = 1, 2, \ldots, n)$ on its diagonal, $\mathbf{1}_N \in \mathbb{R}^N$ denotes an N-dimensional column vector with all components 1, 0 is used to denote zero matrices of any size with zero vectors and zero number as special cases. $I_N \in \mathbb{R}^{N\times N}$ represents the identity matrix of size N. The symbol $\otimes$ is used to represent the Kronecker product. The notation asterisk $*$ represents the elements below the main diagonal of a symmetric matrix.

II. PRELIMINARIES

An undirected graph $G = \mathcal{G}(\mathcal{V}, \mathcal{E}, W)$ consists of a node set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, an edge set $\mathcal{E} \subseteq \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$, and a symmetric adjacency matrix $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ with $w_{ij} \geq 0$ and $w_{ii} = 0$, where $w_{ij} > 0$ if and only if $(v_j, v_i) \in \mathcal{E}$. The node index belongs to a finite index set $\mathcal{I}_N = \{1, 2, \ldots, N\}$. The neighboring set of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The degree of node v_i is defined as $\deg_{\rm in}(v_i) = \sum_{j\in\mathcal{N}_i} w_{ij}$ and the degree matrix of G as $D = {\rm diag}\{\deg_{\rm in}(v_1), \deg_{\rm in}(v_2), \ldots, \deg_{\rm in}(v_N)\}$. The matrix $L = D - W$ is called the Laplacian matrix associated with G. If there does not exist an isolated node in G, then G is said to be connected.

The following lemma shows basic properties of the Laplacian matrix of an undirected graph.

Lemma 1 [33]: Let $L \in \mathbb{R}^{N\times N}$ be the Laplacian matrix of an undirected graph G, then 1) L at least has a zero eigenvalue, and 1_N is an associated eigenvector; that is, $L1_N = 0$. 2) If G is connected, then 0 is a simple eigenvalue of L, and all the other $N - 1$ eigenvalues are positive; that is, $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$, where λ_i $(i \in \mathcal{I}_N)$ denotes the eigenvalue of L.

A high-dimensional multi-agent system composed of N homogeneous agents is considered. The interaction topology can be described by an undirected graph G, each node stands for an agent, the edge between any two nodes denotes the interaction channel between them, and the weight of the edge corresponds to the interaction strength. Each agent can be described as general linear dynamics:

$$\dot{\boldsymbol{x}}_i(t) = A\boldsymbol{x}_i(t) + B\boldsymbol{u}_i(t) \tag{1}$$

where $i = 1, 2, \ldots, N$, $A \in \mathbb{R}^{d\times d}$ and $B \in \mathbb{R}^{d\times m}$ are constant matrices, $\boldsymbol{x}_i(t)$ is the state of agent i, $\boldsymbol{u}_i(t)$ is the control input of agent i. The state $\boldsymbol{x}(t)$ is marked as $\boldsymbol{x}(t) = [\boldsymbol{x}_1^T(t), \boldsymbol{x}_2^T(t), \ldots, \boldsymbol{x}_N^T(t)]^T$. Then, the multi-agent system can be rewritten as

$$\dot{\boldsymbol{x}}(t) = (I_N \otimes A)\,\boldsymbol{x}(t) - (I_N \otimes B)\,\boldsymbol{u}(t). \tag{2}$$

Consider the following consensus protocol

$$\boldsymbol{u}_i(t) = K\sum_{j\in\mathcal{N}_i} w_{ij}\left(\boldsymbol{x}_j(t-\tau(t)) - \boldsymbol{x}_i(t-\tau(t))\right) \tag{3}$$

where $i, j \in \{1, 2, \ldots, N\}$, $K \in \mathbb{R}^{m\times d}$ is a control gain matrix, $\mathcal{N}_i$ represents the neighboring set of agent i, w_{ij} is the interaction strength of the edge from agent j to agent i and $\tau(t)$ denotes the time-varying delay during the information communication. The time-varying delay $\tau(t)$ satisfies $0 \leq \tau(t) \leq \tau_{\max}$ and $|\dot{\tau}(t)| \leq \ell < 1$, where $\tau_{\max}$ and ℓ are known positive constants. In global form, the consensus protocol is

$$\boldsymbol{u}(t) = -\left(L \otimes K\right)\boldsymbol{x}(t-\tau(t)). \tag{4}$$

Then, the dynamics of multi-agent system (2) with protocol (4) can be described by

$$\dot{\boldsymbol{x}}(t) = (I_N \otimes A)\,\boldsymbol{x}(t) - (L \otimes BK)\,\boldsymbol{x}(t-\tau(t)) \tag{5}$$

where $\boldsymbol{x}(t) = \boldsymbol{x}(0)$, $t \in [-\tau_{\max}, 0]$.

Let $\boldsymbol{\delta}_{ij}(t) = \boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)$ represents the state error between agent j and agent i, define the following cost function for multi-agent system (5) with symmetric and positive definite matrices $Q_x \in \mathbb{R}^{d\times d}$ and $Q_u \in \mathbb{R}^{m\times m}$:

$$J_C = J_{Cx} + J_{Cu} \tag{6}$$

where

$$J_{Cx} = \int_0^{\infty}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\left(\boldsymbol{\delta}_{ij}^T(t)\,Q_x\boldsymbol{\delta}_{ij}(t)\right)dt$$

$$J_{Cu} = \int_0^{\infty}\sum_{i=1}^{N}\boldsymbol{u}_i^T(t)\,Q_u\boldsymbol{u}_i(t)dt.$$

Definition 1: Multi-agent system (5) is said to achieve guaranteed cost consensus if there exists a vector-valued function $\boldsymbol{c}(t)$ such that $\lim_{t\to\infty}(\boldsymbol{x}(t) - \mathbf{1}_N \otimes \boldsymbol{c}(t)) = 0$ and there exists a positive scalar J_C^* such that $J_C \leq J_C^*$, where $c(t)$ is called a consensus function and J_C^* is said to be a guaranteed cost.

Definition 2: Multi-agent system (2) is said to be guaranteed cost consensualizable by control protocol (4) if there exists a control gain matrix K such that the multi-agent system achieves guaranteed cost consensus.

Remark 1: In cost function (6), J_{Cx} and J_{Cu} can be regarded as the consensus regulation performance and control energy consumption for high-dimensional multi-agent systems, respectively. Because consensus problems for multi-agent systems focus on state errors among neighboring agents instead of states of all agents, J_{Cx} is constructed by state errors. The guaranteed cost consensus problem is to find a control gain matrix K such that the cost function J_C has a guaranteed cost upper bound J_C^*.

III. MAIN RESULTS

By Lemma 1, it can be obtained that the eigenvalue $\lambda_1 = 0$ with the associated eigenvector $\mathbf{1}_N/\sqrt{N}$ and $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$. Then, there exists an orthogonal matrix

$$U = \begin{bmatrix} \dfrac{1}{N} & \dfrac{\mathbf{1}_{N-1}^T}{N} \\ \dfrac{\mathbf{1}_{N-1}}{N} & \bar{U} \end{bmatrix}$$

which satisfies that $U^TU = I_N$ and

$$\Lambda = U^TLU = {\rm diag}\{0, \Lambda_\lambda\} = {\rm diag}\{0, \lambda_2, \lambda_3, \ldots, \lambda_N\}.$$

Let

$$\boldsymbol{\kappa}(t) = (U^T \otimes I_d)\boldsymbol{x}(t) = \left[\boldsymbol{\kappa}_c^T(t), \boldsymbol{\kappa}_r^T(t)\right]^T \tag{7}$$

where $\boldsymbol{\kappa}_c(t) \in \mathbb{R}^d$ and $\boldsymbol{\kappa}_r(t) = [\boldsymbol{\kappa}_{r2}^T(t), \boldsymbol{\kappa}_{r3}^T(t), \ldots, \boldsymbol{\kappa}_{rN}^T(t)]^T \in \mathbb{R}^{(N-1)d}$, then multi-agent system (5) can be transformed into

$$\dot{\boldsymbol{\kappa}}_c(t) = A\boldsymbol{\kappa}_c(t) \tag{8}$$

$$\dot{\boldsymbol{\kappa}}_{ri}(t) = A\boldsymbol{\kappa}_{ri}(t) - \lambda_i BK\boldsymbol{\kappa}_{ri}(t-\tau(t)) \tag{9}$$

where $i = 2, 3, \ldots, N$.

For cost function (6), it can be shown that

$$J_{Cx} = \int_0^\infty \boldsymbol{x}^T(t)\left(2L \otimes Q_x\right)\boldsymbol{x}(t)dt \tag{10}$$

$$J_{Cu} = \int_0^\infty \boldsymbol{x}^T(t-\tau(t))(L^2 \otimes K^T Q_u K)\boldsymbol{x}(t-\tau(t))dt. \tag{11}$$

Due to $\Lambda_\lambda = \mathrm{diag}\{\lambda_2, \lambda_3, \ldots, \lambda_N\}$, then

$$\boldsymbol{\kappa}_r^T(t)(\Lambda_\lambda \otimes K^T Q_x K)\boldsymbol{\kappa}_r(t) = \sum_{i=2}^N \left(\lambda_i \boldsymbol{\kappa}_{ri}^T(t) K^T Q_x K \boldsymbol{\kappa}_{ri}(t)\right). \tag{12}$$

Thus, by (7) and (10), it can be seen that

$$J_{Cx} = \sum_{i=2}^N \int_0^\infty 2\lambda_i \boldsymbol{\kappa}_{ri}^T(t) Q_x \boldsymbol{\kappa}_{ri}(t)dt. \tag{13}$$

Similarly, by (7) and (11), one has

$$J_{Cu} = \sum_{i=2}^N \int_0^\infty \lambda_i^2 \boldsymbol{\kappa}_{ri}^T(t-\tau(t))K^T Q_u K\boldsymbol{\kappa}_{ri}(t-\tau(t))dt. \tag{14}$$

Thus, one can obtain that cost function (6) can be rewritten as

$$\begin{aligned} J_C = &\sum_{i=2}^N \int_0^\infty 2\lambda_i \boldsymbol{\kappa}_{ri}^T(t) Q_x \boldsymbol{\kappa}_{ri}(t)dt \\ &+ \sum_{i=2}^N \int_0^\infty \lambda_i^2 \boldsymbol{\kappa}_{ri}^T(t-\tau(t))K^T Q_u K\boldsymbol{\kappa}_{ri}(t-\tau(t))dt. \end{aligned} \tag{15}$$

Before obtaining the main result, the following lemma should be introduced.

Lemma 2 [34]: Let $\boldsymbol{\kappa}(t) \in \mathbb{R}^d$ be a vector-valued function with first-order continuous-derivative entries. Then the following integral inequality holds for any matrices $\Phi_1, \Phi_2 \in \mathbb{R}^{d\times d}$, d-dimensional matrix $W = W^T > 0$, and a scalar function $\tau(t) \geq 0$:

$$-\int_{t-\tau(t)}^t \dot{\boldsymbol{\kappa}}^T(s)W\dot{\boldsymbol{\kappa}}(s)ds \leq \boldsymbol{\chi}^T(t)\Phi_a\boldsymbol{\chi}(t) + \tau_{\max}\boldsymbol{\chi}^T(t)\Phi_b^T W^{-1}\Phi_b\boldsymbol{\chi}(t)$$

where

$$\boldsymbol{\chi}(t) = \begin{bmatrix} \boldsymbol{\kappa}(t) \\ \boldsymbol{\kappa}(t-\tau(t)) \end{bmatrix}$$

$$\Phi_a = \begin{bmatrix} \Phi_1^T + \Phi_1 & -\Phi_1^T + \Phi_2 \\ * & -\Phi_2^T - \Phi_2 \end{bmatrix}, \quad \Phi_b = \left[\Phi_1, \Phi_2\right].$$

The following theorem presents a sufficient condition for guaranteed cost consensus.

Theorem 1: Multi-agent system (5) with an undirected and connected topology G achieves guaranteed cost consensus if there exist d-dimensional matrices $P = P^T > 0$, $R = R^T > 0$, $W = W^T > 0$, Φ_1 and Φ_2 such that

$$\Omega_i = \begin{bmatrix} \Omega_{i11} & \Omega_{i12} \\ * & \Omega_{i22} \end{bmatrix} < 0, \quad i = 2, N$$

where

$$\Omega_{i11} = \begin{bmatrix} \Omega_{111} & \Omega_{i112} & \tau_{\max}A^T W \\ * & \Omega_{122} & -\tau_{\max}\lambda_i K^T B^T W \\ * & * & -\tau_{\max}W \end{bmatrix}$$

$$\Omega_{i12} = \begin{bmatrix} \tau_{\max}\Phi_1^T & 2\lambda_i Q_x & 0 \\ \tau_{\max}\Phi_2^T & 0 & \lambda_i K^T Q_u \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_{i22} = \mathrm{diag}\{-\tau_{\max}W, -2\lambda_i Q_x, -Q_u\}$$

$$\Omega_{111} = A^T P + PA + R + \Phi_1^T + \Phi_1$$

$$\Omega_{i112} = -\lambda_i PBK - \Phi_1^T + \Phi_2$$

$$\Omega_{122} = (\ell - 1)R - \Phi_2^T - \Phi_2$$

Proof: Firstly, let

$$\boldsymbol{x}_c(t) = (U \otimes I_d)\left[\boldsymbol{\kappa}_c^T(t), 0\right]^T \tag{16}$$

$$\boldsymbol{x}_r(t) = (U \otimes I_d)\left[0, \boldsymbol{\kappa}_r^T(t)\right]^T \tag{17}$$

where $0 \in \mathbb{R}^{1\times(N-1)d}$ in (16) and $0 \in \mathbb{R}^{1\times d}$ in (17). Due to U is an orthogonal matrix, then $\boldsymbol{x}_c(t)$ and $\boldsymbol{x}_r(t)$ are linearly independent. By (7), one has

$$\boldsymbol{x}(t) = \boldsymbol{x}_c(t) + \boldsymbol{x}_r(t). \tag{18}$$

By (16), it can be obtained that

$$\boldsymbol{x}_c(t) = \frac{\mathbf{1}_N}{\sqrt{N}} \otimes \boldsymbol{\kappa}_c(t). \tag{19}$$

Thus, it follows that multi-agent system (5) achieves consensus if and only if subsystem (9) is asymptotically stable; that is,

$$\lim_{t\to\infty} \boldsymbol{\kappa}_{ri}(t) = 0, \quad i = 2, 3, \ldots, N. \tag{20}$$

Next, consider the following Lyapunov-Krasovskii function (LKF) candidate

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{21}$$

where

$$V_1(t) = \sum_{i=2}^N \boldsymbol{\kappa}_{ri}^T(t)P\boldsymbol{\kappa}_{ri}(t)$$

$$V_2(t)=\sum_{i=2}^{N}\int_{t-\tau(t)}^{t}\boldsymbol{\kappa}_{ri}^T(t)R\boldsymbol{\kappa}_{ri}(t)ds$$
$$V_3(t)=\sum_{i=2}^{N}\int_{-\tau_{\max}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{\kappa}}_{ri}^T(s)W\dot{\boldsymbol{\kappa}}_{ri}(s)dsd\theta.$$

Then, the time derivative of $V(t)$ along the trajectory of (9) is

$$\begin{aligned}\dot{V}_1(t)=&\sum_{i=2}^{N}\left(\boldsymbol{\kappa}_{ri}^T(t)\left(A^TP+PA\right)\boldsymbol{\kappa}_{ri}(t)\right)\\&-2\sum_{i=2}^{N}\left(\lambda_i\boldsymbol{\kappa}_{ri}^T(t)PBK\boldsymbol{\kappa}_{ri}(t-\tau(t))\right)\end{aligned}\tag{22}$$

$$\begin{aligned}\dot{V}_2(t)\le&\sum_{i=2}^{N}\left(\boldsymbol{\kappa}_{ri}^T(t)R\boldsymbol{\kappa}_{ri}(t)\right)\\&-(1-\ell)\sum_{i=2}^{N}\left(\boldsymbol{\kappa}_{ri}^T(t-\tau(t))R\boldsymbol{\kappa}_{ri}(t-\tau(t))\right)\end{aligned}\tag{23}$$

$$\begin{aligned}\dot{V}_3(t)=&\sum_{i=2}^{N}\left(\tau_{\max}\dot{\boldsymbol{\kappa}}_{ri}^T(t)W\dot{\boldsymbol{\kappa}}_{ri}(t)\right)\\&-\sum_{i=2}^{N}\left(\int_{t-\tau_{\max}}^{t}\dot{\boldsymbol{\kappa}}_{ri}^T(s)W\dot{\boldsymbol{\kappa}}_{ri}(s)ds\right).\end{aligned}\tag{24}$$

From (9), one has

$$\sum_{i=2}^{N}\left(\dot{\boldsymbol{\kappa}}_{ri}^T(t)W\dot{\boldsymbol{\kappa}}_{ri}(t)\right)=\sum_{i=2}^{N}\left(\boldsymbol{\chi}_{ri}^T(t)D_i^TWD_i\boldsymbol{\chi}_{ri}(t)\right)$$

where $\boldsymbol{\chi}_{ri}^T(t)=[\boldsymbol{\kappa}_{ri}^T(t),\boldsymbol{\kappa}_{ri}^T(t-\tau(t))]^T$ and the matrix $D_i=[A,\ -\lambda_iBK]$. By Lemma 2, it can be obtained that

$$\begin{aligned}&\sum_{i=2}^{N}\left(-\int_{t-\tau_{\max}}^{t}\dot{\boldsymbol{\kappa}}_{ri}^T(s)W\dot{\boldsymbol{\kappa}}_{ri}(s)ds\right)\\&\le\sum_{i=2}^{N}\left(-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{\kappa}}_{ri}^T(s)W\dot{\boldsymbol{\kappa}}_{ri}(s)ds\right)\\&\le\sum_{i=2}^{N}\left(\boldsymbol{\chi}_{ri}^T(t)\Phi_a\boldsymbol{\chi}_{ri}(t)+\tau_{\max}\boldsymbol{\chi}_{ri}^T(t)\Phi_b^TW^{-1}\Phi_b\boldsymbol{\chi}_{ri}(t)\right).\end{aligned}$$

Then,

$$\dot{V}(t)\le\sum_{i=2}^{N}\chi_{ri}^T(t)\bar{\Theta}_i\boldsymbol{\chi}_{ri}(t)\tag{25}$$

where $\bar{\Theta}_i=\bar{\Delta}_i+\tau_{\max}D_i^TWD_i+\tau_{\max}\Phi_b^TW^{-1}\Phi_b$ and

$$\bar{\Delta}_i=\begin{bmatrix}\Omega_{111}&\Omega_{i112}\\ *&\Omega_{122}\end{bmatrix}.$$

Moreover, define

$$\dot{\Im}(t)=\dot{V}(t)+\bar{J}_C\tag{26}$$

where

$$\begin{aligned}\bar{J}_C=&\sum_{i=2}^{N}\left(2\lambda_i\boldsymbol{\kappa}_{ri}^T(t)Q_x\boldsymbol{\kappa}_{ri}(t)\right)\\&+\sum_{i=2}^{N}\left(\lambda_i^2\boldsymbol{\kappa}_{ri}^T(t-\tau(t))K^TQ_uK\boldsymbol{\kappa}_{ri}(t-\tau(t))\right).\end{aligned}$$

Note that if $\dot{\Im}(t)\le0$, then $\dot{V}(t)\le0$. Thus,

$$\dot{\Im}(t)\le\sum_{i=2}^{N}\boldsymbol{\chi}_{ri}^T(t)\Theta_i\boldsymbol{\chi}_{ri}(t)\tag{27}$$

where $\Theta_i=\Delta_i+\tau_{\max}D_i^TWD_i+\tau_{\max}\Phi_b^TW^{-1}\Phi_b$ and

$$\Delta_i=\begin{bmatrix}\Delta_{i11}&\Omega_{i112}\\ *&\Delta_{i22}\end{bmatrix}$$
$$\Delta_{i11}=\Omega_{111}+2\lambda_iQ_x,\ \Delta_{i22}=\Omega_{122}+\lambda_i^2K^TQ_uK.$$

Hence, by the Schur complement, if $\Omega_i<0$ $(i=2,3,\dots,N)$, then $\Theta_i<0$ $(i=2,3,\dots,N)$. Thus, $\dot{\Im}(t)\le0$ and $\dot{\Im}(t)=0$ if and only if $\tilde{\boldsymbol{x}}_{ri}(t)\equiv0$ $(i=2,3,\dots,N)$. Then, $\dot{V}(t)\le0$ and $\dot{V}(t)=0$ if and only if $\tilde{\boldsymbol{x}}_{ri}(t)\equiv0$ $(i=2,3,\dots,N)$. Moreover, by the convex property of linear matrix inequalities (LMIs), if $\Omega_i<0$ $(i=2,N)$ then $\Omega_i<0$ $(i=2,3,\dots,N)$. Therefore, if $\Omega_i<0$ $(i=2,N)$, subsystems (9) are asymptotically stable.

From (26) and $\dot{\Im}(t)\le0$, one has

$$\bar{J}_C\le-\dot{V}(t).\tag{28}$$

Since $V(t)>0$ and $\dot{V}(t)\le0$, then $\lim_{t\to\infty}V(t)=0$. Integrating both sides of (28), one has $J_C\le V(0)$, where the fact that $\int_0^\infty\bar{J}_Cdt=J_C$ has been used. Therefore, from Definition 1, $\Omega_i<0$ $(i=2,N)$ can ensure that multi-agent system (5) achieves guaranteed cost consensus and the cost function satisfies $J_C\le V(0)$. The conclusion of Theorem 1 can be obtained. ■

Remark 2: By using the state space decomposition approach, guaranteed cost consensus problems for the high-dimensional multi-agent systems with time delays are converted into guaranteed cost stability problems for $N-1$ time-delayed subsystems. It is worth pointing out that Theorem 1 only needs to judge whether two matrices are negative; that is, the LMI criteria are only dependent on the second smallest eigenvalue λ_2 and the maximum eigenvalue λ_N of L. However, λ_2 and λ_N of L may be difficult to be obtained when the dimension N of L is huge. Fortunately, [33] gives a method to estimate λ_2 and the Gersgorin Disc Theorem in [35] can be used to approximately determine λ_N.

When multi-agent system (5) achieves guaranteed cost consensus, an upper bound of guaranteed cost function (6) and the explicit expression of the consensus function are obtained by the following theorem and corollary, respectively.

Theorem 2: When multi-agent system (5) achieves guaranteed cost consensus with K and d-dimensions matrices $P=P^T>0$, $R=R^T>0$, the guaranteed cost satisfies

$$J_C^*=\boldsymbol{x}^T(0)(\Upsilon\otimes(P+\tau_{\max}R))\boldsymbol{x}(0)$$

where $\Upsilon=I_N-\mathbf{1}_N\mathbf{1}_N^T/N$.

Proof: From (7), it can be obtained that $\boldsymbol{\kappa}_r(t)=([0,I_{N-1}]U^T\otimes I_d)\boldsymbol{x}(t)$. Then, one can obtain that

$$V_1(t)=\boldsymbol{x}^T(t)(\Upsilon\otimes P)\boldsymbol{x}(t)\tag{29}$$

$$V_2(t) = \int_{t-\tau(t)}^{t} \boldsymbol{x}^T(s)(\Upsilon \otimes R)\boldsymbol{x}(s)ds \tag{30}$$

$$V_3(t) = \int_{-\tau_{\max}}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^T(s)(\Upsilon \otimes W)\dot{\boldsymbol{x}}(s)dsd\theta \tag{31}$$

where

$$\Upsilon = \begin{bmatrix} \frac{\mathbf{1}_{N-1}^T\mathbf{1}_{N-1}}{N} & \frac{\mathbf{1}_{N-1}^T\bar{U}^T}{\sqrt{N}} \\ \frac{\bar{U}\mathbf{1}_{N-1}}{\sqrt{N}} & \bar{U}\bar{U}^T \end{bmatrix}.$$

By $UU^T = I$ in (5), one has $\mathbf{1}_{N-1}^T\bar{U}^T/\sqrt{N} = -\mathbf{1}_{N-1}^T/N$ and $\bar{U}\bar{U}^T = I_{N-1} - \mathbf{1}_{N-1}\mathbf{1}_{N-1}^T/N$. Then, $\Upsilon = I_N - \mathbf{1}_N\mathbf{1}_N^T/N$. Due to $\boldsymbol{x}(t) = \boldsymbol{x}(0)$, $t \in [-\tau_{\max}, 0]$, it is obtained that $V_2(0) \le \tau_{\max}\boldsymbol{x}^T(0)(\Upsilon \otimes R)\boldsymbol{x}(0)$ and $V_3(0) = 0$. Then, by $V_1(0) = \boldsymbol{x}^T(0)(\Upsilon \otimes P)\boldsymbol{x}(0)$, one has

$$J_C \le \boldsymbol{x}^T(0)\left((\Upsilon \otimes P) + \tau_{\max}(\Upsilon \otimes R)\right)\boldsymbol{x}(0). \tag{32}$$

Thus, the conclusion of Theorem 2 can be obtained. ■

Corollary 1: When multi-agent system (5) achieves guaranteed cost consensus, the consensus function $\boldsymbol{c}(t)$ satisfies

$$\lim_{t\to\infty}\left(\boldsymbol{c}(t) - e^{At}\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_i(0)\right)\right) = 0.$$

Proof: From (8), one can obtain that $\boldsymbol{\kappa}_c(t) = e^{At}\boldsymbol{\kappa}_c(0)$. By (7), one has

$$\boldsymbol{\kappa}_c(0) = [I_d, 0, \ldots, 0](U^T \otimes I_d)\boldsymbol{x}(0) = \frac{1}{\sqrt{N}}(\mathbf{1}_N^T \otimes I_d).$$

Thus, it can be obtained that

$$\boldsymbol{\kappa}_c(t) = e^{At}\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{x}_i(0)\right). \tag{33}$$

From the proof of Theorem 1, if multi-agent system (3) achieves guaranteed cost consensus, then $\lim_{t\to\infty}(\boldsymbol{x}(t) - \boldsymbol{x}_c(t)) = 0$. Thus, from Definition 1 and (19), one can see that consensus function $\boldsymbol{c}(t)$ satisfies

$$\lim_{t\to\infty}\left(\boldsymbol{c}(t) - \frac{1}{\sqrt{N}}\boldsymbol{\kappa}_c(t)\right) = 0. \tag{34}$$

Then, by (33) and (34), the conclusions can be obtained. ■

Remark 3: Theorem 2 shows that the guaranteed cost is related to the initial states $\boldsymbol{x}(0)$ and the maximum time delay $\tau_{\max}$. Moreover, one can see that Corollary 1 implies that the time-varying delay and the guaranteed cost function do not impact on the consensus function.

In Theorem 1, it is difficult to give feasible LMI criteria for obtaining guaranteed cost consensus since there exists nonlinear term (PBK and K^TB^TW) when K is unknown. In the sequel, the result presents a design method of the guaranteed cost consensus for multi-agent system (5), where gives an approach to determine the control gain matrix.

Theorem 3: Multi-agent system (2) is said to be guaranteed cost consensualizable by consensus protocol (4) if there exist d-dimensional matrices $\tilde{P} = \tilde{P}^T > 0$, $\tilde{R} = \tilde{R}^T > 0$, $\tilde{W} = \tilde{W}^T > 0$ and $\tilde{K} \in \mathbb{R}^{m\times d}$ such that

$$\tilde{\Omega}_i = \begin{bmatrix} \tilde{\Omega}_{i11} & \tilde{\Omega}_{i12} \\ * & \tilde{\Omega}_{i22} \end{bmatrix} < 0, \quad i = 2, N$$

where

$$\tilde{\Omega}_{i11} = \begin{bmatrix} \tilde{\Omega}_{i111} & \tilde{\Omega}_{i112} & \tilde{\Omega}_{i113} \\ * & (\ell-3)\tilde{R} & -\tau_{\max}\lambda_i\tilde{K}^TB^T \\ * & * & -\tau_{\max}\tilde{W} \end{bmatrix}$$

$$\tilde{\Omega}_{i12} = \begin{bmatrix} 0 & 2\lambda_i\tilde{P}Q_x & \lambda_i\tilde{K}^TQ_u & \tilde{P} \\ \tau_{\max}\tilde{W} & 0 & \lambda_i\tilde{K}^TQ_u & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{i22} = \operatorname{diag}\{-\tau_{\max}\tilde{W}, -2\lambda_iQ_x, -Q_u, -\tilde{R}\}$$

$$\tilde{\Omega}_{i111} = A\tilde{P} + \tilde{P}^TA^T - \lambda_iB\tilde{K} - \lambda_i\tilde{K}^TB^T + (\ell-1)\tilde{R}$$

$$\tilde{\Omega}_{i112} = \tilde{P} - \lambda_iB\tilde{K} + (\ell-2)\tilde{R}$$

$$\tilde{\Omega}_{i113} = \tau_{\max}\tilde{P}A^T - \tau_{\max}\lambda_i\tilde{K}^TB^T.$$

In this case, the control gain matrix of multi-agent system (5) satisfies $K = \tilde{K}\tilde{R}^{-1}$ and the guaranteed cost function has an upper bound

$$J_C^* = \boldsymbol{x}^T(0)\Big(\Upsilon \otimes \left(\tilde{P}^{-1} + \tau_{\max}\tilde{R}^{-1}\right)\Big)\boldsymbol{x}(0)$$

where $\Upsilon = I_N - \mathbf{1}_N\mathbf{1}_N^T/N$.

Proof: The method of changing variables is used to determine K. By the Schur complement, $\Omega_i < 0$, $i = 2, N$ in Theorem 1 is equivalent to

$$\Psi_i = \begin{bmatrix} \Psi_{i11} & \Omega_{i12} \\ * & \Omega_{i22} \end{bmatrix} < 0, \quad i = 2, N \tag{35}$$

where

$$\Psi_{i11} = \begin{bmatrix} \Omega_{111} & \Omega_{i112} & \tau_{\max}A^T \\ * & \Omega_{122} & -\tau_{\max}\lambda_iK^TB^T \\ * & * & -\tau_{\max}W^{-1} \end{bmatrix}.$$

Let

$$\tilde{A}_i = \begin{bmatrix} A & -\lambda_iBK \\ I_d & -I_d \end{bmatrix}$$

$$S = \begin{bmatrix} P & 0 \\ \Phi_1 & \Phi_2 \end{bmatrix}, \quad \Lambda_R = \begin{bmatrix} R & 0 \\ 0 & (\ell-1)R \end{bmatrix}$$

then Ψ_i can be rewritten as

$$\Psi_i = \begin{bmatrix} \Psi_{i11} & \tau_{\max}D_i^T & \tau_{\max}\Phi_b^T & \Psi_{i14} & \Psi_{i15} \\ * & -\tau_{\max}W^{-1} & 0 & 0 & 0 \\ * & * & -\tau_{\max}W & 0 & 0 \\ * & * & * & -2\lambda_iQ_x & 0 \\ * & * & * & * & -Q_u \end{bmatrix}$$

with $\Psi+_{i11} = S^T\tilde{A}_i + \tilde{A}_i^TS + \Lambda_R$, $\Psi_{i14} = [2\lambda_iQ_x, 0]^T$ and $\Psi_{i15} = [0, \lambda_iQ_uK]^T$. Now, consider the case in which $\Phi_1 = -P$ and $\Phi_2 = R$. In this case, S is invertible and

$$S^{-1} = \begin{bmatrix} P^{-1} & 0 \\ R^{-1} & R^{-1} \end{bmatrix}.$$

Pre- and post-multiplying $\Psi_i < 0$ $(i = 2, N)$ by

$$\Pi^T = \operatorname{diag}\{S^{-T}, I_d, W^{-T}, I_d, I_m\}$$

and

$$\Pi = \mathrm{diag}\{S^{-1}, I_d, W^{-1}, I_d, I_m\}$$

respectively, one has

$$\begin{aligned}\tilde{\Psi}_i &= \Pi^T \Psi_i \Pi \\ &= \begin{bmatrix} \tilde{\Psi}_{i11} & \tilde{\Psi}_{i12} & \tilde{\Psi}_{i13} & \tilde{\Psi}_{i14} & \tilde{\Psi}_{i15} \\ * & -\tau_{\max} W^{-1} & 0 & 0 & 0 \\ * & * & -\tau_{\max} W^{-1} & 0 & 0 \\ * & * & * & -2\lambda_i Q_x & 0 \\ * & * & * & * & -Q_u \end{bmatrix} \\ &< 0\end{aligned}$$

where

$$\tilde{\Psi}_{i11} = \tilde{A}_i S^{-1} + S^{-T} \tilde{A}_i^T + S^{-T} \Lambda_R S^{-1}$$

$$\tilde{\Psi}_{i12} = \tau_{\max} S^{-T} D_i^T$$

$$\tilde{\Psi}_{i13} = [0, \tau_{\max} W^{-1}]^T$$

$$\tilde{\Psi}_{i14} = [2\lambda_i Q_x P^{-1}, 0]^T$$

$$\tilde{\Psi}_{i15} = [\lambda_i Q_u K R^{-1}, \lambda_i Q_u K R^{-1}]^T.$$

Setting $\tilde{P} = P^{-1}$, $\tilde{R} = R^{-1}$, $\tilde{W} = W^{-1}$ and $\tilde{K} = KR^{-1}$, one has $\tilde{\Omega}_i < 0$ $(i = 2, N)$. From Theorem 1, if $\tilde{\Omega}_i < 0$ $(i = 2, N)$ are feasible, then multi-agent system (5) can achieve consensus. Therefore, the conclusion of Theorem 3 can be obtained. ■

Remark 4: Theorems 1 and 3 present the LMI conditions for guaranteed cost consensus and consensualization respectively. The feasibility of these LMI conditions can be checked by using the MATLAB's LMI Toolbox.

IV. NUMERICAL SIMULATIONS

A high-dimensional multi-agent supporting system is considered, where it is composed of eight agents labeled from 1 to 8. The dynamics of each agent is described as (1) with A and $\boldsymbol{B}$. In the cost function (6), $Q_x = 0.6I_2$ and $Q_u = 0.4$ are given. The interaction topology G is given in Fig. 1, where the weights of edges of the interaction topology are 1. Then, it can be obtained that $\lambda_2 = 0.5858$ and $\lambda_N = 4.7321$. Let the time-varying delay satisfies $\tau(t) = 0.05 + 0.04\sin(t)$, then $\tau_{\max} = 0.09$ and $\ell = 0.04$. The simulation step is $T_s = 0.001$ s. The initial states of all agents are

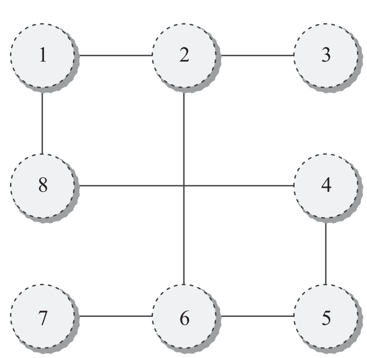

Fig. 1. The interaction topology G.

$$\boldsymbol{x}_1(0) = [19, -14]^T,\ \boldsymbol{x}_2(0) = [-12, 27]^T$$

$$\boldsymbol{x}_3(0) = [-7, 16]^T,\quad \boldsymbol{x}_4(0) = [13, -9]^T$$

$$\boldsymbol{x}_5(0) = [-18, 26]^T,\ \boldsymbol{x}_6(0) = [5, 24]^T$$

$$\boldsymbol{x}_7(0) = [11, -12]^T,\ \boldsymbol{x}_8(0) = [-2, 19]^T.$$

Two cases with different system matrix (A, B) are considered as

Case 1:

$$A = \begin{bmatrix} 0 & 1 \\ -6.2 & -0.8 \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Case 2:

$$A = \begin{bmatrix} -0.5 & 0 \\ -1 & 0 \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ 1.2 \end{bmatrix}.$$

For Case 1, (A, B) are stable. From Theorem 3 and $P = \tilde{P}^{-1}$, $R = \tilde{R}^{-1}$, $W = \tilde{W}^{-1}$, $K = \tilde{K}R$, it can be obtained that $K = [-0.0019, 0.2387]$ and

$$P = \begin{bmatrix} 18.8334 & 1.1682 \\ 1.1682 & 3.0228 \end{bmatrix}$$

$$R = \begin{bmatrix} 8.4464 & 0.3792 \\ 0.3792 & 0.6010 \end{bmatrix}$$

$$W = \begin{bmatrix} 1.7615 & 0.0641 \\ 0.0641 & 0.4972 \end{bmatrix}.$$

In Figs. 2 and 3, the state trajectories of the multi-agent system are shown, where the trajectories marked by circles denote the curves of the consensus function obtained by Corollary 1. It is clear that the multi-agent system achieves guaranteed cost consensus. In this case, the guaranteed cost is $J_C^* = 26\,840.4245$ and the consensus function satisfies $\lim_{t\to\infty}(\boldsymbol{c}(t) - e^{At}[1.1250, 9.6250]^T) = 0$. Fig. 4 shows the trajectories of the cost function.

For Case 2, (A, B) are unstable. Then, one has $K = [-0.2019,\ 0.2839]$ and

$$P = \begin{bmatrix} 16.4035 & -3.2391 \\ -3.2391 & 4.8327 \end{bmatrix}$$

$$R = \begin{bmatrix} 7.3635 & -1.7655 \\ -1.7655 & 1.7549 \end{bmatrix}$$

$$W = \begin{bmatrix} 2.9528 & -1.7730 \\ -1.7730 & 2.0839 \end{bmatrix}.$$

In Figs. 5 and 6, the state trajectories of the multi-agent system are shown, and Fig. 7 shows the trajectories of the cost function. In this case, $J_C^* = 32\,497.6638$ and the consensus function satisfies $\lim_{t\to\infty}(c(t) - e^{At}[1.1250, 9.6250]^T) = 0$.

From the simulation results of Case 1 and Case 2, three aspects should be pointed out. Firstly, one can see that the high-dimensional multi-agent supporting system is able to achieve the guaranteed cost consensus by the proposed approach. Secondly, the runtime may be different when the multi-agent system achieves the guaranteed cost consensus.

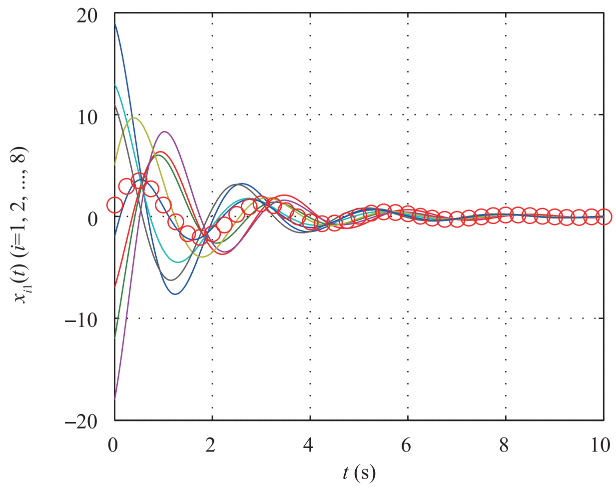

Fig. 2. Case 1: State trajectories of $x_{i1}(t)$ $(i = 1, 2, \ldots, 8)$.

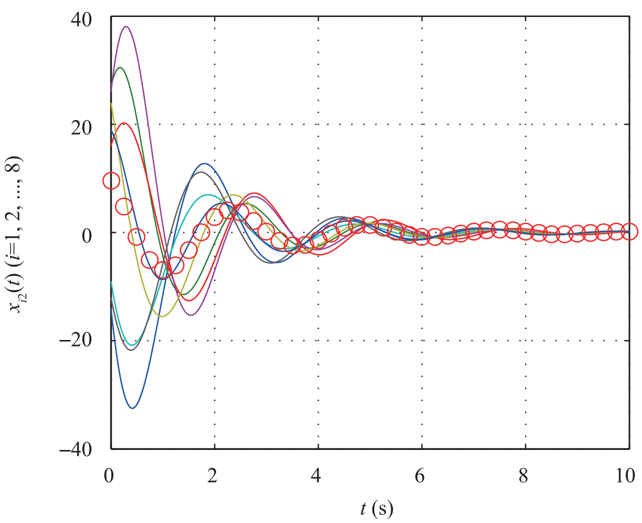

Fig. 3. Case 1: State trajectories of $x_{i2}(t)$ $(i = 1, 2, \ldots, 8)$.

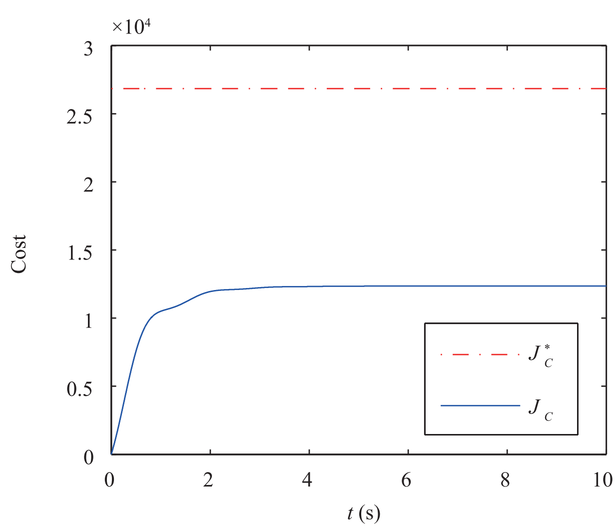

Fig. 4. Case 1: Trajectories of the guaranteed cost function.

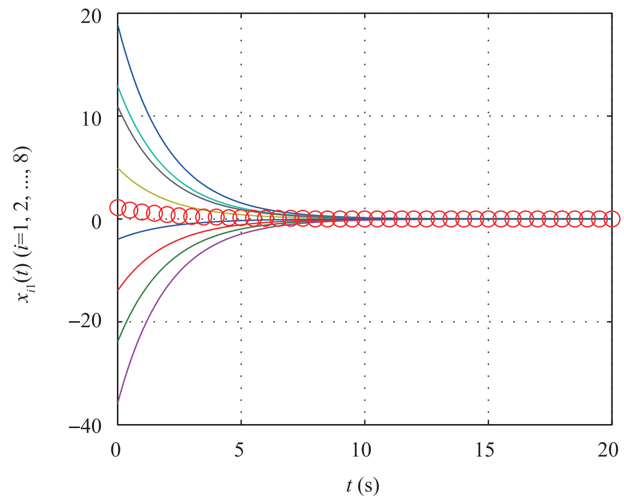

Fig. 5. Case 2: State trajectories of $x_{i1}(t)$ $(i = 1, 2, \ldots, 8)$.

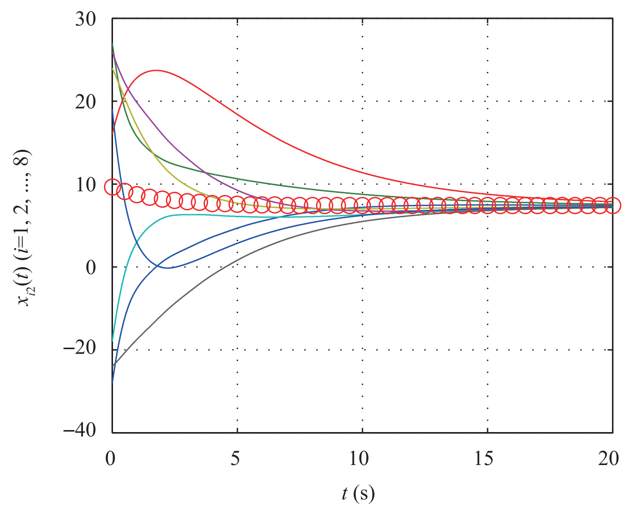

Fig. 6. Case 2: State trajectories of $x_{i2}(t)$ $(i = 1, 2, \ldots, 8)$.

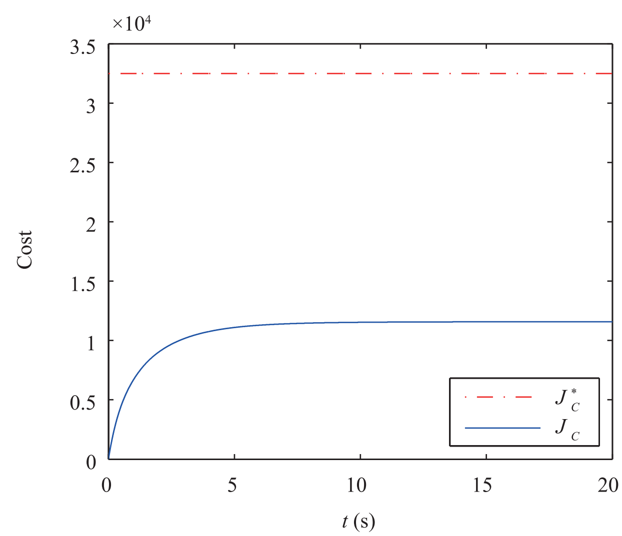

Fig. 7. Case 2: Trajectories of the guaranteed cost function.

The runtime of the unstable cases is 15 s in the above example, while one of the stable cases is 5 s. Thirdly, there exists some levels of the conservatism for the guaranteed cost function, where the conservatism can be depicted by $\Delta J = J_C^* - J_C$ and J_C is used to represent the actual guaranteed cost.

V. Conclusion

The guaranteed cost consensus analysis and design for high-dimensional multi-agent systems with time-varying delays were studied in the current paper. In order to simultaneously consider the control effort and consensus errors, guaranteed cost consensus problems for multi-agent systems were introduced. Sufficient conditions for guaranteed cost consensus problems were presented and an upper bound of the cost function was determined. Moreover, the analysis approach in the current paper is based on the fact that the Laplacian matrix is symmetric and only real Laplacian eigenvalues exist, and the Laplacian matrix associated with a directed topology is asymmetric and its nonzero eigenvalues may be complex numbers. In the future, the influence of directed topologies should be deeply studied.

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