Sampled-data Observer Design for a Class of Stochastic Nonlinear Systems Based on the Approximate Discrete-time Models

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***Abstract*—In this paper, we studied the approximate sampled-data observer design for a class of stochastic nonlinear systems. Euler-Maruyama approximation was investigated in this paper because it is the basis of other higher precision numerical methods, and it preserves important structures of the nonlinear systems. Also, the form of Euler-Maruyama model is simple and easy to be calculated. The results provide a reference for sampled-data observer design method for such stochastic nonlinear systems, and may be useful to many practical control applications, such as tracking control in mechanical systems. And the effectiveness of the approach is demonstrated by a simulation example.**

***Index Terms*—Approximation model, exponentially bounded, sampled-data observer, stochastic nonlinear.**

I. Introduction

THE state observer design is always an interesting issue of control theory and engineering application, especially for stochastic nonlinear systems, due to its nonlinearity and stochastic disturbance [1]−[3]. Sampled-data observer for stochastic nonlinear systems can be used in computer controlled systems while some states are not convenient to be measured directly, such as the speed observer based on position measurements [4], [5]. This can reduce cost and improve reliability of systems by replacing some sensors [6]. Usually, sampled-data observer is designed based on the exact discrete-time model of original systems. Actually, the exact discrete-time model for continuous-time stochastic nonlinear systems is impossible to be obtained in most cases. Designing a sampled-data observer based on the approximate discrete-time models for original systems is a practical and effective method, which has become more and more popular [7]−[10].

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Controller design method based on approximate discrete-time plant models was pursued in many control applications, including but not limited to input-to-state stabilization [11], optimal control [12], sampled-data observer design [7], receding horizon control [13], [14] and filtering [15]. Stochastic differential equations (SDEs) play a significant role in the description and analysis of a lot of practical systems. Compared with ordinary differential equations and partial differential equations, SDEs are far more difficult to be solved. On the other hand, it can be relatively easy to simulate such systems using computer-based algorithms such as the Monte Carlo method. The Euler-Maruyama method obtained a lot of attention and research during the past decades, including the construction method, convergence, stability and so on [16]−[19]. In recent years, many modified Euler-Maruyama methods for various stochastic systems have been researched and developed, as well as their convergence, stability and convergence rates [20]−[24].

It is worth to point out that stochastic differential equations (SDEs) play a significant role in the description and analysis of a lot of practical systems, for example, automatic control systems, financial systems, biological systems and mechanical systems. To the best of the authors' knowledge, sampled-data observer design for a class of stochastic nonlinear systems based on their approximate discrete-time models is still an open problem. In this paper, we mainly focus on the velocity observation and tracking control problem of mechanical systems and motion control systems, for example, the spring pendulum hung from a stochastically vibrating ceiling which can be seen as a telescopic manipulator in a random environment [25], [26]. Usually, the velocity observation for mechanical systems or motion control systems is both costly and mechanically difficult [27]. On the other hand, the position transducer has been commonly used in industry, so, more fast and convenient estimation for velocity with tolerable accuracy is a significant and practical problem, which motivates our work. Specifically, the sampled-data observer which is designed based on the Euler-Maruyama model has been studied to ascertain whether it is an effective state observer for the exact model, or in what sense the design guarantees the convergence when it is applied to the exact models. We will analyze the convergence of observer error when the approximate sampled-data observer servers as a state observer for the exact system under some conditions. The results provide a reference for sampled-data observer design method for such stochastic nonlinear systems.

The remainder of this paper is organized as follows. In Section II, a class of stochastic nonlinear systems and their approximate sampled-data observers are introduced and the problem under consideration is formulated, some necessary definitions and lemmas are given. The main results are then discussed in Section III. The simulation results are shown in Section IV and Section V concludes the paper.

Notations: Throughout this paper, $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{N}$ denote the sets of real, nonnegative real and natural numbers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional real space and $n\times m$-dimensional real matrix space respectively. Given a matrix A, denote by $\|A\|$ its operator norm, and $|\cdot|$ denoting the Euclidean norm in $\mathbb{R}^n$. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. Let $w(t) = (w_1(t), \ldots, w_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. $E\{\cdot\}$ is the mathematical expectation of random variable to the given probability space. Also, let $L^p_{\mathcal{F}_t}(\Omega; \mathbb{R}^n)$ denote the family of all $\mathcal{F}_t$-measurable random variables $\xi : \Omega \to \mathbb{R}^n$ such that $E|\xi|^p < \infty$. All the matrices are assumed to have compatible dimensions if they are not explicitly specified.

II. Preliminaries and Problem Formulation

Let us consider a class of second-order stochastic nonlinear mechanical systems with the following state-space form:

$$\begin{aligned} dx_1 &= f_1(x_1, x_2)dt \\ dx_2 &= f_2(x_1, x_2, u)dt + g_2(x_1, x_2)dw \\ y &= l(x_1) \end{aligned} \tag{1}$$

where $x_1, x_2 \in \mathbb{R}^n$ can usually be the generalized coordinates and velocity vector, respectively; and u is a torque input; $y \in \mathbb{R}^m$ is measurement output. The system is supposed to be disturbed by white noise, where w is an r-dimensional standard Wiener process.

The model can be presented in the following compact form:

$$\begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2, u) \end{bmatrix} dt + \begin{bmatrix} 0 \\ g_2(x_1, x_2) \end{bmatrix} dw$$

which can be rewritten as

$$\begin{aligned} dx &= f(x, u)dt + g(x)dw \\ y &= h(x) \end{aligned} \tag{2}$$

where $x := [x_1^T, x_2^T]^T$, $f(x, u) := [f_1^T, f_2^T]^T$, $g(x) := [0, g_2^T]^T$, $h(x) := l(x_1)$. We need to design a sampled-data observer for the velocity vector x_2 for the original system (1), while only the measurement output relative to position vector is available. Many sampled-data observer design procedure is based on the availability of the exact discrete-time model for the original systems [28], [29]. It is not difficult to make the exact discretization for linear systems directly. However, for nonlinear systems, the exact discrete-time model is impossible to obtain in most practical cases. As an effective and realistic method, a family of approximate discrete-time models can be obtained by different numerical integration methods. Some researchers consider the approximate models for nonlinear systems instead of exact discrete-time models [9], [10].

Assume that a zero-order holder with sampling period T is applied to systems (2), and the state measurement $x(k) := x(kT)$ are available at sampling instants $kT, k \in \mathbb{N}$. We can write the solution of system (2) in discrete-time form that starts from the initial state $x(k) := x(kT)$ while keeping the control signal $u_T(t) = u(kT) =: u(k),\ \forall t \in [kT, (k+1)T],\ k \in \mathbb{N}$, which is said to be the exact discrete-time model of system (2):

$$\begin{aligned} x(k+1) &= x(k) + \int_{kT}^{(k+1)T} f(x(\tau), u_T(k))d\tau \\ &\quad + \int_{kT}^{(k+1)T} g(x(\tau))dw(\tau) \triangleq F_T^e(x(k), u_T(k)) \\ y(k) &= h(x(k)). \end{aligned} \tag{3}$$

In this paper, we consider the Euler-Maruyama approximation for stochastic differential (2):

$$\begin{aligned} x^{\rm em}(k+1) &= x(k) + f(x(k), u_T(k)) \cdot T + g(x(k)) \cdot \Delta w(k) \\ &\triangleq F_T^a(x(k), u_T(k)) \\ y(k) &= h(x(k)) \end{aligned} \tag{4}$$

where $\Delta w(k) = w((k+1)T) - w(kT), k \in \mathbb{N}$, and we have $E\{\Delta w(k)\} = 0, E\{(\Delta w(k))^2\} = T$.

Based on the Euler-Maruyama model (4), a full state sampled-data observer for system (2) can be formulated as:

$$\hat{x}(k+1) = \hat{x}(k) + f(\hat{x}(k), u_T(k)) \cdot T + K_T(\hat{x}(k), y(k)) \tag{5}$$

where $K_T(\hat{x}(k), y(k))$ is the correction term, and satisfies $K_T(x(k), y(k)) = 0$ in general. Many useful observer design methods for such a discrete-time stochastic system (4) can be found in [1], [30], [31]. Now we aim to analyze under what conditions the design (5) can be an effective state observer of the exact model (3), or in what sense the design guarantees the convergence when it is applied to the exact models. The following lemma will be useful in the analysis of main results:

Lemma 1 [31]: Consider a discrete-time stochastic process ξ_k, we say it is exponentially bounded in mean square sense if there exists a Lyapunov function $V(\xi_k)$ and real scalars $0 < c_1 < c_2, 0 < c_3 < 1$, $\lambda \geq 0$, such that

$$\begin{aligned} c_1\|\xi_k\|^2 \leq V(\xi_k) \leq c_2\|\xi_k\|^2 \\ E\{V(\xi_{k+1})\} - V(\xi_k) \leq -c_3V(\xi_k) + \lambda. \end{aligned} \tag{6}$$

More specifically, we have

$$E\{\|\xi_k\|^2\} \leq \frac{c_2}{c_1}\|\xi_0\|^2(1-c_3)^k + \frac{\lambda}{c_1c_3}. \tag{7}$$

III. Convergence Analysis

In this section, we will analyze the convergence of observer error when the approximate sampled-data observer (5) serves as a state observer for the exact system (3). For our analysis, we define the observer errors: $e = \hat{x} - x, \tilde{e} = \hat{x} - x^{\rm em}$. Then, we have the following definition:

Definition 1: We can say that the observer (5) is an exponentially bounded observer for system (4) in mean square sense, if there exist real scalars $\alpha_1 \geq 1, \alpha_2 > 0, \lambda \geq 0$, and $T^* > 0$, such that for each $T \in (0, T^*]$, we have

$$\mathbf{E}\{\|\tilde{e}(k)\|^2\} \leq \alpha_1\|\tilde{e}(0)\|^2e^{-\alpha_2kT} + \lambda. \tag{8}$$

Before the main results about the convergence properties of observer error e are given, we present the following lemma firstly, which quantizes the model error between the exact discrete-time model and its Euler-Maruyama model, as a consistency condition in mean square.

Lemma 2: For the exact discrete-time model (3) and its Euler-Maruyama model (4), if there exist constants $L^* > 0$ and $K > 0$, such that for all $x, y \in \mathbb{R}^n$, the following hold

$$|f(x,u_T(x))-f(y,u_T(y))| \leq L^*|x-y| \tag{9}$$
$$|f(x,u_T(x))-f(y,u_T(x))| \vee |g(x)-g(y)| \leq K|x-y| \tag{10}$$

then, there exist positive constants $c_5 > 0,\ T^* > 0$ such that for all $T \in (0, T^*)$, the one-step consistency between the two models can be presented as following inequality:

$$E\big\|F_T^e(x,u_T(x)) - F_T^a(x,u_T(x))\big\|^2 \leq c_5 T\, E\|x\|^2. \tag{11}$$

Proof: Let $x(t;x_0,t_0)$ be the solution of system (2), then we can write the following equation on $t_0 \leq t \leq T'$:

$$x(t;x_0,t_0) = x_0 + \int_{t_0}^{t} f(x(\tau),u_T(\tau))d\tau + \int_{t_0}^{t} g(x(\tau))dw(\tau). \tag{12}$$

Consider the solution (2), we can write for $t \in [t_0, t_0+T]$ that,

$$\begin{aligned}E\|x(t)\|^2 =& E\Big\{|x(t_0)|^2 + 2|x(t_0)||\int_{t_0}^{t} f(x(\tau),u)d\tau| \\ &+ |\int_{t_0}^{t} f(x(\tau),u)d\tau|^2 + |\int_{t_0}^{t} g(x(\tau))d\omega(\tau)|^2\Big\} \\ \leq& E\Big\{|x(t_0)|^2 + 2|x(t_0)||\int_{t_0}^{t} L^*|x(\tau)|d\tau| \\ &+ |\int_{t_0}^{t} f(x(\tau),u)d\tau|^2 + |\int_{t_0}^{t} g^2(x(\tau))d\tau|\Big\}\end{aligned} \tag{13}$$

then by Cauchy inequality (or Jensen inequality), we have

$$|\int_{t_0}^{t} f(x(\tau),u)d\tau|^2 \leq L^{*2}T\int_{t_0}^{t}|x(\tau)|^2 d\tau.$$

Notice that $2|x(t_0)||x(\tau)| \leq |x(t_0)|^2 + |x(\tau)|^2$, so

$$\begin{aligned}E\|x(t)\|^2 \leq& E\Big\{|x(t_0)|^2 + L^*\int_{t_0}^{t}(|x(t_0)|^2 + |x(\tau)|^2)d\tau \\ &+ L^{*2}T\int_{t_0}^{t}|x(\tau)|^2 d\tau + K^2\int_{t_0}^{t}|x(\tau)|^2 d\tau\Big\} \\ \leq& E\Big\{(1+L^*T)|x(t_0)|^2 \\ &+ (L^* + L^{*2}T + K^2)\int_{t_0}^{t}|x(\tau)|^2 d\tau\Big\}.\end{aligned} \tag{14}$$

By the Gronwall inequality, we can obtain that

$$E\,\|x(t)\|^2 \leq e^{(2L^*+L^{*2}T+K^2)T}\|x(t_0)\|^2, \quad t \in [t_0, t_0+T].$$

According to Hölder inequality, when $p \geq 1$, we have

$$\big(E\|x(t)\|\big)^p \leq E\big\{\|x(t)\|^p\big\}.$$

Thus we can write

$$\begin{aligned}&\big(E\|x(t)\|\big)^2 \leq e^{(2L^*+L^{*2}T+K^2)T}\|x(t_0)\|^2 \\ \Rightarrow& E\|x(t)\| \leq M\|x(t_0)\|, t \in [t_0, t_0+T], \forall x(t_0), t_0 \geq 0\end{aligned}$$

where $M := e^{(2L^*+L^{*2}T^*+K^2)T^*/2}$.

Now, let us consider one-step model error in mean square between the exact discrete-time model (3) and its Euler-Maruyama model (4), named one-step consistency:

$$\begin{aligned}&E\big\|F_T^e(x,u_T(x)) - F_T^a(x,u_T(x))\big\|^2 \\ =& E\Big\|\int_0^T\big[f(x(\tau),u_T) - f(x,u_T)\big]d\tau \\ &+ \int_0^T\big[g(x(\tau)) - g(x)\big]d\omega(\tau)\Big\|^2 \\ \leq& E\Big|\Big(\int_0^T\big[f(x(\tau),u_T) - f(x,u_T)\big]d\tau\Big)^2\Big| \\ &+ E\Big|\Big(\int_0^T\big[g(x(\tau)) - g(x)\big]d\omega(\tau)\Big)^2\Big| \\ \leq& K(M-1)^2(T+1)T\,E\,\|x\|^2\end{aligned} \tag{15}$$

denote $c_5 := K(M-1)^2(T^*+1) > 0$, which completes the proof. ■

Now we aim to analyze under what conditions the design (5) can be an effective state observer of the exact model (3), or in what sense the design guarantees the convergence when it is applied to the exact models. To state our results conveniently, we have the following assumption:

Assumption 1: For each $x_0 \in \mathbb{R}^n$, there exist a compact set $\mathfrak{X} \in \mathbb{R}^n$ and a positive number $\Delta_x > 0$, $T^* > 0$ such that for $T \in (0, T^*]$ and $t > 0, k > 0$, we have $x(t) \in \mathfrak{X}$, $\|x(k)\|^2 \leq \Delta_x$.

Remark 1: Assumption 1 is satisfied by many engineering applications, e.g., pose estimation of a robot moving on earth's surface, state estimation for aircraft guidance and control, etc [3].

Theorem 1: For any compact set $\mathfrak{X} \in \mathbb{R}^n$ and $T^* > 0$, suppose that the approximate sampled-data observer (5) is an exponentially bounded observer for system (4) in mean square, and the model error between the exact discrete-time model (3) and its Euler-Maruyama model (4) satisfies the consistency condition (11), then the observer error between (5) and (3) is exponentially bounded in mean square.

Proof: Consider the stochastic nonlinear system (2) and sampled-data observer (5) designed based on its discrete-time approximation (4). Let us define new variables: $e = \hat{x} - x, \tilde{e} = \hat{x} - x^{\text{em}}$. We have

$$\begin{aligned}e(k+1) &= \hat{x}(k+1) - x^{\text{em}}(k+1) + x^{\text{em}}(k+1) - x(k+1) \\ &= \tilde{e}(k+1) + F_T^a(x,u_T(x)) - F_T^e(x,u_T(x)).\end{aligned}$$

Denoting $R_T(x) = F_T^a(x,u_T(x)) - F_T^e(x,u_T(x))$, we can obtain that $\|R_T(x)\|^2 \leq c_5 T\|x\|^2$.

Suppose that (5) is an exponential bounded observer in mean square for (4), then we have

$$E\left\{\|\tilde{e}(k)\|^2\right\} \leq \alpha_1\|\tilde{e}(0)\|^2 e^{-\alpha_2 kT} + \lambda$$

where $\alpha_1 \geq 1, \alpha_2 > 0, \lambda \geq 0$, and assume there exists a $T^* > 0$, such that for each $T \in (0, T^*]$, we have

$$e^{-\alpha_2 T} < \frac{1}{2}. \tag{16}$$

Without loss of generality, the positive-definite function can be chosen as $V(x) = \|x\|^2$, then, we can write

$$\begin{aligned} & V(e(k+1)) - V(e(k)) \\ & = \|e(k+1)\|^2 - \|e(k)\|^2 \\ & \leq 2\|\tilde{e}(k+1)\|^2 + 2\|R_T(x)\|^2 - \|e(k)\|^2 \\ & \leq -(1 - 2e^{-\alpha_2 T})\|e(k)\|^2 + 2c_5 T \Delta_x + 2(1 - e^{-\alpha_2 T})\lambda \end{aligned}$$

thus, we have

$$E\{V(e(k+1))\} - V(e(k)) \leq -c_3 V(e(k)) + \lambda^*$$

where $c_3 := 1 - 2e^{-\alpha_2 T} \in (0,1)$, $\quad \gamma^* := 2c_5 T\Delta_x + 2(1 - e^{-\alpha_2 T})\lambda \geq 0$.

It can be obtained directly that

$$E\{\|e(k)\|^2\} \leq \frac{c_2}{c_1}\|e(0)\|^2(1-c_3)^k + \frac{\lambda^*}{c_1 c_3} \tag{17}$$

hence it can be said that the observer error it is exponentially bounded in mean square. ■

IV. Numerical Simulation

Second-order stochastic nonlinear equation has been used to describe and analyse many practical, especially financial and motion control systems. For simplicity, let us consider the Van der Pol oscillator [8] driven by Wiener processes:

$$\begin{aligned} & dx_1 = x_2 dt \\ & dx_2 = [-x_1 + \epsilon(1 - x_1^2)x_2]dt + 0.5dw \\ & y = x_1 \end{aligned} \tag{18}$$

where $\epsilon = 0.8$. Let $x = [x_1, x_2]^T$,

$$f(x) = \begin{bmatrix} x_2 \\ -x_1 + \epsilon(1 - x_1^2)x_2 \end{bmatrix}.$$

We have the Euler-Maruyama approximations

$$\begin{aligned} & x[k+1] = x(k) + f(x) \cdot T + g(x)\Delta w \\ & y[k] = [1, 0]x[k] \end{aligned} \tag{19}$$

and the approximate sampled-data observer:

$$\hat{x}(k+1) = \hat{x}(k) + f(\hat{x}(k), u_T(k)) \cdot T + K_T(\hat{x}(k), y(k)).$$

Let

$$\begin{aligned} l_1(\hat{x}, y) &= (5 - \epsilon(1 - y^2[k]))T \\ l_2(\hat{x}, y) &= (5 - \epsilon(\hat{x}_1[k] + y[k])\hat{x}_2[k] \\ &\quad + (5 + \epsilon(1 - y^2[k]))\epsilon(1 - y^2[k]))T \end{aligned}$$

and

$$K_T(\hat{x}(k), y(k)) = \begin{bmatrix} l_1(\hat{x}, y) \\ l_2(\hat{x}, y) \end{bmatrix} (y - \hat{x}_1[k]).$$

Let the system initial state $x[0] = [1, 0]^T$, observer initial state $z[0] = [0, 1]^T$. Take the value of T as 0.03, 0.1 and 0.3, respectively, and the simulation results are shown in Figs. 1–3. Through running the simulation, the results indicate that the error of sampled-data observer grows as the sampling period increases.

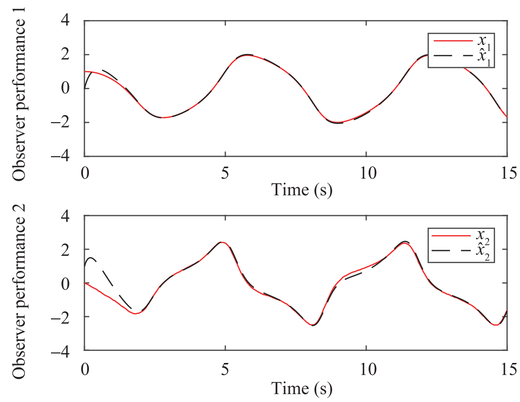

Fig. 1: The output of observer with $T = 0.03$.

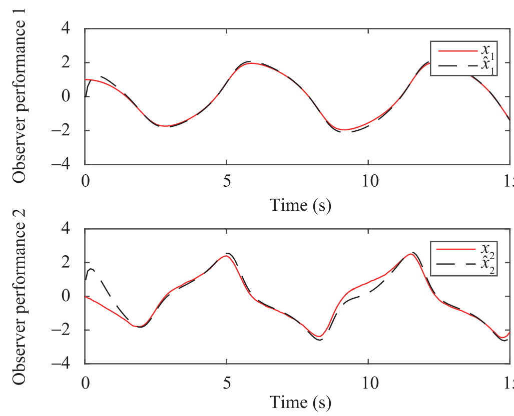

Fig. 2: The output of observer with $T = 0.1$.

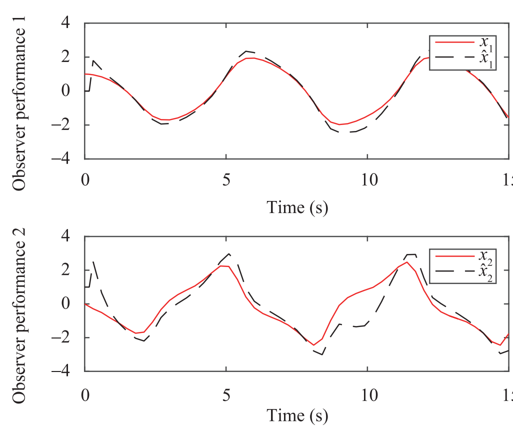

Fig. 3: The output of observer with $T = 0.3$.

V. Conclusion

In this paper, we have studied the approximate sampled-data observer design for a class of stochastic nonlinear systems. Euler-Maruyama approximation was investigated in this paper because it is the basis of other higher precision numerical methods, and it preserves important structures of the nonlinear systems. Also, the form of Euler-Maruyama model is simple and easy to be calculated. The results provide a reference for sampled-data observer design method for such stochastic nonlinear systems, and may be useful to many practical control applications, such as tracking control in mechanical system-s. We believe that more accurate numerical approximation method will bring better observer performance. Some further questions like this will be studied in our future work.

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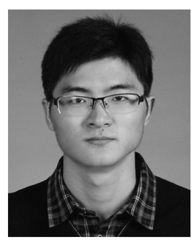

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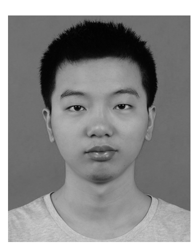

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