The Initial Guess Estimation Newton Method for Power Flow in Distribution Systems

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***Abstract*—With the increasing integration of distributed generations (DGs), there is a demand for DGs to play a more important role on the voltage regulation. Meanwhile, the high penetration of DGs could raise a technical problem that the distribution system may operate with bi-directional power flow, leading to the inadequacy of the traditional power flow. Considering this new scenario in distribution system power flow, the convergence theorem is proposed, which contributes to develop a novel selection method of the initial guess closed to the convergent solution. Moreover, to ensure the fast rate of power flow convergence, the theorem of the maximum iterations estimation is also proposed. Based on the two proposed theorems, an Initial Guess Estimation Newton method is proposed, considering different operational status of DGs and initial guess sensitivity simultaneously. Based on the standard node systems, Tongliao grid, and 69 system of USA, three simulation examples are provided to illustrate the effectiveness of the proposed method.**

***Index Terms*—Convergent theorem, high penetration distributed generations (DGs), initial Guess estimation Newton method, maximum iteration times.**

I. Introduction

THE distribution system, as a critical link of power grid, contributes to guaranteeing the power supply quality, improving operation efficiency, and interacting with consumers. Recently, the distribution system has undergone a profound change due to the high penetration of distributed generations (DG) units. Renewable generation provides many advantages to power grid, such as sustainable and environmentally friendly energy utilization with resource plug-and-play; it lends some hindrance to existing methods used for conventional power grid. Power flow analysis is one of the most fundamental and most heavily used tools in distribution system studies, such as network optimization, voltage control, state estimation and others [1]. Therefore it is as the core module for the distribution system, the power flow calculation (PFC) still faces serious challenges.

Many studies focus on distribution power flow with presence of DGs. The current solution can be roughly divided into the following categories: 1) the first solution is committed to finding the methods to handle DG units as PQ (in which active and reactive power are given) or PV (in which active power and voltage are given) nodes in PFC [2]−[7]. In [2]−[4], whether DGs should be modeled as PV or PQ depend on their operational modes and control characteristics. DGs are modeled as PQ nodes when they run in constant power factor mode; and modeled as PV nodes when they run in constant voltage mode. DGs, represented by PV node, use the reactive power injected to the specified node by a dummy node and dummy branch to maintain the specified voltage value [5]. The novel power flow analysis in islanded micro-grids taking DGs into consideration [6] is proposed, which adapts the real characteristics of the islanded micro-grid operation. In grid-connected micro-grids, DG units are required to share the load demands and keep the system frequency and voltages within their limits, which are controlled as PV or PQ buses. In [7], authors assume that all types of DGs have the capability to operate in PQ mode.The aforementioned methods have the common point that they modeled DGs as PQ buses. Although they establish models for various types of DGs, the calculation still attributes to PQ node power flow method. 2) The second is to find the power point or the incidence matrix (node voltage, loop current, incidence matrix between injected currents) to divide the weakly meshed distribution system into the several radial systems. The main reason can be summarized as follows: some DG units cannot be directly handled as PQ nodes such as the fuel cell power plant, micro-turbine, and synchronous generator [8]−[12]; The power flow issue cannot be addressed in a radial power system with DGs modeled as PV nodes by using the backward/forward sweep method or the direct approach [13]−[15].

However, these two solutions have encountered the problems of the increasing penetration of DG units, respectively. For the first solution, when they are operated in droop control or V-f (voltage-frequency) mode, DGs, modeled as PQ nodes, often cause divergence with practical condition [16], [17]. It is well known that PQ nodes cannot take part in the voltage regulation, however, the DGs with droop control are bound to join the voltage regulation and form loops into the power systems when amounts of DGs are connected to the grid. As a result, DGs cannot be all modeled as PQ nodes in distribution

Manuscript received November 3, 2015; accepted April 1, 2016. This work was supported by National Natural Science Foundation of China (NSFC) Key Program (61573094), and the Fundamental Research Funds for the Central Universities (N140402001). Recommended by Associate Editor Chengdong Li.

Citation: Q. Y. Sun, L. Liu, D. Z. Ma, and H. G. Zhang, "The initial guess estimation newton method for power flow in distribution systems," *IEEE/CAA Journal of Automatica Sinica*, vol. 4, no. 2, pp. 231−242, Apr. 2017.

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Digital Object Identifier 10.1109/JAS.2017.7510514

grid with high penetration of renewable generation. For the second solution, transmission lines have made systems operate closer to their limits and the typically radial topological structure of distribution system has also been changed due to the high penetration of DG units. The changed distribution system has the following characteristics: meshed structure; high R/X (resistance/reactance) ratios; multiphase and unbalanced operation; and distributed generations. These characteristics cause the traditional power flow methods used in transmission systems, such as the Gauss-Seidel, the Forward/Backward sweep method (FB), Newton-Raphson and fast decoupled, to fail to calculate the power flow [18]−[24].

In [19], it develops two matrices: the bus-injection to branch-current matrix and the branch-current to bus voltage matrix, to obtain the power flow solution with a simple matrix multiplication. On this basis, the method has been widely used in [20]−[24]. Some limitations exist in the second method, such as the need of data format, data manipulations, and all parameters of branches and nodes, which weaken its utilization in the high penetration distribution system.

In other words, as abundant DGs are connected to grid, the distribution system loses its radial or weakly meshed characteristic. As a result, there is no direct mathematical relationship (matrix) between branch parameters and control variables, leading to a direct approach method, forward/backward sweep method has been invalidated in distribution system power flow calculation. So the two main solutions aforementioned only are appropriate for the system with few DGs.

Aiming at the power flow problem in distribution system with the high penetration of DGs, it is necessary and urgent to find a new solution. Due to the increase of DG integration, the structure of distribution system is changing from one-generation radial system to multi-generation meshed system. The power flow can no longer be unidirectional flow. In this case, the structure of distribution system with the high penetration of DGs can be similar to the multi-loop power system. Therefore, Newton-Like (NL) methods can be adopted to calculate the power flow.

The meshed structure, with the higher resistance/reactance (R/X) ratio of feeders, makes distribution systems ill conditioned. Hence, the Fast Decoupled Newton method becomes unsuitable for most distribution power flow problems. Because the Jacobian matrix ceases to be diagonally dominant, its inverse makes the system ill-conditioned, and further causes convergence problems in power flow solutions [25], [26]. To overcome this problem, the literatures [27], [28] employ a coordinate transformation in Y-matrix (admittance matrix) of the fast decoupled method. Active power and reactive power of all load buses are embedded in the Y-bus matrix as the constant diagonal elements, and the new specified P-Q values of the load buses are redefined. However, the ill conditioned case-system results in the divergence of the NL method. To improve convergence in such case, it requires choosing suitable initial voltage.

The NL method presents an advantage on quadratic convergence rate and modest computer storage request over other methods [29], which is the reason for its widely use in power flow study. However, several bottlenecks appear with respect to its development, especially faced with DGs integration into the power system. Firstly, a higher requirement for the selection of initial guess is imperative in the NL method [26], [30]. Secondly, iterations of power flow calculation change a lot due to the different selection of initial guess. In addition, the improper selection of the initial guess may further cause the result of power flow divergent [3], [31], [32]. Therefore, the selection of initial guess comes to be realized as one of the fundamental problems in NL flow calculation. The methods in [33] modify the Jacobian matrix and adjust the incremental voltage. Reference [34], [35] present different techniques to improve the convergence of power flow by adjusting the Jacobian iteratively within the NL method and increasing the distance to the voltage stability limit through adjusting the reactive nodal powers injections. These adjustments incorporated within the NL method can obtain a faster rate of convergence, a larger region of convergence or both. Although the NL method is improved by modifying partial calculation process, the method may diverge when the initial operating point is far from the solution. In other words, initial guess sensitivity issues remain unsolved. To overcome this problem, this paper proposes a convergence theorem of NL power flow and a maximum iterations estimation theorem. With these two theorems, preliminary estimation can be obtained before power flow calculation, and on this basis, the convergent solution can be easily obtained from the power flow equations.

To sum up, the main contributions and benefits of this paper are: 1) the proposed convergence theorem contributes to develop a novel selection method of the initial guess; the proposed maximum iterations estimation theorem can ensure the rate of convergence; Proposed two theorems can be used to determine the convergence before calculation and directly select optimal initial guess from the feasible region. 2) the proposed Initial Guess Estimation Newton method, which contains a novel selection method of the initial guess, maximum iterations estimation, and power flow calculation, can handle power flow calculation of distribution systems with high penetration of DGs.

This paper first proposes the convergence theorem of the NL power flow method and its proof in Section II. And the iterations estimation theorem and the proof are proposed in Section III. Subsequently, the workflow of the proposed power flow method which can solve the power flow calculation with the high penetration of DGs, can be obtained in Section IV. In Section V, simulation results are carried out in practical distribution systems with various DGs. Analysis and results have been verified the excellence and effectiveness of the proposed method. Finally, the conclusion is obtained in Section VI.

II. The Convergence Analysis of NL Power Flow Algorithm

A. Mechanism Model of the Newton-like Power Flow

The mission of power flow calculation is, mainly, to determine operating state of the system, such as bus voltage, power distribution, and power loss, for given operating conditions. The Newton method, which is strict to the selection of the initial guess, has been extensively used in power flow calculations. It can be briefly summarized as the following.

$$\begin{pmatrix} \Delta P^k \\ \Delta Q^k \end{pmatrix} = J_k \begin{pmatrix} \Delta\theta^k \\ \Delta V^k \end{pmatrix} \tag{1}$$

where ΔP^k, ΔQ^k respectively represent the active and reactive power injection vectors at iteration k; $\Delta\theta^k,\ \Delta V^k$ respectively represent the voltage angle and magnitude correction vectors at iteration k. The nodal active and reactive powers can be calculated from the following expressions:

$$P_a^{\mathrm{cal}} = V_a \sum_{b=1}^{n} V_b (G_{ab}\cos\theta_{ab} + B_{ab}\sin\theta_{ab}) \tag{2}$$

$$Q_a^{\mathrm{cal}} = V_a \sum_{b=1}^{n} V_b (G_{ab}\sin\theta_{ab} - B_{ab}\cos\theta_{ab}) \tag{3}$$

where $\theta_{ab} = \theta_a - \theta_b$, and n represents the number of nodes in the system. In (1) the Jacobian matrix at iteration k can be given by

$$J_k = \begin{pmatrix} \frac{\partial P^{\mathrm{cal}}}{\partial V} & \frac{\partial P^{\mathrm{cal}}}{\partial \theta} \\ \frac{\partial Q^{\mathrm{cal}}}{\partial V} & \frac{\partial Q^{\mathrm{cal}}}{\partial \theta} \end{pmatrix}. \tag{4}$$

Based on the Jacobian matrix above, a body of techniques has been proposed to improve the convergence of power flow by adjusting the Jacobian iteratively within the NL method. In other words, the Jacobian matrix is still the key point of convergence. Although the classical Newton's method is improved by modifying Jacobian matrix, the initial guess sensitivity issues remain unsolved. Therefore, if the initial guess can be selected from the solvable region, the convergent solution can be easily obtained.

B. The Affine Invariants Based Newton-Like Power Flow Calculation Convergence Theorem

Newton iterative program can be shown as:

$$x^{k+1} = x^k - \left[F'(x^k)\right]^{-1} F(x^k), \quad k = 0,1,2,\ldots. \tag{5}$$

In (5), $G = AF$ can be obtained by the affine transformation of F,where $A \in L(\mathbb{R}^n)$ is a non-singular matrix. On that basis, $\left[G'(x)\right]^{-1} G(x) = \left[AF'(x)\right]^{-1} (AF(x)) = \left[F'(x)\right]^{-1} F(x)$ can be obtained, which indicates that the sequence of iterations $\{x^k\}$, in the affine transformation is invariant. As a result, the convergence and divergence of $\{x^k\}$ remain unchanged. Similarly, for the conditions of NL convergence theorem, the convergence domain and its initial selection range get extended with the affine matrix A properly selected to meet the theorem condition of the mapping $G = AF$. Besides, detailed proof of affine invariant based convergence theorem can be summarized as follows:

Lemma 1: Set $A, C \in L(\mathbb{R}^n)$, A is nonsingular, $\left\|A^{-1}\right\| \le \alpha$, $\|A - C\| \le \beta$ and $\alpha\beta < 1$ then [38]

$$\left\|C^{-1}\right\| \le \frac{\alpha}{1-\alpha\beta}. \tag{6}$$

Theorem 1: For power flow equation $F(V) = 0$, voltage initial guess is V_0, which satisfies the following conditions:

1) the initial voltage value $V_0 \in D_0$, makes $J_0 = F'(V_0)$ nonsingular and $\left\|J_0^{-1}\right\| \le \beta$, $\|J_m - J_n\| \le \gamma \|V_m - V_n\|$.

2)
$$\left\|J_0^{-1}\left[J_m - J_n\right]\right\| \le \omega \|V_m - V_n\| \tag{7}$$

$$\left\|{J_0}^{-1} F(V_0)\right\| \le \eta \tag{8}$$

where ω, η are constants.The power flow calculation convergence operator ρ satisfies the following conditions:

$$\rho = \omega\eta \le \frac{1}{2}. \tag{9}$$

If we let $t^* = (1 - \sqrt{1-2\rho})/\omega,\ t^{**} = (1 + \sqrt{1-2\rho})/\omega$, the sequence $\{V_k\} \subset S(V_0, t^*) \subset D_0$ of Newton iteration $x^{k+1} = x^k - \left[F'(x^k)\right]^{-1} F(x^k),\ k = 0,1,2,\ldots$ can converge to the unique solution V^* which satisfies $F(V) = 0$ in region $S(V_0, t^{**}) \cap D_0$. In other words, the solution sequence of the NL method $\{V_k\}$ converges to V^*.

Proof : By the quadratic optimal function, w equals to the minimum of the product of β and γ.

$$j(x) = \frac{1}{2}\gamma x^2 - \frac{1}{\beta}x + \frac{\eta}{\beta} = 0 \tag{10}$$

the roots of equation are

$$x^* = \frac{1 - \sqrt{1-2\beta\gamma\eta}}{\beta\gamma}, \quad x^{**} = \frac{1+\sqrt{1-2\beta\gamma\eta}}{\beta\gamma}. \tag{11}$$

Substitute (9) into (11)

$$x^* = \frac{1-\sqrt{1-2\rho}}{\rho}\eta, \quad x^{**} = \frac{1+\sqrt{1-2\rho}}{\rho}\eta$$

the iterative sequence of the solution of NL methods can be expressed as x_k , and then

$$x_{k+1} = x_k - \frac{j(x_k)}{j'(x_k)}, \quad k = 0,1,2,\ldots;\ x_0 = 0.$$

It is easy to verify $x_k \to x^*$, when $k \to \infty$. The following we just need to prove $\left\|V_{(k)} - V^*\right\| \le |x^* - x_k|$.

$$\eta_k = x_{k+1} - x_k = -\frac{j(x_k)}{j'(x_k)},\ \beta_k = -\frac{1}{j'(x_k)},\ \rho_k = \beta_k\eta_k\gamma$$

when $k = 0$, we have $\eta_0 = \eta,\ \beta_0 = \beta,\ \rho_0 = \rho$. Take the derivative of (10) with respect to x, one has

$$j'(x) = \gamma x - \frac{1}{\beta}. \tag{12}$$

By substituting x_{k+1} and x_k to (12) , then it can be described as follows:

$$j'(x_{k+1}) - j'(x_k) = \gamma(x_{k+1} - x_k) = \gamma\eta_k. \tag{13}$$

Consider a function as follows:

$$\begin{aligned} &j(x_{k+1}) - [j(x_k) + j'(x_k)(x_{k+1} - x_k)] \\ &= \frac{1}{2}\gamma x_{k+1}^2 - \frac{1}{\beta}x_{k+1} - \frac{1}{2}\gamma x_k^2 + \frac{1}{\beta}x_k - (\gamma x_k - \frac{1}{\beta})(x_{k+1} - x_k) \\ &= \frac{1}{2}\gamma x_{k+1}^2 - \frac{1}{2}\gamma x_k^2 - \gamma x_k(x_{k+1} - x_k) \\ &= \frac{1}{2}\gamma(x_{k+1} - x_k)^2 = \frac{1}{2}\gamma{\eta_k}^2. \end{aligned} \tag{14}$$

From (13), it follows that

$$j'(x_{k+1}) = \gamma\eta_k + j'(x_k) = \gamma\eta_k - \frac{1}{\beta_k} = \frac{\gamma\eta_k\beta_k - 1}{\beta_k} = \frac{\rho_k - 1}{\beta_k}. \tag{15}$$

By substituting $\beta_k = -1/j'(x_k)$, $\rho_k = \beta_k \eta_k \gamma$ into (14), (15), one has

$$\beta_{k+1} = -\frac{1}{j'(x_{k+1})} = \frac{\beta_k}{1-\rho_k} \tag{16}$$

$$\eta_{k+1} = -\frac{j(x_{k+1})}{j'(x_{k+1})} = \frac{1}{2}\frac{\gamma\beta_k\eta_k^2}{1-\rho_k} = \frac{1}{2}\frac{\rho_k}{1-\rho_k}\eta_k \tag{17}$$

$$\rho_{k+1} = \gamma\beta_{k+1}\eta_{k+1} = \frac{\gamma\beta_k}{1-\rho_k}\cdot\frac{\rho_k\eta_k}{2(1-\rho_k)} = \frac{\rho_k^2}{2(1-\rho_k)^2}. \tag{18}$$

Note that we have the following inequalities,

$$0 < \rho \le \frac{1}{2}, \quad 1 < \frac{1-\sqrt{1-2\rho}}{\rho} \le 2.$$

As the iterative equation of NL method, it can be derived

$$\left\|V_{(1)} - V_{(0)}\right\| = \left\|[J_0]^{-1}F(V_{(0)})\right\| \le \eta_0 = |x_1 - x_0|. \tag{19}$$

According to the satisfied conditions, and voltage amplitude $V_{(1)}$ which satisfies $V_{(1)} \in S(V_0,\delta)$, it can be obtained as follows:

$$\begin{aligned}
&\left\|F\left(V_{(1)}\right)\right\| \\
&= \left\|F\left(V_{(1)}\right) - F\left(V_{(0)}\right) - F'(v)\left(V_{(1)} - V_{(0)}\right)\right\| \\
&\le \frac{1}{2}\gamma\left\|V_{(1)} - V_{(0)}\right\|^2 \le \frac{1}{2}\gamma\eta_0^2 \\
&v \in (V_{(0)}, V_{(1)}); \|J_1 - J_0\| \le \gamma\left\|V_{(1)} - V_{(0)}\right\| \le \gamma\eta_0.
\end{aligned} \tag{20}$$

Since $\left\|{J_0}^{-1}\right\| \le \beta$ and $\|J_1 - J_0\| \le \gamma\left\|V_{(1)} - V_{(0)}\right\| \le \gamma\eta_0$ the existence of ${J_1}^{-1}$ can be obtained from Lemma 1, and

$$\left\|{J_1}^{-1}\right\| \le \frac{\beta_0}{1-\rho_0} = \beta_1. \tag{21}$$

Note the definition of voltage amplitude $V_{(1)}$, the following inequalities can be deduced from (9) and (19).

$$\begin{aligned}
\left\|V_{(2)} - V_{(1)}\right\| &\le |x_2 - x_1| = \eta_1 \\
&\le \frac{1-\sqrt{1-2\rho_1}}{\rho_1}\eta_1 = \left(\frac{1-\sqrt{1-2\rho}}{\rho} - 1\right)\eta.
\end{aligned} \tag{22}$$

If $\exists\delta \ge (1-\sqrt{1-2\rho})/\rho$, the following inequality can be obtained

$$\begin{aligned}
\left\|V_{(2)} - V_{(0)}\right\| &\le \left\|V_{(2)} - V_{(1)}\right\| + \left\|V_{(1)} - V_{(0)}\right\| \\
&\le \left(\frac{1-\sqrt{1-2\rho}}{\rho} - 1\right)\eta + \eta \\
&= \frac{1-\sqrt{1-2\rho}}{\rho}\eta \le \delta.
\end{aligned}$$

From (22), it can be obtained that voltage amplitude $V_{(2)} \in S(V_0,\delta)$. By using the mathematical induction, for $\forall k \in Z, V_{(k)} \in S(V_0,\delta)$, one has

$$\begin{cases}
\left\|F\left(V_{(k)}\right)\right\| \le \frac{1}{2}\gamma\eta_{k-1}^2 \\
\left\|{J_k}^{-1}\right\| \le \beta_k \\
\left\|{J_k}^{-1}F\left(V_{(k)}\right)\right\| \le \eta_k \\
\left\|V_{(k+1)} - V_{(k)}\right\| \le x_{k+1} - x_k.
\end{cases} \tag{23}$$

For any positive integer m, n, k, which satisfies $m = n + k$. Using the fourth equation of (23), one has

$$\begin{aligned}
&\left\|V_{(m)} - V_{(n)}\right\| \le \left\|V_{(n+k)} - V_{(n+k-1)}\right\| + \cdots + \\
&\left\|V_{(n+1)} - V_{(n)}\right\| \le |x_{n+k} - x_{n+k-1}| + \cdots + \\
&|x_{n+1} - x_n| \le |x_m - x_n|.
\end{aligned} \tag{24}$$

Due to the convergence of x_k , it can be derived $\left\{V_{(k)}\right\}$ is the Cauchy convergence sequence. Therefore, $\left\{V_{(k)}\right\}$ has the limit and it can be supposed as V^* when $k \to \infty$. (24) can be described as

$$\left\|V^* - V_{(k)}\right\| \le |x^* - x_k|. \qquad \blacksquare$$

Using the first equation of (23), note $\eta_k \to 0$, it can be concluded that $F(V^*) = 0$. Thus, we can draw a conclusion that V^* is the solution of the power flow equation $F(V) = 0$.

Remark 1: The above-mentioned Theorem 1 provides the evaluation whether the initial guess can get a convergent solution. The convergence theorem $\rho \le 1/2$ is a sufficient condition for the power flow solvability. If $\rho \le 1/2$, the power flow can converge to the true value V^*. Otherwise, other initial guess should be tried repeatedly. Therefore, the theorem helps to reduce the difficulty of the initial guess selection and unnecessary calculation.

III. The Convergence Rate Analysis of NL Power Flow Algorithm

Convergence theorem of NL power flow algorithm is proposed to judge whether initial guess is suitable for the power flow. Aimed at the iterative estimate of NL power flow algorithm, an estimation theorem of the maximum iterations is proposed as follows.

Theorem 2: If convergence operator of flow calculation is $\rho \le 1/2$ and

$$\max\left\{\left[\Delta V_i^{(k)}\right]\right\} < \varepsilon \tag{25}$$

where $\Delta V_i^{(k)}$ represents the voltage difference between two iterations at the bus i and ε represents the iteration precision, then the maximum iterations of power flow equations can be expressed as:

$$k = \text{Int} \times \left[\log_2\left(\log_\theta \frac{\varepsilon\theta}{\eta + \varepsilon\theta}\right)\right] + 1, \ \theta = \frac{1-\sqrt{1-2\rho}}{1+\sqrt{1-2\rho}}. \tag{26}$$

Proof : Using the Taylor formula, the power system balance equation can be expressed as:

$$\begin{aligned}
f(x^*) &= f(x_k) + f'(x_k)(x^* - x_k) + \frac{\gamma}{2}(x^* - x_k)^2 \\
&= f(x_k) + f'(x_k)(x_{k+1} - x_k) \\
&\quad + f'(x_k)(x^* - x_{k+1}) + \frac{\gamma}{2}(x^* - x_k)^2
\end{aligned} \tag{27}$$

where $\gamma = f''(\xi), \ \xi \in (x_k, x^*)$, combining with (18), it can be derived

$$\begin{aligned}
x^* - x_{k+1} &= -\frac{\gamma}{2f'(x_k)}(x^* - x_k)^2 \\
&= \frac{\gamma\beta_k}{2}(x^* - x_k)^2 = \frac{\rho_k}{2\eta_k}(x^* - x_k)^2.
\end{aligned} \tag{28}$$

Taking k times iteration to (28), for $k = 0, 1, 2, \ldots,$

$$x^* - x_{k+1} = \prod_{j=0}^{k} \left(\frac{\rho_{k-j}}{2\eta_{k-j}} \right)^{2^j} (x^* - x_0)^{2^{k+1}}, \quad x_0 = 0. \quad (29)$$

From (29) , it follows that

$$x^* - x_{k+1} = \prod_{j=0}^{k} \left(\frac{\rho_{k-j}}{2\eta_{k-j}} \right)^{2j} (x^*)^{2^{k+1}}. \quad (30)$$

According to (17) and (18) one has

$$\frac{\rho_j}{2\eta_j} = \frac{\eta_j}{\eta_{j-1}^2}, \quad j = 1, 2, \ldots, n.$$

Substitute it into (29), we have

$$\begin{aligned} x^* - x_{k+1} &= \prod_{j=0}^{k-1} \left(\frac{\eta_{k-j}}{\eta_{k-j-1}^2} \right)^{2^j} \left(\frac{\rho}{2\eta} \right)^{2^k} (x^*)^{2^{k+1}} \\ &= \frac{\eta_k}{\eta^{2^k}} \left(\frac{\rho}{2\eta} \right)^{2^k} \left(\frac{1 - \sqrt{1 - 2\rho}}{\rho} \right)^{2^{k+1}} \eta^{2^{k+1}} \\ &= \eta_k \left(\frac{\rho}{2} \right)^{2^k} \left(\frac{1 - \sqrt{1 - 2\rho}}{\rho} \right)^{2^{k+1}}. \end{aligned} \quad (31)$$

In addition, let

$$\theta_k = \frac{1 - \sqrt{1 - 2\rho_k}}{1 + \sqrt{1 - 2\rho_k}} \quad (32)$$

then the following holds:

$$\theta_k = \theta_{k-1}^2 = \cdots = \theta^{2^k}, \quad \rho_k = \frac{2\theta_k}{(1 + \theta_k)^2}.$$

From (17), the above can be expressed as

$$\eta_k = \frac{\theta^{2^{k-1}}}{1 + \theta^{2^k}} \eta_{k-1} = \prod_{j=0}^{k-1} \frac{\theta^{2^j}}{1 + \theta^{2^{j+1}}} \eta = \frac{\theta^{2^k - 1}}{\sum_{j=0}^{2^{k-1}-1} \theta^{2j}} \eta. \quad (33)$$

Substitute (33) into (31) and note $(1 - \sqrt{1 - 2\rho}\,)/\rho = 1 + \theta$, it can be derived

$$\|x_k - x^*\| \le \frac{\theta^{2^k - 1}}{\sum_{j=0}^{2^{k-1}-1} \theta^{2j}} \eta. \quad (34)$$

Substitute (34) into $\left\| V^* - V_{(k)} \right\| \le x^* - x_k$, it can be concluded as following:

$$\left\| V_{(k+1)} - V_{(k)} \right\| < \sqrt{\varepsilon} = \frac{\theta^{2^{k+1} - 1}}{\sum_{j=0}^{2^k - 1} \theta^{2j}} \eta$$

namely

$$k = \text{Int} \times \left[\log_2 \left(\log_\theta \frac{\varepsilon\theta}{\eta + \varepsilon\theta} \right) \right] + 1. \qquad \blacksquare$$

Thus, the iterations of the power flow can be estimated by above analysis.

Remark 2: The initial guess, calculated by the Theorem 1, can make power flow convergent. However, whether the initial guess is proper also depends on further judgment about the iterations that the power flow takes. From such consideration, the Theorem 1, which can get the same iterations with the actual calculation, is proposed not only to obtain a faster rate of convergence, but to avoid redundant calculation at the same time. By combining the two theorems, a proper initial guess can be obtained for both convergence and fast rate of convergence.

IV. The Workflow of the Initial Guess Estimation Newton Method

In the distribution system with the high penetration of DGs, the convergence theorem and the estimation of maximum iterations have been applied. This section proposed an Initial Guess Estimation Newton method, containing the proposed convergence theorem, maximum iterations estimation, and power flow calculation. The distribution system power flow calculation process is as following:

Step 1: Determine node types according to the modes of DG integration to power grid, and input data of all nodes.

Step 2: Set initial values of every node (PV, PQ, $V\theta$) according to node types.

Step 3: Determine initial Jacobian matrix J_0.

Step 4: Judge whether initial guess led power flow achieves convergence with certain convergence factor. If $\rho \le 0.5$, then calculate the next step; otherwise, adjust the affine matrix and re-select initial guess to return to Step 2.

Step 5: Form node admittance matrix.

Step 6: Obtain constant terms $\Delta P^{(0)} \Delta Q^{(0)} (\Delta V^2)^{(0)}$ of modified equations.

Step 7: Determine coefficient matrix of modified equations and solve modified equations $\Delta e^{(t)} \Delta f^{(t)}$.

Step 8: Modify voltage vectors of each node: $e^{(1)} = e^{(0)} - \Delta e^{(0)}$, $f^{(1)} = f^{(0)} - \Delta f^{(0)}$

Step 9: Examine the convergence. If power flow is convergent, on this basis, power flows of branches can be determined and outputted; otherwise, start another iteration with initial guess $e^{(1)}$, $f^{(1)}$.

The proposed power flow method can handle high penetration of DGs and simultaneously avoid the initial guess sensitivity issues. The flowchart of the proposed algorithm is shown in Fig. 1.

V. Simulation Results and Verification of Practical Distribution Network

A. The Power Flow Instance Analysis of Tongliao Power Grid

In order to validate the effectiveness of the proposed method, Tongliao power grid of China's Inner Mongolia autonomous region, is provided in simulation. The structure and data of Tongliao shares highly practical importance and

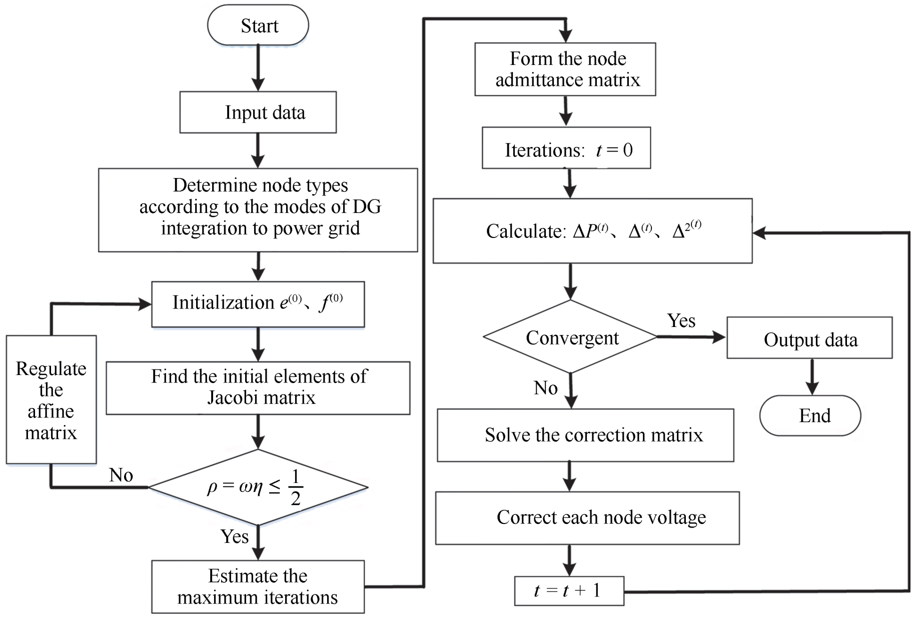

Fig. 1. The power flow calculation process.

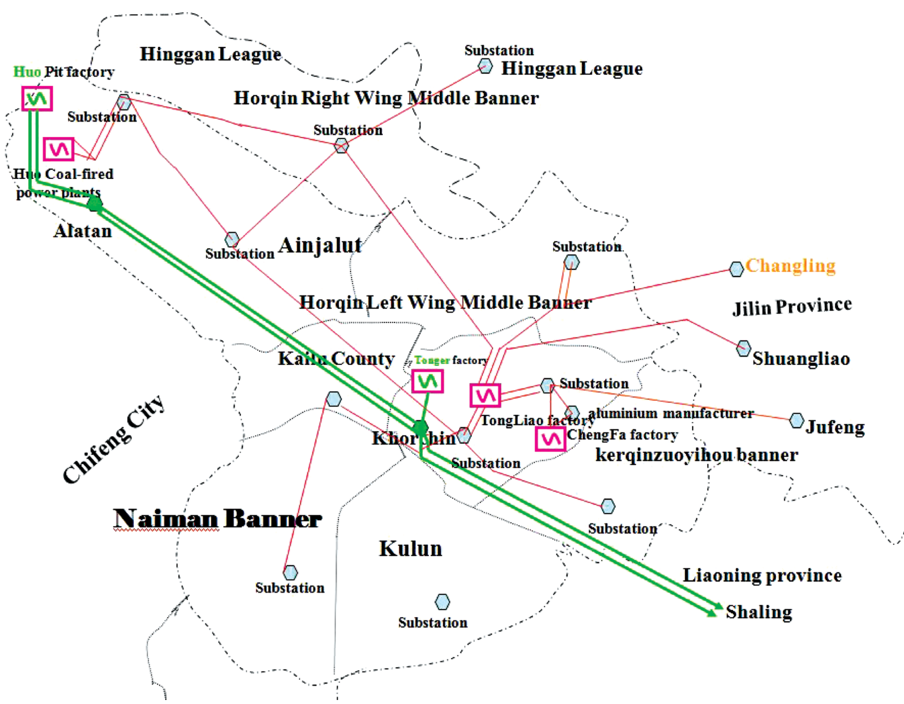

Fig. 2. The Tongliao system structure diagram.

extensive representation in the field of power system in China, where six main power plants of 66 kV voltage class and above are involved to a total 1811.8 MW installed capacity. Besides, there are 155 transmission lines of a total length of 4745.32 Km with 66 kV voltage class and above, 129 substations with 66 kV voltage class and above, and 248 main transformers of total 3905.90 MVA. The base capacity is 10 MW, and reference voltage is 66 kV. Through the analysis of Tongliao 220 kV power grid, the network structure is a typical meshed network. The system structure diagram is shown in Fig. 2.

The primary substation of network of Naiman Banner, which supplies power to the fifteen 66 kV-distribution substations with radiant network structure, is represented as the research object for power flow calculation. The detailed wiring diagram of Naiman distribution network is shown in Fig. 3. The main parameters of Naiman Banner are shown in Table I. In the minimum load operation mode, the node voltage will rise as the charging reactive power is not consumed completely; while in the maximum load operation mode, the reactive power load of entire power grid is rather heavy. When the voltage is short of reactive power support, the whole system of the substation voltage is on the lower side. The long distance between the substations and power plants causes serious voltage drop. Therefore, the NL method with flat start may lead the power flow to divergent solution.

TABLE I
THE PARAMETERS OF NAIMAN DISTRIBUTION SYSTEM ON A TYPICAL SUMMER WORKING DAY

No.	Bus	Bus voltage	P	Q	DG P	DG Q
1	Daqin	62.73	8.12	1.69		
2	Zhendong	63.21	16.89	3.58		
3	Yilong	63.21	8.33	1.90		
4	Huanghua	62.79	16.73	4.00		
5	Xinglong	62.37	1.68	0.34	170	50
6	Wulan	61.215	15.05	3.66	100	70
7	Xinzhen	60.69	2.10	0.44		
8	Baiyinchang	60.48	8.82	1.82	200	100
9	Nanwan	60.165	2.55	0.52		
10	Qinglongshan	59.22	3.82	0.77	200	100
11	Baiyin	58.8	2.45	0.52	150	50
12	Baixiantong	61.425	13.45	3.50		
13	Pingan	60.69	2.07	0.42		
14	Dongming	58.695	6.03	1.45	200	50
15	Desheng	60.69	2.07	0.42		
16	Zhian	59.43	3.80	0.77		

B. Simulation Examples of Convergence Theorem of NL Power Flow Algorithm

In order to validate the effectiveness of the proposed Theorem 1, the IEEE test system with 4, 14, 30, 57, 118 buses and Naiman power grid are provided in this section. When the initial guess is $V_i = 1 + j0$, the convergence operators of flow calculation ρ can be obtained as shown in Table II.

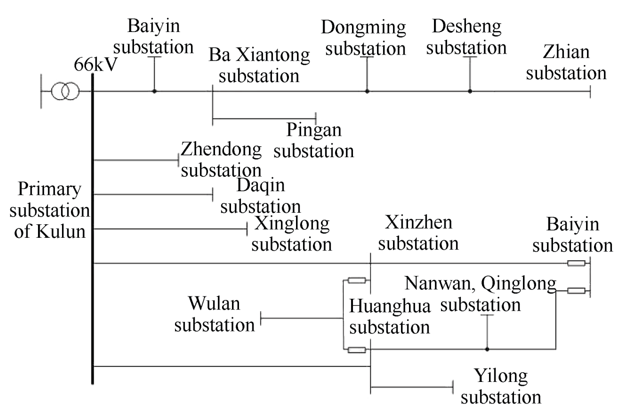

Fig. 3. Wiring diagram of Naiman distribution grid.

TABLE II
THE STANDARD BUS SIMULATION OF THEOREM 1

Standard Bus	Convergence operator ρ	Convergence
IEEE4	0.2738	convergence
IEEE14	0.2976	convergence
IEEE30	0.3254	convergence
IEEE57	0.3569	convergence
IEEE118	0.3825	convergence

The IEEE 4 node system is taken as an example to illustrate how to calculate ρ. The initial voltage magnitude and angle of voltage are set to 1 and 0 respectively. And J_0 , the initial Jacobian matrix,can be expressed as follows:

$$J_0 = \begin{bmatrix} -44.92 & 0 & 25.84 & -8.98 & 0 \\ 0 & -42.47 & 16.63 & 0 & -7.89 \\ 25.84 & 16.63 & -42.47 & 5.16 & 3.32 \\ 8.98 & 0 & -5.16 & -44.74 & 0 \\ 0 & 8.49 & -3.32 & 0 & -39.24 \end{bmatrix}.$$

The inverse matrix of J_0 can be obtained as follows:

$$J_0{}^{-1} = \begin{bmatrix} -0.04 & -0.01 & -0.03 & 0.004 & -0.0002 \\ -0.01 & -0.03 & -0.02 & 0 & 0.004 \\ -0.03 & -0.02 & -0.05 & 0 & -0.0003 \\ -0.004 & 0 & 0 & -0.021 & 0 \\ 0 & -0.004 & 0 & 0 & -0.024 \end{bmatrix}.$$

Then the norm of $J_0{}^{-1}$ can be obtained as $\left\| J_0{}^{-1} \right\| = 0.093$. As

$$F(V_{(0)}) = \begin{bmatrix} -1.7000002 \\ 2.302371 \\ 0.467392 \\ -0.960000 \\ 2.844353 \end{bmatrix}$$

$$\left\| J_0{}^{-1} F(V_{(0)}) \right\| = 0.1961.$$

When $m = 2, n = 1$, then $\eta \geq 0.1961$.

$$J_1 = \begin{bmatrix} -43.3 & 0 & 24.9 & -6.84 & 0 \\ 0 & -46.2 & 18.0 & 0 & -11.7 \\ 25.3 & 17.8 & -43.1 & 4.00 & 4.08 \\ 10.1 & 0 & 6.03 & -41.3 & 0 \\ 0 & 7.37 & -3.10 & 0 & -49.2 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} -43.2 & 0 & 24.8 & -6.76 & 0 \\ 0 & -45.94 & 17.95 & 0 & -11.45 \\ 25.27 & 17.76 & -43.04 & 3.99 & 4.03 \\ 10.16 & 0 & -6.03 & -41.19 & 0 \\ 0 & 7.45 & -3.11 & 0 & -48.4 \end{bmatrix}.$$

The results can be obtained by following calculation:

$$\| J_2 - J_1 \| = 1.2121 \left\| V_{(2)} - V_{(1)} \right\| = 0.0810.$$

When $\eta = 0.1961$, $\omega = 1.3617897$, $\rho = \omega\eta = 0.26705 < 0.5$.

With the calculation of the standard node data as shown in Table II, all of the convergence operators of flow calculation ρ are less than 1/2. This result, which accords with actual situation, can be inferred from the content of Theorem 1, where all of power flow has the convergent solution.

In order to verify the validity of the proposed Theorem 1 under the condition of unsolvable power flow, a prevalent problem in the smart distribution grid which is the changing of radial network into a meshed one, is taken into consideration. According to the actual situation, the network parameters of standard nodes are modified. In the simulation of IEEE node system, the reactive power of the system is compensated, and the reactive power of load can shrink by $9\,\%-30\,\%$, R/X is enlarged appropriately with a scope of $1.8-3$.

Under this circumstance, the system has no convergent solution by flat start. When the initial guess is selected to around 0.7, the power flow can obtain the convergent solution. Due to the complexity of the Naiman network, the system cannot obtain the power flow solution with flat start either.

Thus, the initial guess is selected as 0.85. The simulation results can be obtained and shown in Table III.

TABLE III
CONVERGENCE AND DIVERGENCE WITH DIFFERENT INITIAL GUESS
(CONVERGENCE-✓, DIVERGENCE-×)

Standard bus	Initial guess	Convergence operator ρ	Convergence based on theorem 1	The actual convergence
IEEE4	0.71	0.0893	✓	✓
	1	1.986	×	×
IEEE14	0.75	0.1059	✓	✓
	1	4.263	×	×
IEEE30	0.74	0.1045	✓	✓
	1	2.6354	×	×
IEEE57	0.72	0.2972	✓	✓
	1	3.1865	×	×
IEEE118	0.72	0.3042	✓	✓
	1	2.5077	×	×
Naiman	0.86	0.232	✓	✓
	1	9.832	×	×

C. Simulation Cases of Iterative Estimation Theorem of NL Power Flow Algorithm

In order to verify the validity of the Theorem 2, modified IEEE test system with 4, 14, 30, 57, 118 buses and Naiman power grid are provided to test and to demonstrate the effect.

TABLE IV
THE SIMULATION RESULTS FOR ILLUSTRATING THEOREM 1 AND 2

Standard bus	Initial guess	Iterations	Iterations based on Theorem 2	Convergence operator
IEEE4	0.67	5	5	0.1710
	0.71	3	3	0.0893
	0.87	4	4	0.1056
IEEE14	0.67	12	12	0.2655
	0.75	8	8	0.1059
	0.81	14	15	0.2957
IEEE30	0.68	12	12	0.3647
	0.74	9	9	0.1045
	0.83	10	10	0.234
IEEE57	0.69	9	9	0.3078
	0.72	8	8	0.2972
	0.81	10	11	0.3818
IEEE118	0.66	10	11	0.4146
	0.72	7	8	0.3042
	0.85	14	14	0.7573
Naiman	0.75	6	7	0.332
	0.86	5	5	0.232
	0.9	8	8	0.413

From the results shown in Table IV, the PFCs have a faster convergence rate when the initial guess is 0.7. The difference between iterations calculated by Theorem 2 and practical iterations is 0 or 1. Therefore, the Theorem 2, which provides a theoretical basis of iterations estimated after the initial guess selection based on Theorem 1, can be used for estimating the iterations. In addition, the convergence operator of flow calculation ρ plays an important role in evaluating the NL power flow algorithm.

In the IEEE 118-node system simulation, the range of the initial guess V_0 is (0.4, 1.1). The relationship among the convergence operator ρ, iterations k and the initial guess is shown in Fig. 4. The selected initial guess which makes the convergence operator of flow calculation $\rho \leq 1/2$ is a sufficient condition of the power flow solvability. As shown in Fig. 4, when the convergence operator of flow calculation meets $\rho \leq 1/2$, NL power flow will have a convergent solution; when $\rho \leq 1/2$, the iterative rate can be the fastest if the initial guess making ρ the least is taken into the power flow equation. When the initial value satisfies $0.6 < V_0 < 0.8$, the power flow has a faster convergence rate and less iterations.

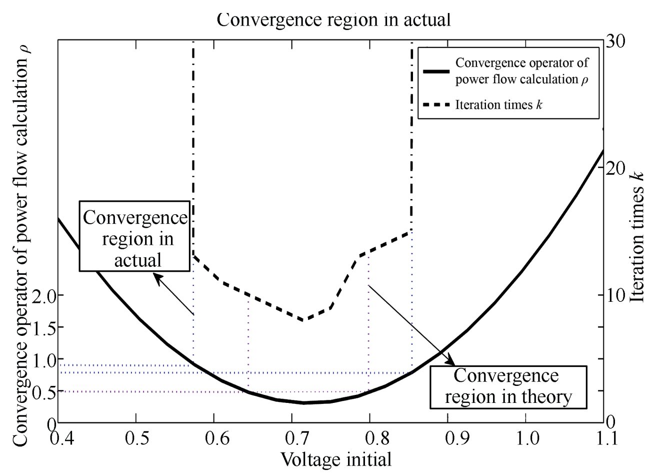

Fig. 4. The relationship among the convergence operator of flow calculation, iterations and the initial guess.

D. Simulation Cases of PG&E69 Node System With the High Penetration of DGs

In this section, the USA $PG\&E69$ (Pacific Gas and Electric company) node system will be used as a test instance, which could verify the power flow method algorithm can be used to calculate the power flow of distribution system with massive DGs.

Case 1: Considered that NL is sensitive to initial values, the simulation is set to test the proposed selection method of initial guess by using the proposed method for $PG\&E69$ node system. All nodes are viewed as PQ nodes during the simulation. With such assumption, some common methods, such as the forward/backward sweep method, can be used for power flow calculation, although this kind of transformation is not in conformity with practical systems. The result of the forward/backward sweep method after transformation can provide comparison with the one of proposed method. Furthermore, the validity of proposed methods can be verified. The node voltage comparison of the two methods is shown in detail in Fig. 5. In terms of the Fig. 5, the node voltage obtained by the proposed method, coincides with the ones obtained by the forward/backward sweep method, which provides to verify the validity of the proposed method.

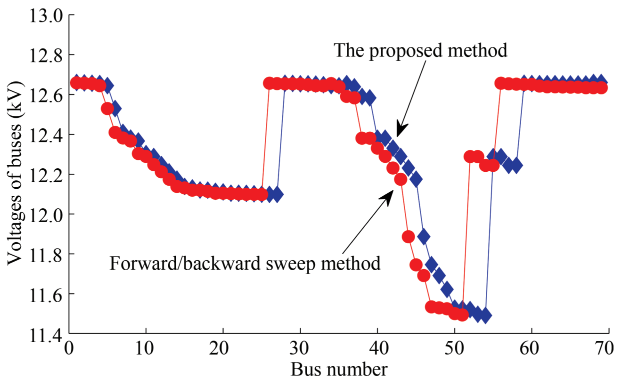

Fig. 5. Bus voltages comparison by using two methods.

Case 2: To validate the effectiveness of the solvability of power flow where high penetration of DGs are connected into the power distribution system, meanwhile to check the effectiveness of proposed algorithm in Section IV, PG & E69 node system of USA is used in simulation, with the installation of DG at given nodes. With respect to the ratio of DG penetration, different countries have different technical requirements. This paper sets the ratio, according to the common concerns of most countries, at least 30 %, that is, 20 DGs are connected to the test distribution system. Those grid-connected DGs can be divided, evenly in number, into four typical kinds: photovoltaic generation as PV or PQ nodes connected to the distribution system, fuel cells, five micro turbines which consume non-renewable energy, as PQ or PV nodes, and double-fed induction generators as PQ or PV nodes. The structure of US PG & E69 node system with DGs is shown in Fig. 6, and the grid-connection parameters of different DG node types are shown in Table V.

With the initial guess selection method, the simulation results of PG & E69 node system of USA are shown in Table VI ($\rho = 0.4$). By contrast, the iteration errors of different methods in this system, are shown as Fig. 7. In Fig. 7, the blue

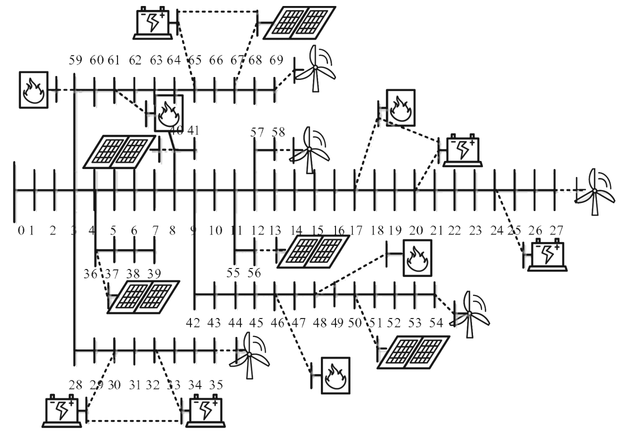

Fig. 6. The structure of US PG & E69 node system with DGs.

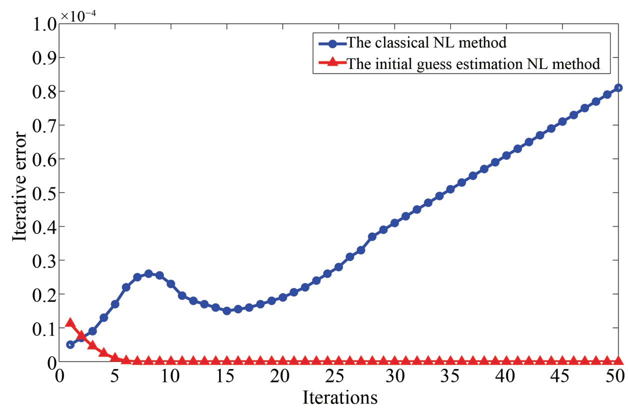

Fig. 7. The iteration error of different methods.

TABLE V
THE GRID-CONNECTION PARAMETERS OF DIFFERENT DG NODE TYPES

Location	DG type	Grid-connected equipment	Grid-connected mode	Suitable model	The parameter of DGs
67	Photovoltaic	Inverter	Droop control	PV	P = 600 kW, V = 12.66 kV
40	Photovoltaic	Inverter	VF	PV	P = 400 kW, V = 12.66 kV
50	Photovoltaic	Inverter	VF	PV	P = 400 kW, V = 12.66 kV
36	Photovoltaic	Inverter	PQ	PQ	P = 150 kW
56	Photovoltaic	Inverter	PQ	PQ	P = 100 kW
17	Combustion engines	Squirrel cage induction generator	AGC	PQ	P = 200 kW, Q = 100 kvar
48	Combustion engines	Squirrel cage induction generator	AGC	PQ	P = 100 kW, Q = 50 kvar
61	Combustion engines	Synchronous generator	AGC	PV	P = 300 kW, V = 12.66 kV
46	Combustion engines	Synchronous generator	AGC	PV	P = 350 kW, V = 12.66 kV
59	Combustion engines	Synchronous generator	AGC	PV	P = 400 kW, V = 12.66 kV
65	Fuel cells	Inverter	VF	PV	P = 400 kW, V = 12.66 kV
20	Fuel cells	Inverter	VF	PV	P = 350 kW, V = 12.66 kV
24	Fuel cells	Inverter	PQ	PQ	P = 100 kW, Q = 50 kvar
30	Fuel cells	Inverter	PQ	PQ	P = 100 kW, Q = 50 kvar
33	Fuel cells	Inverter	PQ	PQ	P = 150 kW, Q = 50 kvar
69	DFIG	Rectifier+Inverter	PQ	PQ	P = 200 kW, Q = 100 kvar
27	DFIG	Rectifier+Inverter	PQ	PQ	P = 200 kW, Q = 100 kvar
54	DFIG	Rectifier+Inverter	PQ	PQ	P = 100 kW, Q = 50 kvar
35	DFIG	Rectifier+Inverter	VF	PV	P = 300 kW, V = 12.66 kV
58	DFIG	Rectifier+Inverter	droop control	PV	P = 350 kW, V = 12.66 kV

TABLE VI
THE NODE VOLTAGE RESULTS OF USA PG & E69 SYSTEM WITH DGS

Bus	Node voltage	Bus	Node voltage
1	12.66000	36	12.6395454189406
2	12.6515705194103	37	12.5892212711898
3	12.6307291434544	38	12.5832449278076
4	12.6314575506151	39	12.3808288006677
5	12.6449255784602	40	12.6600103862626
6	12.5292616750488	41	12.2882393905248
7	12.4093251189490	42	12.2305961410115
8	12.3803274283270	43	12.1747431610678
9	12.3664582413513	44	11.7467006934176
10	12.304088861627	45	11.6884649603822
11	12.285610395844	46	12.6600873356179
12	12.245025247999	47	12.5524155129464
13	12.211508607491	48	11.5242409212403
14	12.174396539135	49	11.4996707733901
15	12.138531716067	50	12.6600381296615
16	12.131936405013	51	12.3295547716032
17	12.030637127931	52	12.2474970568231
18	12.209502641966	53	12.1557507119100
19	12.406515127796	54	12.1032946845128
20	12.660355784202	55	12.2518879449342
21	12.444669715376	56	12.6494557471175
22	12.304033418243	57	12.6425191498275
23	12.300072705481	58	12.6600241205472
24	12.249952333614	59	12.6600355816604
25	12.100469373439	60	12.3356321812796
26	12.095548211463	61	12.6600000000000
27	12.085548211463	62	12.2858605974177
28	12.657827620725	63	12.3039766653827
29	12.655693412691	64	12.3676010715207
30	12.649929977780	65	12.6600000000000
31	12.656826223203	66	12.4099517129080
32	12.652548251722	67	12.6600000000000
33	12.648226900821	68	12.5292444816913
34	12.644776047585	69	12.3766026058628
35	12.642485921134		

spot represents the iterative error of traditional NL method without using initial guess selection method. Its error starts increasing after 17th iteration, that is, the power flow tends to diverge. The iterative error of this system using the proposed method, is shown as the red triangle. Its error almost comes to zero after 6th iteration, which shows the convergence of power flow. The comparison highlights the importance and improvement on convergence contributed by the proposed method. In other words, the effectiveness of the proposed theorem can be verified.

E. The Simulation Contrast of the Proposed Method and FB Method Using the PG&E69 Node System Involving the High Penetration of DGs

In order to prove that the proposed method has the capability to deal with the power flow problems of numerous DGs in distribution system, with improvements over other methods to solve the integration of DGs, the simulation contrast of two methods has been carried out. The forward/backward sweep method (FB), as a widely-agreed power flow calculation method to handle DGs, is employed to test and compare with the proposed method. The FB method assumes that all types of DGs have to operate in PQ mode, while it is unnecessary to require all DGs to operate in PQ mode for the proposed Initial Guess Estimation Newton method. The proposed method can deal with all modes of DGs no matter what mode DGs are operated in, PQ or PV.

The USA PG & E69 node system is employed in the comparison simulation. The results of simulation, including the data for twenty DGs connected to the PG & E69 system and the error analysis, are shown in the Table VII. Numerical simulation demonstrates that the results of proposed method which are closer to the actual value shows an average $0.152\,\%$ error. By contrast, the results of FB method which are far from the actual value, shows an average $0.88804\,\%$ error. There is a big deviation between the maximum error and minimum error in the FB method. It also suggests that a large voltage fluctuation exist in FB power flow method. (The traditional Newton method without using the initial guess selection method cannot calculate the power flow of this system since the results are divergent. It has been verified in Fig. 7).

TABLE VII
$PG\&E69$ SYSTEM POWER FLOW RESULTS OF THE PROPOSED ALGORITHM COMPARED WITH FB RESULTS

Bus	PG & E69 system			Error analysis (%)	
	Actual	FB	Proposed	FB	Proposed
67	12.657640	12.325430	12.660000	2.62	0.0186
40	12.655443	12.354530	12.660010	2.38	0.0361
50	12.656475	12.457328	12.660038	1.573	0.028
36	12.627211	12.557234	12.639545	0.554	0.098
56	12.643217	12.617355	12.649456	0.205	0.049
17	11.993418	11.721107	12.030637	2.27	0.3103
48	11.536827	11.464724	11.524209	0.625	0.109
61	12.657349	12.631894	12.660000	0.2011	0.0209
46	12.655624	12.621135	12.660087	0.2725	0.0353
59	12.654833	12.625748	12.649456	0.2298	0.0425
65	12.651205	12.629152	12.660000	0.1743	0.0695
20	12.654947	12.630018	12.660356	0.197	0.0427
24	12.204403	12.053607	12.249952	1.236	0.3732
30	12.650620	12.631174	12.649930	0.1537	0.00545
33	12.651314	12.630678	12.648269	0.1631	0.0241
69	12.301629	12.115807	12.376603	1.511	0.6095
27	11.981624	11.810024	12.085548	1.432	0.867
54	12.081472	11.887069	12.103295	1.609	0.1806
35	12.651076	12.622701	12.642486	0.2243	0.0679
58	12.563709	12.637258	12.660036	0.1300	0.0500
Max				2.62	0.867
Min				0.1300	0.00545
Avg.				0.88804	0.152

VI. CONCLUSION

This paper proposed an initial Guess estimation Newton method to solve the power flow distribution and direction problems caused by the high penetration DGs. Two theorems were proposed to prove the convergence problems caused by the initial guess selection and reduce the redundant computation of power flow. Based on the theoretical and simulation results, the proposed methods have been verified to apply in typical distribution systems, especially systems with various DGs.

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