Letter

Set Stabilization of Large-Scale Stochastic Boolean Networks: A Distributed Control Strategy

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Dear Editor,

This letter deals with the set stabilization of stochastic Boolean control networks (SBCNs) by the pinning control strategy, which is to realize the full control for systems by imposing control inputs on a fraction of agents. The pinned agents are determined based on the information on the network structure, rather than the whole state transition, of an SBCN. Regarding each pinned agent, a mode-dependent pinning controller and a more economical mode-independent pinning controller are designed to stabilize SBCNs towards a given state set. Finally, a 90-nodal T-cell receptor signaling network is presented to illustrate the theoretical validity of the obtained results.

SBCN is a stochastic and controlled counterpart of Kauffman's Boolean networks (BNs) [1] to model gene regulatory networks under uncertain gene interactions and therapeutic interventions. In the SBCN model, the expression of genes is quantified as states "1" (ON) and "0" (OFF), and the state update obeys logical functions, randomly selected from a candidate set at each time step. Beyond biological systems, the SBCN model has attracted considerable attention in potential games [2], multi-agent systems [3], networked knowledge [4], and smart home [5]. Stabilization, as an essential issue in SBCNs, reveals the ability of systems to reach and stay at a desired attractor (fixed state or state set) [6]–[8]. As discovered in the large granular lymphocyte [9], the full-zero steady state manifests programmed cell death, while some states indicate healthy and beneficial situations.

Recent efforts towards addressing SBCN-related problems are mainly based on the algebraic state space representation (ASSR) approach [10]. The main idea of this approach is to convert the logical function into an algebraic form with a state transition matrix of exponential dimension. Although this approach facilitates the analysis and control of SBCNs, it encounters a high computational complexity, which restricts the application to small-scale networks. Lately, a remarkable framework is established in [11]–[13] to handle the (set) stabilization, controllability, and their minimal node control problems of large-scale BNs by integrating the information on the network structures. It not only mitigates the uncertainties in the nodal dynamics but also maintains the time complexity of designing pinning controllers for BNs at a relatively low level. However, these works focus on deterministic BNs, rather than a more general SBCN model.

Motivated by the above discussion, this letter aims to design a distributed pinning control scheme to stabilize SBCNs towards a given state set from the network-structural viewpoint. Key novelties of this scheme are three aspects: 1) The pinned agent is determined by finding the arcs from the non-functional agent set to the functional agent set, the minimum feedback arc set, and the agent with an undesired steady state. Compared with the pinned agent selection in [14] and [15], our method only uses the information of the n-nodal network structure instead of the 2^n-nodal state transition, which extremely reduces the time complexity. 2) Mode-dependent pinning controller to stabilize SBCNs is proven to exist. To reduce the control cost, a mode-independent pinning controller is further designed. 3) A biological example is presented to reveal that this control design scheme is applicable to therapeutic interventions for large-scale T-cell receptor signaling networks.

Notations: Throughout this letter, $\mathbb{B} := \{0,1\}$; $+_{\mathbb{B}}$ is the modular addition operation over the field $\mathbb{B}$; $\mathbb{N}_{[l:r]} := \{l, l+1, \ldots, r\}$; δ_n^k is the k-th column of the n-dimensional identity matrix; $\delta_n[k_1, k_2, \ldots, k_m] = [\delta_n^{k_1}\ \delta_n^{k_2} \ldots \delta_n^{k_m}]$; $\ltimes$ is the semi-tensor product of matrices; $\otimes$ is the Kronecker product of matrices; $\Phi_n := [\delta_n^1 \ltimes \delta_n^1\ \delta_n^2 \ltimes \delta_n^2 \ldots \delta_n^n \ltimes \delta_n^n]$; $\mathsf{Col}_i(A)$ is the i-th column of matrix A; $|S|$ is the cardinality of set S.

Problem statement: Consider the SBCN with n agents and m modes as

$$x_i(t+1) = \begin{cases} u_i^{\sigma(t)}(t) \odot_i^{\sigma(t)} f_i^{\sigma(t)}\left([x_j(t)]_{j \in \mathcal{N}_i^{\sigma(t)}}\right), & i \in \Lambda \\ f_i^{\sigma(t)}\left([x_j(t)]_{j \in \mathcal{N}_i^{\sigma(t)}}\right), & i \notin \Lambda \end{cases} \tag{1}$$

where $x_i \in \mathbb{B}$ is the state variable of agent i; $\mathcal{N}_i^{\sigma(t)} \subseteq \mathbb{N}_{[1:n]}$ is the set of in-neighbors for agent i; $\sigma(t) \in \mathbb{N}_{[1:m]}$ is the switching signal; $f_i^{\sigma(t)} : \mathbb{B}^{|\mathcal{N}_i^{\sigma(t)}|} \to \mathbb{B}$ is a logical function; $u_i^{\sigma(t)} \in \mathbb{B}$ is the control input; $\odot_i^{\sigma(t)} \in \{\vee, \wedge, \dagger, \leftrightarrow, \ldots\}$ is the binary Boolean operator that couples control input $u_i^{\sigma(t)}$ and original dynamic equation $f_i^{\sigma(t)}$. Here, the set of agents that should be imposed controller is denoted by Λ and is hereafter called pinned agent set.

Many dynamics issues, including synchronization, observability, output tracking, and stabilization to a single state, can be transformed into a set stabilization problem [16]. Before proceeding further, we define set stabilization of SBCN (1) under pinning control.

Definition 1 [16]: Given a state set $\mathbf{S} \subseteq \mathbb{B}^n$, SBCN (1) is said to achieve the finite-time $\mathbf{S}$-stabilization with probability one by pinning control if there exists an integer $T > 0$ such that $\Pr\{x(t) := (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbf{S}\} = 1$ holds for all $t \geq T$.

Main results: Given a desired state set $\mathbf{S} \subseteq \mathbb{B}^n$, the feasible pinning controller to $\mathbf{S}$-stabilize SBCN (1) can be designed by two steps:

Step 1: Determine the pinned agent set Λ. Regarding the desired state set $\mathbf{S} \subseteq \mathbb{B}^n$, we first divide the agent set $\mathbb{N}_{[1:n]}$ into two disjoint subsets Ω and Ω^c as

$$\Omega := \left\{ i \in \mathbb{N}_{[1:n]} \mid x \in \mathbf{S}, x +_{\mathbb{B}} \delta_n^i \in \mathbf{S} \right\}, \quad \Omega^c := \mathbb{N}_{[1:n]} \setminus \Omega. \tag{2}$$

It indicates that the agent in Ω could be in an arbitrary state, that is, 1 or 0, which does not affect the $\mathbf{S}$-stabilization of SBCN (1). In reverse, agents in Ω^c should maintain their states to realize the $\mathbf{S}$-stabilization of SBCN (1). In this case, we denote by Ξ_1 the arc set, which contains the arcs from the agents in Ω to the agents in Ω^c. After that, collect the ending vertices of arcs in Ξ_1 into the set

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$$\Lambda_1 := \left\{i \in \Omega^c \mid \mathcal{N}_i^{\sigma(t)} \cap \Omega \neq \emptyset,\ \sigma(t) \in \mathbb{N}_{[1:m]}\right\}. \quad (3)$$

Next, we denote the network structure of SBCN (1) by a digraph $\mathsf{G} := (\mathsf{V}, \mathsf{E})$ with $\mathsf{V} := \mathbb{N}_{[1:n]}$ and $\mathsf{E} := \{e_{j\to i} \mid j \in \mathcal{N}_i^{\sigma(t)}, \sigma(t) \in \mathbb{N}_{[1:m]}, i \in \mathbb{N}_{[1:n]}\}$. Regarding Ω^c, we further denote a sub-digraph of G by $\mathsf{G}_{\Omega^c} := (\mathsf{V}_{\Omega^c}, \mathsf{E}_{\Omega^c})$ with $\mathsf{V}_{\Omega^c} := \Omega^c$ and $\mathsf{E}_{\Omega^c} := \mathsf{E} \cap (\Omega^c \times \Omega^c)$. Here, we find the minimum feedback arc set in G_{Ω^c} as Ξ_2, which contains the minimum number of arcs whose deletion can make G_{Ω^c} acyclic. The algorithm for finding the minimum feedback arc set in a given digraph please refer to [13]. Subsequently, let the agent set that contains the ending vertices of arcs in Ξ_2 be

$$\Lambda_2 := \left\{i \in \Omega^c \mid e_{j\to i} \in \Xi_2\right\}. \quad (4)$$

Finally, for agent $i \in \Omega^c$, its in-neighbor set becomes $\hat{\mathcal{N}}_i^{\sigma(t)}$ after deleting the in-arcs in $\Xi_1 \cup \Xi_2$. It indicates that $\left(\bigcup_{\kappa=1}^m \hat{\mathcal{N}}_i^\kappa\right) \cap \Omega = \emptyset$. Denote the steady state of each agent $i \in \Omega^c$ in mode $\kappa \in \mathbb{N}_{[1:m]}$ by $\alpha_i^\kappa \in \mathbb{B}$, which satisfies $\alpha_i^\kappa = f_i^\kappa\left([\alpha_j^\kappa]_{j\in\hat{\mathcal{N}}_i^\kappa}\right)$. Accordingly, we denote by

$$\Lambda_3^\kappa := \Omega^c \setminus \left\{i \in \Omega^c \mid x \in \mathbf{S}, (\delta_n^i)^T x = \alpha_i^\kappa\right\} \quad (5)$$

the set of agents whose steady state is unequal to the desired state. Combined with the above results, the pinned agent set is obtained as

$$\Lambda = \Lambda_1 \cup \Lambda_2 \cup \left(\bigcup_{\kappa=1}^m \Lambda_3^\kappa\right). \quad (6)$$

Step 2: Construct control input $u_i^{\sigma(t)}$ and logical coupling $\odot_i^{\sigma(t)}$. Here, the mode-dependent control input $u_i^{\sigma(t)}$ is in the feedback form

$$u_i^{\sigma(t)}(t) = h_i^{\sigma(t)}\left([x_j(t)]_{j\in\mathcal{N}_i^{\sigma(t)}}\right) \quad (7)$$

where $h_i^{\sigma(t)}$ is a logical function. As proved in [14], regarding each pinned agent $i \in \Lambda$, feasible mode-dependent pinning controllers (that is, $u_i^{\sigma(t)}$ and $\odot_i^{\sigma(t)}$) always exist.

By resorting to the ASSR approach [10], the algebraic form of dynamics (1) for $i \in \Lambda$ is derived as

$$\delta_2^{2-x_i(t+1)} = \mathsf{M}_i^{\sigma(t)}\mathsf{H}_i^{\sigma(t)}(I_{2^{|\mathcal{N}_i^{\sigma(t)}|}} \otimes \mathsf{F}_i^{\sigma(t)})\Phi_{2^{|\mathcal{N}_i^{\sigma(t)}|}} \ltimes_{j\in\mathcal{N}_i^{\sigma(t)}} \delta_2^{2-x_j(t)}$$

where $\mathsf{M}_i^{\sigma(t)}$, $\mathsf{H}_i^{\sigma(t)}$, and $\mathsf{F}_i^{\sigma(t)}$ are the structure matrices of $\odot_i^{\sigma(t)}$, $h_i^{\sigma(t)}$, and $f_i^{\sigma(t)}$, respectively. Note that $\mathsf{M}_i^{\sigma(t)}$ and $\mathsf{H}_i^{\sigma(t)}$ can uniquely determine $\odot_i^{\sigma(t)}$ and $h_i^{\sigma(t)}$, respectively. Furthermore, after deleting the in-arcs in $\Xi_1 \cup \Xi_2$ and shifting the steady state of agents in Λ_3, the dynamics for $i \in \Lambda$ are turned to

$$x_i(t+1) = \oslash_i^{\sigma(t)} \hat{f}_i^{\sigma(t)}\left([x_j^{\sigma(t)}]_{j\in\hat{\mathcal{N}}_i^{\sigma(t)}}\right) \quad (8)$$

where $\hat{f}_i^{\sigma(t)}$ is a logical function, and $\oslash_i^{\sigma(t)}$ is a conditional operator, which is assigned NOT operator "$\neg$" if $i \in \Lambda_3^{\sigma(t)}$ and vanishes otherwise. Its algebraic form is derived as

$$\delta_2^{2-x_i(t+1)} = \hat{\mathsf{F}}_i^{\sigma(t)} \ltimes_{j\in\hat{\mathcal{N}}_i^{\sigma(t)}} \delta_2^{2-x_j(t)} = \hat{\mathsf{F}}_i^{\sigma(t)} W_i^{\sigma(t)} \ltimes_{j\in\mathcal{N}_i^{\sigma(t)}} \delta_2^{2-x_j(t)}$$

where $\hat{\mathsf{F}}_i^{\sigma(t)}$ is the structure matrix of $\neg\hat{f}_i^{\sigma(t)}$ for $i \in \Lambda_3^{\sigma(t)}$ and of $\hat{f}_i^{\sigma(t)}$ for others, and $W_i^{\sigma(t)}$ is the dummy matrix, whose construction can be referred to [13]. In what follows, we present the feasible pinning control design scheme for $\mathbf{S}$-stabilizing SBCN (1).

Theorem 1: Given a state set $\mathbf{S} \subseteq \mathbb{B}^n$, SBCN (1) is finite-time $\mathbf{S}$-stabilized with probability one under the pinning controller with Λ determined as (6) and $\odot_i^{\sigma(t)}$, $h_i^{\sigma(t)} (i \in \Lambda)$ solved from the equations

$$\mathsf{M}_i^{\sigma(t)}\mathsf{H}_i^{\sigma(t)}(I_{2^{|\mathcal{N}_i^{\sigma(t)}|}} \otimes \mathsf{F}_i^{\sigma(t)})\Phi_{2^{|\mathcal{N}_i^{\sigma(t)}|}} = \hat{\mathsf{F}}_i^{\sigma(t)} W_i^{\sigma(t)}. \quad (9)$$

Proof: Under such designed pinning controllers, the dynamic equation of pinned agent $i \in \Lambda$ turns to (8). In this case, the in-neighbor set for agent $i \in \Lambda$ turns to $\hat{\mathcal{N}}_i^{\sigma(t)}$, which is obtained from $\mathcal{N}_i^{\sigma(t)}$ by deleting the in-arcs in $\Xi_1 \cup \Xi_2$. Deleting the in-arcs in Ξ_1 eliminates the impact of agents in Ω on the controlled agent i. Without loss of generality, assume $p \in \mathcal{N}_i^{\sigma(t)} \cap \Omega$. It holds

$$f\left([x_j(t)]_{j\in\mathcal{N}_i^{\sigma(t)}\setminus\{p\}}, \neg x_p(t)\right) \neq f\left([x_j(t)]_{j\in\mathcal{N}_i^{\sigma(t)}}\right). \quad (10)$$

From the definition of Ω in (2), there exists a series of entries $a_j \in \mathbb{B}$, $j \in \mathbb{N}_{[1:n]} \setminus \{p\}$, such that $\eta_p^0 := (a_1, a_2, \ldots, a_p = 0, \ldots, a_n)^T \in \mathbf{S}$ and $\eta_p^1 := (a_1, a_2, \ldots, a_p = 1, \ldots, a_n)^T \in \mathbf{S}$. Starting from state η_p^0 (or η_p^1), the next state of SBCN (1) obeying mode $\kappa \in \mathbb{N}_{[1:m]}$ is defined as $x^\kappa(1; \eta_p^0)$ (or $x^\kappa(1; \eta_p^1)$). If $x^\kappa(1; \eta_p^0) \in \mathbf{S}$ (or $x^\kappa(1; \eta_p^1) \in \mathbf{S}$), it must hold $x^\kappa(1; \eta_p^1) \notin \mathbf{S}$ (or $x^\kappa(1; \eta_p^0) \notin \mathbf{S}$) owing to $i \in \Omega^c$ and (10). It leads to the observation that SBCN (1) will be unstable although it reaches a stable state η_p^1 (or η_p^0) in $\mathbf{S}$.

Furthermore, deleting the arcs in Ξ_2 makes G_{Ω^c} acyclic. Since G_{Ω^c} is the union of m network structures corresponding to m modes of SBCN (1), then, for each mode $\kappa \in \mathbb{N}_{[1:m]}$, its network structure induced by the agents in Ω^c, denoted by $\mathsf{G}_{\Omega^c}^\kappa$, is also acyclic. Note that $\mathsf{G}_{\Omega^c} = \bigcup_{\kappa=1}^m \mathsf{G}_{\Omega^c}^\kappa$. By referring to [17], one can draw the conclusion that, regarding each mode $\kappa \in \mathbb{N}_{[1:m]}$, agent $i \in \Omega^c$ is stabilized at a fixed state, termed as $\alpha_i^\kappa \in \mathbb{B}$. If $i \in \Lambda_3^{\sigma(t)}$, steady state $\alpha_i^{\sigma(t)}$ is not the value of the i-th entry of a state in $\mathbf{S}$ and should be shifted to $\neg\alpha_i^{\sigma(t)}$. Therefore, in mode $\sigma(t)$, the dynamic equation for $i \in \Lambda_3^{\sigma(t)}$ has an additional NOT operator. ■

Theorem 1 provides a way to solve all feasible mode-dependent pinning controllers, which exist for any SBCN (1). To save switching devices, we prioritize imposing the following mode-independent pinning controller on $i \in \Lambda$:

$$\begin{cases} u_i^{\sigma(t)}(t) = u_i(t) := h_i\left([x_j(t)]_{j\in\mathcal{N}_i}\right), \\ \odot_i^{\sigma(t)} = \odot_i, \end{cases} \quad \forall\, \sigma(t) \in \mathbb{N}_{[1:m]} \quad (11)$$

where $\mathcal{N}_i := \bigcup_{\kappa=1}^m \mathcal{N}_i^\kappa$, h_i is a logical function, and $\odot_i$ is the binary Boolean operator. Next, we explore the existence condition of feasible mode-independent pinning controllers to $\mathbf{S}$-stabilize SBCN (1).

Theorem 2: To transform the dynamics of agent $i \in \Lambda$ into (8) such that SBCN (1) is $\mathbf{S}$-stabilized, mode-independent pinning controller is feasible for agent $i \in \Lambda$ if and only if it holds

$$\mathbb{N}_{[1:2^{|\mathcal{N}_i|}]} \in \{\Theta_i^a \cup \Theta_i^b \mid a, b \in \mathbb{N}_{[1:4]}\} \quad (12)$$

with

$$\begin{aligned} \Theta_i^1 &:= \{j \mid \mathsf{Col}_j(\mathsf{F}_i^\kappa \hat{W}_i^\kappa) = \mathsf{Col}_j(\hat{\mathsf{F}}_i^\kappa \check{W}_i^\kappa), \kappa \in \mathbb{N}_{[1:m]}\} \\ \Theta_i^2 &:= \{j \mid \mathsf{Col}_j(\mathsf{F}_i^\kappa \hat{W}_i^\kappa) \neq \mathsf{Col}_j(\hat{\mathsf{F}}_i^\kappa \check{W}_i^\kappa), \kappa \in \mathbb{N}_{[1:m]}\} \\ \Theta_i^3 &:= \{j \mid \mathsf{Col}_j(\hat{\mathsf{F}}_i^\kappa \check{W}_i^\kappa) = \delta_2^1, \kappa \in \mathbb{N}_{[1:m]}\} \\ \Theta_i^4 &:= \{j \mid \mathsf{Col}_j(\hat{\mathsf{F}}_i^\kappa \check{W}_i^\kappa) = \delta_2^2, \kappa \in \mathbb{N}_{[1:m]}\} \end{aligned} \quad (13)$$

where $\hat{W}_i^\kappa$ and $\check{W}_i^\kappa$ are dummy matrices.

Proof: According to Theorem 1, a mode-independent pinning controller is feasible to transform the dynamics of agent $i \in \Lambda$ into (8) if and only if the following equations are solvable:

$$\mathsf{M}_i\mathsf{H}_i[I_{2^{|\mathcal{N}_i|}} \otimes (\mathsf{F}_i^\kappa \hat{W}_i^\kappa)]\Phi_{2^{|\mathcal{N}_i|}} = \hat{\mathsf{F}}_i^\kappa \check{W}_i^\kappa,\ \kappa \in \mathbb{N}_{[1:m]},\ i \in \Lambda \quad (14)$$

where M_i and H_i are the structure matrices of $\odot_i$ and h_i. The relation between the columns of matrices $\mathsf{F}_i^\kappa \hat{W}_i^\kappa$ and $\hat{\mathsf{F}}_i^\kappa \check{W}_i^\kappa$ has four possible cases, indicated by Θ_i^1, Θ_i^2, Θ_i^3, and Θ_i^4. They, respectively, induce $\mathsf{M}_i =: [\mathsf{M}_i^1, \mathsf{M}_i^2]$ with M_i^1 and M_i^2 valued from $\delta_2[1,2]$, $\delta_2[2,1]$, $\delta_2[1,1]$, and $\delta_2[2,2]$. Therefore, equations (14) are solvable if and only if all the columns can be fully contained by at most two of the cases indicated by Θ_i^1, Θ_i^2, Θ_i^3, and Θ_i^4, that is, Condition (12). ■

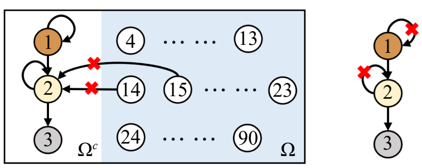
Remark 1: Regarding SBCN (1), its distributed pinning control design based on network structure makes the time complexity be $O(|\mathbf{S}|n + \sum_{i\in\Omega^c} |\mathcal{N}_i||\Lambda| + n^2 + |\Lambda|2^{\max_{i\in\Lambda}|\mathcal{N}_i|})$. It breaks $O(2^n)$-barrier in traditional pinning control design methods (see, e.g., [14] and [15]), and thus can be applied to large-scale systems.

Biological example: Consider a 90-nodal T-cell receptor signaling network, whose logical rule governing the state update of each node is shown in [18]. Since costimulatory molecule CD28 (x_1), coreceptors CD4 (x_2), and TCRlig (x_3) have an impact on the activation of key transcription factors in T cells [18], their desired states are ON, and the target state set is assigned as $\mathbf{S} = \{(1,1,1,x_4,\ldots,x_{90})^T \mid x_4,\ldots,x_{90} \in \mathbb{B}\}$. Besides, camk2 ($x_{13}$), camk4 ($x_{14}$), and rlk ($x_{80}$) are Kinase molecules [18]. Their different combinations of states can induce 8 possible modes of an SBCN.

The process of determining the pinned nodes is illustrated in Fig. 1. First, by (2), one can calculate $\Omega = \mathbb{N}_{[4:90]}$ and $\Omega^c = \{1,2,3\}$. As indicated in Fig. 1(a), it derives $\Xi_1 = \{e_{14\to2}, e_{15\to2}\}$, and thus $\Lambda_1 = \{2\}$. Furthermore, one can obtain sub-digraph G_{Ω^c} as Fig. 1(b). It derives $\Xi_2 = \{e_{1\to1}, e_{2\to2}\}$, and thus $\Lambda_2 = \{1,2\}$. The dynamics of agents 1, 2, 3 are, respectively, established as

$$x_1^+ = x_1,\ x_2^+ = \begin{cases} \neg x_{15} \wedge [x_1 \vee (\neg x_1 \wedge x_2)], & \sigma = 1,2,5,6,\\ \neg x_{15} \wedge x_1 \wedge x_2, & \sigma = 3,4,7,8, \end{cases} x_3^+ = x_2$$

where x^+ represents the state at the next time step. Since $f_1(x_1 = 1) = 1$, $f_2(x_1 = 1, x_2 = 1, x_{15} = 0) = 1$, $f_2(x_1 = 1, x_2 = 1, x_{15} = 1) = 0$, and $f_3(x_2 = 1) = 1$, one obtains $\Lambda_3^\kappa = \{2\}, \kappa \in \mathbb{N}_{[1:8]}$. Therefore, the pinned node set is $\Lambda = \{1,2\}$.

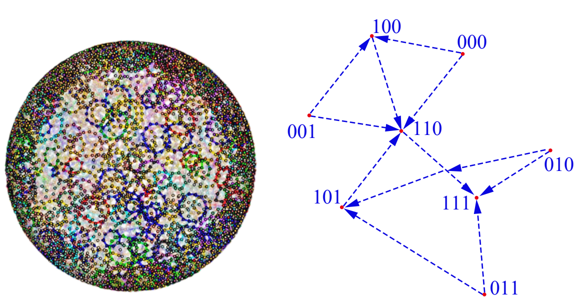


(a) (b)

Fig. 1. Illustration of pinning controlled T-cell receptor signaling network. (a) Sub-digraph G; (b) Sub-digraph G_{Ω^c}.

According to Theorem 1, we calculate that $\mathsf{F}_1^\kappa = \delta_2[1\ 2]$ and $\hat{\mathsf{F}}_1^\kappa W_1^\kappa = \delta_2[1\ 1]$ for all $\kappa \in \mathbb{N}_{[1:8]}$; $\mathsf{F}_2^\kappa = \delta_2[2,1,2,1,2,1,2,2]$ and $\hat{\mathsf{F}}_2^\kappa W_2^\kappa = \delta_2[1,1,1,1,1,1,1,1]$ for $\kappa = 1,2,5,6$, while $\mathsf{F}_2^\kappa = \delta_2[2,1,2,2,2,2,2,2]$ and $\hat{\mathsf{F}}_2^\kappa W_2^\kappa = \delta_2[1,1,1,1,2,2,2,2]$ for $\kappa = 3,4,7,8$. By Theorem 2, mode-independent pinning controller is feasible for Agent 1 owing to $\Theta_1^3 = \mathbb{N}_{[1:2]}$ but is infeasible for Agent 2 since Condition (12) does not hold for $\Theta_2^1 = \{2,6\}$, $\Theta_2^2 = \{1,3\}$, $\Theta_2^3 = \{1,2,3,4\}$, and $\Theta_2^4 = \emptyset$. By solving (9), one can obtain a feasible solution as $\mathsf{M}_1 = \delta_2[1,1,1,2]$ and $\mathsf{H}_1 = \delta_2[2,1]$, which correspond to $\odot_1 = \vee$ and $u_1 = \neg x_1$. Then, by solving (14), one can obtain a feasible solution as, for $\kappa = 1,2,5,6$, $\mathsf{M}_2^\kappa = \delta_2[1,1,1,2]$ and $\mathsf{H}_2^\kappa = \delta_2[1,2,1,2,1,1,1,1]$, that is, $\odot_2^\kappa = \vee$ and $u_2^\kappa = \neg x_1 \vee x_{15}$; for $\kappa = 3,4,7,8$, $\mathsf{M}_2^\kappa = \delta_2[1,2,2,1]$ and $\mathsf{H}_2^\kappa = \delta_2[2,1,2,2,1,1,1,1]$, that is, $\odot_2^\kappa = \leftrightarrow$ and $u_2^\kappa = \neg x_1 \vee (\neg x_{15} \wedge x_2)$.

Under the above-designed pinning controller, the state transition graph of SBCN (1) is derived as Fig. 2. Fig. 2(a) indicates the global state transition with 398 cyclic attractors, and Fig. 2(b) shows that $(x_1(t), x_2(t), x_3(t))$ starting from any state will evolve into $(1,1,1)$.



(a) Whole transition (b) Part transition

Fig. 2. Illustration of pinning controlled T-cell receptor signaling network.

Conclusion: This letter has investigated the problem of stabilizing SBCNs via a distributed pinning control strategy. The pinned agent set has been determined based on the network structure, and subsequently, mode-dependent and mode-independent pinning controllers have been constructed. A biological example indicates that stabilizing a 90-nodal T-cell receptor signaling network to $\mathbf{S}$ can only impose controllers on two agents. Besides, the maximum in-degree of all agents being 4 contributes to the time complexity of controller design here being $O(n^2)$, which is much less than $O(2^n)$ in [14] and [15].

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References

[1] S. Kauffman, "Metabolic stability and epigenesis in randomly constructed genetic nets," *J. Theoretical Biology*, vol. 22, no. 3, pp. 437–467, 1969.

[2] G. Zhao, Y. Wang, and H. Li, "A matrix approach to the modeling and analysis of networked evolutionary games with time delays," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 4, pp. 818–826, 2016.

[3] L. Lin, J. Cao, S. Zhu, and P. Shi, "Synchronization analysis for stochastic networks through finite fields," *IEEE Trans. Autom. Control*, vol. 67, no. 2, pp. 1016–1022, 2022.

[4] J. Lü, G. Wen, R. Lu, Y. Wang, and S. Zhang, "Networked knowledge and complex networks: An engineering view," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 8, pp. 1366–1383, 2022.

[5] M. Kabir, M. Hoque, B.-J. Koo, and S.-H. Yang, "Mathematical modelling of a context-aware system based on Boolean control networks for smart home," in *Proc. 18th IEEE Int. Symp. Consumer Electronics*, 2014, pp. 1–2.

[6] X. Li, D. Ho, and J. Cao, "Finite-time stability and settling-time estimation of nonlinear impulsive systems," *Automatica*, vol. 99, pp. 361–368, 2019.

[7] S. Zhu, J. Lu, Y. Lou, and Y. Liu, "Induced-equations-based stability analysis and stabilization of Markovian jump Boolean networks," *IEEE Trans. Autom. Control*, vol. 66, no. 10, pp. 4820–4827, 2020.

[8] X. Li, S. Song, and J. Wu, "Exponential stability of nonlinear systems with delayed impulses and applications," *IEEE Trans. Autom. Control*, vol. 64, no. 10, pp. 4024–4034, 2019.

[9] R. Zhang, M. V. Shah, J. Yang, S. B. Nyland, X. Liu, J. K. Yun, R. Albert, and T. P. Loughran, "Network model of survival signaling in large granular lymphocyte leukemia," *Proc. National Academy of Sciences*, vol. 105, no. 42, pp. 16308–16313, 2008.

[10] D. Cheng, H. Qi, and Z. Li, *Analysis and Control of Boolean Networks: A Semi-Tensor Product Approach*. London, UK: Springer-Verlag, 2011.

[11] S. Zhu, J. Lu, L. Sun, and J. Cao, "Distributed pinning set stabilization of large-scale Boolean networks," *IEEE Trans. Autom. Control*, vol. 68, no. 3, pp. 1886–1893, 2023.

[12] S. Zhu, J. Lu, S.-I. Azuma, and W. Zheng, "Strong structural controllability of Boolean networks: Polynomial-time criteria, minimal node control, and distributed pinning strategies," *IEEE Trans. Autom. Control*, vol. 68, no. 9, pp. 5461–5476, 2023.

[13] S. Zhu, L. Lin, J. Cao, J. Lam, and S.-i. Azuma, "Toward stabilizable Boolean networks: Minimal node control," *IEEE Trans. Autom. Control*, vol. 69, no. 1, pp. 174–188, 2024.

[14] F. Li and L. Xie, "Set stabilization of probabilistic Boolean networks using pinning control," *IEEE Trans. Neural Networks and Learning Systems*, vol. 30, no. 8, pp. 2555–2561, 2019.

[15] L. Wang, M. Fang, Z.-G. Wu, and J. Lu, "Necessary and sufficient conditions on pinning stabilization for stochastic Boolean networks," *IEEE Trans. Cyber.*, vol. 50, no. 10, pp. 4444–4453, 2019.

[16] Y. Guo, P. Wang, W. Gui, and C. Yang, "Set stability and set stabilization of Boolean control networks based on invariant subsets," *Automatica*, vol. 61, pp. 106–112, 2015.

[17] F. Robert, *Discrete Iterations: A Metric Study*. London, UK: Springer-Verlag, 2012.

[18] J. Saez-Rodriguez, L. Simeoni, J. A. Lindquist, *et al.*, "A logical model provides insights into T cell receptor signaling," *PLoS Computational Biology*, vol. 3, no. 8, p. e163, 2007.