Letter

Prescribed-Time Nash Equilibrium Seeking for Pursuit-Evasion Game

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Dear Editor,

This letter is concerned with prescribed-time Nash equilibrium (PTNE) seeking problem in a pursuit-evasion game (PEG) involving agents with second-order dynamics. In order to achieve the prior-given and user-defined convergence time for the PEG, a PTNE seeking algorithm has been developed to facilitate collaboration among multiple pursuers for capturing the evader without the need for any global information. Then, it is theoretically proved that the prescribed-time convergence of the designed algorithm for achieving Nash equilibrium of PEG. Eventually, the effectiveness of the PTNE method was validated by numerical simulation results.

A PEG consists of two groups of agents: evaders and pursuers. The pursuers aim to capture the evaders through cooperative efforts, while the evaders strive to evade capture. PEG is a classic noncooperative game. It has attracted plenty of attention due to its wide application scenarios, such as smart grids [1], formation control [2], [3], and spacecraft rendezvous [4]. It is noteworthy that most previous research on seeking the Nash equilibrium of the game, where no agent has an incentive to change its actions, has focused on asymptotic and exponential convergence [5]–[7]. For instance, the distributed Nash equilibrium seeking strategies were investigated for the noncooperative games under leader-following consensus protocol [6]. Building on the consensus based distributed Nash equilibrium seeking strategies, node-based and edge-based control laws were proposed [8].

In real-world scenarios, achieving Nash equilibrium in a finite time is valuable, practical, and desirable. Therefore, there are some solid works done research on games with finite-time Nash equilibrium seeking [9]–[12]. A distributed algorithm has been designed to enable players to achieve Nash equilibriums in non-cooperative games within a finite time frame [13]. For a two-pursuer one-evader game, capture can occur in finite time only by defining a nonzero capture radius and for a subset of initial game states [14]. Compared to asymptotic convergence, finite-time convergence can accelerate the Nash equilibrium seeking algorithm and has been shown to perform better against disturbances [15]. However, the finite-time Nash equilibrium seeking influenced by initial status of systems. This critical limitation also affects the performance of finite-time distributed optimization, as the initial status of evaders may be difficult to estimate.

In response to the issue of convergence rate heavily relying on the players' initial conditions, the fixed-time [16]–[18] Nash equilibrium seeking algorithms were employed for noncooperative games [19]. The first-order fixed-time algorithm was designed to achieve exact convergence to the Nash equilibrium of the game in [19]. However, the fixed-time bounds can be defined by the system designer under an appropriate tuning of the parameters of the algorithms. Building on these excellent cornerstone results, the control algorithm designed in this letter is motivated by two main objectives. Firstly, notice that the existing Nash equilibrium seeking algorithms are established over asymptotic convergence [6] or fixed-time convergence [19]. For PEG, the evaders should be captured within a prescribed-time which means the convergence time can be pre-designed. Secondly, it is interesting yet challenging to develop a Nash equilibrium seeking algorithm for PEG with second-order dynamics.

Motivated by the above outstanding achievements, this letter aims to develop a PTNE seeking algorithm for PEG. The main contributions of this letter are formulated as follows.

1) Second-order differential equations are used to model a wide range of physical phenomena, such as mechanical, electrical systems, and biological processes. Therefore, analyzing systems with second-order dynamics is crucial for designing and optimizing processes and devices. Different from [20], this letter discusses the Nash equilibrium seeking problem of PEG. Notably, our analysis emphasizes incorporating velocity within the payoff function, setting our work apart.

2) Compared to [21], we have addressed the Nash equilibrium problem of PEG with second-order dynamics. Inspired by the asymptotic convergence [5], [6], [8] and fixed-time convergence methods [17], [18], we design a PTNE seeking algorithm that enables pursuers to capture the evader within a user-defined convergence time.

Problem formulation: For a pursuit-evasion game G_{pe} with $\mathcal{N} = \{\mathcal{N}_p, \mathcal{N}_e\}$ agents, where $\mathcal{N}_p$ is the pursuer set and $\mathcal{N}_e$ is the evader set with the evader following a specific strategy. The motion of agents can be described by second-order dynamic equations as follows:

$$\begin{cases}\dot{\mathbf{x}}(t) = \mathbf{v}(t)\\ \dot{\mathbf{v}}(t) = \mathbf{u}(t)\end{cases} \tag{1}$$

where $\mathbf{u}(t) = \{u_i, i \in \mathcal{N}\}$, $\mathbf{x}(t) = \{x_i, i \in \mathcal{N}\}$, and $\mathbf{v}(t) = \{v_i, i \in \mathcal{N}\}$ are the control input, position, and velocity. In this letter, we consider n pursuers and 1 evader. When considering the scenario with m evaders, we can decompose it into the problem of m sets of multiple pursuers and 1 evader. For $i = \{1,2,\dots,n\}$, we define u_i, x_i, and v_i as the control input, position, and velocity of the i-th pursuer. Additionally, we denote u_{n+1}, x_{n+1}, and v_{n+1} as the control input, position, and velocity of the evader, respectively.

In this letter, the communication graph of the agents can be defined as $\mathcal{G} = \{\mathcal{O},\mathcal{E}\}$, where $\mathcal{O} = \{1,2,\dots,n+1\}$ represents the nodes set and $\mathcal{E}$ represents the edges set. An edge $(i, j)(i \neq j)$ means that vehicles i and j can obtain information from each other. A graph is called an undirected graph if $(i, j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$ for any $i, j \in \mathcal{O}$. The graph is connected when there is a path between any two nodes. $\mathcal{A} = (a_{ij})_{(n+1)\times(n+1)}$ is the adjacency matrix, where $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. $\mathcal{L}^{PE} = (l_{ij})_{(n+1)\times(n+1)}$ is the Laplacian matrix, where $l_{ii} = \sum_{j=1}^{n+1} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$.

Notations: Denote $\otimes$ as Kronecker product. sign$(\cdot)$ is the sign function. For a vector $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$, $\mathbf{sign}(\mathbf{x}) = (\mathrm{sign}(x_1), \mathrm{sign}(x_2), \dots, \mathrm{sign}(x_n))^T$. Denote $\mathbf{1}_n = (1,1,\dots,1)^T \in \mathbb{R}^n$. Let $\lambda_{\max}(\Gamma)(\lambda_{\min}(\Gamma))$ represent the largest(smallest) eigenvalue of the matrix Γ. C^n represents a class of functions that are n-th continuously differentiable. Here, $\Omega \in \mathbb{R}^n$ is the closed set. diag$(x_1, x_2, \dots, x_n)$ represents a diagonal matrix with diagonal elements $x_1, x_2, \dots, x_n$.

This letter focuses on constructing an algorithm for achieving the PTNE of PEG with agents exhibiting second-order dynamical characteristics. When reaching the PTNE of PEG, multiple pursuers can cooperatively capture the evader within a prescribed-time. Moreover, the convergence time can be preassigned offline for any initial state.

For the PEG, where $i \in \{1,2,\dots,n\}$, the pursuers select their strategies based on interactions with other agents to optimize the payoff function $J_i(x_i, \mathbf{x}_{-i}, v_i, \mathbf{v}_{-i})$ in consideration of the evader's strategy. The strategy of i-th pursuer is $\mathbf{e}_i = (x_i, v_i)^T$. $\mathbf{e}_{-i} = \{\mathbf{x}_{-i}, \mathbf{v}_{-i}\}$ are the strategies of all the agents except i-th agent, where $\mathbf{x}_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{n+1})$ and $\mathbf{v}_{-i} = (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_{n+1})$. $(\mathbf{e}_i, \mathbf{e}_{-i}) = (x_i, \mathbf{x}_{-i}, v_i, \mathbf{v}_{-i})$ is the action profile of all the agents. The payoff function of pursuer $i \in \mathcal{N}_p$ $J_i(\mathbf{e}_i, \mathbf{e}_{-i})$, $J_i : \Omega_i \to \mathbb{R}^{n_i}$ is shown as follows:

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Citation: L. Xue, J. Ye, Y. Wu, J. Liu, and D. C. Wunsch, "Prescribed-time Nash equilibrium seeking for pursuit-evasion game," *IEEE/CAA J. Autom. Sinica*, vol. 11, no. 6, pp. 1518–1520, Jun. 2024.

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Digital Object Identifier 10.1109/JAS.2023.124077

$$J_i(\mathbf{e}_i,\mathbf{e}_{-i}) = \frac{1}{2}\sum_{j=1}^{n+1} a_{ij}\|\mathbf{e}_i-\mathbf{e}_j\|^2 = \frac{1}{2}\sum_{j=1}^{n+1} a_{ij}\left\|\binom{x_i}{v_i}-\binom{x_j}{v_j}\right\|^2. \tag{2}$$

Remark 1: For the PEG, the second-order system is closer to the real physical world, which is relevant in applications such as missile interception. Hence, the payoff function (2) considers the agents' positions and velocities, making the PTNE seeking problem more challenging.

Definition 1 (Nash equilibrium): For a game $G_{pe}(\mathcal{N}_p, J_i, \Omega_i)$, an action profile $(\mathbf{e}_i^*,\mathbf{e}_{-i}^*) = (x_i^*,\mathbf{x}_{-i}^*,v_i^*,\mathbf{v}_{-i}^*)$ is a Nash equilibrium of the game G_{pe} if

$$J_i(\mathbf{e}_i^*,\mathbf{e}_{-i}^*) \le J_i(\mathbf{e}_i,\mathbf{e}_{-i}^*),\ \ \forall \mathbf{e}_i \in \Omega_i,\ \ \forall i \in \mathcal{N}_p. \tag{3}$$

Lemma 1 [22]: For each $i \in \mathcal{N}_p, \mathbf{e}_i \in \Omega_i$, the payoff function $J_i(\mathbf{e}_i,\mathbf{e}_{-i}) = J_i(x_i,\mathbf{x}_{-i},v_i,\mathbf{v}_{-i})$ is C^2 in its definition domain, strictly convex and radially unbounded in $\mathbf{e}_i$ for every $\mathbf{e}_{-i}$.

For the Nash equilibrium, no agent has any motivation to unilaterally deviate from their actions. After inspection, the payoff functions meet the conditions in Lemma 1, which indicates that Nash equilibrium $(\mathbf{e}_i^*,\mathbf{e}_{-i}^*)$ exists uniquely.

Furthermore, the Nash equilibrium satisfies

$$\nabla_i J_i(\mathbf{e}_i^*,\mathbf{e}_{-i}^*) = \mathbf{0}_{n_i} \tag{4}$$

where $\nabla_i J_i(\mathbf{e}_i,\mathbf{e}_{-i}) = \frac{\partial J_i(\mathbf{e}_i,\mathbf{e}_{-i})}{\partial \mathbf{e}_i} \in \mathbb{R}^{n_i}$. They represent the gradient of agent i's different payoff in terms of its own action.

Assumption 1: The undirected communication graph $\mathcal{G}$ including n pursuers and one evader is connected.

The Laplacian matrix of n pursuers and one evader is $\mathcal{L}^{PE} = \begin{bmatrix}\mathcal{L}^{pe} & \mathcal{L}^1\\ \mathcal{L}^2 & \sum_{j=1}^{n+1} a_{(n+1)j}\end{bmatrix}$, where $\mathcal{L}^{pe} \in \mathbb{R}^{n\times n}$, $\mathcal{L}^1 \in \mathbb{R}^{n\times 1}$ and $\mathcal{L}^2 \in \mathbb{R}^{1\times n}$. The Laplacian matrix of n pursuers is $\mathcal{L}^p$, where $\mathcal{L}^p \in \mathbb{R}^{n\times n}$.

$$\mathcal{L}^{pe} = \mathcal{L}^p + \mathrm{diag}(a_{1(n+1)}, a_{2(n+1)}, \ldots, a_{n(n+1)}) \triangleq \mathcal{L}^p + \mathcal{M}. \tag{5}$$

Assumption 2: The derivative of evader's control input is bounded. That is, there is a positive constant ϖ satisfying $|\dot{u}_{n+1}| \le \varpi$.

A high-gain function is presented as follows:

$$\theta_\iota(t) = \begin{cases} \dfrac{\dot{\eta}_\iota(t)}{\eta_\iota(t)} = \dfrac{c}{\iota T - t}, & 0 \le t < \iota T\\[2mm] \dfrac{c}{\iota T}, & t \ge \iota T \end{cases} \tag{6}$$

where $\eta(t) = (\frac{\iota T}{\iota T - t})^c$, $\iota = 1,2$, and $c > 1$.

Remark 2: Inspired by [20] and [21], the high-gain function (6) is applied in this work to achieve PTNE of PEG. It is worth mentioning that achieving PTNE can hold in any initial value situation, which is the effect of a high gain function. In the numerical example, we will also demonstrate the results with different initial values.

Lemma 2 [21]: Consider a system described by

$$\dot{x}(t) = f(t, x(t)). \tag{7}$$

There exists a Lyapunov function V such that

$$\dot{V} \le -(b + d\theta_\iota(t))V \tag{8}$$

where $b \ge 0$ and $d > 0$, $\theta_\iota(t)$ is declared in (6), then the origin of system (7) is globally prescribed-time stable with the prescribed-time ιT. It obtains

$$\begin{cases} V \le \eta_\iota(t)^{-d}\exp(-bt)V(0), & t < \iota T\\ V = 0, & t \ge \iota T. \end{cases} \tag{9}$$

Lemma 3 [23]: Under Assumption 1, $\mathcal{L}^{pe}$ in (5) is positive definite and symmetric. And we can find a vector $\delta = (\delta_1,\delta_2,\ldots,\delta_n)^T (\delta_i \in \mathbb{R}$, $i = 1,2,\ldots,n)$ satisfies $\mathcal{L}^{pe}\delta = 1_n$. Let $\delta_{\min} = \min(\delta_1,\delta_2,\ldots,\delta_n)$. Let $\Delta = \mathrm{diag}(1/\delta_1, 1/\delta_2, \ldots, 1/\delta_n)$, $\Gamma = \frac{\Delta\mathcal{L}^{pe} + (\mathcal{L}^{pe})^T\Delta}{2}$, then Γ and Δ are positive definite.

For achieving a PTNE of PEG with second-order dynamics, the pursuers have to capture the evader within prescribed-time with distributed cooperative characteristics. To do this, a control algorithm is designed to optimize pursuers' strategies, enabling them to operate with distributed and cooperative characteristics.

Main results: In this section, for the evader with a specific strategy that meets Assumption 2, a control algorithm ensuring prescribed-time convergence of PEG is given as follows:

$$\begin{aligned} u_i &= -\theta_2\mathbf{k}\frac{\partial J_i(\mathbf{e}_i,\mathbf{e}_{-i})}{\partial \mathbf{e}_i} + y_i, \quad i \in \{1,\ldots,n\}\\ \dot{y}_i &= -(b_1 + d_1\theta_1)\zeta_i - \mu\,\mathrm{sign}(\zeta_i) \end{aligned} \tag{10}$$

where $\mathbf{k} = \left(k_1\theta_2, k_2\right)$, $k_1 > 0$, $k_2 > 0$, $b_1 \ge 0$, $d_1 > 0$ and $\mu > 0$ that are designed later, $\zeta_i = \sum_{s\in\mathcal{N}_p} a_{is}(y_i - y_s) + a_{i(n+1)}(y_i - y_{n+1})$, y_i is agent i's estimate of evader's input control u_{n+1} and $y_{n+1} = u_{n+1}$.

Next, we state the following main result.

Theorem 1: Suppose Assumptions 1 and 2 hold. For a given time $T_p = 2T$ and any initial $\mathbf{x}(0)$ and $\mathbf{v}(0)$, the proposed PTNE seeking algorithm in (10) guarantees that all pursuers converge to the PTNE in T_p, provided that there are parameters $\beta > 0$, $\gamma > 0$, $\rho > 0$, $b_2 \ge 0$ and $d_2 > 0$, which meet with the following inequalities:

$$\mu \ge \varpi \tag{11a}$$

$$\beta^2 - \gamma(k_1\gamma + k_2\beta)\lambda_{\min}(\Gamma)\delta_{\min} < 0 \tag{11b}$$

$$\kappa_1 < 0 \tag{11c}$$

$$\kappa_2 < 0 \tag{11d}$$

$$\kappa_3 = \frac{c\kappa_1}{2T} + \frac{b_2(k_1\gamma + k_2\beta)}{2}\lambda_{\min}(\Gamma)\delta_{\min} + \frac{b_2\beta\rho}{2} < 0 \tag{11e}$$

$$\kappa_4 = \frac{c\kappa_2}{2T} + \frac{b_2\gamma}{2} + \frac{b_2\beta}{2\rho} < 0 \tag{11f}$$

where $\kappa_1 = (-k_1\beta + \frac{k_1\gamma + k_2\beta}{c} + \frac{d_2(k_1\gamma + k_2\beta)}{2})\lambda_{\min}(\Gamma)\delta_{\min} + \frac{\beta\rho}{2c} + \frac{\beta\rho d_2}{2}$ and $\kappa_2 = -k_2\gamma\lambda_{\min}(\Gamma)\delta_{\min} + \beta + \frac{d_2\gamma}{2} + \frac{\beta}{2c\rho} + \frac{d_2\beta}{2\rho}$.

Proof: See Section II in the Supplementary Material. ■

Remark 3: To select suitable control parameters for (10) based on Theorem 1, the process of parameter tuning can be broken down into three steps. First, provide the communication topology graph $\mathcal{G}$ and Laplacian matrix. Next, compute the parameters $\lambda_{\min}(\Gamma)$ and $\delta_{\min}$. Finally, by initializing μ, c, α, β, γ, b_2 and d_2, we can obtain control parameters k_1 and k_2 through (11b) to (11f).

Numerical example: The PEG constituted by a platoon of agents with 1 evader and 3 pursuers is considered. The communications architecture is shown in Fig. 1.

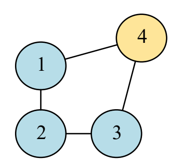

Fig. 1. The communication architecture PEG.

The initial states, velocities, and estimates for the agents in the first group are set as follows: $\mathbf{x}(0) = (1,2,3,4)^T$, $\mathbf{v}(0) = (1,2,3,4)^T$, and $\mathbf{y}(0) = (-1,2,3,0)^T$. Given the preceding discussion, the Nash equilibrium is achieved when $x_1 = x_2 = x_3 = x_4$ and $v_1 = v_2 = v_3 = v_4$. The simulation runs from $t = 0$ s to $t = 4$ s, with a prescribed-time set to $T_p = 2T = 2$ s. Moreover, the control input for the evader is defined as $u_4(t) = 2\sin(t)$. Let $\beta = 1$, $\gamma = 1$, $\rho = 2$, $b_2 = 0$, and $d_2 = 0.1$. The parameters in (10) are set as $b_1 = 0$, $d_1 = 1$, $\mu = 2$, $c = 3$, $k_1 = 5.3$, and $k_2 = 2.5$.

With the provided parameters, all the agents' position trajectories, velocity trajectories and estimate trajectories are depicted in Figs. 2 and 3, respectively. Fig. 2 shows that pursuers are able to pursue the evader within the prescribed-time. Fig. 3 indicates that all pursuers maintain consistency with the evader's velocity. Fig. 4 shows that within 1 s, each agent's estimate of u_4 reaches the true value. As to the second group of initial states $\mathbf{x}'(0) = (5,10,15,20)^T$, velocities $\mathbf{v}'(0) = (5,10,15,20)^T$, and estimates $\mathbf{y}'(0) = (1,2,3,0)^T$, the results are presented in Figs. 5−7. It can be seen that the PTNE seeking algorithm converges within the prescribed-time for this group as well. Hence, the proposed method has been numerically verified.

Conclusion: In this letter, the PTNE of PEG which contains agents with second-order dynamics has been investigated. A distributed algorithm has been presented for pursuers to capture the evader within the prescribed-time. By adaptively adjusting the parameters of the control scheme, the PTNE was obtained. Then, the effectiveness

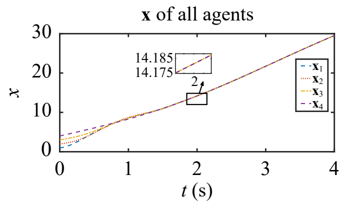

Fig. 2. x_i under $\mathbf{x}(0)$, $\mathbf{v}(0)$, $\mathbf{y}(0)$.

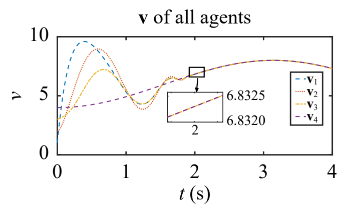

Fig. 3. v_i under $\mathbf{x}(0)$, $\mathbf{v}(0)$, $\mathbf{y}(0)$.

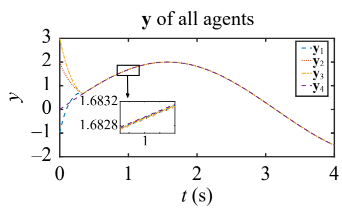

Fig. 4. y_i under $\mathbf{x}(0)$, $\mathbf{v}(0)$, $\mathbf{y}(0)$.

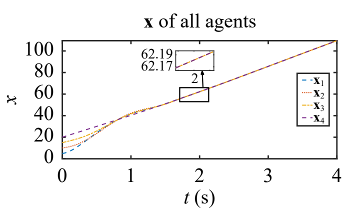

Fig. 5. x_i under $\mathbf{x}'(0)$, $\mathbf{v}'(0)$, $\mathbf{y}'(0)$.

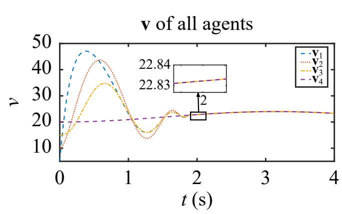

Fig. 6. v_i under $\mathbf{x}'(0)$, $\mathbf{v}'(0)$, $\mathbf{y}'(0)$.

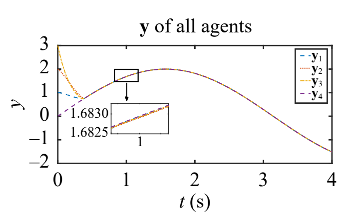

Fig. 7. y_i under $\mathbf{x}'(0)$, $\mathbf{v}'(0)$, $\mathbf{y}'(0)$.

of the designed method was verified by the numerical examples. In the future, the PTNE of PEG under joint switching communication architecture will be considered.

Acknowledgments: This work was supported in part by the Natural Science Foundation of Jiangsu Province of China (BK20231417, BK20210214), the National Natural Science Foundation of China (62273094, 62203114), and the "Zhishan" Scholars Programs of Southeast University.

Supplementary material: The supplementary material associated with this letter are accessible through the following links: https://maifile.cn/est/a3106980374449/pdf.

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