Design and Robust Performance Evaluation of a Fractional Order PID Controller Applied to a DC Motor

J. Viola, L. Angel, and J. M. Sebastian

***Abstract*—This paper proposes a methodology for the quantitative robustness evaluation of PID controllers employed in a DC motor. The robustness analysis is performed employing a 2^3 factorial experimental design for a fractional order proportional integral and derivative controller (FOPID), integer order proportional integral and derivative controller (IOPID) and the Skogestad internal model control controller (SIMC). The factors assumed in experiment are the presence of random noise, external disturbances in the system input and variable load. As output variables, the experimental design employs the system step response and the controller action. Practical implementation of FOPID and IOPID controllers uses the MATLAB stateflow toolbox and a NI data acquisition system. Results of the robustness analysis show that the FOPID controller has a better performance and robust stability against the experiment factors.**

***Index Terms*—Factorial experimental design, fractional-order PID controller, robustness analysis, SIMC PID controller.**

I. Introduction

THE robustness of a control strategy is the capacity to reach the desired operation specifications despite the presence of external disturbances and parametric uncertainties of the model guaranteeing a robust performance and robust stability.

The classic control strategies as the proportional, integral and derivative (PID) controller are widely implemented to control different systems due to its simplicity in the tuning and implementation. The PID controller provides good features in the face of disturbances that affect the system dynamics, as long as the magnitude of the disturbances does not further alter the operation point of the feedback system. Some advanced control strategies such as quantitative feedback theory (QFT) [1], [2], H-infinity [3], [4], loop shaping [5] and adaptive control [6] can be used for designing control systems to deal process with robustness problems, however, their implementation is quite complex.

In the recent years and given a better understanding of the fractional calculus [7], [8], a control strategy known as fractional control has been presented suggesting the use of non-integer operators for the modeling and design of control systems [9]−[11]. Some applications of fractional control [12]−[19] exhibit a good performance in the face of external disturbances and parametric uncertainties, with good results in the control of industrial processes. Although the effect of the disturbances has been analyzed considering performance indexes as the integral square error (ISE) or the integral of time multiply by absolute error (ITAE) when the fractional-control strategy is compared against the classic PID control [20]−[23], a quantitative analysis, capable to identify which of these disturbances further affects the stability and the performance of each control strategy, has not been conducted.

This paper aims to perform a robustness analysis comparing the IOPID, SIMC PID [24] and FOPID applied to the speed control of a motor-generator set. The robustness analysis aims to describe how the presence of disturbances affects the performance and stability of each one of the PID control strategies.

Usually the performance and robust stability evaluation of a fractional control system is based on the graphical analysis of the unit step response [25] or the graphical analysis of the frequency response of the sensitivity and complementary sensitivity functions of the system [9]−[11]. The proposed method in this paper quantifies the effect of the external disturbances (random noise, external perturbation and the presence of uncertain load) and its possible combinations on the output variables of the system (unit step response and the control action of the closed loop system) using statistical methods and the analysis of variance (ANOVA). This methodology gives the percentage of incidence of each perturbation over the system outputs.

A factorial experimental design 2^3 is used as tool for the robustness analysis which utilizes as input factors the presence of random noise in the feedback loop, the existence of an external disturbance to the process input and the presence of uncertain load. The effect of these factors in the stability and the performance of each control strategy is measured using the root mean square error (RMSE) criterion and the standard deviation of the step response and the root mean square (RMS) value and the standard deviation of the control action of the system. The factorial experimental design has been implemented for the behavior analysis of biological [26], chemical [27], [28] and communications [29], [30] processes

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J. Viola and L. Angel are with the Universidad Pontificia Bolivariana, Bucaramanga 681004, Colombia (e-mail: jairo.viola@upb.edu.co; luis.angel@upb.edu.co).

J. M. Sebastian is with the Universidad Politecnica de Madrid, Centre for Automation and Robotics (CAR) UPM-CSIC, Madrid 28006, Spain (e-mail: jsebas@etsii.upm.es).

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as its application in control of processes is very limited.

The main contribution of this paper is the proposal of a new method for a quantitative robustness evaluation of the control strategies FOPID, IOPID and SIMC PID using a factorial experimental design 2^k. As additional contributions of this paper are the design and digital implementation of the FOPID controller and its validation in a real prototype.

This paper is structured as follows: the first part presents the basic concepts of the fractional calculus and its application to the PID control theory. Then, the principal aspects of the SIMC PID and IOPID controllers are introduced. Subsequently the design techniques for tuning the FOPID, SIMC PID and IOPID controllers are established. It is followed by the description of the system to control (motor-generator), the model identification, the design of the FOPID, SIMC PID and IOPID controllers and their digital implementation employing the stateflow toolbox from MATLAB. Later the robustness analysis for the implemented control systems is performed using a factorial experimental design 2^3. For this analysis, the experimental data is initially acquired from each PID control system considering the effect of every factor (presence of random noise in the feedback loop, the existence of an external disturbance to the process input and the presence of uncertain load) in the system step response and in the control signal. For the step response the RMSE (root mean square error) value and the standard deviation (SD OUT) are computed, whereas for the control signal the RMS (root mean square) value and the standard deviation (SD CONTROL) will be determined. Then, the experiments design is performed using the DESIGN EXPERT software and based on it, the percentage of incidence of every factor to the control strategy is established. Finally, the analysis of the results and the obtained conclusions are exposed.

II. Preliminary Concepts

A. FOPID Controller

In the theory of fractional calculus [7], [8], notation used to represent the derivative and integral of fractional order is given by (1), where α represents the fractional order of the derivative and integral, which are non-integers. $f(t)$ represents the function to derive or integrate.

$$D_t^\alpha f(t) = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} f(t), & a > 0 \\ f(t), & a = 0 \\ \dfrac{d^\alpha}{dt^\alpha} = I^\alpha f(t), & a < 0. \end{cases} \quad (1)$$

FOPID controller transfer function is based on the widespread definition presented in (1) which can be represented by the Riemann-Lieuville (R-L) and Grunwald-Letnikov (G-L) approximations.

R-L definition for $\alpha > 0$ is given by:

$$D_t^\alpha f(t) = \frac{1}{1-\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (2)$$

for $(n-1 < \alpha < n)$ and where $\Gamma()$ is the gamma function.

The definition of G-L is given by (3)

$$D_t^\alpha f(t) = \lim_{t\to 0} (\frac{1}{h^\alpha}) \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} f(t-jh) \quad (3)$$

where

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}.$$

Assuming zero initial conditions, the Laplace transform for the fractional-order derivative of $f(t)$, is defined in (4).

$$LD^{\pm\alpha} f(t) = s^{\pm\alpha} F(s). \quad (4)$$

According to (1), it is possible to define the transfer function of the FOPID controller through the following integral differential equation:

$$u(t) = k(e(t) + \frac{1}{T_i} D^{-\lambda} e(t) + T_d D^\mu e(t)) \quad (5)$$

where k is the proportional constant, T_i is the integral time constant, T_d is the derivative time constant, λ is the order (non-integer) of the integral term, μ corresponds to the order (non-integer) of the derivative term, $e(t)$ represents the error signal and $u(t)$ is the control action signal. Based on (5) and applying the Laplace transform of fractional order defined in (4), the transfer function of the FOPID controller can be expressed as:

$$u(s) = \frac{u(s)}{e(s)} = k\left(1 + \frac{1}{T_i} s^{-\lambda} + T_d s^\mu\right). \quad (6)$$

As presented in (6), the FOPID controller has five tuning parameters, which gives a greater flexibility in the design stage but becomes more complex obtaining the parameters of the controller.

B. SIMC PID Controller

The SIMC tuning technique is based on the internal model control (IMC) theory and proposes a simple methodology for the tuning of the PI and PID industrial controllers. This design methodology is used specially for the control of systems of first and second order with delay. As shown in [24], the principal advantages of this design technique are the simplicity in the determination of the parameters of the PID controllers for first and second order systems and the robustness of the control system facing changes in the set point, presence of external disturbances and random noise. According to [24], the structure of the SIMC PID controller is given by the interacting form of the PID controller.

$$G_c(s) = k'\left(1 + \frac{1}{T_i s}\right)(1 + T_d' s). \quad (7)$$

In order to use the parallel form of the PID controller, the transformation factor $f = 1 + T_d'/T_i'$ is used, so the controller parameters are defined as:

$$\begin{aligned} k &= k' f \\ T_i &= T_i' f \\ T_d &= \frac{T_d'}{f}. \end{aligned} \quad (8)$$

III. TECHNIQUES FOR CONTROLLERS DESIGN

A. Design of FOPID Controller in the Frequency Domain

The design methodology in the frequency domain for fractional controllers starts from a linear model of the system $G_p(s)$ and the transfer function of the controller $G_c(s)$, which must satisfy the following operations specifications:

1) Phase margin (pm):

$$\arctan(G_c(jw)G_p(jw)) = -\pi + pm. \tag{9}$$

2) Gain crossover frequency (w_c):

$$|G_c(s)G_p(s)| = 0 \text{ dB}. \tag{10}$$

3) Robustness against variation of plant gain:

$$\frac{d}{dw}\arctan(G_c(jw)G_p(jw)) = 0. \tag{11}$$

4) Rejection of high frequency noise:

$$\left|\frac{G_c(jw)G_p(jw)}{1+G_c(jw)G_p(jw)}\right| = B \text{ dB}. \tag{12}$$

5) Rejection of output disturbances:

$$\left|\frac{1}{1+G_c(jw)G_p(jw)}\right| = A \text{ dB}. \tag{13}$$

6) Controller saturation: It is considered due to the actuators in the real systems have physical limitations that cannot be exceeded. For the motor-generator system, the control action is limited between ± 10 V.

The desired design specifications for the FOPID controller are the phase margin (pm), the gain cross over frequency (w_C), the magnitude of the sensitivity function (A) and the magnitude of the complementary sensitivity function (B). In order to solve the equation system described in (9)$-$(13), it is necessary to use optimization methodologies.

B. Design of SIMC PID Controller

For the second-order system with delay defined in (14), Reference [24] proposes the design of a PID controller with the constants defined by (15).

$$G_p(s) = \frac{k_p}{(T_1 s+1)(T_2 s+1)}e^{-\theta s} \tag{14}$$

$$k' = \frac{1}{k_p}\frac{1}{T_c+\theta}$$

$$T_i' = \min\left[T_1, 4(T_c+\theta)\right]$$

$$T_d' = T_2. \tag{15}$$

As shown in (15), T_c is the only tuning parameter. According to [24], it is suggested to make T_c the same as the delay time θ, guaranteeing a robust behavior and a great following to changes in the set point. Once the parameters have been determined using (15), it is possible to obtain the parameters of the parallel PID controller employing (8).

C. Design of the IOPID Controller in the Frequency Domain

Constants from the IOPID controller can be obtained through design techniques in the frequency domain solving the equation system established by (9)$-$(11).

IV. DESCRIPTION AND IDENTIFICATION OF THE SYSTEM

A. System to be Controlled

The system to be controlled is shown in Fig. 1. It corresponds to a DC motor-generator set formed of two DC motors Bodine brand with 2500 RPM and nominal voltage ± 130 V DC. In Fig.1, motors are shown using the label 1. One motor works as motor and the other motor is used as generator and is mechanically coupled to the first motor and a taco-generator 2 with ± 7 V output. The power driver 3 with ± 10 V input regulates the motor tension. The generator's maximum voltage output is ± 90 V with a current of 2.3 A. Control system is composed of the data acquisition card NI DAQ 6008 4 and the MATLAB-Simulink interface 5. The resistive loads shown in 6 are employed as uncertain load.

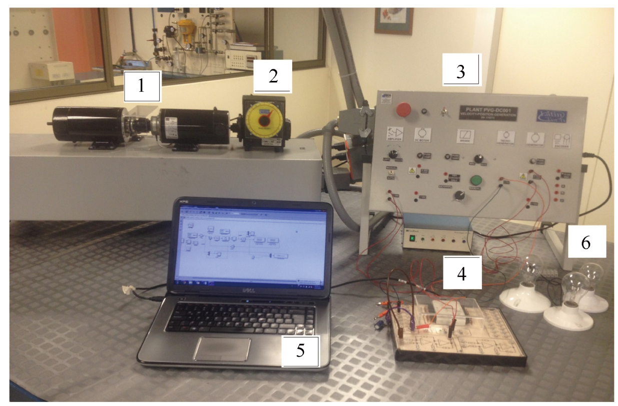

Fig. 1. Motor-generator system using the MATLAB-Simulink control scheme.

B. System Identification

Data acquisition card NI-DAQ 6008 from National Instruments is employed to get the identification data of the motor-generator system. Fig. 2 shows the input and output data employed in the identification process. Using a staggered signal as input, the motor is driven into different speeds including changes in direction of rotation. Taco-generator voltage ± 7 V is used as output signal. Acquired data is introduced in the MATLAB identification toolbox to obtain

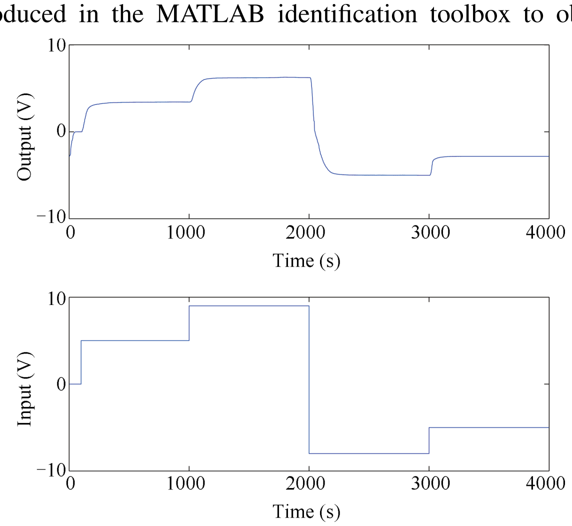

Fig. 2. Staggered input signal for the identification and system response.

an approximated linear model from the system, which corresponds to a second-order system with delay given by (16). The identificated model has a fit of 90 %.

$$G_p(s) = \frac{0.63606}{(42.77s+1)(7.45s+1)}e^{-0.61s}. \quad (16)$$

V. Design and Implementation of the PID Controllers

From the identified second order model of the system (16) PID controllers design is performed. The proposed design specifications for the feedback system are a gain margin of 10 dB, a gain crossing frequency 0.033 rad/s, a phase margin of 60, the magnitude of the sensitivity function of $A = -20$ dB, the magnitude of the complementary sensitivity function of $B = -20$ dB and saturation limits ±10 V. The constants of the FOPID, SIMC PID and IOPID controllers are obtained using the design techniques described in Section III. The constants for every PID controller are shown in Table I.

TABLE I
Controllers Constants

Controller	k	T_i	T_d	λ	μ
FOPID	0.133	2.5	0.76	0.89	0.44
SIMC PID	0.33	4.7	0.6	1	1
IOPID	0.145	5.98	0.005	1	1

A. Controllers Discretization

1) FOPID Controller: In order to get the practical implementation of a controller in a specific hardware, it is necessary to find its discrete model expressed in terms of a difference equation. There are approximations that employ numerical differentiation techniques that allow approximating the derivative operator. Among these techniques the most commonly used are, inter alia, forward difference, backward difference and bilinear transform. However, for fractional controllers it is necessary to redefine the discretization techniques previously described so as to convert the non-integer order term to the discrete domain.

According to [31] the fractional derivative with order can be represented in a discrete form, using Tustin definition as:

$$s^\beta = \left(\alpha^\beta \frac{1-z^{-1}}{1+z^{-1}}\right)^\beta \quad (17)$$

with

$$\alpha^\beta = \frac{w_c}{\tan(\frac{w_c T}{2})}$$

where T is the sampling time and w_c is the gain crossover frequency chosen for FOPID tuning. However, in order to determine the controller difference equation, a Taylor series approximation for (17) is required. This approximation is given by:

$$\left(\alpha^\beta \frac{1-z^{-1}}{1+z^{-1}}\right)^\beta = \alpha^\beta \sum_{k=0}^{N} f_k(\beta) w^k \quad (18)$$

where $w = z^{-1}$, N is the order of truncation of the Taylor series and $f_k(\beta)$ is:

$$f_k(\beta) = \frac{1}{k!}\frac{d^k}{dw^k}\left(\frac{1-w}{1+w}\right)^\beta \Bigg|_{w=0}.$$

In this paper, s^β operator has an order of truncation N = 6, whose $f_k(\beta)$ coefficients are shown in Table II.

TABLE II
Controllers Constants

Constant	$f_k(\beta)$	Coefficient
a_6	$f_0(\beta)$	1
a_5	$f_1(\beta)$	-2β
a_4	$f_2(\beta)$	$-2\beta^2$
a_3	$f_3(\beta)$	$-\left(\frac{4}{3}\beta^3 + \frac{2}{3}\beta\right)$
a_2	$f_4(\beta)$	$+\left(\frac{2}{3}\beta^4 + \frac{4}{3}\beta^2\right)$
a_1	$f_5(\beta)$	$-\left(\frac{4}{15}\beta^5 + \frac{4}{3}\beta^3 + \frac{2}{5}\beta\right)$
a_0	$f_6(\beta)$	$-\left(\frac{4}{45}\beta^6 + \frac{8}{9}\beta^4 + \frac{46}{45}\beta^2\right)$

In order to reduce the infinite gain of integral term $s^{-\lambda}$ of the FOPID controller, it is necessary to rewrite this term as:

$$s^{-\lambda} = \frac{1}{s}s^{1-\lambda}. \quad (19)$$

Applying Taylor series expansion in (19), integral term can be approximated as:

$$s^{-\lambda} = \alpha^{-\lambda}\frac{z+1}{z-1}\sum_{k=0}^{N} f_k(1-\lambda)w^{-k} \quad (20)$$

where $w = z^{-1}$, $(z+1)/(z-1)$ corresponds to the Tustin approximation for the integral term $1/s$ and N to the truncation order. For the evaluation of the parameter $f_k(1-\lambda)$, coefficients shown in Table II are used with $\beta = 1-\lambda$. For the derivative term, coefficients of Table II are used with $\beta = \mu$. Replacing (18) and (20) in (6), FOPID discrete structure is given by:

$$\frac{u(z)}{e(z)} = k_p + k_i\alpha^{-\lambda}\frac{z+1}{z-1}\sum_{k=0}^{N} f_k(1-\lambda)w^{-k} + k_d\left(\alpha^\mu \sum_{k=0}^{N} f_k(\mu)w^k\right). \quad (21)$$

Expressing (20) as:

$$u(z) = u_p(z) + u_i(z) + u_d(z) \quad (22)$$

with

$$u_p(z) = ke(z)$$
$$u_i(z) = \left[k_i\alpha^{-\lambda}\frac{z+1}{z-1}\sum_{k=0}^{N} f_k(1-\lambda)w^{-k}\right]e(z)$$
$$u_d(z) = \left[k_d\left(\alpha^\mu \sum_{k=0}^{N} f_k(\mu)w^k\right)\right]e(z). \quad (23)$$

Replacing the Taylor series approximation coefficients shown in Table II at (23), the terms $u_p(k)$, $u_i(k)$ and $u_d(k)$ in (24) are given by:

$$\begin{aligned} u_p(k) &= ke(k)\\ u_i(k) &= \left[\frac{k_i}{\alpha^\lambda}(e(k)-e(k-1))\right]\\ &\quad -(a_5-1)u_1(k-1)-(a_4-a_5)u_i(k-2)\\ &\quad -(a_3-a_4)u_i(k-3)-(a_2-a_3)u_i(k-4)\\ &\quad -(a_1-a_2)u_i(k-5)-(a_0-a_1)u_i(k-6)\\ &\quad +a_0u_i(k-7)\\ u_d(k) &= k_d\alpha^\mu[(e(k)-a_5e(k-1))\\ &\quad +a_4e(k-2)+a_3e(k-3)+a_2e(k-4)\\ &\quad +a_1e(k-5)+a_0e(k-6). \end{aligned} \tag{24}$$

The structure presented in (23) allows to employ the anti-windup technique in the integral term and facilitates the implementation of the control law into a specific hardware.

2) IOPID and SIMC PID Controllers: For the practical implementation of these controllers, the classic numeric differentiation techniques of the derivative operator were used such as Tustin, forward difference or backward difference; in which the difference equation is obtained using the same methodology described in the previous section.

B. Controller's Practical Implementation

Response of the FOPID, SIMC PID and IOPID controllers to a staggered input signal is shown in Fig. 3. This input signal corresponds to variations in the speed of the motor from 0 RPM to 2500 RPM with changes in direction of rotation. As Fig. 3 (a) shows, FOPID controller performs a better following task, IOPID controller reaches the maximum overshoot and the SIMC PID controller has maximum settling time. From Fig. 3 (b), the control action of FOPID controller reaches its steady-state value in less time. Meanwhile, the control action of the SIMC PID controller is further affected by the random noise.

Fig. 4 presents the implementation of the feedback control system utilizing MATLAB-Simulink. It is possible to observe: the input ports (feedback signal obtained from the tacogenerator) and output from the data acquisition card (control signal to manipulate the motor driver) D1, the controller implemented through the stateflow toolbox D2, the interface to export data for later analysis D3, and the generator of perturbations to the input of the process D4. The practical implementation of the FOPID, SIMC PID and IOPID controllers is developed employing the stateflow toolbox. As an example, Fig. 5 shows the digital implementation of the FOPID controller according to (23) and (24). It is important to highlight the antiwind-up scheme for the integral action (dotted line), the high-frequency filter for the derivative action and the saturation limits of the actuator.

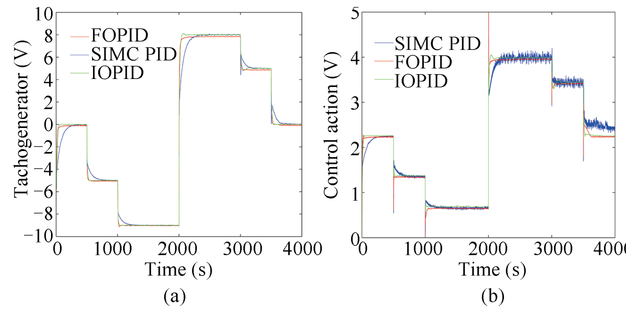

Fig. 3. Response to step input and control action from the FOPID, SIMC PID and IOPID controllers.

VI. Factorial Experimental Design 2^3 and Robustness Analysis

Robustness of a control system is defined as the capacity of the controller to ensure robust performance and robust stability of the system given the presence of external disturbances and random noise. This paper proposes the robustness analysis of the control system presented in the Section IV using the FOPID, SIMC PID and IOPID controllers and considering its dynamical behavior given the presence of external disturbances. For this robustness analysis, a factorial experimental design 2^k with $k = 3$ factors [32] is proposed. The following factors have been taken into consideration: the presence of random noise in the feedback loop (A), the existence of external disturbance in the input of the process (B) and uncertainty in the load (C). Two levels have been considered for each factor (presence or absence). The robustness of the

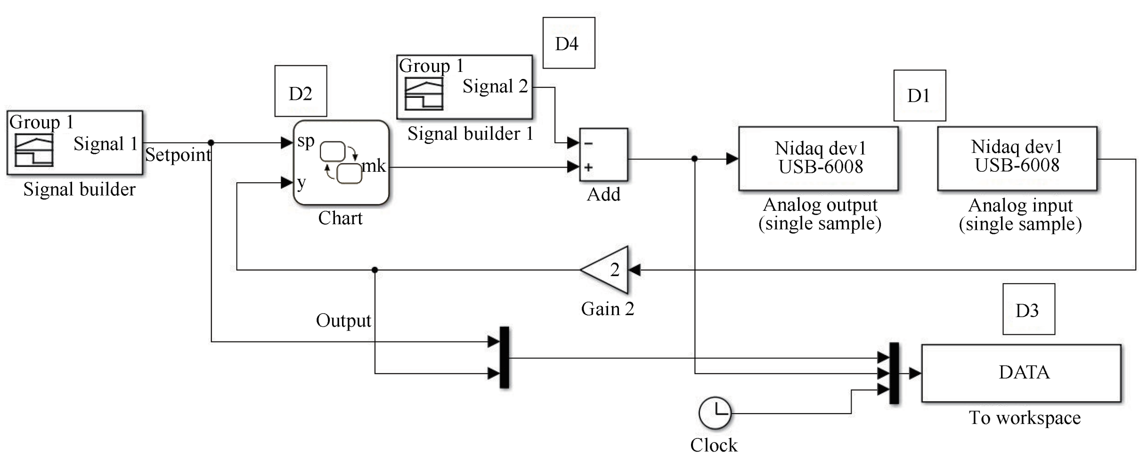

Fig. 4. Feedback control system implemented in MATLAB-Simulink.

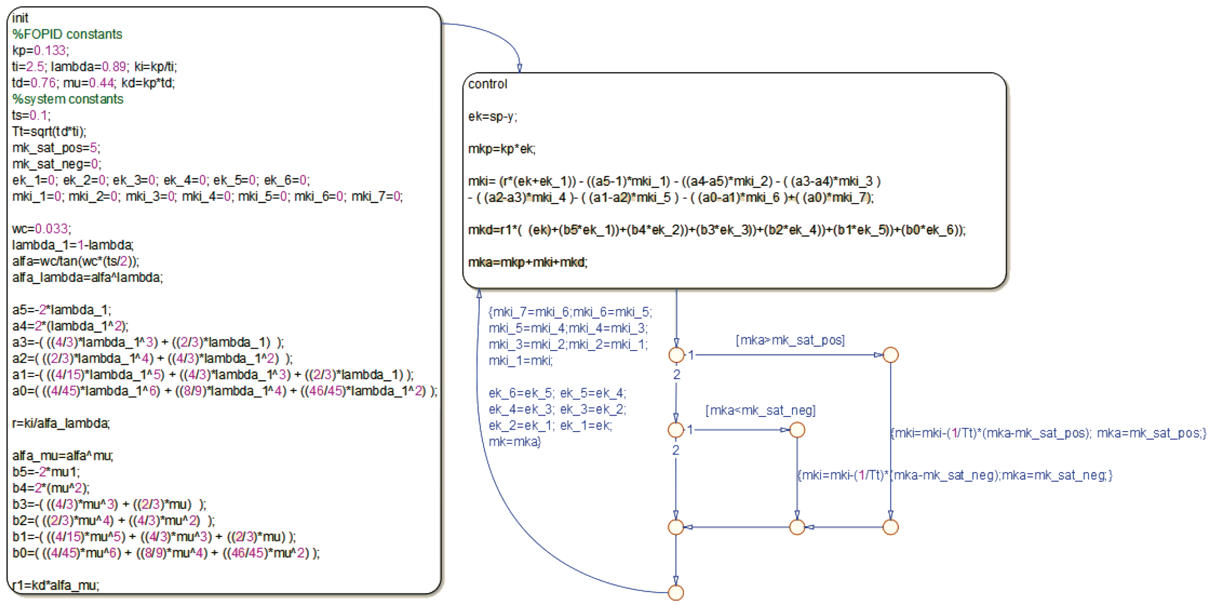

Fig. 5. FOPID controller implemented in stateflow.

TABLE III
FACTORS AND OUTPUT VARIABLES WHEN THE FOPID, SIMC PID AND IOPID CONTROLLERS ARE EMPLOYED

Factors			FOPID				IOPID				SIMC PID			
A	B	C	RMSE	SD OUT	RMSE CONTROL	SD CONTROL	RMSE	SD OUT	RMSE CONTROL	SD CONTROL	RMSE	SD OUT	RMSE CONTROL	SD CONTROL
0	0	0	0.15	6.45	3.46	1.29	0.42	6.38	3.53	1.28	0.22	6.49	3.55	1.27
0	0	1	0.16	6.44	3.48	1.28	0.42	6.38	3.55	1.29	0.24	6.49	3.56	1.3
0	1	0	0.16	6.43	3.43	1.36	0.45	6.36	3.52	1.39	0.23	6.49	3.55	1.33
0	1	1	0.17	6.43	3.47	1.33	0.46	6.36	3.54	1.4	0.242	6.49	3.55	1.37
1	0	0	0.23	6.46	3.47	1.25	0.64	6.42	3.54	1.26	0.36	6.51	3.52	1.27
1	0	1	0.23	6.46	3.5	1.27	0.64	6.42	3.56	1.27	0.35	6.53	3.57	1.31
1	1	0	0.23	6.45	3.47	1.28	0.59	6.37	3.54	1.36	0.34	6.51	3.54	1.37
1	1	1	0.22	6.45	3.5	1.35	0.58	6.39	3.55	1.39	0.34	6.54	3.53	1.39

system will be principally analyzed through the dynamic behavior of the system step response and controller action.

Specifically, the experimental design has as output variables, the RMSE value and the standard deviation (SD OUT) for the response of the system given a step input. Moreover, the RMS value (RMS CONTROL) and the standard deviation (SD CONTROL) will be considered as output variables for the control action. Two replicas of the experiment have been taken into account to obtain a better representation of the information from the statistical standpoint. Table III presents the effect of every factor and their combinations, over the output variables previously mentioned, when the FOPID, SIMC PID and IOPID controllers are employed.

A. Box-plot

A box-plot is created based on Table III in order to perform a preliminary statistical analysis of the data. The box-plot for the output variables from the FOPID, SIMC PID and IOPID controllers with all the factors and their possible combinations are shown in Fig. 6. According to Fig. 6, the output variables RMSE, RMS CONTROL and SD CONTROL are less affected for the presence of the analyzed factors when the FOPID controller is used. The evidence is that the previously mentioned

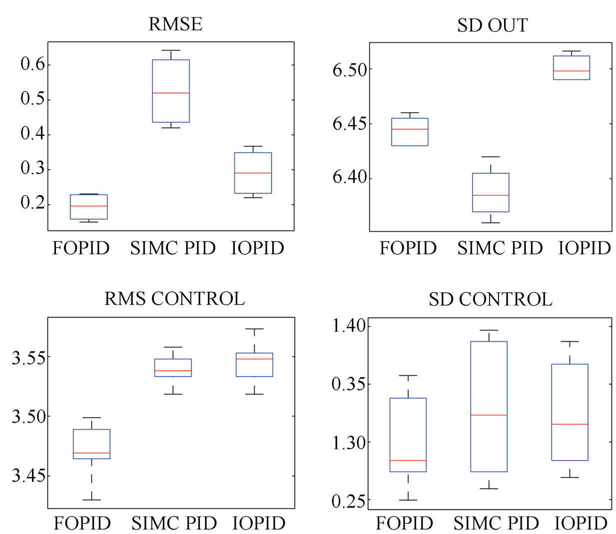

Fig. 6. Box-plot of the FOPID, SIMC PID and IOPID controllers.

output variables have the lowest value of the median and the lowest dispersion when the FOPID controller is utilized. The output variables RMSE and SD CONTROL are further affected by the presence of the analyzed factors when the SIMC PID controller is employed. The output variables RMS

CONTROL and SD OUT are further affected by the presence of the analyzed factors when the IOPID controller is used, which is reflected in a highest median and a biggest dispersion.

B. Factorial Experimental Design 2^3

In order to establish quantitatively which of the analyzed factors and their possible combinations affect to each output variable when the FOPID, SIMC PID and IOPID controllers are employed, a factorial experimental design 2^3 is performed using the data shown in Table III. The quantitative robustness analysis using the experimental design methodology is considered to be as one of the principal contributions of this paper. The analysis of variance (ANOVA) is performed using the DESIGN EXPERT software and based on it is possible to find the P value of every factor over each output variable when a specific controller is used. A confidence interval of 95 percent ($\alpha = 0,05$) was employed to perform the ANOVA. A factor have a significant incidence over an output variable if the P value is lower than α. Tables IV−VII present the obtained results from the factorial experimental design for each one of the output variables.

TABLE IV
ANOVA FOR THE RMSE OUTPUT VARIABLE WHEN THE FOPID, SIMC PID AND IOPID CONTROLLERS ARE EMPLOYED

	P value		
Factor	FOPID	SIMC PID	IOPID
A-noise	0.007	0.95	0.47
B-load	0.08	0.001	0.6
C-disturbance	0.0001	0.0001	0.001
AB	0.33	0.2	0.27
AC	0.01	0.003	0.02
BC	0.03	0.0001	0.01
ABC	0.24	0.04	0.70

TABLE V
ANOVA FOR THE SD OUT OUTPUT VARIABLE WHEN THE FOPID, SIMC PID AND IOPID CONTROLLERS ARE EMPLOYED

	P value		
Factor	FOPID	SIMC PID	IOPID
A-noise	0.51	0.25	0.03
B-load	0.007	0.0001	0.001
C-disturbance	0.0001	0.0001	0.001
AB	0.47	0.62	0.98
AC	0.33	0.95	0.14
BC	0.27	0.009	0.01
ABC	0.57	0.87	0.8

TABLE VI
ANOVA FOR THE RMS CONTROL OUTPUT VARIABLE WHEN THE FOPID, SIMC PID AND IOPID CONTROLLERS ARE EMPLOYED

	P value		
Factor	FOPID	SIMC PID	IOPID
A-noise	0.0004	0.0003	0.01
B-load	0.4255	0.003	0.27
C-disturbance	0.0007	0.0008	0.07
AB	0.15	0.72	0.19
AC	0.86	0.53	0.87
BC	0.08	0.29	0.54
ABC	0.52	0.69	0.48

TABLE VII
ANOVA FOR THE SD CONTROL OUTPUT VARIABLE WHEN THE FOPID, SIMC PID AND IOPID CONTROLLERS ARE EMPLOYED

	P value		
Factor	FOPID	SIMC PID	IOPID
A-noise	0.08	0.0005	0.01
B-load	0.0001	0.0001	0.0001
C-disturbance	0.03	0.0003	0.02
AB	0.36	0.13	0.46
AC	0.01	0.99	0.98
BC	0.60	0.85	0.14
ABC	0.17	0.44	0.26

C. Regression Models

Based on the ANOVA, it is possible to obtain the regression model for every output variable when each one of the controllers is used. The general regression model is shown in (25) and Table VIII presents the regression coefficients for every output variable when the FOPID, SIMC PID and IOPID controllers are employed, where y is the output variable, μ is the global mean of the output variable, k_i for $i = 1$ to 7, correspond to the coefficients of the regressor and A, B and C correspond to the factors of the experiment.

$$\begin{aligned} y = \mu + k_1\mathrm{A} + k_2\mathrm{B} + k_3\mathrm{C} + k_4\mathrm{AB} \\ + k_5\mathrm{AC} + k_6\mathrm{BC} + k_7\mathrm{ABC}. \end{aligned} \tag{25}$$

The interpretation of the ANOVA for every output variable and its correspondent regresor is presented below. Table IV shows that the RMSE output variable is affected by the factors A, C, AC and BC when the FOPID controller is employed. The factor that presents the most significant incidence is C (disturbance) due to this factor has the lowest P value. With the SIMC PID controller, the factors of incidence are B, C, AC, BC and ABC, with B (load) and C (disturbance) as the most significant factors. In the case of the IOPID controller the factors of incidence are C, AC and BC, where C (disturbance) is the most significant. Based on the data from Table VIII, the curve of the regression coefficients for the RMSE output variable and for each one of the controllers is shown in Fig. 7.

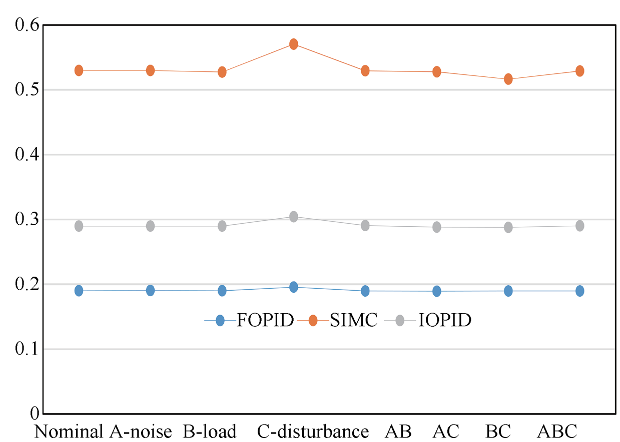

Fig. 7. Regression coefficients for the output variable RMSE using the FOPID, SIMC PID and IOPID controllers.

TABLE VIII

FACTORS AND OUTPUT VARIABLES WHEN THE FOPID, SIMC PID AND IOPID CONTROLLERS ARE EMPLOYED

	RMSE			SD OUT			RMS CONTROL			SD CONTROL		
Factor	FOPID	SIMC PID	IOPID	FOPID	SIMC PID	IOPID	FOPID	SIMC PID	IOPID	FOPID	SIMC PID	IOPID
μ	0.19	0.53	0.29	6.45	6.39	6.5	3.47	3.54	3.54	1.3	1.33	1.33
k_1	2.85E−3	−4E−5	−0.43E−3	6.31E−4	−1.1E−3	−1.13E−3	0.01	7.6E−3	0.011	6.6E−3	7.9E−3	0.01
k_2	−1.87E−4	−5.4E−3	−1E−3	−3.25E−3	−0.012	−2.28E−3	−1.68E−3	−5.18E−5	−3.78E−3	0.029	0.053	0.047
k_3	0.033	0.09	0.06	0.01	0.017	9.66E−3	0.011	6.53E−3	−6.6E−3	−8.43E−3	−8.6E−3	9.9E−3
k_4	−8.25E−4	−1E−3	2.2E−3	−6.93E−4	−0.87E−4	−6.25E−6	3.18E−3	−4.56E−4	−4.54E−3	3.18E−3	2.3E−3	2.71E−3
k_5	−2.5E−3	−3.1E−3	−5.1E−3	−9.47E−4	5.1E−5	−7.31E−4	3.56E−4	8.1E−4	5.4E−4	0.01	6.2E−6	8.12E−5
k_6	−2.1E−3	−0.02	−6.36E−3	−1E−3	−3.26E−3	−1.5E−3	3.95E−3	1.38E−3	-2.01E−3	1.8E−3	2.6E−4	5.78 E−3
k_7	−1E−3	−1.8E−3	7.56E−4	−5.31E−4	−1.5E−4	1.8E−4	1.33E−3	5.06E−4	−2.4E−3	5E−3	1.1E−3	−4.24E−3

Every point in the curve represents the level of incidence of each factor and their combinations over an output variable employing the FOPID, SIMC PID and IOPID controllers. It is possible to notice that the factor with more significance in all the controllers is C (disturbance). Besides, the SIMC PID controller is the most affected by C (disturbance), meanwhile, the FOPID controller is the less affected. The controller with the lowest nominal RMSE value and the smallest dispersion is the FOPID controller despite the presence of the factors and their combinations. In nominal operation conditions, the FOPID controller presents the lowest mean of the RMSE output variable (0.19) while the SIMC PID controller has the highest mean value (0.53).

Table V shows that SD OUT output variable is affected by the factors B and C when the FOPID controller is used, with C (disturbance) as the factor with the most significant incidence. With the SIMC PID controller, the factors of incidence are B, C and BC with B (load) and C (disturbance) as the most significant ones. In the case of the IOPID controller the factors of incidence are A, B, C and BC, with C (disturbance) as the most significant. Based on the data presented in Table VIII, the curve of the regression coefficients for the SD OUT output variable and for each one of the controllers is shown in Fig. 8.

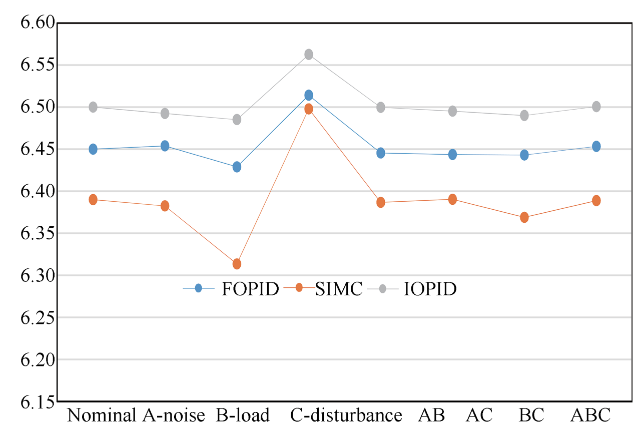

Fig. 8. Regression coefficients for the output variable SD OUT using the FOPID, SIMC PID and IOPID controllers.

It is possible to notice that the factors with most significance in all the controllers are C (disturbance) and B (load). Besides, the SIMC PID controller is the most affected by C (disturbance) while the FOPID and IOPID controllers are affected in a similar way. The SIMC PID controller has a smallest nominal value of SD OUT and the biggest dispersion in the presence of the factors and their combinations. In nominal operation conditions the SIMC PID controller has the lowest mean value of the SD OUT output variable (6.39), meanwhile, the IOPID controller presents the highest mean value (6.50).

Based on Table VI, is possible to notice that the RMS CONTROL output variable is affected by the factors A and C when the FOPID controller is employed, with A (noise) as the factor with the most significant incidence. With the SIMC PID controller the factors of incidence are A, B and C with A (noise) as the most significant one. In the case of the IOPID controller the most significant factor is A (noise). From Table VIII, the curve of the regression coefficients for the RMS CONTROL output variable and for each one of the controllers is shown in Fig. 9.

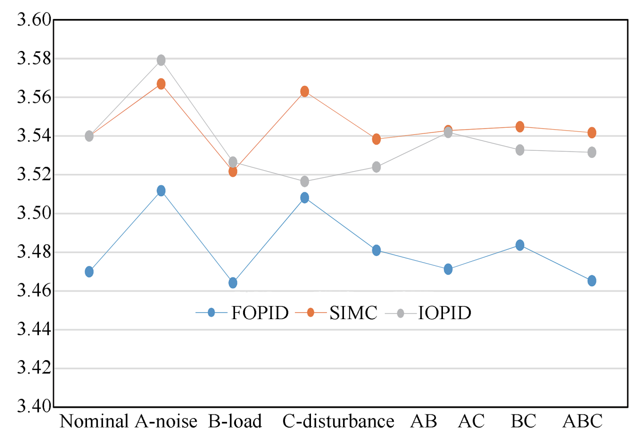

Fig. 9. Regression coefficients for the output variable RMS CONTROL using the FOPID, SIMC PID and IOPID controllers.

It is possible to observe that the factors with more significance in all the controllers are A (noise) and C (disturbance). Besides, the FOPID controller is the most affected by the mentioned factors, while the SIMC PID controllers is the less affected one due to it presents a smallest dispersion. In nominal operation conditions, the FOPID controller has the lowest mean of the RMS CONTROL output variable (3.47) and the SIMC PID and IOPID controllers have a similar mean value (3.54).

According to Table VII is possible to observe that SD CONTROL output variable is influenced by the factors B, C and AC when the FOPID controller is employed, with B (load) as the factor that presents the most significant incidence. For the SIMC PID and IOPID controllers, the factors of incidence are A, B and C with B (load) as the most significant one.

Based on Table VIII, the curve of the regression coefficients for the SD CONTROL output variable and for each one of the controllers is shown in Fig. 10. It is possible to notice that the factor with more significance in all the controllers is B (load). Besides, the SIMC PID controller is the most affected by the previously mentioned factor, while the FOPID controller is the less affected and presents a smaller dispersion in the presence of the factors and their combinations. In nominal operation conditions, the FOPID controller presents a lower mean value in the SD CONTROL output variable (1.3) and the SIMC PID and IOPID controllers have a similar mean value (1.33).

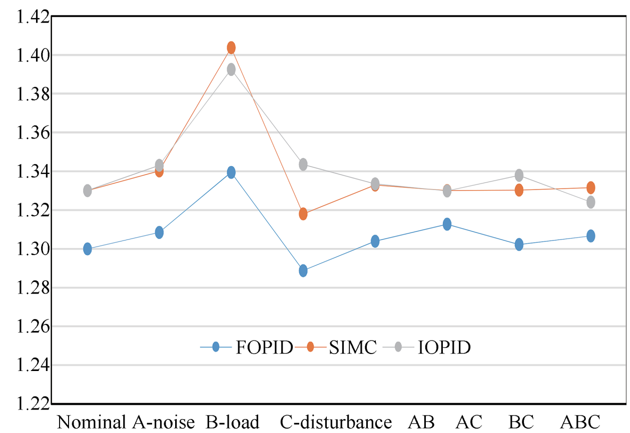

Fig. 10. Regression coefficients for the output variable SD CONTROL using the FOPID, SIMC PID and IOPID controllers.

D. Robustness Analysis

For the robustness analysis of each controller, the results from the experimental design obtained in Figs. 7−10 will be employ, as well as the regression coefficients presented in Table VIII. The robustness of every controller will be evaluated from the level of incidence of each factor over the output variables used in the experiment. The level of incidence is calculated subtracting to each point of the presented curves in Figs. 7−10 the mean value of every output variable for each controller. In order to quantify the level of incidence in percentage terms, the maximum value of the variations for every output variable has been taken as an incidence of 100 %. Fig. 11−14 show the level of incidence in percentage terms for the output variables RMSE, SD OUT, RMS CONTROL and SD CONTROL.

As Fig. 11 shows, the FOPID controller presents the lowest level of incidence (14 %) for the RMSE output variable and in the presence of the factor C (disturbance), while the SIMC PID controller shows the highest incidence (100 %).

For the SD out output variable shown in Fig. 12, the FOPID and IOPID controllers present a similar level of incidence (60 %) in the presence of the factor C (disturbance), meanwhile the SIMC PID presents the highest level of incidence (100 %). This indicates that the FOPID controller shows a greater performance and robust stability than the IOPID and SIMC PID controllers facing the presence of external disturbances.

As Fig. 13 shows, for the RMS CONTROL output variable facing the presence of the factors A (noise) and C (disturbance), the FOPID controller presents the highest level of

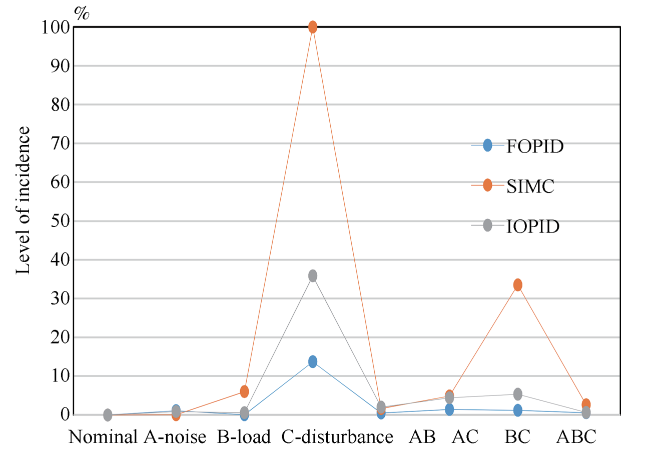

Fig. 11. Level of incidence for the output variable RMSE using the FOPID, SIMC PID and IOPID controllers.

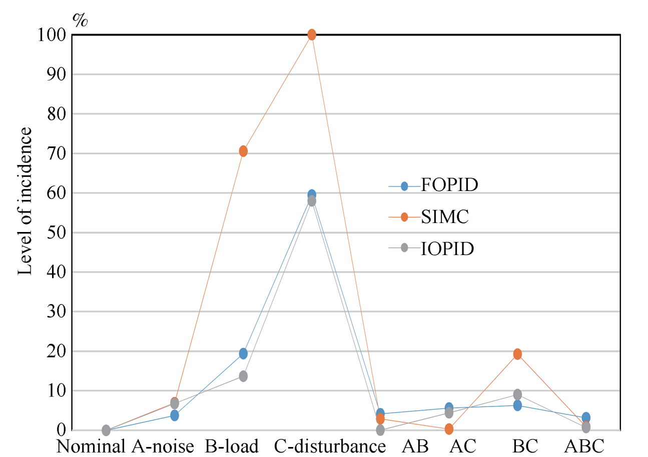

Fig. 12. Level of incidence for the output variable SD OUT using the FOPID, SIMC PID and IOPID controllers.

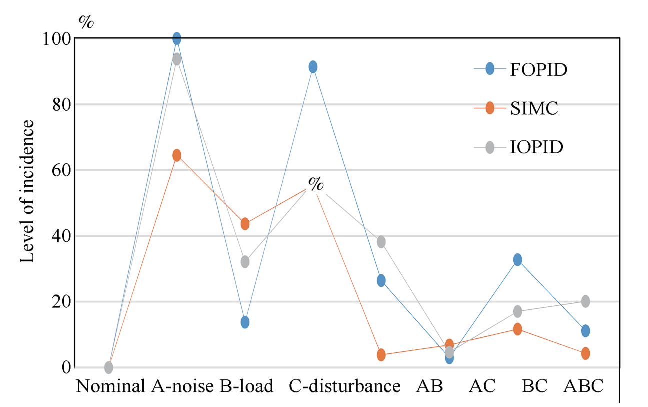

Fig. 13. Level of incidence for the output variable RMS CONTROL using the FOPID, SIMC PID and IOPID controllers.

incidence (100 % and 91 %) while the SIMC PID controller exhibits the lowest level of incidence (64 % and 56 %). For the factor B (load) is possible to notice that the FOPID controller has the lowest level of incidence (13 %) and the SIMC PID controller has the highest level of incidence (43 %). This means that despite the FOPID controller presents the highest level of incidence in the presence of the factors A (noise) and C (disturbance), these levels do not exceed the minimum values reached by the SIMC PID and IOPID controllers as is presented in Fig. 9.

For the SD CONTROL output variable, Fig. 14 shows that the FOPID controller presents the lowest level of incidence (53 %) facing the presence of the factor B (load), while the SIMC PID exhibits the highest level of incidence (100 %). Given the presence of the factors A (noise) and C (disturbance), the level of incidence of these controllers is similar (13 % and 18 %). This indicates that the FOPID controller presents a better performance and robust stability than the IOPID and SIMC PID controllers in the presence of uncertainties in the load.

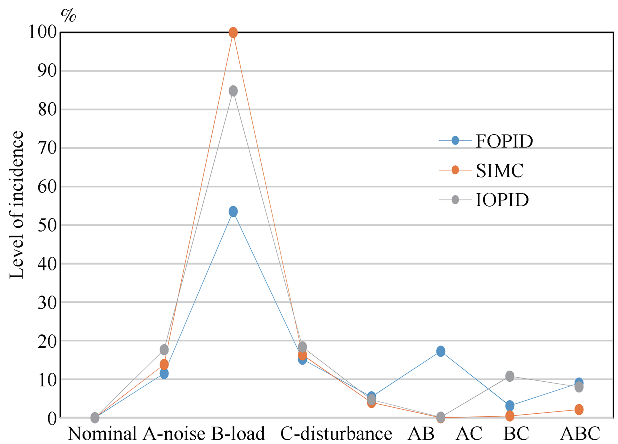

Fig 14. Level of incidence for the output variable SD CONTROL using the FOPID, SIMC PID and IOPID controllers.

VII. Conclusions

This paper has presented a comparative robustness study using a factorial experimental design among the FOPID, SIMC PID and IOPID controllers employed to control a motor-generator system. The factors used to measure the robustness were the presence of random noise in the feedback loop, the existence of external disturbance in the input of the system and uncertainty in the load. The robustness analysis was performed studying the level of incidence of the factors over the system step response and the control action. This level of incidence was measured through the output variables RMSE, SD OUT, RMS CONTROL and SD CONTROL. The results of the statistical robustness analysis show that the step response is further affected by the presence of C (disturbance) with the FOPID controller as the one with the lowest level of incidence. This allows concluding that the FOPID controller is more robust facing the presence of external disturbances. The control action is further affected for the factors A (noise), B (load) and C (disturbance) for all the controllers. The FOPID controller presents the highest level of incidence due to the factors A (noise) and C (disturbance), however is the less affected by the factor B (load). This means that the control action from the FOPID controller is more robust facing the presence of uncertainty in the load.

Based on the robustness analysis, it is possible to conclude that the FOPID controller, despite is not a robust control technique, is an alternative for the control of systems with uncertain load and presence of external disturbances, which is reflected in a smaller control effort.

Finally, the experiments design methodology used this paper for the robustness analysis of the control system, makes possible to quantify and have numeric evidence of the real performance of the controllers given the presence of different factors that affect the behavior of the control system.

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J. Viola received the engineering degree electronics engineering from Universidad Pontificia Bolivariana, Bucaramanga, Colombia, in 2013. His current research interests include process control, advanced control strategies, industrial robotics and fractional control. Corresponding author of this paper.

L. Angel received the Ph.D. degree from Universidad Politecnica de Madrid, Spain in 2005. He is currently an associate professor with the Universidad Pontificia Bolivariana, and his research interests include pattern recognition, fractional control applied to industrial robotics control and advanced process control.

J. M. Sebastian received the B. S. degree in electrical engineering, the M. S. degree in control engineering, and the Ph. D. degree in computer vision from the Universidad Politecnica de Madrid, Spain, in 1979, 1982, and 1987, respectively. Since 1982, he has been a professor and researcher at the Universidad Politecnica de Madrid, Industrial Engineering School, Control Department. He teaches computer vision and control engineering and his research interests include computer vision, visual servoing, and distance learning.