Robust Tracking Control for Self-balancing Mobile Robots Using Disturbance Observer

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***Abstract*—In this paper, a robust tracking control scheme based on nonlinear disturbance observer is developed for the self-balancing mobile robot with external unknown disturbances. A desired velocity control law is firstly designed using the Lyapunov analysis method and the arctan function. To improve the tracking control performance, a nonlinear disturbance observer is developed to estimate the unknown disturbance of the self-balancing mobile robot. Using the output of the designed disturbance observer, the robust tracking control scheme is presented employing the sliding mode method for the self-balancing mobile robot. Numerical simulation results further demonstrate the effectiveness of the proposed robust tracking control scheme for the self-balancing mobile robot subject to external unknown disturbances.**

***Index Terms*—Disturbance observer, robust tracking control, self-balancing mobile robot, sliding mode control (SMC).**

I. INTRODUCTION

As it is well known, stabilization of non-holonomic wheeled mobile robots has received much attention due to their wide usefulness in various applications [1]. Moreover, the tracking control scheme of the wheeled mobile robot has been extensively studied in the past decades [2], [3]. The self-balancing mobile robot is a special kind of wheeled mobile robots which form a class of nonlinear, coupled and under-actuated systems. As a result, the modeling, control and application problems of the self-balancing mobile robot have received increasing attention from researchers and have been widely investigated [4], [5]. Recently, some various and efficient control schemes have been proposed for the self-balancing mobile robot. In [6], the modeling and control design were studied for self-balancing two-wheeled vehicles. An adaptive backstepping self-balancing control was developed for a two-wheeled electric scooter in [7]. In [8], the application of fuzzy control scheme was studied for a self-balancing two-wheeled vehicle. Adaptive neural network control was proposed for a self-balancing two-wheeled scooter in [9]. In [10], an adaptive robust self-balancing and steering control was developed for a two-wheeled human transportation vehicle. Neural network-based motion control was proposed for an under-actuated wheeled inverted pendulum model in [11]. In [12], a robust adaptive motion/force control was presented for wheeled inverted pendulums.

Due to the stronger robustness against large uncertainties, nonlinearities, and bounded external disturbances, the sliding mode control (SMC) has been widely applied in the control of uncertain nonlinear systems [13]. Generalized SMC was proposed for multi-input nonlinear systems in [14]. In [15], a non-singular terminal SMC was proposed for the nonlinear second-order systems with input saturation. Up to now, the SMC has been applied to various practical systems [16]. In [17], a robust multi-input and multi-output (MIMO) water level control was developed for interconnected twin-tanks using second order SMC. A higher-order sliding mode three-axis solar pressure satellite attitude control system was designed in [18]. In [19], a trajectory tracking SMC was designed for under-actuated autonomous underwater vehicles. Extended state observer-based adaptive SMC was studied for the differential-drive mobile robot with uncertainties in [20]. In [21], a sliding-mode velocity control was designed for a two-wheeled self-balancing vehicle. Although different SMC schemes have been extensively studied in the practical systems, the SMC needs to be further developed for the self-balancing mobile robot and the dynamic information of the unknown disturbance should be fully utilized.

To further improve the disturbance attenuation performance, the disturbance observer can be introduced to handle the unknown disturbance for the self-balancing mobile robot. Since the disturbance compensation method using disturbance observer has more robustness against unknown disturbances than the conventional control methods, many efficient disturbance observer based control (DOBC) schemes have been developed in the recent decades [22]−[29]. The disturbance attenuation and rejection problem was studied for a class of MIMO nonlinear systems using the DOBC technique in [26]. In [29], a robust autopilot design was proposed for bank-to-turn missiles using disturbance observers. Now, the disturbance observer has been introduced to design the SMC in order to further enhance the robust control performance. For uncertain nonlinear systems with disturbances, the DOBC scheme using terminal SMC was developed to improve the anti-disturbance ability [30]. In [31], a SMC scheme was designed for a class of uncertain nonlinear systems based on disturbance observer to tackle system uncertainty and external disturbance. A prediction-accuracy-enhanced continuous-time model predictive control was proposed for disturbed systems

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via a disturbance observer in [32]. At the same time, some disturbance observer based control schemes have been developed for various robots [33]. In [34], a nonlinear DOBC design was given for a robotic exoskeleton incorporating fuzzy approximation. Nonlinear disturbance observer-based dynamic surface control was proposed for mobile wheeled inverted pendulum in [35]. In [36], a robust tracking control scheme was developed for wheeled mobile robots with skidding and slipping using fuzzy disturbance observer. However, to improve the tracking performance, the robust tracking control scheme based on the disturbance observer is needed to be further refined for the self-balancing mobile robot with unknown disturbances.

Inspired by the above discussion, a disturbance observer based robust tracking control scheme is proposed for self-balancing mobile robot with unknown disturbances in this paper. The organization of this paper is as follows. Section II details the problem formulation and preliminaries. The disturbance observer based robust tracking controller is designed in Section III. Numerical simulation studies are presented in Section IV to demonstrate the effectiveness of the developed robust tracking control method of the self-balancing mobile robot with unknown disturbances, followed by some concluding remarks in Section V.

II. Problem Statement and Preliminaries

A. Nonlinear Model of Self-balancing Mobile Robots

A self-balancing mobile robot made by Googol Technology Consulting Inc is employed in this paper. In general, the pitch channel and the yaw channel are coupled for the studied self-balancing mobile robot. To facilitate designing of the tracking control scheme of the self-balancing mobile robot, we introduce the decoupling matrix to render the pitch channel and the yaw channel decouple [37]. Considering the external disturbance forces, the nonlinear dynamic model of self-balancing mobile robot is given as follows [37]:

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{x}_2 \\ \dot{\bar{x}}_2 &= F(\bar{x}) + G(\bar{x})u + D \end{aligned} \tag{1}$$

where $\bar{x}_1 = [x, \theta, \delta]^T$, $\bar{x}_2 = [\dot{x}, \dot{\theta}, \dot{\delta}]^T$ and $\bar{x} = [\bar{x}_1^T, \bar{x}_2^T]^T$. x is the linear displacement of chassis, θ is the pitch angle and δ is the yaw angle of self-balancing mobile robot, respectively. $u = [C_\theta, C_\delta]^T$ is the control input vector. C_θ and C_δ are the pitch torque and the yaw torque, respectively. $D = [d_1, d_2, d_3]^T$ is the unknown external disturbance torque vector. $F(\bar{x}) = [f_1(\bar{x}), f_2(\bar{x}), 0]^T$ and $G(\bar{x}) = [g_{11}(\bar{x}), 0; g_{21}(\bar{x}), 0; 0, g_{32}(\bar{x})]$ are the system function vector and the control input matrix, respectively. The detailed expressions of $F(\bar{x})$ and $G(\bar{x})$ are given by [37]

$$B = M_p^2 L^2 \cos^2\theta - (M_p L^2 + J_p)\left(M_p + \frac{2J_r}{R^2} + 2M_r\right)$$

$$f_1 = \frac{(M_p^2 g L^2 \sin\theta\cos\theta - (M_p L^2 + J_p) M_p L \dot{\theta} \sin\theta)}{B}$$

$$f_2 = \frac{\left(M_p^2 L^2 \dot{\theta}^2 \sin\theta\cos\theta - M_p g L \sin\theta\left(M_p + \frac{2J_r}{R^2} + 2M_r\right)\right)}{B}$$

$$g_{11} = \frac{-2\left(M_p L^2 + J_p + M_p L R\cos\theta\right)}{RB}$$

$$g_{21} = -2\frac{\left(M_p L\cos\theta + \left(M_p + \frac{2J_r}{R^2} + 2M_r\right)R\right)}{RB}$$

$$g_{32} = \frac{2\bar{D}}{\left(M_r R + \frac{J_r}{R}\right)\bar{D}^2 + 2J_\delta R}$$

where M_p is the mass of self-balancing mobile robot. L is the distance between the chassis and the center of gravity of robot body. J_p is the moment of inertia of the body around the center of gravity. J_r is the equivalent rotary inertia of wheel, speed reducer and motor rotor. R is the wheel radius. M_r is the wheel mass. g is the acceleration of gravity. $\bar{D}$ is the distance between two wheels and J_δ is the moment of inertia of the vehicle body around the Y axis.

From (1), we know that the self-balancing mobile robot is an under-actuated system. Since the pitch channel and the yaw channel are decoupled by using the decoupling matrix, the system (1) can be transformed to two subsystems. One subsystem can be described as

$$\dot{X} = \phi(X) + \Gamma(X)C_\theta + D_0 \tag{2}$$

where $X = [\dot{x}, \dot{\theta}]^T$, $\phi(X) = [f_1, f_2]^T$, $\Gamma(X) = [g_{11}, g_{21}]^T$ and $D_0 = [d_1, d_2]^T$.

Another subsystem is expressed as

$$\ddot{\delta} = g_{32} C_\delta + d_3. \tag{3}$$

For the studied self-balancing mobile robot, the two wheels are coaxial and they are driven by two DC motors in differential mode. The nonholonomic self-balancing mobile robot is considered in this paper. It is always assumed in the literature that the system is subject to a "pure" rolling without slipping constraint. Then, the non-holonomic constraints of self balancing vehicle are as follows [38]:

$$\begin{aligned} & y\cos\delta - x\sin\delta = 0 \\ & x\cos\delta + y\sin\delta + \frac{\bar{D}\delta}{2} = R\psi_1 \\ & x\cos\delta + y\sin\delta - \frac{\bar{D}\delta}{2} = R\psi_2 \end{aligned} \tag{4}$$

where x and y are positions in the transverse and vertical coordinates. ψ_1 and ψ_2 are angular velocities of the left and right wheels, respectively.

To design the robust tracking control scheme, we define the pose of the self-balancing mobile robot as $q_0 = [x, y, \delta]^T$. Then, we have [38]

$$\dot{q}_0 = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \cos\delta & 0 \\ \sin\delta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{5}$$

where $v = R(\dot{\psi}_1 + \dot{\psi}_2)/2$ and $\omega = R(\dot{\psi}_1 - \dot{\psi}_2)/2$. v is the forward speed of the chassis and ω is the yaw rate of self-balancing mobile robot.

Define the reference pose as $q_r = [x_r, y_r, \delta_r]^T$ and the reference velocity is defined as $z_r = [v_r, \omega_r]^T$. The reference trajectory is chosen as [38]

$$\dot{q}_r = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\delta}_r \end{bmatrix} = \begin{bmatrix} \cos\delta_r & 0 \\ \sin\delta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix}. \tag{6}$$

A smooth velocity control law $z_c = f_c(q_e, z_r)$ can be designed according to $\lim_{t\to\infty} q_e = 0$ where q_e is the tracking error and is defined as follows [38]:

$$q_e = \begin{bmatrix} x_e \\ y_e \\ \delta_e \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \delta_r - \delta \end{bmatrix}. \tag{7}$$

Then, we can obtain [38]

$$\dot{q}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} v_r\cos\delta_e - v_c + y_e\omega_c \\ v_r\sin\delta_e - x_e\omega_c \\ \omega_r - \omega_c \end{bmatrix} \tag{8}$$

where v_c is the desired forward speed of the chassis and ω_c is the desired yaw rate of self-balancing mobile robot.

B. Design of Desired Velocity Control Law

A virtual velocity control should be designed to track the kinematic model of the self-balancing mobile robot (1). Namely, under the given virtual velocity control, q_e is stable under the assumption $v_r > 0$.

To design the desired velocity control law z_c, we assume that the yaw angle δ of self-balancing mobile changes in the interval $(-\pi/2, \pi/2)$ and define a new variable as [39]

$$\bar{x}_e = x_e - \lambda_1\varphi(\omega_c)y_e \tag{9}$$

where λ_1 is a positive design parameter, and $\varphi(\omega_c) = \arctan(\omega_c)$. From (9), we can know $x_e \to 0$ when $\bar{x}_e \to 0$ and $y_e \to 0$.

The designed desired velocity control law should guarantee the convergence of the tracking error q_e. To obtain the desired velocity control law, we choose the following Lyapunov function candidate:

$$V_0 = \frac{1}{2}\bar{x}_e^2 + \frac{1}{2}y_e^2 + 2\left(1 - \cos\frac{\delta_e}{2}\right). \tag{10}$$

Then, we obtain

$$\dot{V}_0 = \bar{x}_e\dot{\bar{x}}_e + y_e\dot{y}_e + \sin\frac{\delta_e}{2}\dot{\delta}_e. \tag{11}$$

Invoking (8) and (9) yields

$$\begin{aligned} \dot{\bar{x}}_e &= \dot{x}_e - \lambda_1\dot{\varphi}(\omega_c)y_e\dot{\omega}_c - \lambda_1\varphi(\omega_c)\dot{y}_e \\ &= v_r\cos\delta_e - v_c + y_e\omega_c - \frac{\lambda_1 y_e}{1+\omega_c^2}\dot{\omega}_c \\ &\quad - \lambda_1\varphi(\omega_c)(v_r\sin\delta_e - x_e\omega_c). \end{aligned} \tag{12}$$

Considering (9) and (12), we have

$$\begin{aligned} \bar{x}_e\dot{\bar{x}}_e &= \bar{x}_e v_r\cos\delta_e - \bar{x}_e v_c + y_e x_e\omega_c \\ &\quad - \lambda_1\varphi(\omega_c)y_e^2\omega_c - \frac{\lambda_1\bar{x}_e y_e}{1+\omega_c^2}\dot{\omega}_c \\ &\quad - \lambda_1\varphi(\omega_c)v_r\bar{x}_e\sin\delta_e + \lambda_1\varphi(\omega_c)\bar{x}_e x_e\omega_c. \end{aligned} \tag{13}$$

Substituting (8) and (13) into (11), we obtain

$$\begin{aligned} \dot{V}_0 &= \bar{x}_e v_r\cos\delta_e - \bar{x}_e v_c - \lambda_1\varphi(\omega_c)y_e^2\omega_c - \frac{\lambda_1\bar{x}_e y_e}{1+\omega_c^2}\dot{\omega}_c \\ &\quad - \lambda_1\varphi(\omega_c)v_r\bar{x}_e\sin\delta_e + \lambda_1\varphi(\omega_c)\bar{x}_e x_e\omega_c \\ &\quad + y_e v_r\sin\delta_e + \sin\frac{\delta_e}{2}(\omega_r - \omega_c). \end{aligned} \tag{14}$$

Considering the following fact

$$\sin\delta_e = 2\sin\frac{\delta_e}{2}\cos\frac{\delta_e}{2} \tag{15}$$

we have

$$\begin{aligned} \dot{V}_0 &= \bar{x}_e v_r\cos\delta_e - \bar{x}_e v_c - \lambda_1\varphi(\omega_c)y_e^2\omega_c - \frac{\lambda_1\bar{x}_e y_e}{1+\omega_c^2}\dot{\omega}_c \\ &\quad - \lambda_1\varphi(\omega_c)v_r\bar{x}_e\sin\delta_e + \lambda_1\varphi(\omega_c)\bar{x}_e x_e\omega_c \\ &\quad + \sin\frac{\delta_e}{2}\left(\omega_r - \omega_c + 2y_e v_r\cos\frac{\delta_e}{2}\right). \end{aligned} \tag{16}$$

According to (16), the desired velocity control law is designed as

$$z_c = \begin{bmatrix} \omega_c \\ v_c \end{bmatrix} \tag{17}$$

where $\omega_c = \omega_r + 2y_e v_r\cos(\delta_e/2) + \beta_1\sin(\delta_e/2)$ and $v_c = \beta_2\bar{x}_e + v_r\cos\delta_e - \lambda_1 y_e\dot{\omega}_c/(1+\omega_c^2) - \lambda_1\varphi(\omega_c)v_r\sin\delta_e + \lambda_1\varphi(\omega_c)x_e\omega_c$. $\beta_1 > 0$ and $\beta_2 > 0$ are positive design constants.

Substituting (17) into (16) yields

$$\dot{V}_0 = -\beta_1\sin^2\frac{\delta_e}{2} - \beta_2\bar{x}_e^2 - \lambda_1\varphi(\omega_c)y_e^2\omega_c. \tag{18}$$

According to the property of the arctan function and the definition of $\varphi(\omega_c)$, we know that

$$-\varphi(\omega_c)\omega_c \le 0. \tag{19}$$

Thus, we have

$$-\lambda_1\varphi(\omega_c)y_e^2\omega_c \le 0. \tag{20}$$

Invoking (18) and (20), we have

$$\dot{V}_0 \le -\beta_1\sin^2\frac{\delta_e}{2} - \beta_2\bar{x}_e^2 \le 0. \tag{21}$$

From (21), we can obtain that $\bar{x}_e \to 0$, $y_e \to 0$ and $\delta_e \to 0$ when $t \to \infty$. In accordance with (9), we know $x_e \to 0$ when $\bar{x}_e \to 0$ and $y_e \to 0$. Namely, the pose of self-balancing mobile robot can track the reference pose q_r under the desired velocity control law (17).

Now, the control object of this paper is to design the torque input C_θ and C_δ such that $v \to v_c$, $\omega \to \omega_c$ and $\theta \to 0$ as $t \to \infty$.

III. Robust Tracking Control Based on Nonlinear Disturbance Observer

In this section, the nonlinear disturbance observer is firstly designed. Then, the robust tracking control scheme considering the unknown disturbance will be designed for the self-balancing mobile robot. To proceed with the design of disturbance observer based tracking control scheme for the self-balancing mobile robot (1), the following lemma is required:

Lemma 1 [40]: For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\gamma_1(\|x\|) \leq V(x) \leq \gamma_2(\|x\|)$, such that $\dot{V}(x) \leq -\kappa V(x) + c$, where $\gamma_1, \gamma_2 : \mathbb{R}^n \to \mathbb{R}$ are class K functions, κ and c are positive constants, then the solution $x(t)$ is uniformly bounded.

A. Nonlinear Disturbance Observer Design

In order to further enhance the robustness performance of tracking control scheme, the nonlinear disturbance observer can be developed to estimate the system unknown disturbance $D(t)$. From a practical view, we can assume

$$\|\dot{D}\| \leq \varepsilon_1, \quad \|\ddot{D}\| \leq \varepsilon_2 \tag{22}$$

where ε_1 and ε_2 are positive constants. Throughout the paper, $\|\cdot\|$ denotes the 2-norm of a vector.

To estimate the unknown disturbance of the self-balancing mobile robot, the nonlinear disturbance observer is designed as [41], [42]

$$\begin{aligned}
\hat{D} &= \xi_1 + K_1(\bar{x}_2)\\
\dot{\xi}_1 &= -L_1(\bar{x}_2)(F(\bar{x}) + G(\bar{x})u + \hat{D}) + \hat{\dot{D}}\\
\hat{\dot{D}} &= \xi_2 + K_2(\bar{x}_2)\\
\dot{\xi}_2 &= -L_2(\bar{x}_2)(F(\bar{x}) + G(\bar{x})u + \hat{D})
\end{aligned} \tag{23}$$

where $\hat{D}$ and $\hat{\dot{D}}$ are the estimations of the compound disturbance D and the derivation of the compound disturbance $\dot{D}$, respectively. $K_1(\bar{x}_2)$ and $K_2(\bar{x}_2)$ are the design parameters of the developed nonlinear disturbance observer. The design parameters $L_1(\bar{x}_2)$ and $L_2(\bar{x}_2)$ should satisfy $L_1(\bar{x}_2) = \frac{\partial K_1(\bar{x}_2)}{\partial \bar{x}_2}$ and $L_2(\bar{x}_2) = \frac{\partial K_2(\bar{x}_2)}{\partial \bar{x}_2}$. ξ_1 and ξ_2 are the intermediate state variables of the developed nonlinear disturbance observer.

Define $\tilde{D} = D - \hat{D}$ and $\tilde{\dot{D}} = \dot{D} - \hat{\dot{D}}$. Invoking (23), we have

$$\begin{aligned}
\dot{\tilde{D}} &= \dot{D} - \dot{\hat{D}} = \dot{D} - \dot{\xi}_1 - L_1(\bar{x}_2)\dot{\bar{x}}_2\\
&= - L_1(\bar{x}_2)\tilde{D} + \tilde{\dot{D}}
\end{aligned} \tag{24}$$

$$\begin{aligned}
\dot{\tilde{\dot{D}}} &= \ddot{D} - \dot{\hat{\dot{D}}} = \ddot{D} - \dot{\xi}_2 - L_2(\bar{x}_2)\dot{\bar{x}}_2\\
&= - L_2(\bar{x}_2)\tilde{D} + \ddot{D}.
\end{aligned} \tag{25}$$

Define $\rho = [\tilde{D}^T, \tilde{\dot{D}}^T]^T$. Invoking (24) and (25) yields

$$\dot{\rho} = L(\bar{x}_2)\rho + \Upsilon\ddot{D} \tag{26}$$

where $L(\bar{x}_2) = \begin{bmatrix} -L_1(\bar{x}_2) & I_3 \\ -L_2(\bar{x}_2) & 0 \end{bmatrix}$ and $\Upsilon = \begin{bmatrix} 0 \\ I_3 \end{bmatrix}$.

In order to analyze the stability of the disturbance estimate error ρ, the Lyapunov function candidate is chosen as [33]

$$V_o = \frac{1}{2}\rho^T\rho. \tag{27}$$

Differentiating V_o and invoking (26), we have

$$\begin{aligned}
\dot{V}_o &= \rho^T\dot{\rho} = \rho^T(L(\bar{x}_2)\rho + \Upsilon\ddot{D})\\
&\leq \rho^T(L(\bar{x}_2) + 0.5\|\Upsilon\|^2 I_6)\rho + 0.5\varepsilon_2^2.
\end{aligned} \tag{28}$$

The above nonlinear disturbance observer design procedure for the self-balancing mobile robot can be summarized in the following theorem.

Theorem 1: Considering the self-balancing mobile robot (1) with the unknown disturbance, the nonlinear disturbance observer is designed according to (23). Then, the disturbance estimate error $\tilde{D}$ is bounded for the developed nonlinear disturbance observer.

Proof: From (28), we know that the disturbance estimate ρ is bounded according to Lemma 1 if the design parameters $L_1(\bar{x}_2) = \frac{\partial K_1(\bar{x}_2)}{\partial \bar{x}_2}$ and $L_2(\bar{x}_2) = \frac{\partial K_2(\bar{x}_2)}{\partial \bar{x}_2}$ are chosen such that $L(\bar{x}_2) + 0.5\|\Upsilon\|^2 I_6$ is a negative definite matrix. ■

Remark 1: In our developed nonlinear disturbance observer, the design function vectors $K_1(\bar{x}_2)$ and $K_2(\bar{x}_2)$ should be carefully chosen to guarantee the convergence of the disturbance estimate error. In order to facilitate the analysis, the design functions $K_1(\bar{x}_2)$ and $K_2(\bar{x}_2)$ can be chosen as the linear function vectors to obtain the constant matrices L_1 and L_2.

B. Tracking Control Scheme Design

Based on the output of the developed nonlinear disturbance observer, the robust tracking control scheme can be designed using the sliding mode control method.

Firstly, the robust tracking control scheme will be designed for the first subsystem. Define

$$\sigma_1 = \dot{x} - v_c \tag{29}$$

$$\sigma_2 = \dot{\theta} + \lambda_2\theta \tag{30}$$

where $\lambda_2 > 0$ is a design parameter.

The sliding mode surface of the first subsystem is chosen as

$$\sigma = \sigma_1 + \lambda_3\sigma_2 \tag{31}$$

where $\lambda_3 > 0$ is a design parameter.

Considering (2), we have

$$\begin{aligned}
\dot{\sigma} = {}& f_1 + g_{11}C_\theta + d_1 - \dot{v}_c\\
&+ \lambda_3(f_2 + g_{21}C_\theta + d_2 + \lambda_2\dot{\theta}).
\end{aligned} \tag{32}$$

According to (32), the robust tracking control scheme of the first subsystem is designed as

$$C_\theta = \frac{-k_1\sigma - k_2\text{sign}(\sigma) + \chi - \hat{d}_1 - \lambda_3\hat{d}_2}{g_{11} + \lambda_3 g_{21}} \tag{33}$$

where $\chi = -f_1 - \lambda_3 f_2 + \dot{v}_c + \lambda_2\lambda_3\dot{\theta}$. k_1 and k_2 are the positive design constants.

Substituting (33) into (32) yields

$$\dot{\sigma} = -k_1\sigma - k_2\text{sign}(\sigma) + d_1 - \hat{d}_1 + \lambda_3(d_2 - \hat{d}_2). \tag{34}$$

Define $\tilde{d}_1 = d_1 - \hat{d}_1$ and $\tilde{d}_2 = d_2 - \hat{d}_2$. Then, we have

$$\dot{\sigma} = -k_1\sigma - k_2\text{sign}(\sigma) + \tilde{d}_1 + \lambda_3\tilde{d}_2. \tag{35}$$

From (35), we obtain

$$\begin{aligned} \sigma\dot{\sigma} &= -k_1\sigma^2 - k_2|\sigma| + \sigma\tilde{d}_1 + \lambda_3\sigma\tilde{d}_2 \\ &\leq -(k_1 - 0.5 - 0.5\lambda_3^2)\sigma^2 - k_2|\sigma| + \|\rho\|^2. \end{aligned} \tag{36}$$

For the second subsystem, the sliding mode surface is chosen as

$$s = \omega - \omega_c \tag{37}$$

where $\omega = \dot{\delta}$.

Invoking (3) and (37) yields

$$\dot{s} = \dot{\omega} - \dot{\omega}_c = g_{32}C_\delta + d_3 - \dot{\omega}_c. \tag{38}$$

Using the output of the nonlinear disturbance observer, the robust tracking control scheme of the second subsystem is designed as

$$C_\delta = \frac{-k_3 s - k_4\text{sign}(s) - \hat{d}_3 + \dot{\omega}_c}{g_{32}} \tag{39}$$

where k_3 and k_4 are positive design constants.

Substituting (39) into (38) yields

$$\dot{s} = -k_3 s - k_4\text{sign}(s) + d_3 - \hat{d}_3. \tag{40}$$

Define $\tilde{d}_3 = d_3 - \hat{d}_3$. Then, we obtain

$$\dot{s} = -k_3 s - k_4\text{sign}(s) + \tilde{d}_3. \tag{41}$$

Considering (41), we have

$$\begin{aligned} s\dot{s} &= -k_3 s^2 - k_4|s| + s\tilde{d}_3 \\ &\leq -(k_3 - 0.5)s^2 - k_4|s| + 0.5\|\rho\|^2. \end{aligned} \tag{42}$$

The robust tracking control scheme design for the self-balancing mobile robot with the unknown disturbance can be summarized in the following theorem.

Theorem 2: Considering the self-balancing mobile robot (1) with the unknown disturbance, the nonlinear disturbance observer is designed according to (23) and the robust control scheme is designed as (33) and (39). Then, the tracking error is bounded under the developed disturbance observer based robust tracking control scheme.

Proof: To analyze the closed-loop system stability, consider the following Lyapunov function candidate:

$$V = V_o + \frac{1}{2}\sigma^2 + \frac{1}{2}s^2 = \frac{1}{2}\sigma^2 + \frac{1}{2}s^2 + \frac{1}{2}\rho^T\rho. \tag{43}$$

Consider (28), (36) and (42). Differentiating V yields

$$\begin{aligned} \dot{V}_o &= \sigma\dot{\sigma} + s\dot{s} + \rho^T\dot{\rho} \\ &\leq -(k_1 - 0.5 - 0.5\lambda_3^2)\sigma^2 - k_2|\sigma| + \|\rho\|^2 \\ &\quad - (k_3 - 0.5)s^2 - k_4|s| + 0.5\|\rho\|^2 \\ &\quad + \rho^T(L(\bar{x}_2) + 0.5\|\Upsilon\|^2 I_6)\rho + 0.5\varepsilon_2^2 \\ &\leq -(k_1 - 0.5 - 0.5\lambda_3^2)\sigma^2 - (k_3 - 0.5)s^2 \\ &\quad + \rho^T(L(\bar{x}_2) + (1.5 + 0.5\|\Upsilon\|^2)I_6)\rho + 0.5\varepsilon_2^2 \\ &\leq -\kappa V + C \end{aligned} \tag{44}$$

where

$$\kappa = \min\begin{pmatrix} k_1 - 0.5 - 0.5\lambda_3^2 \\ k_3 - 0.5 \\ \lambda_{\min}(L(\bar{x}_2) + (1.5 + 0.5\|\Upsilon\|^2)I_6) \end{pmatrix} > 0 \tag{45}$$

$$C = 0.5\varepsilon_2^2 > 0. \tag{46}$$

From (44), we know that all the closed-loop system signals are bounded under the developed robust controller (33) and (39) according to Lemma 1. Thus, the sliding mode surfaces σ, s and the disturbance estimate error ρ are bounded. From (29), (30), (31) and (37), we obtain that the self-balancing mobile robot can track the desired trajectory under the effect of unknown disturbance. ■

IV. Simulation Results

The simulation results are presented to illustrate the effectiveness of the proposed disturbance observer based robust tracking control scheme of the self-balancing mobile robot. The used self-balancing mobile robot is produced by Googol Technology Consulting Inc. The main parameters of the studied self-balancing mobile robot are given in Table I.

TABLE I
System Parameters of the Self-balancing Mobile Robot

Symbol	Name	Value
M_r	Wheel mass	0.42 kg
M_p	Car body mass	20.82 kg
R	Wheel radius	0.09 m
L	Distance between the chassis and the center	0.2 m
$\bar{D}$	Distance between two wheels	0.438 m

In the simulation study, the unknown time-varying disturbances are considered as follows:

$$d_1(t) = -0.2\sin(0.6t)\text{Nm}$$

$$d_2(t) = -0.5\cos(0.8t)\text{Nm}$$

$$d_3(t) = -0.3\sin(0.5t)\text{Nm}.$$

In the nonlinear disturbance observer design, the function vectors $K_1(x)$ and $K_2(x)$ are chosen as $K_1(x) = [5\dot{x}, 5\dot{\theta}, 5\dot{\delta}]^T$ and $K_2(x) = [10\dot{x}, 5\dot{\theta}, 15\dot{\delta}]^T$. Thus, we can obtain $L_1 = \text{diag}\{5, 5, 5\}$ and $L_2 = \text{diag}\{10, 5, 15\}$. For the tracking control scheme design, all the other design parameters are chosen as $\lambda_1 = 0.185$, $\lambda_2 = 5$, $\lambda_3 = 0.85$, $\beta_1 = 1$, $\beta_2 = 1$, $k_1 = 10$, $k_2 = 1$, $k_3 = 10$, $k_4 = 1$. The nonlinear disturbance observer is designed according to (23) and the robust control scheme is designed as (33) and (39). To illustrate the effectiveness of the proposed disturbance observer based robust tracking control scheme, two cases are given. In Case 1, we set the desired trajectory as a line signal. In Case 2, we set the desired trajectory as a circle signal.

A. Simulation Results of Case 1

The desired tracking signal comprises two components: $v_r = 0.2\,\text{m/s}$ and $\omega_r = 0\,\text{rad/s}$. The initial conditions are chosen as $\hat{D} = [0, 0, 0]^T$ and $q_{r0} = [0.28, 0.52, 0]^T$. The tracking control result of the self-balancing mobile robot under the proposed control scheme is shown in Fig. 1 and the pitch angle and yaw angle responses are given in Fig. 2. Although the self-balancing mobile robot is subject to the unknown disturbance, the position efficiently tracks the corresponding desired line signal. At the same time, the zero requirement of pitch angle is guaranteed. Fig. 3 shows the pose errors of the self-balancing mobile robot and they are convergent and bounded. The disturbance estimate errors of the designed nonlinear disturbance observer are given in Fig. 4 which are also convergent. From Figs. 1−4, we can see that the developed disturbance observer-based robust tracking control scheme is effective for tracking desired line trajectory of the self-balancing mobile robot.

B. Simulation Results of Case 2

The desired tracking signals are chosen as $v_r = 0.2\,\text{m/s}$ and $\omega_r = 0.1\,\text{rad/s}$. In the simulation study, the initial conditions are chosen as $\hat{D} = [0, 0, 0]^T$ and $q_{r0} = [0.5, -0.2, \pi/3]^T$. The tracking control result of the self-balancing mobile robot is shown in Fig. 5. The position of the self-balancing mobile robot effectively tracks the corresponding desired circle tracking trajectory even in the presence of the unknown disturbance. In Fig. 6, the pitch angle and yaw angle responses are presented. From Fig. 6, we can note that the pitch angle converges to zero and satisfies the requirement. Fig. 7 shows that the pose errors of the self-balancing mobile robot are convergent and bounded, and Fig. 8 shows that the disturbance estimate errors of the developed disturbance observer are convergent. These results further validate that the developed disturbance observer based tracking control scheme is effective for tracking the circle trajectory of the self-balancing mobile robot with unknown disturbances.

Based on these simulation results of Case 1 and Case 2, we verify that the proposed disturbance observer based robust tracking control scheme is valid for the self-balancing mobile with the unknown disturbance.

V. Conclusion

For the self-balancing mobile with external unknown bounded disturbances, a robust tracking control scheme has been proposed. The desired velocity control law has been designed using the Lyapunov analysis method and the arctan

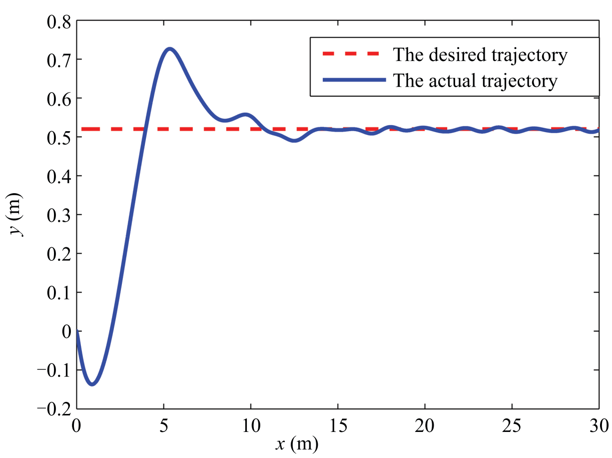

Fig. 1. Tracking control result of the line trajectory.

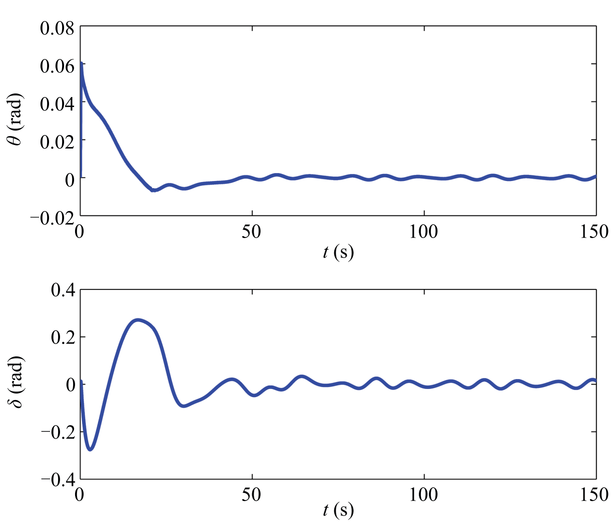

Fig. 2. Pitch angle and yaw angle responses of Case 1.

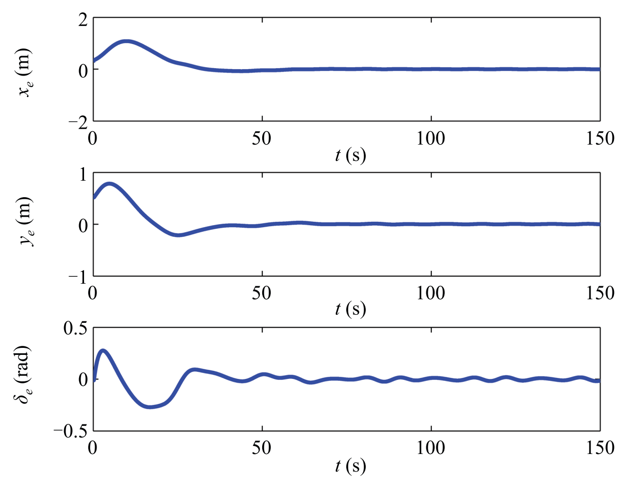

Fig. 3. The pose errors of Case 1.

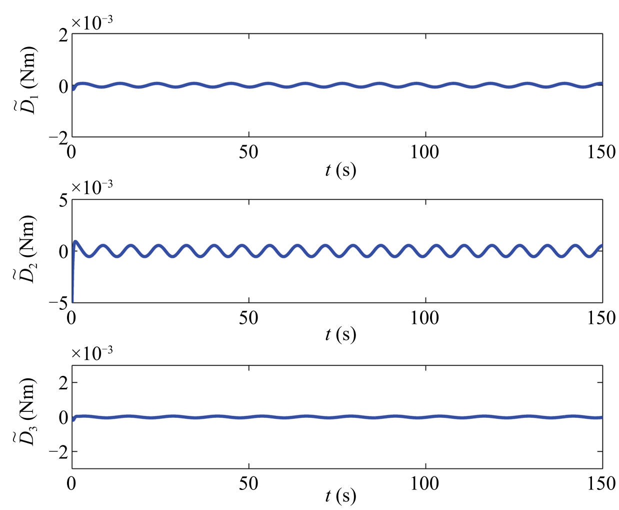

Fig. 4. The disturbance estimate errors of Case 1.

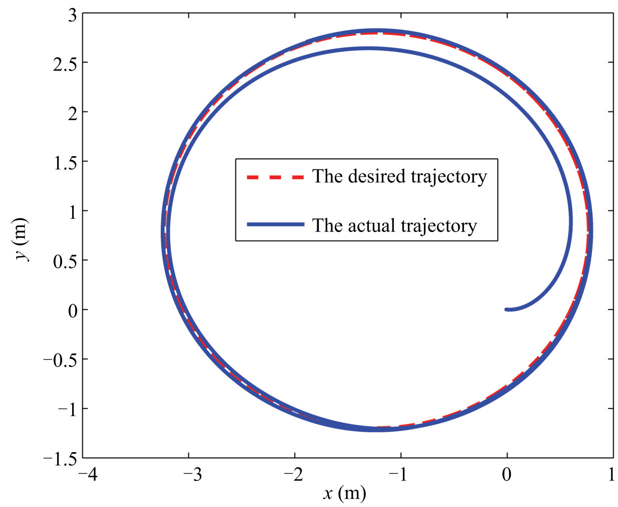

Fig. 5. Tracking control result of the circle trajectory.

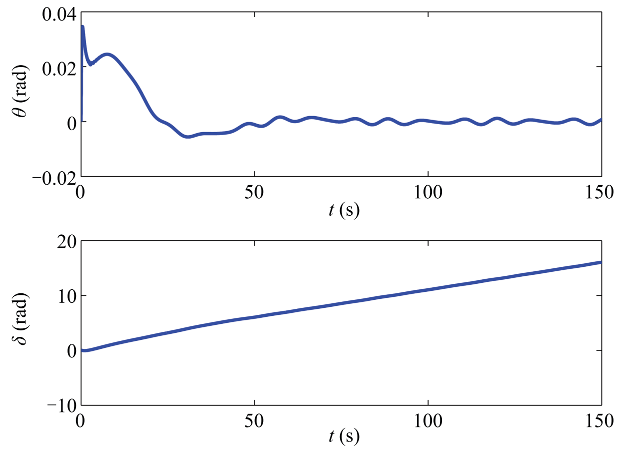

Fig. 6. Pitch angle and yaw angle responses of Case 2.

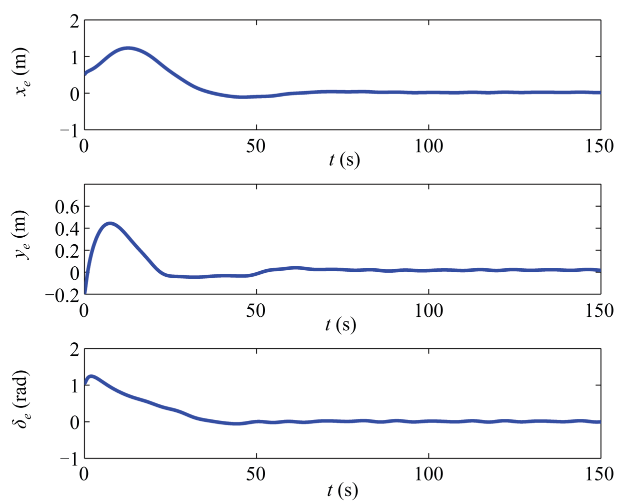

Fig. 7. The pose errors of Case 2.

function. At the same time, a nonlinear disturbance observer has been developed to tackle the unknown disturbance. By using the output of the nonlinear disturbance observer, the tracking control scheme has been designed using the sliding mode technique to guarantee that all the closed-loop signals are ultimately uniformly bounded. The self-balancing mobile robot produced by Googol Technology Consulting Inc has been used to illustrate the effectiveness of the developed disturbance observer based control scheme by numerical simulations and the simulation results have shown that a good tracking performance has been achieved. In future work, the experimental study will be done for the self-balancing mobile robot.

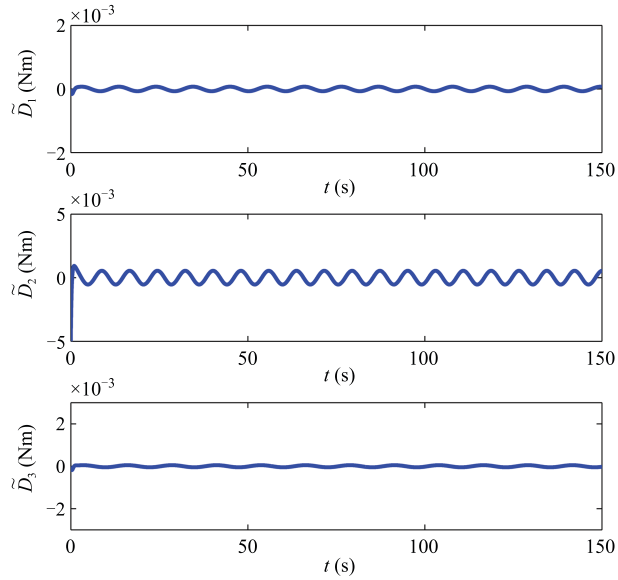

Fig. 8. The disturbance estimate errors of Case 2.

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