A Novel Multi-agent Formation Control Law With Collision Avoidance

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***Abstract*—In this paper a stable formation control law that simultaneously ensures collision avoidance has been proposed. It is assumed that the communication graph is undirected and connected. The proposed formation control law is a combination of the consensus term and the collision avoidance term (CAT). The first order consensus term is derived for the proposed model, while ensuring the Lyapunov stability. The consensus term creates and maintains the desired formation shape, while the CAT avoids the collision. During the collision avoidance, the potential function based CAT makes the agents repel from each other. This unrestricted repelling magnitude cannot ensure the graph connectivity at the time of collision avoidance. Hence we have proposed a formation control law, which ensures this connectivity even during the collision avoidance. This is achieved by the proposed novel adaptive potential function. The potential function adapts itself, with the online tuning of the critical variable associated with it. The tuning has been done based on the lower bound of the critical variable, which is derived from the proposed connectivity property. The efficacy of the proposed scheme has been validated using simulations done based on formations of six and thirty-two agents respectively.**

***Index Terms*—Consensus, collision avoidance, formation control, graph theory, stability.**

I. INTRODUCTION

THE consensus based formation control is one of the emerging area of research in multi-agent coordination. A good survey on consensus based formation can be found in [1]. Some of the approaches to formation control can be roughly categorized as leader-follower [2], virtual leader [3]−[6], and graph theory based approaches [7].

Using graph-theory, there are broadly two approaches to achieve formation: displacement based and distance based. Examples of the distance based formation can be found in [8]−[10], while examples of displacement based formation control can be found in [11], [12]. In all such formation control algorithms, agents mutually share their information state with an objective to converge to a common value called consensus or agreement that results in the desired formation.

The consensus problem for directed and undirected networks of dynamic agents for fixed and switching topologies has been discussed by Saber and Murray [13]. Two consensus protocols along with the convergence analysis for networks with and without delays have been presented in this work. Sepulchre [14] has derived a linear consensus law, where an agent updates its current estimate of the consensus value towards a weighted average of its neighbor's estimates at each time-step. The author has shown that the approach can also be extended to the nonlinear space defined on arbitrary Riemannian manifolds. ChengandSavkin [11] have discussed the consensus based decentralized motion coordination control, where the agents move in a desired pattern starting from any initial position. Finite time consensus problem has been discussed by Zhao *et al.* [15], in which authors have derived observer based control algorithms for multi-agent systems. Saber [16] has discussed the flocking behavior with reference to Reynolds rules, where agents achieve velocity consensus while avoiding collision.

A stable artificial potential field based control law has been presented by Tanner *et al.* [17], where agents are placed in arbitrarily changing connected network. Cortés [12] has addressed the issue of globally stable formation in a fixed sensing network, but this algorithm cannot handle the collision avoidance among agents. Oh and Ahn [8] have designed a formation control for single-integrator agent based on the inter-agent distance. It is shown that the proposed control law achieves the local asymptotic stability of infinitesimally rigid formations. Huang *et al.* [9] have derived the formation control law for the single integrator agents that has been shown to be asymptotically stable. Zavlanos and Pappas [18] have considered the problem of controlling the network of agents, while ensuring the connectivity property of the network during the motion. Mastellone *et al.* [19] have addressed the problem of coordinated tracking of multiple non-holonomic robots, where robots follow a desired trajectory, while maintaining a specific formation. Hokayem *et al.* [20] have proposed a scheme for the coordination of the multiple Lagrangian systems, that guarantees the collision avoidance among agents. Decentralized potential filed based multi-agent navigation has been studied by Dimarogonas and Frazzoli in [21]. They provide the analysis of the proposed algorithm, with the combined use of primal and dual Lyapunov function. Do [22] have designed a cooperative controller for N mobile ellipsoidal shape agents to track the desired trajectory, while ensuring collision

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avoidance. Tian and Wang [23] have proposed a distance based formation law based on constructive perturbation method and conventional gradient law, that can ensure global stability. Su *et al.* [24] and Chen *et al.* [25] have proposed flocking control law where issues such as connectivity preservation during collision avoidance have been addressed. In summary, distance based formation control that is based on the artificial potential function approach cannot ensure global stability. In contrast, the displacement based approach can ensure global stability based on certain assumptions on graph connectivity property. However, collision avoidance is not an integral part of such an analysis to the best of our knowledge.

Given this status, in this paper, an integrated approach to both consensus and collision avoidance has been studied for the first time within displacement based formation strategy. In this work, we assume that the agents are connected, which is a more relaxed condition over a fully connected communication graph. A novel formation control law has been derived that ensures global asymptotic stability and collision avoidance simultaneously. The formation control law comprises of a consensus term and a collision avoidance term. The consensus term is based on an undirected and connected communication graph. This term creates and maintains the desired formation. The collision avoidance term is based on a proposed adaptive potential function. The potential function adapts by online tuning of a critical variable in the collision avoidance term (CAT). This ensures that the agents do not collide with each other while maneuvering. In addition to this the adaptive potential function based CAT allows agents to have bounded repulsion, thus ensuring that the communicating graph remains intact. The consensus term and the CAT function simultaneously to achieve formation while avoiding collision and preserving the existing communication graph. Extensive simulations of the proposed approach have been done on formations of 6 and 32 agents, respectively. Simulation results show that agents maintain formation and preserve the existing communication graph, while avoiding collision.

The proposed work is different from the works found in [24], [25] as follows:

1) The proposed work is about formation control, while these two works focus on flocking.

2) Connectivity preservation in [24], [25] are done by using different connectivity preservation terms while the proposed work relaxed the connectivity preservation term and ensures it by the novel design of the CAT using connectivity property.

3) While designing the CAT, we have proposed the novel adaptive potential function, which adapts itself based on the online tuning of the critical variable associated with it. The tuning has been done with the lower bound of the critical variable, which has been derived from the connectivity property.

4) The proposed displacement based formation control law ensures global asymptotic stability of the final configuration, while the potential function based flocking in [24], [25] ensures local stability of the final configuration.

It can be noted that in the existing literature for formation control, the consensus term and the collision avoidance term do not function simultaneously. Also in most cases the potential function based collision avoidance results in excessive repulsion between the agents, which may lead to a break in the communication link among the agents. In this paper, the consensus term and the CAT function simultaneously to ensure a stable formation. Since the CAT is based on an adaptive potential function, the repulsion among the agents is bounded. This maintains the connectivity among the agents.

This paper has been organized as follows. In Section II, preliminary concepts about graph theory and connected graph are provided. The problem definition is given in Section III. In Section IV, the consensus term for formation and collision avoidance law, and stability of the multi-agent system under the proposed control law are discussed. In Section V, a lower bound on a critical variable associated with the CAT is derived. Simulation results are presented in Section VI. The paper is summarized in Section VII.

II. Preliminaries

The communication network for agents considered for this paper, is represented by a connected undirected graph. This graph can be mathematically denoted as, $G = (\nu, \varepsilon, A)$, where ν is the vertex set and ε is the edge set. A is the adjacency matrix of G with the elements a_{ij}. For an undirected graph, agent i and j can communicate with each other so, $a_{ij} = a_{ji}$. Another important matrix parameter in graph theory is the Laplacian matrix, $L = D - A$, where D is the diagonal matrix with the diagonal entries defined as $d_i = \sum_{j \neq i} a_{ij}$. In a connected graph, a path always exists between any two vertices, where in the completely connected graph the direct path exist between any two vertices. The pictorial representation of the completely connected and the connected graph has been shown in Figs. 1, and 2, respectively. Thus the connected graph is more relaxed topology as compared to a fully connected graph.

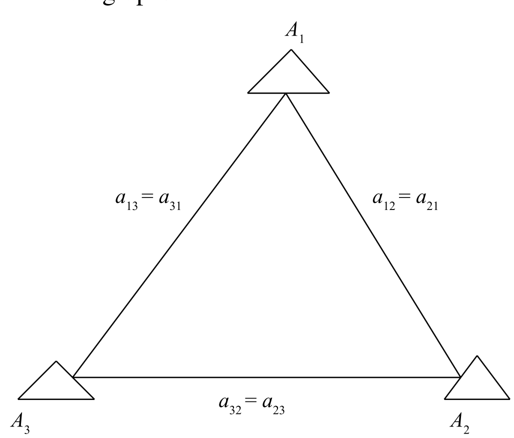

Fig. 1. The completely connected undirected graph of three agents.

III. Problem Definition

A multi-agent system consisting of n agents has been considered. The communication topology of this multi-agent system is represented by a connected graph G, as defined in Section II.

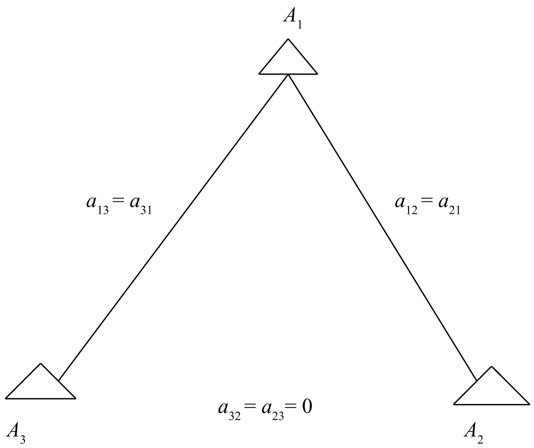

Fig. 2. The connected undirected graph for three agents.

Each agent in the system is represented by a single integrator model:

$$\dot{p_i}= u_i, \quad \text{for} \quad i = 1, 2 \dots, n \tag{1}$$

where, $p_i \in \mathbb{R}^d$, is the position of the ith agent in the inertial frame, and $u_i \in \mathbb{R}^d$ is the control law. In this paper, $d = 2$. It is assumed that the agents communicate with each other within their fixed interaction range (IR), which depends on the onboard sensor. The IR is defined as a circular region with radius $r > 0$, centered at the agent. The Neighbor of the ith agent is defined as (N_i), which is the set of all pre-selected agents located within IR of the ith agent. In this paper, it is assumed that members of N_i are fixed as per the initial configuration, i.e. neighborhood topology does not change.

The desired formation shape is given as $Z^* = [(z_1^*)^T \dots (z_n^*)^T]^T \in (\mathbb{R}^d)^n$ in the inertial frame. It is assumed that this desired formation shape is translation invariant, i.e., the desired formation among the agents may take place anywhere in the plane and is maintained over time. It is assumed that the local frame attached to each agent is always parallel to the inertial frame. To avoid collision, we have assumed an onboard proximity sensor, which has a fixed sensing range with the radius, r_{out}. Hence another set, collision avoidance neighbor, N_i^c has been formed. The members of this set will be the jth agents, which are within the proximity sensing range of ith agent.

Given above conditions, the control problem is to derive a suitable position based formation control law that simultaneously ensures collision avoidance and connectivity even in the time of collision avoidance, while guaranteeing global asymptotic stability.

IV. Formation Control Law

The proposed formation control law, u_i has two parts: a consensus term, and a CAT. This control law is represented as

$$u_i = f_i^c + f_i^{ca} \tag{2}$$

where f_i^c is the the consensus term, and f_i^{ca} is the CAT.

A. The Consensus Based Control Law

A specific formation can be achieved by maintaining desired distances among the agents. The distance constraint is given as

$$\|p_j - p_i\|_{j\in N_i} = k_{ij}, \quad \text{for } i = 1, \dots, n \tag{3}$$

where k_{ij} is the desired distance between ith and jth agents. The stress function $\zeta(P) : (\mathbb{R}^d)^n \to \mathbb{R}$ [26] for the connected graph to satisfy the distance constraint (3) is taken as

$$\zeta(P) = \sum_{j\in N_i} [f_e(p_i, p_j) - k_{ij}]^2 \tag{4}$$

where $f_e(p_i, p_j) = \|p_j - p_i\|_2$ is the Euclidean distance between jth and ith agents and $P = [p_1^T, p_2^T, \dots, p_n^T]^T \in (\mathbb{R}^d)^n$ is the coordinate matrix. On optimizing the majorization function of $\zeta(P)$ [26], a linear equation of the form $AX = Y$ is obtained

$$(L(G) \otimes I_d)P = (L(G) \otimes I_d)Z^* \tag{5}$$

where $L(G)$ is the Laplacian matrix, and $Y = (L(G) \otimes I_d)Z^* \in (\mathbb{R}^d)^n$. Let $Y = [b_1^T, b_2^T, \dots, b_n^T]^T$ where $b_i \in \mathbb{R}^d$. The elements of the adjacency matrix are given below

$$a_{ij} = \begin{cases} 1, & j \in N_i \\ 0, & j \notin N_i \end{cases}. \tag{6}$$

Detailed computation of (5) is given in [12] and [27]. Using Jacobi over relaxation method given in [12], we can find the formation control law as

$$p_i(l + 1)=(1 - h)p_i(l) + h\frac{1}{d_i}(\sum_{j\in N_i} a_{ij}p_j(l) + b_i) \tag{7}$$

where $p_i(l)$ and $p_j(l)$ are the positions of ith and jth agents at lth iteration respectively. The ith diagonal element of the Laplacian matrix L is, $d_i = \sum_{\forall j\neq i} a_{ij}$. Selecting the control input $\bar{u}_i(l)$ as

$$\bar{u}_i(l) = -hp_i(l) + h\frac{1}{d_i}(\sum_{j\in N_i} a_{ij}p_j(l) + b_i). \tag{8}$$

Equation (7) can be re-written as

$$p_i(l + 1) = p_i(l) + \bar{u}_i(l). \tag{9}$$

It turns out that (9) is the discrete time version of the single integrator model given in (1). Using (7)−(9), dynamics of each agent takes the form

$$\dot{p_i} = -hp_i + \frac{h}{d_i}(\sum_{j\in N_i} a_{ij}p_j + b_i). \tag{10}$$

B. Consensus and Stability

Equation (10) can be written as

$$\begin{aligned}\dot{p_i}&=-\frac{hd_i}{d_i}p_i + h\frac{1}{d_i}(\sum_{j\in N_i} a_{ij}p_j + b_i)\\ &=-h\frac{1}{d_i}(d_ip_i - \sum_{j\in N_i} a_{ij}p_j - b_i)\\ &=-h\frac{1}{d_i}(d_i\begin{pmatrix}p_{ix}\\ p_{iy}\end{pmatrix} - \sum_{j\in N_i} a_{ij}\begin{pmatrix}p_{jx}\\ p_{jy}\end{pmatrix} - b_i).\end{aligned} \tag{11}$$

Given that $b = [b_1^T, b_2^T, \ldots, b_n^T] = (L(G) \otimes I_d)Z^*$, for ith agent

$$b_i = d_i \begin{pmatrix} z_{ix}^* \\ z_{iy}^* \end{pmatrix} - \sum_{j \in N_i} a_{ij} \begin{pmatrix} z_{jx}^* \\ z_{jy}^* \end{pmatrix} \quad (12)$$

where $d_i = \sum_{\forall j \neq i} a_{ij}$. Error, e_i of ith agent is defined as, $e_i = p_i - z_i^*$. Using (11) and (12), the consensus term can be derived as

$$\dot{e}_i = -h\frac{1}{d_i}\sum_{j \in N_i} a_{ij}(e_i - e_j). \quad (13)$$

Equation (13) can be written in the matrix form as

$$\begin{aligned} \dot{e} &= -K_g(L(G) \otimes I_d)e \\ &= -(KL(G) \otimes I_d)e \end{aligned} \quad (14)$$

where $K_g = K \otimes I_d$, $K = \text{diag}(h/d_i)$, $e = [e_1^T, e_2^T, \ldots, e_n^T]^T \in (\mathbb{R}^d)^n$. Here diag($\cdot$) denotes the diagonal entries of the $n \times n$ matrix, and $\otimes$ denotes the Kronecker product. Given that error coordinates are independent of each other, it is sufficient to prove the stability of the single dimension version of (14). This representation looks as

$$\dot{e}_s = -(KL(G))e_s \quad (15)$$

where $e_s = [e_{x1} \quad e_{x2} \quad \ldots \quad e_{xn}]^T \in \mathbb{R}^n$. Let $M = KL(G)$.

Elements of this matrix M can be written as

$$m_{ij} = \begin{cases} \sum_{j \neq i} k_i a_{ij}, & \text{when} \quad i = j \\ -k_i a_{ij}, & \text{when} \quad i \neq j \end{cases}$$

where k_i is the diagonal term of the matrix K. It can be seen that M is the Laplacian matrix with row sum zero. As the Laplacian matrix M is not symmetric, the error dynamics in (15) appears to represent a digraph (directed graph). Let us define the Lyapunov potential, $V_L(e_s)$ as:

$$V_L(e_s) = e_s^T \tilde{P} e_s \quad (16)$$

where $\tilde{P} > 0 \in \mathbb{R}^{n \times n}$ is the diagonal matrix. Taking the time derivative of (16) and using (14) we can write

$$\begin{aligned} \dot{V}_L(e_s) &= 2e_s^T \tilde{P} \dot{e_s} \\ &= -2e_s^T \tilde{P} M e_s \\ &= -e_s^T(\tilde{P}M + M^T\tilde{P})e_s. \end{aligned} \quad (17)$$

From Lemma 6 of [28], we have:

$$\tilde{P}M + M^T\tilde{P} = Q \geq 0, \quad \text{for } M = KL \text{ and } \tilde{P} > 0.$$

Hence, $\dot{V}_L = -e_s^T Q e_s \leq 0$. By LaSalle's invariance principle [29], the trajectory will converge to the largest invariant set, $S = \{e_s \in \mathbb{R}^n | \dot{V}_L \equiv 0\}$. By Lemmas 8 and 9 of [28], we have $S = \{e_s \in \mathbb{R}^n | e_s = \alpha \mathbf{1}_n \quad \forall \alpha \in \mathbb{R}\}$. Hence, $e_s(t) \to \alpha \mathbf{1}_n$ for some $\alpha \in \mathbb{R}$ as $t \to \infty$. Since each error term e_i converges to same value, as per the (13), the $\dot{e}_i$ will converge to zero ensuring global asymptotic stability.

C. Collision Avoidance for Multi-agent System

Consider two regions around an agent as shown in Fig. 3. Here, one region is enclosed by an outer boundary, $r_{\text{out}} < \min(k_{ij})$ and the other region is enclosed by an inner boundary, $r_{\text{in}} < r_{\text{out}}$. It is shown in this figure that the collision avoidance region (CAR) for each agent is the region enclosed by r_{out}. When an agent is inside the CAR of another agent, a strictly decreasing smooth potential function [30] comes into action to avoid collision. Further, it has to be ensured that no agent enters the inner boundary, r_{in} of another agent. The function $g(x)$ which helps to vary the potential function [16] smoothly, is defined as

$$g(x) = \begin{cases} k_1, & x \in (0, r_{\text{in}}) \\ k_2(x), & x \in [r_{\text{in}}, r_{\text{out}}] \\ 0, & x \notin (0, r_{\text{out}}] \end{cases} \quad (18)$$

where $x = \|p_j - p_i\|$, $k_1 = 1$. Here, $k_2(x)$ is defined as

$$k_2(x) = \frac{1}{2}\left[1 + \cos\left(\pi\frac{x - r_{\text{in}}}{r_{\text{out}} - r_{\text{in}}}\right)\right]. \quad (19)$$

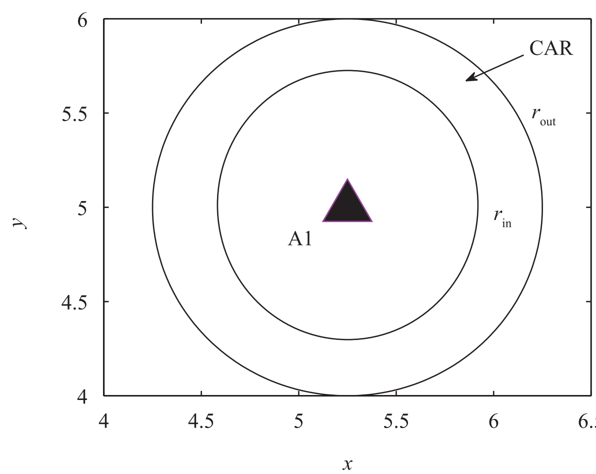

Fig. 3. Partition of two regions of an agent, A1 for collision avoidance.

The action function, $\phi(x)$ is defined as

$$\phi(x) = g(x)\tilde{\phi}(x) \quad (20)$$

where

$$\tilde{\phi}(x) = \frac{-x}{\sigma_{ij} + x^2} \quad (21)$$

where σ_{ij} is to be tuned online by each agent at the time of collision avoidance. This action function is activated when an agent is within the CAR of another. To generate a gradient based CAT, a smooth collective potential function [27], [16] is defined as

$$V(p) = \frac{1}{2}\sum_i \sum_{j \in N_i^c} \psi_c(\|p_j - p_i\|) \quad (22)$$

where $\psi_c(x)$ is the proposed novel adaptive repulsive potential with finite cut-off at r_{out}. This function is defined as

$$\psi_c(x) = \int_{r_{\text{out}}}^{x} \phi(s)ds. \quad (23)$$

The strictly decreasing nature of the function, $\psi_c(x)$ is shown in Fig. 4. In (2), the collision avoidance term, f_i^{ca} is defined as

$$f_i^{ca} = -\bigtriangledown_{p_i} V(p) \tag{24}$$

where $\bigtriangledown_{p_i}$ is the gradient along p_i. Finally, using (20)−(24), the collision avoidance part f_i^{ca} can be written as

$$f_i^{ca} = \sum_{j\in N_i^c} \phi(\|p_j - p_i\|)\frac{p_j - p_i}{\|p_j - p_i\|}. \tag{25}$$

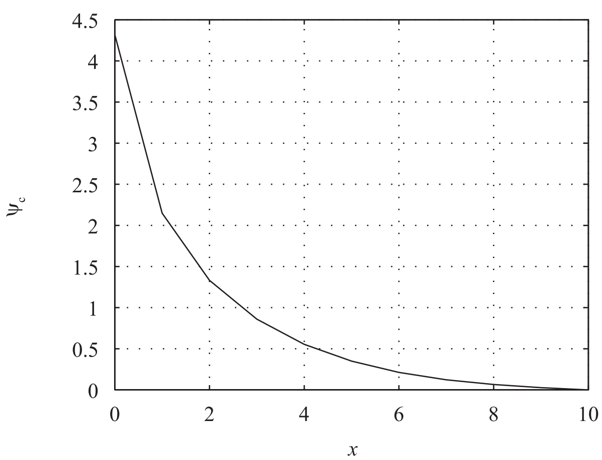

Fig. 4. Strictly decreasing potential function.

By combining the collision avoidance law in (25) with (2) and the consensus term in (13), the formation control law is established as:

$$\begin{aligned} u_i = &\sum_{j\in N_i^c} \phi(\|p_j - p_i\|)\frac{p_j - p_i}{\|p_j - p_i\|} \\ &+h\frac{1}{d_i}\sum_{j\in N_i} a_{ij}(e_j - e_i) \end{aligned} \tag{26}$$

where the first part represents the CAT and the second part represents the consensus term. Due to the CAT, agents will repel each other at the time of collision avoidance, which may weaken the connectivity assumption. To ensure connectivity even during the collision avoidance the novel adaptive potential function (22) is proposed. The potential function adapts itself by online tuning of the critical variable, σ_{ij} associated with (21) based on the lower bound has been derived in the next section. This derivation takes care of maintaining the connectivity property which guarantees global asymptotic stability as derived in the beginning of this section.

V. Derivation of the Lower Bound of Variable σ_{ij}

At the time of collision avoidance, each agent calculates $\tilde{\phi}(x)$, given in (21), and the variable σ_{ij} is updated online based on the lower bound σ_{ijlb}.

A. Geometric Interpretation and Connectivity Between Agents

Let us assume that two agents A_1 and A_2 are connected and are situated in the CAR of each other as shown in Fig. 5. CARs for each agent are shown by the circles, drawn by thick black dash dot with radius $r_{\rm out}$. The interaction regions (IR) for each agent are shown by the bigger dotted circles. The safety region (SR) for a particular agent pair is shown by the red color boundary. If any of these two agents cross this safety region, then, the connection between them will be lost. The safety region is shown separately in Fig. 6, where c is the center. The maximum radius of SR is $r/2$.

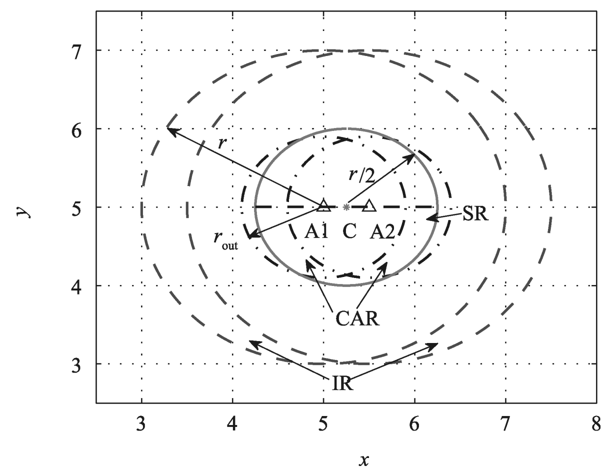

Fig. 5. Interaction region (IR) (blue), collision avoidance region (CAR) (black), and safety region (SR) (solid red).

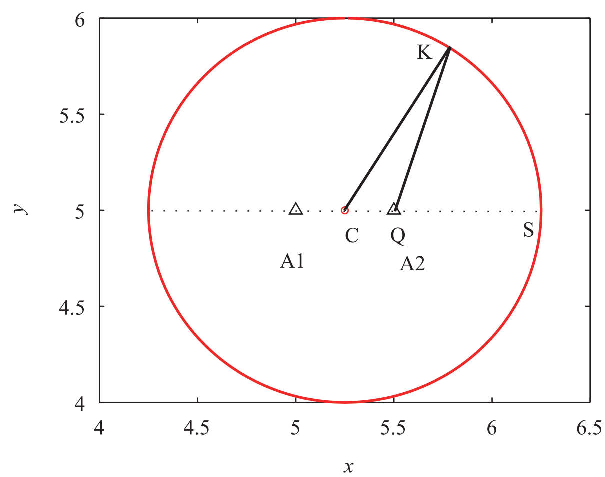

Fig. 6. Safety region (SR).

Lemma 1: Consider $C_{\rm BN}$ be the set of all the points on a circle centered at point $\mathrm{C} \in \mathbb{R}^{\rm d}$. $L_{\rm CS}$ is the set of all the points on the straight line, CQS, where $q \in L_{\rm CS} \setminus \{\mathrm{C}\}$, and $\mathrm{S} = C_{\rm BN} \bigcap L_{\rm CS} \in \mathbb{R}^{\rm d}$, is a point on the circle. Then, inequality $dist_{\rm QS} < dist_{\rm QK}$ is always satisfied, where $k \in C_{\rm BN} \setminus \{\mathrm{S}\}$ and the $dist_{ij}$, represents the distance between, agent i and j.

Proof: With reference to Fig. 6, let the set $C_{\rm BN}$ contain all boundary points. Let the set $L_{\rm CS}$ contain all points on the straight line CQS. Using the triangular inequality we have,

$$dist_{\rm CQ} + dist_{\rm QK} > dist_{\rm CK}. \tag{27}$$

From Fig. 6, let $dist_{\rm CK} = \tilde{r}$. Then $dist_{\rm CQ} + dist_{\rm QS} = \tilde{r}$. Equation (27) can be written as:

$$\tilde{r} - dist_{\rm QS} + dist_{\rm QK} > \tilde{r}. \tag{28}$$

This inequality leads to the desired proof as:

$$dist_{\mathrm{QS}} < dist_{\mathrm{QK}}. \tag{29}$$

■

According to Lemma 1, if the agent A_2 is repelled by $dist_{\mathrm{QS}}$, then agents A_2 and A_1 will no more be connected. If, during the collision avoidance, this minimum distance property is not violated, then, the network will remain connected even during the collision avoidance. This is ensured by developing an online tuning of the parameter σ_{ij} associated with the action function.

Property 1 (Property of Connectivity): Agents pair needs to stay within their SR to communicate with each other. Hence considering the minimum distance repelling within SR as stated in Lemma 1, the ith agent should satisfy the following constraint:

$$\|\dot{p}_i\| < (\tilde{r} - d_{ic}) \tag{30}$$

where $\tilde{r} = r/D_r^{ij}$, $D_r^{ij} > 2$, and d_{ic} is the distance of the ith agent from the center C of the circular region SR. In Fig. 6, d_{ic} = CQ.

Theorem 1: For real and positive σ_{ij}, D_r^{ij} is

$$D_{rlb}^{ij} < D_r^{ij} < D_{rub}^{ij} \tag{31}$$

$$D_{rlb}^{ij} = \varphi_1(n_1, n_2, n_3, n_4, d_{ic}, \beta_i, x) \tag{32}$$

where

$$\begin{aligned} n_1 &= p_{ix} - p_{jx} \\ n_2 &= \sum_{j \in N_i} a_{ij} e_{jx} - d_i e_{ix} \\ n_3 &= p_{iy} - p_{jy} \\ n_4 &= \sum_{j \in N_i} a_{ij} e_{jy} - d_i e_{iy} \\ \beta_i &= \frac{h}{\mathcal{N}(N_i) d_i} \end{aligned}$$

where $\mathcal{N}$ is the cardinality of N_i, $p_i = (p_{ix} \quad p_{iy})^T$, and error $e_i = (e_{ix} \quad e_{iy})^T$. The upper bound D_{rub}^{ij} is,

$$D_{rub}^{ij} = \min(ub_1, ub_2) \tag{33}$$

where

$$ub_1 = \varphi_2(n_2, n_4, d_{ic}, \beta_i) \tag{34}$$
$$ub_2 = \varphi_3(n_1, n_2, n_3, n_4, d_{ic}, \beta_i) \tag{35}$$

where φ_1, φ_2, φ_3 are some functions.

Proof: Using (20), (21), (26) in (30), we can write as

$$\left\| \sum_{j \in N_i^c} \lambda_{ij}^{'} + \frac{\mathcal{N}(N_i) h}{\mathcal{N}(N_i) d_i} \sum_{j \in N_i} a_{ij}(e_j - e_i) \right\| < \tilde{r} - d_{ic} \tag{36}$$

where $\lambda_{ij}^{'} = \alpha_{ij}(p_i - p_j)$, $\alpha_{ij} = g(x)/\sigma_{ij} + x^2$. The jth member of N_i^c, may be the neighbor of the ith agent, $j \in N_i$, or may not be the neighbor, $j \notin N_i$. Hence, (36) can be written as:

$$\left\| \sum_{j \notin N_i} \lambda_{ij}^{'} + \sum_{j \in N_i} \lambda_{ij}^{'} + \frac{\mathcal{N}(N_i) h}{\mathcal{N}(N_i) d_i} \sum_{j \in N_i} a_{ij}(e_j - e_i) \right\| < \tilde{r} - d_{ic} \tag{37}$$

$$\Rightarrow \left\| \sum_{j \notin N_i} \lambda_{ij}^{'} + \sum_{j \in N_i} (\lambda_{ij}^{'} + \lambda_i^{''}) \right\| < \tilde{r} - d_{ic} \tag{38}$$

where $\lambda_i^{''} = h/(\mathcal{N}(N_i) d_i) \sum_{j \in N_i} a_{ij}(e_j - e_i)$. Using triangular inequality, equation (38) can be written as

$$\left\| \sum_{j \notin N_i} \lambda_{ij}^{'} \right\| + \left\| \sum_{j \in N_i} (\lambda_{ij}^{'} + \lambda_i^{''}) \right\| < \tilde{r} - d_{ic} \tag{39}$$

the (39) holds, for $j \in N_i$, if following holds

$$\left\| \sum_{j \in N_i} (\lambda_{ij}^{'} + \lambda_i^{''}) \right\| < \frac{\tilde{r} - d_{ic}}{2} \tag{40}$$

and

$$\left\| \sum_{j \notin N_i} \lambda_{ij}^{'} \right\| < \frac{\tilde{r} - d_{ic}}{2}. \tag{41}$$

Using the triangular inequality in (40) and (41), we can write as

$$\sum_{j \in N_i} \left\| (\lambda_{ij}^{'} + \lambda_i^{''}) \right\| < \frac{\tilde{r} - d_{ic}}{2} \tag{42}$$

and

$$\sum_{j \notin N_i} \left\| \lambda_{ij}^{'} \right\| < \frac{\tilde{r} - d_{ic}}{2} \tag{43}$$

respectively. Inequality (42) holds for $j \in N_i$, if following holds:

$$\left\| (\lambda_{ij}^{'} + \lambda_i^{''}) \right\| < \frac{\tilde{r} - d_{ic}}{2\mathcal{N}(N_i)} \tag{44}$$

$$\Rightarrow \left\| \lambda_{ij}^{'} + \frac{h}{\mathcal{N}(N_i) d_i} \sum_{j \in N_i} a_{ij}(e_j - e_i) \right\| < \frac{\tilde{r} - d_{ic}}{2\mathcal{N}(N_i)}. \tag{45}$$

Squaring both sides, we have,

$$(\alpha_{ij} n_1 + \beta_i n_2)^2 + (\alpha_{ij} n_3 + \beta_i n_4)^2 - \left(\frac{\tilde{r} - d_{ic}}{2\mathcal{N}(N_i)} \right)^2 < 0 \tag{46}$$

where

$$\alpha_{ij} = \frac{g(\|p_j - p_i\|)}{\sigma_{ij} + (\|p_j - p_i\|)^2} \tag{47}$$

$$\beta_i = \frac{h}{\mathcal{N}(N_i) d_i}. \tag{48}$$

Variable α_{ij} of the quadratic inequality in (46) can vary within the upper and lower bounds, which can be defined as

$$\alpha_{ijlb} < \alpha_{ij} < \alpha_{ijub}. \tag{49}$$

As from (47), we know that σ_{ij} is inversely proportional to the α_{ij}, and we are interested in finding the lower bound of σ_{ij}. Hence we have considered the upper bound α_{ijub}, which is the maximum of the two roots of quadratic equality of (46) defined as

$$\alpha_{ijeq} = \frac{-2(n_1n_2+n_3n_4)\beta_i}{2(n_1^2+n_3^2)} \pm T_1 \tag{50}$$

where

$$T_1 = \frac{\sqrt{4(n_1^2+n_3^2)(\dfrac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)})^2 - 4\beta_i^2(n_1n_4-n_2n_3)^2}}{2(n_1^2+n_3^2)}$$

using (49), we can write the following inequality,

$$\frac{g(\|p_j-p_i\|)}{\sigma_{ij}+(\|p_j-p_i\|)^2} < \alpha_{ijub} \tag{51}$$

$$\Rightarrow \sigma_{ij} > \frac{g(\|p_j-p_i\|)}{\alpha_{ijub}} - (\|p_j-p_i\|)^2. \tag{52}$$

So σ_{ij} has a lower bound which is given by

$$\sigma_{ijlb} = \frac{g(\|p_j-p_i\|)}{\alpha_{ijub}} - (\|p_j-p_i\|)^2.$$

For $\sigma_{ij} \in \mathbb{R}^+$, σ_{ijlb} and α_{ijub} should be real positive. So the following inequalities hold

$$\sigma_{ijlb} > 0 \tag{53}$$

$$2(n_1n_2+n_3n_4)\beta_i < \sqrt{T_2-T_3} \tag{54}$$

and

$$T_2 - T_3 > 0 \tag{55}$$

where

$$T_2 = 4(n_1^2+n_3^2)\left(\frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)}\right)^2$$
$$T_3 = 4\beta_i^2(n_1n_4-n_2n_3)^2.$$

The expression of α_{ijub} can be found from (50), and using it in (53), we can write as

$$\frac{g(\|p_j-p_i\|)}{(\|p_j-p_i\|)^2} > \frac{-2(n_1n_2+n_3n_4)\beta_i+\sqrt{T_2-T_3}}{2(n_1^2+n_3^2)} \tag{56}$$

$$\Rightarrow \left\{\frac{2g(\|p_j-p_i\|)(n_1^2+n_3^2)}{(\|p_j-p_i\|)^2} + 2(n_1n_2+n_3n_4)\beta_i\right\}^2$$
$$> 4(n_1^2+n_3^2)\left(\frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)}\right)^2 - 4\beta_i^2(n_1n_4-n_2n_3)^2 \tag{57}$$

$$\Rightarrow \left(\frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)}\right)^2 < T_4+T_5 \tag{58}$$

$$\Rightarrow D_r^{ij} > \frac{r}{d_{ic}+\mathcal{N}(N_i)\sqrt{T_4+T_5}} = \varphi_1 \tag{59}$$

where

$$T_4 = \frac{\left\{\dfrac{2g(\|p_j-p_i\|)(n_1^2+n_3^2)}{(\|p_j-p_i\|)^2} + 2(n_1n_2+n_3n_4)\beta_i\right\}^2}{4(n_1^2+n_3^2)}$$

$$T_5 = \frac{4\beta_i^2(n_1n_4-n_2n_3)^2}{4(n_1^2+n_3^2)}$$

and

$$\tilde{r} = \frac{r}{D_r^{ij}}$$

from (59), we have the lower bound, $D_{rlb}^{ij} = \varphi_1$.

Using (54), we have,

$$(n_1^2+n_3^2)\left(\frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)}\right)^2$$
$$> (n_1n_2+n_3n_4)^2\beta_i^2 + \beta_i^2(n_1n_4-n_2n_3)^2. \tag{60}$$

Dividing both the sides by $(n_1^2+n_3^2)$ and taking the square root, we get,

$$\left(\frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)}\right) > \sqrt{\frac{(n_1n_2+n_3n_4)^2\beta_i^2+\beta_i^2(n_1n_4-n_2n_3)^2}{(n_1^2+n_3^2)}} \tag{61}$$

$$\Rightarrow D_r^{ij} < \frac{r}{d_{ic}+2\mathcal{N}(N_i)\sqrt{\beta_i^2(n_2^2+n_4^2)}} = \varphi_2 \tag{62}$$

using (55), we have,

$$(n_1^2+n_3^2)\left(\frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i)}\right)^2 > \beta_i^2(n_1n_4-n_2n_3)^2 \tag{63}$$

$$\Rightarrow D_r^{ij} < \frac{r}{d_{ic}+2\mathcal{N}(N_i)\sqrt{T_5}} = \varphi_3. \tag{64}$$

From (62) and (64), we can obtain the upper bounds as, $ub_1 = \varphi_2$, and $ub_2 = \varphi_3$. Actual upper bound will be

$$D_{rub}^{ij} = \min(ub_1, ub_2). \tag{65}$$

■

Using the Property 1, and assuming that the difference between upper bound, D_{rub}^{ij} and lower bound, D_{rlb}^{ij} is greater than 2, D_r^{ij} can be selected from Theorem 1 as:

$$D_{rnew}^{ij} = \begin{cases} D_{rlb}^{ij}+\delta_l, & \text{if } D_{rlb}^{ij} < 2 \\ D_{rlb}^{ij}+\delta_g, & \text{if } D_{rlb}^{ij} \geq 2. \end{cases} \tag{66}$$

Using (52), the tuning law for the critical variable, σ_{ij} is given as:

$$\sigma_{ijnelw} = \sigma_{ijlb} + \delta_a \tag{67}$$

where δ_a, δ_l and δ_g, are selected heuristically. Inequality (43) holds for $j \notin N_i$, if following holds:

$$\left\|\lambda_{ij}^{'}\right\| < \frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i^c \setminus N_i)}. \tag{68}$$

Simplifying the above equation, we get following relation:

$$\sigma_{ij} > \frac{g(x)}{T_6} - x^2 = \sigma_{ijlb} \tag{69}$$

where

$$T_6 = \frac{\tilde{r}-d_{ic}}{2\mathcal{N}(N_i^c \setminus N_i)}\sqrt{\frac{1}{n_1^2+n_3^2}}$$

at the time of collision avoidance, the tuning law is same as in (67) for the agents $j \notin N_i$ with the derived lower bound in (69).

VI. Simulation Results

We have considered two different systems, one with 6-agents and another with 32-agents. We have assumed that agents can communicate in a connected graph. Critical Variable, σ_{ij}, for the CAT is tuned based on (67). The values of δ_l, δ_g, δ_a, and h are given in Table I. The simulations are done using MATLAB.

TABLE I
Heuristic Values

Constant	Value
δ_l	2.0
δ_g	$D_{rub}^{ij} - D_{rlb}^{ij}/2.0$
δ_a	0.1
h	0.005

A. Hexagon Formation With Six Agents

We have considered a 6-agent system. Each agent is subjected to the control law as defined in (26). The initial position for one such simulation is given in Table II, and the initial configuration is shown in Fig. 7. The outer region $r_{\text{out}} = 1.5$ unit, the inner region $r_{\text{in}} = 0.5$ unit and the radius of the IR is taken as $r = 4.5$ unit. Trajectories of six agents and the final formation have been shown in Fig. 8. Fig. 9 illustrates the minimum inter agent distance for each agent. It can be observed that this distance measure never goes below r_{in}. This guarantees no collision. Update of the critical variable, σ_{ij}, for those agents that appear in each other's CAR is shown in Fig. 10. By looking at this figure and the previous figure for inter-agent distances, one can see that this critical variable assumes a small value, when inter-agent distance becomes small within the collision avoidance region, which ensures the increase in the repelling magnitudes of agents at the time of collision avoidance. But our proposed scheme ensures the appropriate repelling magnitude for the connectivity. Figs. 11 and 12 show error in coordinate values, e_{ix} and e_{iy}, respectively. It is observed that all agents are converging towards some constant equilibrium point, which confirms the achievement of the desired formation with six agents. It also confirms the consensus among agents.

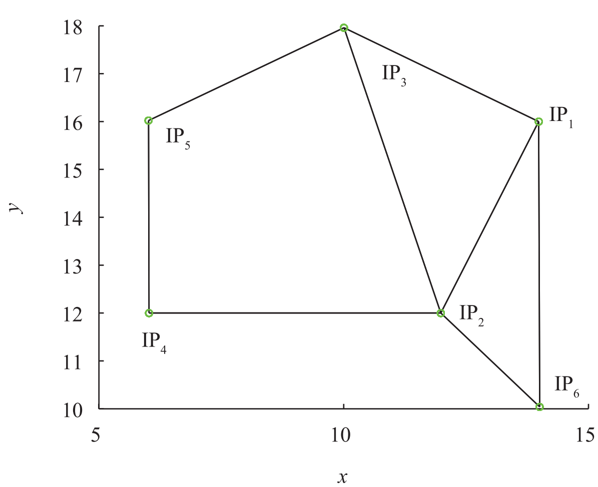

Fig. 7. Agents initial configuration.

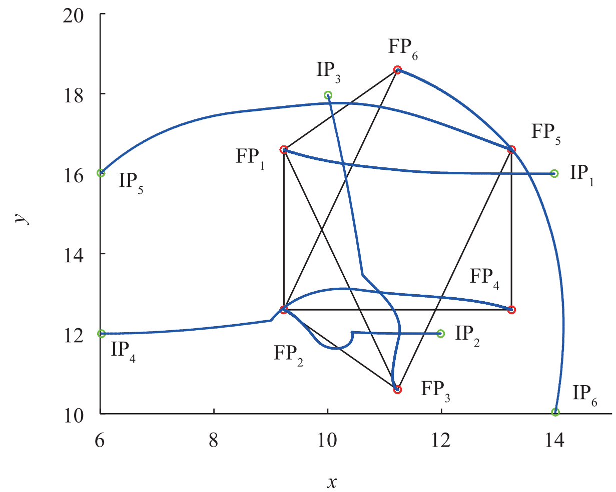

Fig. 8. Evolution trajectories of all agents and the hexagon formation.

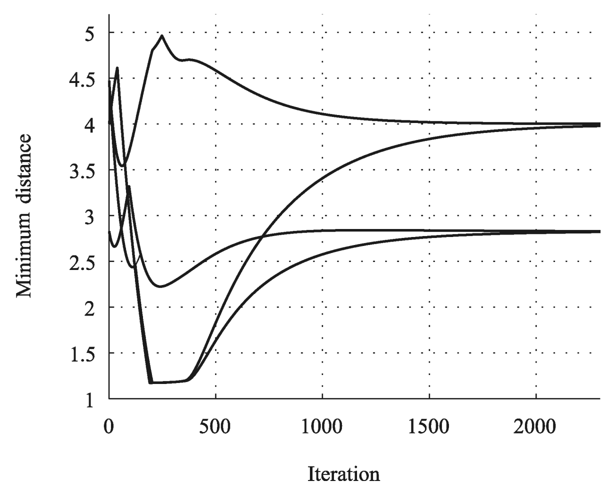

Fig. 9. Minimum inter-agent distances for neighboring agents.

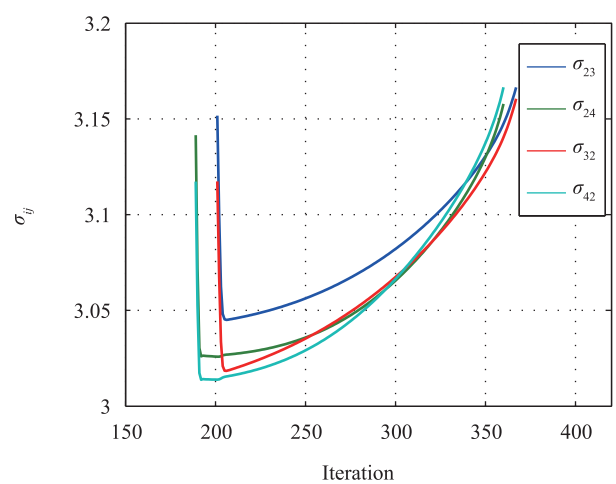

Fig. 10. The agent i computes the variable σ_{ij}, here $i = 2, 3, 4$.

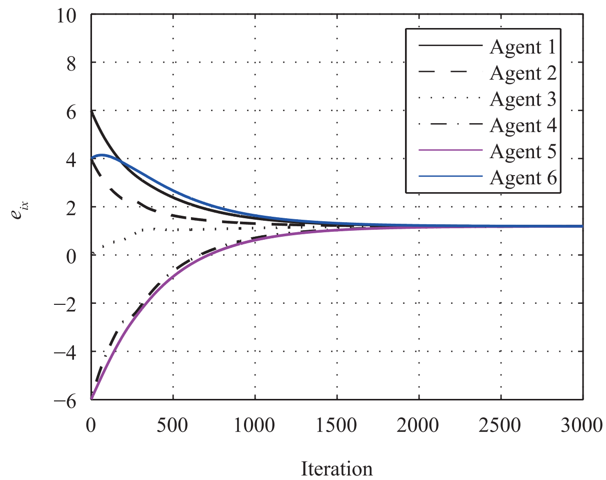

Fig. 11. Convergence in x direction for six agents, $i = 1, \ldots, 6$.

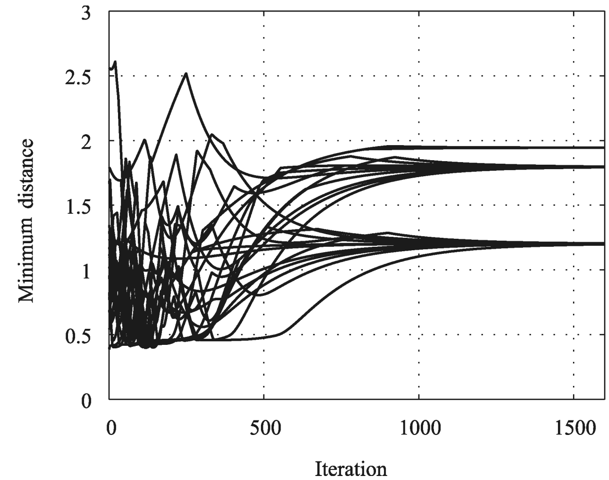

Fig. 14. Minimum inter-agent distances among agents.

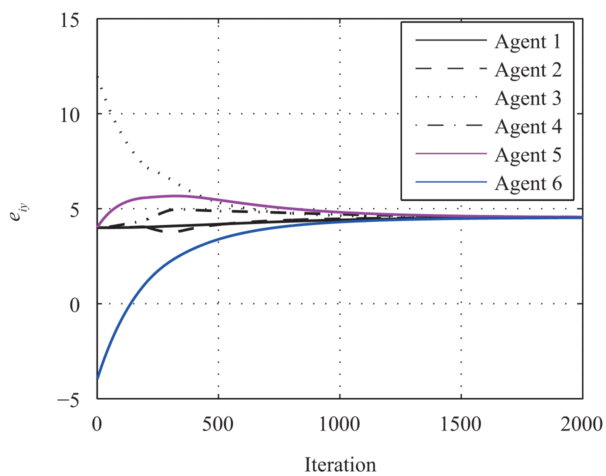

Fig. 12. Convergence in y direction for six agents, $i = 1, \ldots, 6$.

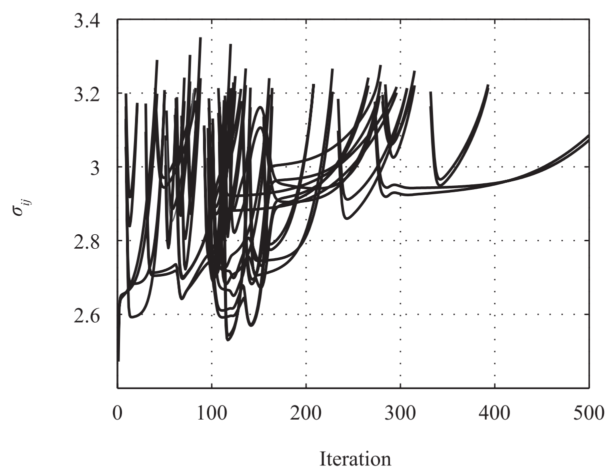

Fig. 15. Agent i computes σ_{ij} for jth agents which are in its CAR.

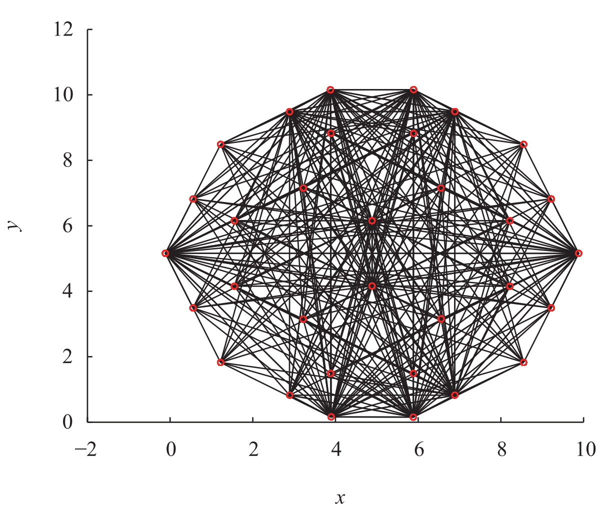

Fig. 13. A planar formation with 32 agents.

TABLE II
INITIAL POSITION FOR SIX AGENT SYSTEM

Agent	Initial position
1 st	$[14 \quad 16]^T$
2 nd	$[12 \quad 12]^T$
3 rd	$[10 \quad 18]^T$
4 th	$[6 \quad 12]^T$
5 th	$[6 \quad 16]^T$
6 th	$[14 \quad 10]^T$

B. Formation With 32 Agents

For 32 agents, initial positions are randomly generated and the values of δ_l, δ_g, δ_a, h are given in Table I, where the outer region $r_{\text{out}} = 0.5$, $r_{\text{in}} = 0.1$ and the radius of (IR) is taken as $r = 10.5$ unit. The final formation for 32 agents is shown in Fig. 13. Minimum inter agent distance for each agent is shown in Fig. 14. This confirms that the inter agent distances never go less than r_{in}. Hence, we can conclude that there is no collision. Fig. 15 shows the critical variable, σ_{ij} for the i agent. Figs. 16 and 17 show e_{ix} and e_{iy}, respectively. It is observed that all agents have converged to some constant equilibrium point. This confirms the generation of the desired formation and hence the consensus among agents is assured.

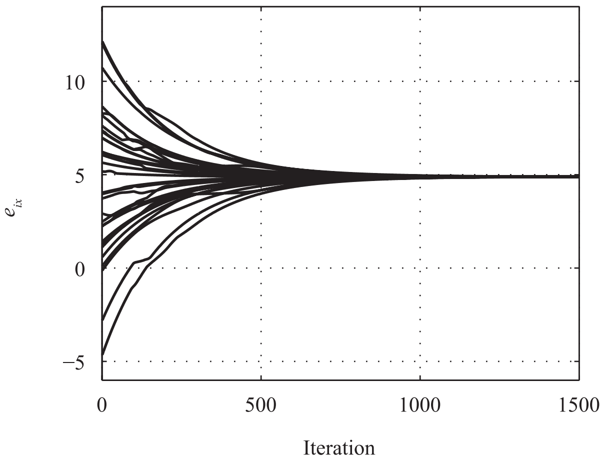

Fig. 16. Convergence in x direction for 32 agents, $i = 1, \ldots, 32$.

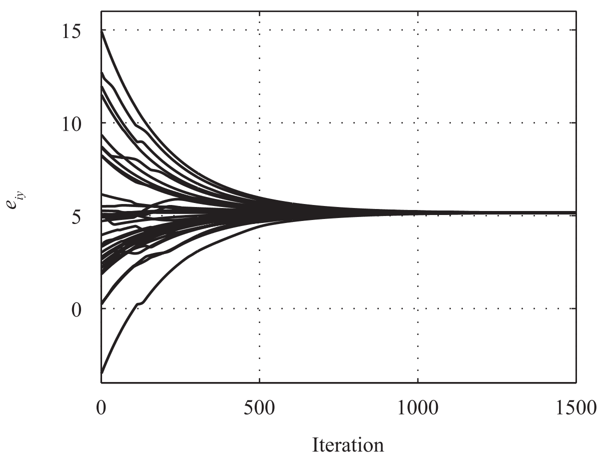

Fig. 17. Convergence in y direction for 32 agents, $i = 1, \ldots, 32$.

VII. Conclusion

In this paper, we have proposed a novel formation control law, which has the following interesting features:

1) The underlying communication graph in the multi-agent system is undirected and connected.

2) For the single integrator type of agents, the formation control law consists of two terms: the error consensus term and the CAT.

3) The novel error consensus term is derived, which is found by optimizing the majorization function of the stress function. It is shown that agents error dynamics as derived based on this error consensus term, is globally asymptotically stable.

4) The innovative design of the CAT produces the bounded repelling magnitude to ensure the graph connectivity during collision avoidance. It is possible by the proposed novel self-adaptive potential function. The self-adaptive nature of this function is achieved by the proposed on-line tuning of the critical variable associated with it. The tuning is done using the newly achieved lower bound of that variable. Lower bound is derived from the proposed connectivity property.

5) It is shown that the critical variable σ_{ij} is inversely proportional to the roots of the quadratic inequality —- the inequality derived to ensure the connectivity property during the collision avoidance.

6) Simulations are done on six agent system and thirty two agent system respectively. The plot of minimum inter agent distances for both the systems show that agents maintain safe distances between them to avoid collision. Through out the simulations, agents maintain the connectivity among them as confirmed from the achievement of the final desired configuration. It has been shown that the variables e_{ix}, and e_{iy} converge towards some constant value, confirming the generation of the desired formation.

7) This work naturally leads us to explore the case of directed graph. The effect of double-integrator model for agents will also be explored for fully-connected, connected and directed graph based networks.

References

[1] W. Ren, R. W. Beard, and E. M. Atkins, "A survey of consensus problems in multi-agent coordination," in *Proc. the 2005 American Control Conf. (ACC)*, Portland, OR, USA, 2005, pp. 1859−1864.

[2] R. R. Nair, L. Behera, V. Kumar, and M. Jamshidi, "Multisatellite formation control for remote sensing applications using artificial potential field and adaptive fuzzy sliding mode control," *IEEE Syst. J.*, vol. 9, no. 2, pp. 508−518, Jun. 2015.

[3] Z. Yao, Y. D. Song, and W. C. Cai, "Neuro-adaptive virtual leader based formation control of multi-unmanned ground vehicles," in *Proc. 2010 the 11th Int. Conf. Control Automation Robotics & Vision (ICARCV)*, Singapore, 2010, pp. 615−620.

[4] W. Cai, L. G. Weng, R. Zhang, M. Zhang, and Y. D. Song, "Virtual leader based formation control of multiple unmanned ground vehicles (UGVs): Control design, simulation and real-time experiment," in *Advances in Cooperative Control and Optimization*, P. M. Pardalos, R. Murphey, D. Grundel, and M. J. Hirsch, Eds. Berlin, Heidelberg: Springer, 2007, pp. 221−230.

[5] X. X. Yang, G. Y. Tang, Y. Li, and P. D. Wang, "Formation control for multiple autonomous agents based on virtual leader structure," in *Proc. 2012 the 24th Chinese Control and Decision Conf. (CCDC)*, Taiyuan, China, 2012, pp. 2833−2837.

[6] J. L. Peng, T. W. Dai, Q. S. Fang, and F. H. Zhao, "A flexible formation control algorithm based on virtual leader," in *Proc. 2012 the 31st Chinese Control Conf. (CCC)*, Hefei, China, 2012, pp. 6411−6414.

[7] W. Ren and R. W. Beard, *Distributed Consensus in Multi-Vehicle Cooperative Control*, London: Springer, 2008.

[8] K. K. Oh and H. S. Ahn, "Formation control of mobile agents based on inter-agent distance dynamics," *Automatica*, vol. 47, no. 10, pp. 2306−2312, Oct. 2011.

[9] H. Huang, C. B. Yu, and X. K. Wang, "Control of triangular formations with a time-varying scale function," in *Proc. 30th American Control Conf. (ACC)*, San Francisco, CA, 2011, pp. 4828−4833.

[10] D. V. Dimarogonas and K. H. Johansson, "On the stability of distance-based formation control," in *Proc. 47th IEEE Conf. Decision and Control (CDC)*, Cancun, Mexico, 2008, pp. 1200−1205.

[11] T. M. Cheng and A. V. Savkin, "Decentralized control of multi-agent systems for swarming with a given geometric pattern," *Computers & Mathematics with Applications*, vol. 61, no. 4, pp. 731−744, Feb. 2011.

[12] J. Cortés, "Global and robust formation-shape stabilization of relative sensing networks," *Automatica*, vol. 45, no. 12, pp. 2754−2762, Dec. 2009.

[13] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Automat. Contr.*, vol. 49, no. 9, pp. 1520−1533, Sep. 2004.

[14] R. Sepulchre, "Consensus on nonlinear spaces," *Annu. Rev. Contr.*, vol. 35, no. 1, pp. 56−64, Apr. 2011.

[15] Y. Zhao, Z. S. Duan, G. G. Wen, and Y. J. Zhang, "Distributed finite-time tracking control for multi-agent systems: An observer-based approach," *Syst. Contr. Lett.*, vol. 62, no. 1, pp. 22−28, Jan. 2013.

[16] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Trans. Automat. Contr.*, vol. 51, no. 3, pp. 401−420, Mar. 2006.

[17] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. Automat. Contr.*, vol. 52, no. 5, pp. 863−868, May 2007.

[18] M. M. Zavlanos and G. J. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Trans. Rob.*, vol. 23, no. 4, pp. 812−816, Aug. 2007.

[19] S. Mastellone, D. M. Stipanović, C. R. Graunke, K. A. Intlekofer, and M. W. Spong, "Formation control and collision avoidance for multi-agent non-holonomic systems: Theory and experiments," *Int. J. Rob. Res.*, vol. 27, no. 1, pp. 107−126, Jan. 2008.

[20] P. F. Hokayem, D. M. Stipanović, and M. W. Spong, "Coordination and collision avoidance for lagrangian systems with disturbances," *Appl. Math. Comput.*, vol. 217, no. 3, pp. 1085−1094, Oct. 2010.

[21] D. V. Dimarogonas and E. Frazzoli, "Analysis of decentralized potential field based multi-agent navigation via primal-dual lyapunov theory," in *Proc. 2010 the 49th IEEE Conf. Decision and Control (CDC)*, Atlanta GA, USA, 2010, pp. 1215−1220.

[22] K. D. Do, "Coordination control of multiple ellipsoidal agents with collision avoidance and limited sensing ranges," *Syst. Contr. Lett.*, vol. 61, no. 1, pp. 247−257, Jan. 2012.

[23] Y. P. Tian and Q. Wang, "Global stabilization of rigid formations in the plane," *Automatica*, vol. 49, no. 5, pp. 1436−1441, May 2013.

[24] H. S. Su, X. F. Wang, and G. R. Chen, "A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements," *Int. J. Contr.*, vol. 82, no. 7, pp. 1334−1343, Oct. 2009.

[25] Z. Y. Chen, M. C. Fan, and H. T. Zhang, "How much control is enough for network connectivity preservation and collision avoidance?," *IEEE Trans. Cybern.*, vol. 45, no. 8, pp. 1647−1656, Aug. 2015.

[26] I. Borg and P. J. F. Groenen, *Modern Multidimensional Scaling: Theory and Application*, 2nd ed. New York: Springer, 2005.

[27] A. Mondal and L. Behera, "Variable gain gradient based collision free desired formation generation," in *Proc. the 3rd Int. Conf. Advances in Control and Optimization of Dynamical Systems*, Kanpur, India, 2013, pp. 448−454.

[28] H. W. Zhang, F. L. Lewis, and Z. H, Qu, "Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs," *IEEE Trans. Ind. Electron.*, vol. 59, no. 7, pp. 3026−3041, Jul. 2012.

[29] H. K. Khalil, *Nonlinear Systems*. 2nd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 1996.

[30] X. Y. Luo, S. B. Li, and X. P. Guan, "Flocking algorithm with multi-target tracking for multi-agent systems," *Pattern Recognit. Lett.*, vol. 31, no. 9, pp. 800−805, Jul. 2010.

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