Letter

Robust Distributed Model Predictive Control for Formation Tracking of Nonholonomic Vehicles

Zhigang Luo, Bing Zhu, Jianying Zheng, and Zewei Zheng

Dear Editor,

This letter proposes a robust distributed model predictive control (MPC) strategy for formation tracking of a group of wheeled vehicles subject to constraints and disturbances. Formation control has attracted significant interest because of its applications in searching and exploration [1], [2]. The objective of formation tracking is to achieve the reference formation via interactions among agents. In formation tracking, MPC can be applied to treat constrains. In [3], by sharing predictive information with neighbors, each agent uses distributed MPC strategy to generate the desired trajectory without any collision. Based on the leader-follower structure, a distributed MPC is proposed in [4] to mitigate effects caused by replay attacks. In [5], distributed MPC is applied to optimize energy scheduling.

Nonholonomic systems refer to those with non-integrable constraints [6]. For wheeled vehicles, side-slip is un-permitted, and this constraint is non-integrable. Besides, control constraints and external disturbance are suggested to be considered in kinematics of nonholonomic systems. A disturbance observer-based MPC strategy is designed in [7] to estimate unknown disturbances. A velocity integral controller incorporated in MPC is proposed in [8] to guarantee the formation control. Two robust MPC strategies are proposed in [9] for a unicycle robot to track its reference trajectory, where the formation with communications is not considered.

Inspired by [9], a robust distributed MPC for formation tracking of the non-holonomic multi-vehicle systems is proposed in this letter. Subject to the input constraint and external bounded disturbances, each vehicle tracks a virtual time-varying trajectory generated by the virtual structure approach, and exchanges information to achieve the formation. Main contributions include: 1) A modified robust constraint is designed to guarantee feasibility in case of bounded disturbances, and a novel positive invariant terminal set and auxiliary control are designed for individual vehicles to guarantee stability; 2) Coupling costs in individual optimizations are proposed for formation tracking.

Problem statement: Consider multi-vehicle systems

$$\dot{z}_i(t) = [v_i(t)\cos\theta_i(t), v_i(t)\sin\theta_i(t), \omega_i(t)]^T + [n_i^T(t), 0]^T \tag{1}$$

where $z_i(t) \triangleq [p_i(t)^T, \theta_i(t)]^T$ and $u_i(t) \triangleq [v_i(t), \omega_i(t)]^T$ denote the state and control of individual vehicles; $p_i(t) \triangleq [x_i(t), y_i(t)]^T$ represents the position of each vehicle in the inertial frame; $v_i(t)$ and $\omega_i(t)$ denote the linear and angular velocities, respectively; $\theta_i(t)$ denotes the orientation of each vehicle.

Since the disturbance is mainly induced by side-slip, position disturbance is considered and the angular velocity disturbance is neglected in this paper. $n_i(t) \triangleq [n_{xi}(t), n_{yi}(t)]^T$ is the external disturbance and it is bounded by $\|n_i(t)\| \le \eta$. The control input constraint satisfies $u_i(t) \in U_i = \{u_i(t) \| |v_i(t)| \le v_{\max}, |\omega_i(t)| \le \omega_{\max}\}$, where $v_{\max}$ and $\omega_{\max}$ are limits of linear and angular speed, respectively.

A virtual leader is introduced to facilitate the formation tracking.

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Its dynamics can be described by

$$\dot{z}_r(t) \triangleq [\cos\theta_r(t), 0; \sin\theta_r(t), 0; 0, 1][v_r(t); \omega_r(t)]$$

where $z_r(t) = [p_r(t)^T, \theta_r(t)]^T$ denotes its state; the leader position $p_r(t) = [x_r(t); y_r(t)]$; the leader input $u_r(t) = [v_r(t); \omega_r(t)]$ is designed in prior and known to all followers. Each follower can transmit predictive trajectories with its neighbors bilaterally and the communication topology structure is fully connected.

For each follower vehicle, its head state is defined by:

$$z_{hi}(t) = [p_{hi}(t)^T, \theta_{hi}(t)]^T = z_i(t) + l[\cos\theta_i(t); \sin\theta_i(t); 0]$$

where $p_{hi}(t) = [x_{hi}(t); y_{hi}(t)]$ denotes the head position, and l is the distance from the center to the head. For the virtual leader and the followers, Frenet-Serret frames ${}^rO$ and ${}^iO$ can be constructed, as shown in Fig. 1. For each vehicle, there is a virtual structure point $p_{di} = [x_{di}, y_{di}]^T$ in ${}^rO$ fixed on the leader. The objective is to drive the head of each vehicle P_{hi} to the desired virtual structure point p_{di}.

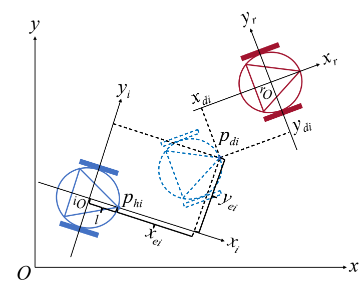

Fig. 1. Configuration of the virtual leader and followers.

Main result: For each follower, the tracking error $p_{ei} = [x_{ei}, y_{ei}]^T$ is defined by deviation between $p_{hi}(t)$ and p_{di} in ${}^iO$:

$$p_{ei} = S(\theta_i)(p_r - p_{hi}) + S^T(\theta_{ei})p_{di} \tag{2}$$

where

$$S(\theta_i) = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix}, \quad \theta_{ei} \triangleq \theta_r - \theta_i.$$

The tracking error dynamics is

$$\dot{p}_{ei}(t) = \begin{bmatrix} 0 & \omega_i(t) \\ -\omega_i(t) & 0 \end{bmatrix} p_{ei}(t) + u_{ei}(t) + n_i(t) \tag{3}$$

where

$$u_{ei}(t) = \begin{bmatrix} -v_i(t) + (v_r - y_{di}\omega_r)\cos\theta_{ei}(t) - x_{di}\omega_r \sin\theta_{ei}(t) \\ -l\omega_i(t) + (v_r - y_{di}\omega_r)\sin\theta_{ei}(t) + x_{di}\omega_r\cos\theta_{ei}(t) \end{bmatrix}.$$

In case of disturbance, the nominal system is added to facilitate the distributed MPC design. The superscript "$\tilde{\cdot}$" is used to denote the nominal term, and nominal error dynamics can be obtained by

$$\dot{\tilde{p}}_{ei}(t) = f(\tilde{p}_{ei}, \tilde{u}_{ei}) = \begin{bmatrix} 0 & \tilde{\omega}_i(t) \\ -\tilde{\omega}_i(t) & 0 \end{bmatrix} \tilde{p}_{ei}(t) + \tilde{u}_{ei}(t) \tag{4}$$

where $\tilde{p}_{ei}(t)$ is the error; $\tilde{u}_{ei}(t)$ is the virtual nominal error input.

Assumption 1: The nominal tracking error system (4) for each vehicle is Lipschitz in $\tilde{p}_{ei}$, i.e., $\|f(\tilde{p}_{ei}^1, \tilde{u}_{ei}) - f(\tilde{p}_{ei}^2, \tilde{u}_{ei})\| \le a\|\tilde{p}_{ei}^1 - \tilde{p}_{ei}^2\|$, where a is the Lipschitz constant.

For each vehicle, the cost function to be minimized is formulated by $J_i(\tilde{p}_{ei}(t_k), \tilde{u}_{ei}(t_k)) = \int_{t_k}^{t_k+T} L_i(\tilde{p}_{ei}(\tau|t_k), \tilde{u}_{ei}(\tau|t_k))d\tau + g_i(\tilde{p}_{ei}(t_k + T|t_k))$ with terminal cost $g_i(\tilde{p}_{ei}(t_k+T|t_k)) = \frac{1}{2}\|\tilde{p}_{ei}(t_k+T|t_k)\|^2$. T is the control horizon, and the stage cost is $L_i(\tilde{p}_{ei}(\tau|t_k), \tilde{u}_{ei}(\tau|t_k)) = \|\tilde{p}_{ei}(\tau|t_k)\|_Q^2 + \|\tilde{u}_{ei}(\tau|t_k)\|_P^2 + \sum_{j\in N_i}\|\tilde{p}_{ij}(\tau\|t_k)\|_H^2$, where $Q = \mathrm{diag}(q_1, q_2)$, $P = \mathrm{diag}(p_1, p_2)$, and $H = \mathrm{diag}(h_1, h_2)$. The formation error cost is constructed by $\|\tilde{p}_{ij}(\tau|t_k)\|_H^2 = \|\tilde{p}_{ei}(\tau\|t_k) - \hat{p}_{ej}(\tau|t_k)\|_H^2$, where $\hat{p}_{ej}(\tau|t_k)$ denotes the assumed position error trajectory of vehicle j.

Definition 1: The terminal set Ω_i is a region where an auxiliary terminal controller $\tilde{u}_i(\tau) = k_i(\tilde{p}_{ei}(\tau))$ exists such that, for any $\tilde{p}_{ei}(t) \in \Omega_i$, it satisfies $\tilde{p}_{ei}(\tau) \in \Omega_i$ and $\tilde{u}_i(\tau) \in U_i$ for $\tau > t$.

Lemma 1: For tracking dynamics (4), let $A = [1, 0; -2y_{di}, x_{di}^2 + y_{di}^2]$, and $u_i(\tau) = k_i(\tilde{p}_{ei}(\tau|t_k)) = [k_1\tilde{x}_{ei} + (v_r - y_{di}\omega_r)\cos\tilde{\theta}_{ei} - x_{di}\omega_r\sin\tilde{\theta}_{ei}; (k_2\tilde{y}_{ei} + (v_r - y_{di}\omega_r)\sin\tilde{\theta}_{ei} - x_{di}\omega_r\cos\tilde{\theta}_{ei})/l]$, with $k_1, k_2 > 0$. Then the terminal region is given by

$$\Omega_i = \{(\tilde{x}_{ei}, \tilde{y}_{ei}) | a_{\min} \le \tilde{x}_{ei} \le a_{\max},\ b_{\min} \le \tilde{y}_{ei} \le b_{\max}\} \tag{5}$$

where $a_{\min} = (-v_{\max} - \sqrt{\|A\|}\min\|u_r\|)/k_1$, $a_{\max} = (v_{\max} - \sqrt{\|A\|}\times\max\|u_r\|)/k_1$, $b_{\min} = (-l\omega_{\max} - \sqrt{\|A\|}\min\|u_r\|)/k_2$ and $b_{\max} = (l\omega_{\max} - \sqrt{\|A\|}\max\|u_r\|)/k_2$.

Proof: Choosing the terminal cost $g_i(\tilde{p}_{ei}(\tau|t_k)) = (\tilde{x}_{ei}^2 + \tilde{y}_{ei}^2)/2$ as the Lyapunov function and taking its derivative yield $\dot{g}_i(\tilde{p}_{ei}(\tau|t_k)) = \tilde{x}_{ei}\dot{\tilde{x}}_{ei} + \tilde{y}_{ei}\dot{\tilde{y}}_{ei}$. With the terminal controller $u_i(\tau)$, it follows that $\tilde{u}_{ei}(\tau) = [-k_1\tilde{x}_{ei}; -k_2\tilde{y}_{ei}]$ and $\dot{g}_i(\tilde{p}_{ei}(\tau|t_k)) = -k_1\tilde{x}_{ei}^2 - k_2\tilde{y}_{ei}^2 \le 0$. Parameters k_1 and k_2 reflect the rate of decay of g_i and can be adjusted as required. This implies that $\tilde{p}_{ei}(\tau) \in \Omega_i$ for any $\tau \ge t$ once $\tilde{p}_{ei}(t) \in \Omega_i$. Since $\tilde{u}_i(\tau) \in U_i$, it follows that $-v_{\max} \le k_1\tilde{x}_{ei} + (v_r - y_{di}\omega_r)\cos\tilde{\theta}_{ei} - x_{di}\omega_r\sin\tilde{\theta}_{ei} \le v_{\max}$ and $-\omega_{\max} \le \frac{1}{l}(k_2\tilde{y}_{ei} + (v_r - y_{di}\omega_r)\sin\tilde{\theta}_{ei} - x_{di}\omega_r\cos\tilde{\theta}_{ei}) \le \omega_{\max}$. Using the triangular inequality, it holds that $a_{\min} \le \tilde{x}_{ei} \le a_{\max}$ and $b_{\min} \le \tilde{y}_{ei} \le b_{\max}$. Thus the terminal controller satisfies the constraints in the terminal region (5). ■

In the distributed MPC, each vehicle i does not have acess to neighboring predictive trajectories. Before solving the optimization, each vehicle transmits the assumed predictive trajectories to neighbors. Define $u_i^*(\tau|t_k)$, $\hat{u}_i(\tau|t_k)$ as the optimal input signal obtained by calculating the optimization problem and assumed input signal, respectively. The assumed control signal $\hat{u}_i(\tau|t_k)$ is formulated by

$$\hat{u}_i(\tau|t_k) = \begin{cases} u_i^*(\tau|t_{k-1}), & \tau \in [t_{k-1}, t_{k-1}+T) \\ k_i(\tilde{p}_{ei}(\tau|t_{k-1})), & \tau \in [t_{k-1}+T, t_k+T). \end{cases} \tag{6}$$

By applying the assumed control input trajectory, the assumed position error trajectory $\hat{p}_{ei}(\tau|t_k)$ for each vehicle can be obtained.

The proposed MPC strategy is to guarantee that tracking errors of individual vehicles converge to a small neighborhood of the origin. The optimization for each vehicle is constructed by

$$\min_{\tilde{u}_{ei}} \quad J_i(\tilde{p}_{ei}(t_k), \tilde{u}_{ei}(t_k)), \quad \tau \in [t_k, t_k+T] \tag{7}$$

$$\text{s.t.} \quad \tilde{z}_i(t_k|t_k) = z_i(t_k),\ \tilde{u}_i(\tau|t_k) \in U_i \tag{8}$$

$$\dot{\tilde{z}}_i(\tau|t_k) = f(\tilde{z}_i(\tau|t_k), \tilde{u}_i(\tau|t_k)) \tag{9}$$

$$\|\tilde{p}_{ei}(\tau|t_k)\| \le re^{t_k+T-\tau},\ \tilde{p}_{ei}(t_k+T|t_k) \in \Omega_\varepsilon \tag{10}$$

where r is a constant satisfying $\Omega_r = \{\tilde{p}_{ei} | \|\tilde{p}_{ei}\| \le r\} \subseteq \Omega_i$, and the terminal region $\Omega_\varepsilon = \{\tilde{p}_{ei} | \|\tilde{p}_{ei}\| \le \varepsilon r\}$ and $0 < \varepsilon < 1$.

Remark 1: Initial assumed predictive trajectories is set to $\hat{p}_{ei}(\tau|t_0) = p_{ei}(t_0|t_0)$ to avoid centralized computation.

Lemma 2: Consider error dynamics (4). Suppose the optimization (7)−(10) is feasible at time t_k. If Algorithm 1 is implemented, and

$$\eta \le (1-\varepsilon)re^{-aT}/\delta, \quad \min\{k_1, k_2\}\delta \ge \ln(1/\varepsilon) \tag{11}$$

then, $\bar{u}_i(\tau|t_{k+1})$ exists at t_{k+1} satisfying the terminal constraint.

Algorithm 1 Robust Distributed MPC Algorithm for Each Vehicle

1: At the initial time $t_k = t_0$, set $\tilde{z}_i(t_k|t_k) = z_i(t_k)$ and $\hat{p}_{ei}(\tau|t_0) = p_{ei}(t_0|t_0)$, send $\hat{p}_{ei}(\tau|t_0)$ to $j \in N_i$, and receive $\hat{p}_{ej}(\tau|t_0)$ from $j \in N_i$.

2: Solve the optimization problem of every vehicle in parallel to obtain the optimal virtual error control sequence $\tilde{u}_{ei}^*(\tau|t_k)$, and calculate the actual control input $\tilde{u}_i^*(\tau|t_k)$ of each vehicle.

3: Apply $u(\tau) = \tilde{u}_i^*(\tau|t_k)$ to the real system over $\tau \in [t_k, t_{k+1})$.

4: Calculate predictive error trajectory $\hat{p}_{ei}(\tau|t_{k+1})$ and send it to $j \in N_i$ and get $\hat{p}_{ej}(\tau|t_{k+1})$ from $j \in N_i$.

5: Let $t_{k+1} = t_k + \delta$ and go to Step 2.

Proof: Assume the optimization problem is feasible at t_k. A feasible control sequence $\bar{u}_i(\tau|t_{k+1})$ for vehicle i at t_{k+1} exists

$$\bar{u}_i(\tau|t_{k+1}) = \begin{cases} \tilde{u}_i^*(\tau|t_k), & \tau \in [t_{k+1}, t_k+T) \\ k_i(\bar{p}_{ei}(\tau|t_{k+1})), & \tau \in [t_k+T, t_{k+1}+T) \end{cases} \tag{12}$$

where $k_i(\bar{p}_{ei}(\tau|t_{k+1}))$ is the terminal controller defined in Lemma 1. According to the invariant set theory, $\bar{u}_i(\tau|t_{k+1})$ at t_{k+1} satisfies the control constraint. Errors exist between (1) and the nominal system due to the disturbance. By using Gronwall-Bellman inequality, the bound of error at t_{k+1} satisfies

$$\left\|p_{ei}(t_{k+1}) - \tilde{p}_{ei}^*(t_{k+1}|t_k)\right\| \le \eta\delta e^{a\delta} \tag{13}$$

where $\tilde{p}_{ei}(t_k|t_k) = p_{ei}(t_k)$, and Assumption 1 holds. The feasible error state over $[t_{k+1}, t_k+T]$ is denoted by $\bar{p}_{ei}(\tau|t_{k+1})$. The bound of difference between $\bar{p}_{ei}(\tau|t_{k+1})$ and $\tilde{p}_{ei}^*(\tau|t_k)$ is then derived by $\eta\delta e^{a(\tau-t_k)}$. Substituting t_k+T into τ yields $\|\bar{p}_{ei}(t_k+T|t_{k+1})\| \le \eta\delta e^{aT} + \|\tilde{p}_{ei}^*(t_k+T|t_k)\|$. From (10) and (11), it follows that $\|\bar{p}_{ei}(t_k+T|t_{k+1})\| \le r$. Taking the derivative of $\|\bar{p}_{ei}(\tau|t_{k+1})\|^2$ and implementing the comparison principle yield $\|\bar{p}_{ei}(t_{k+1}+T|t_{k+1})\| \le \|\bar{p}_{ei}(t_k+T|t_{k+1})\|e^{-\delta\min\{k_1,k_2\}}$. According to (11), it follows that $\|\bar{p}_{ei}(t_{k+1}+T|t_{k+1})\| \le \varepsilon r$, indicating the terminal constraint is satisfied. ■

Lemma 3: Consider error dynamics (4). Assume the optimization (7)−(10) is feasible at t_k. Then, a feasible control $\bar{u}_i(\tau|t_{k+1})$ exists at t_{k+1} such that $\|\bar{p}_{ei}(\tau|t_{k+1})\| \le re^{t_{k+1}+T-\tau}$, if $e^\delta - 1 \ge 1-\varepsilon$ and the condition (11) are satisfied.

Proof: A feasible control sequence of at t_{k+1} is given in (12). For $\tau \in [t_{k+1}, t_k+T)$, according to (11) and Gronwall-Bellman inequality, it follows $\|\bar{p}_{ei}(\tau|t_{k+1})\| \le \left\|\tilde{p}_{ei}^*(\tau|t_k)\right\| + r - \varepsilon r$. Applying the conditions in Lemma 3 and (10) yield $\|\bar{p}_{ei}(\tau|t_{k+1})\| \le re^{t_k+T-\tau} + r(e^\delta - 1)$. To prove $\|\bar{p}_{ei}(\tau|t_{k+1})\| \le re^{t_{k+1}+T-\tau}$, construct $y(\tau) = re^{t_{k+1}+T-\tau} - re^{t_k+T-\tau} - r(e^\delta - 1)$. Since $y(t_k+T) = 0$ and $y(\tau)$ is decreasing with τ, it holds that $y(\tau) \ge 0$ over $[t_{k+1}, t_k+T)$, implying $re^{t_{k+1}+T-\tau} \ge re^{t_k+T-\tau} + r(e^\delta - 1)$. It then follows that $\|\bar{p}_{ei}(\tau|t_{k+1})\| \le re^{t_{k+1}+T-\tau}$. For $\tau \in [t_k+T, t_{k+1}+T)$, Lemma 2 implies that $\|\bar{p}_{ei}(\tau|t_{k+1})\| \le r \le re^{t_{k+1}+T-\tau}$. Consequently, the error constraint is satisfied over $[t_{k+1}, t_{k+1}+T)$. ■

Theorem 1: Assume that the optimization (7)–(10) is feasible for each vehicle initially. If conditions in Lemmas 2 and 3 are satisfied, the optimization problem (7)–(10) is feasible for all $t \ge t_0$.

Proof: Suppose the optimization is feasible at t_k. A feasible control sequence (12) exists. From Lemma 2, it holds that $\bar{u}_i(\tau|t_{k+1}) \in U_i$ and $\bar{p}_{ei}(t_{k+1}+T|t_{k+1}) \in \Omega_\varepsilon$. Meanwhile, constraint (10) can be satisfied according to Lemma 3. Hence, the optimization is feasible at t_{k+1}. By induction, the optimization is feasible $\forall t \ge t_0$. ■

Theorem 2: Consider error dynamics (4). If Theorem 1 holds, and

$$-k_1 + q_{\max} + p_{\max}k_1^2 \le 0, -k_2 + q_{\max} + p_{\max}k_2^2 \le 0 \tag{14}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \beta \le -\mu \tag{15}$$

where $\mu > 0$ is constant, $q_{\max} = \max\{q_1, q_2\}$, $q_{\min} = \min\{q_1, q_2\}$, $p_{\max} = \max\{p_1, p_2\}$, $h_{\max} = \max\{h_1, h_2\}$ and the stability parameters $\beta = q_{\min}\delta\varepsilon^2 r^2$, $\alpha_1 = q\eta^2\delta^2(e^{2aT} - e^{2a\delta})/(2a) + \sqrt{2}q\eta\delta r(e^{2aT} - e^{2a\delta})^{\frac{1}{2}} \times (re^{T-\delta} - r)^{\frac{1}{2}}/a$, $\alpha_2 = \eta\delta e^{aT}r$, $\alpha_3 = \sum_{j\in N_i} rh_{\max}((8r + 4\eta\delta)(e^{2T-\delta} - e^{T+\delta}) + (3e^{2\delta} + \varepsilon)(\eta\delta e^T - \eta\delta e^{T-\delta} + re^\delta - r + \delta\varepsilon r))$, $\alpha_4 = \sum_{j\in N_i} r^2\delta(1+\varepsilon^2)h_{\max}$, then tracking errors converges to $\Omega = \{p_e | \|p_e\| \le \eta\delta e^{a\delta} + \varepsilon r\}$.

Proof: Select the Lyapunov candidate for vehicle i by $V_i(t_k) = \min_{\theta/\theta} J_i(\tilde{p}_{ei}(t_k), \tilde{u}_{ei}(t_k))$. For $k \ge 1$, $\Delta V_i = V_i(t_{k+1}) - V_i(t_k) \le J_i(\bar{p}_{ei}(\tau|t_{k+1}), \bar{u}_{ei}(\tau|t_{k+1})) - J_i(\tilde{p}_{ei}^*(\tau|t_k), \tilde{u}_{ei}^*(\tau|t_k)) = \Delta V_{i1} + \Delta V_{i2} + \Delta V_{i3} + \Delta V_{ij1} + \Delta V_{ij2} + \Delta V_{ij3}$, where

$$\Delta V_{i1} = \int_{t_{k+1}}^{t_k+T} (\|\bar{p}_{ei}(\tau|t_{k+1})\|_Q^2 - \left\|\tilde{p}_{ei}^*(\tau|t_k)\right\|_Q^2)d\tau$$

$$\Delta V_{i2} = \int_{t_k+T}^{t_{k+1}+T} (\|\bar{p}_{ei}(\tau|t_{k+1})\|_Q^2 + \|\bar{u}_{ei}(\tau|t_{k+1})\|_P^2)d\tau + \frac{1}{2}\|\bar{p}_{ei}(t_{k+1}+T|t_{k+1})\|^2 - \frac{1}{2}\left\|\tilde{p}_{ei}^*(t_k+T|t_k)\right\|^2$$

$$\Delta V_{i3} = -\int_{t_k}^{t_{k+1}} (\left\|\tilde{p}_{ei}^*(\tau|t_k)\right\|_Q^2 + \left\|\tilde{u}_{ei}^*(\tau|t_k)\right\|_P^2)d\tau$$

$$\Delta V_{ij1} = \sum_{j\in N_i}\int_{t_{k+1}}^{t_k+T} (\left\|\bar{p}_{ei}(\tau|t_{k+1}) - \hat{p}_{ej}(\tau|t_{k+1})\right\|_H^2 - \left\|p_{ei}^*(\tau|t_k) - \hat{p}_{ej}(\tau|t_k)\right\|_H^2)d\tau$$

$$\Delta V_{ij2} = \sum_{j\in N_i}\int_{t_k+T}^{t_{k+1}+T} \left\|\bar{p}_{ei}(\tau|t_{k+1}) - \hat{p}_{ej}(\tau|t_{k+1})\right\|_H^2 d\tau$$

$$\Delta V_{ij3} = \sum_{j\in N_i} -\int_{t_k}^{t_{k+1}} \left\|p_{ei}^*(\tau|t_k) - \hat{p}_{ej}(\tau|t_k)\right\|_H^2 d\tau.$$

For ΔV_{i1}, according to holder inequality, it holds that

$$\Delta V_{i1} \le \int_{t_{k+1}}^{t_k+T} q\eta\delta e^{a(\tau-t_k)}(2\left\|\tilde{p}_{ei}^*(\tau|t_k)\right\| + \eta\delta e^{a(\tau-t_k)})d\tau \le \alpha_1.$$

For $\tau \in [t_k+T, t_{k+1}+T)$, it holds that $\|\bar{p}_{ei}(\tau|t_{k+1})\|$ is in the termi-

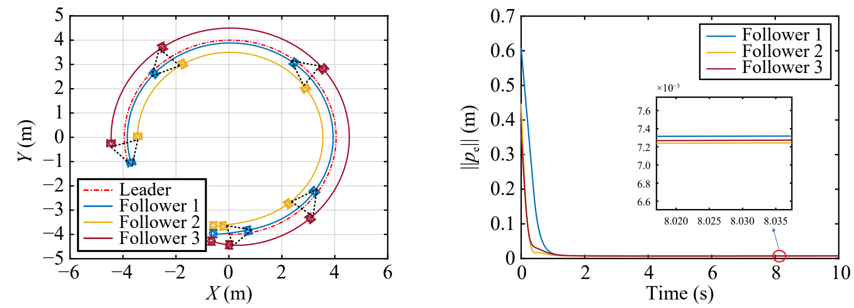

Fig. 2. Trajectories of three non-holonomic agents in formation tracking with the proposed robust distributed MPC (left); Norms of tracking errors (right).

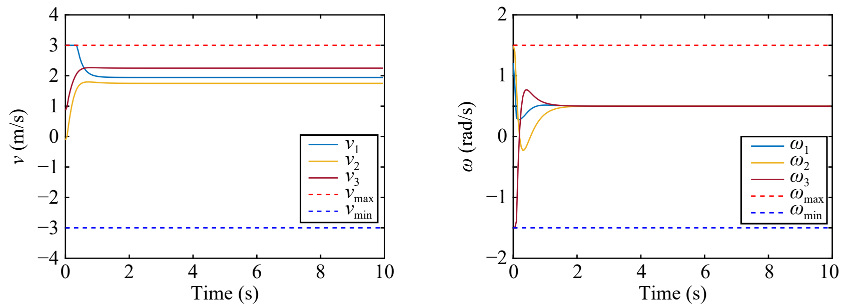

Fig. 3. Linear velocities (left) and angular velocities (right).

nal region Ω_r. Construct a function $s(\tau)=\frac{1}{2}\frac{d}{d\tau}\|\bar{p}_{ei}(\tau|t_{k+1})\|^2+\|\bar{p}_{ei}(\tau|t_{k+1})\|_Q^2+\|\bar{u}_{ei}(\tau|t_{k+1})\|_P^2$. With feasible control $\bar{u}_i(\tau|t_{k+1})$ and (14), it holds $s(\tau)\le 0$. Integrating $s(\tau)$ over $[t_k+T,t_{k+1}+T)$ and taking it into ΔV_{i2} yield

$$\Delta V_{i2}\le\frac{1}{2}\|\bar{p}_{ei}(t_k+T|t_{k+1})\|^2-\frac{1}{2}\left\|\tilde{p}^*_{ei}(t_k+T|t_k)\right\|^2\le\alpha_2.$$

For ΔV_{i3}, assume $\left\|\tilde{p}^*_{ei}(\tau|t_k)\right\|\ge\varepsilon r$ over $[t_k,t_{k+1})$. It holds that

$$\Delta V_{i3}=-\int_{t_k}^{t_{k+1}}\left\|\tilde{p}^*_{ei}(\tau|t_k)\right\|_Q^2 d\tau\le-\beta.$$

For ΔV_{ij1}, ΔV_{ij2} and ΔV_{ij3}, since $\|\bar{p}_{ei}(\tau|t_{k+1})\|\le r$ and $\|\hat{p}_{ej}(\tau|t_{k+1})\|\le\varepsilon r$ over $[t_k+T,t_{k+1}+T)$, it holds that

$$\Delta V_{ij1}\le(3e^{2\delta}+\varepsilon)(\eta\delta e^{T}-\eta\delta e^{T-\delta}+re^{\delta}-r+\delta\varepsilon r))+\sum_{j\in N_i}rh_{\max}((8r+4\eta\delta)(e^{2T-\delta}-e^{T+\delta})\le\alpha_3$$

$$\Delta V_{ij2}\le\sum_{j\in N_i}h_{\max}\int_{t_k+T}^{t_{k+1}+T}\|\bar{p}_{ei}(\tau|t_{k+1})\|^2+\left\|\hat{p}_{ej}(\tau|t_{k+1})\right\|^2 d\tau\le\alpha_4$$

$$\Delta V_{ij3}=\sum_{j\in N_i}\int_{t_k}^{t_{k+1}}-\left\|p^*_{ei}(\tau|t_k)-\hat{p}_{ej}(\tau|t_k)\right\|_H^2 d\tau\le 0.$$

Consequently, it follows from (15) that $\Delta V_i\le\alpha_1+\alpha_2+\alpha_3+\alpha_4-\beta\le-\mu$, indicating that $p^*_{ei}(\tau|t_k)$ enters Ω_ε in finite time. In case that $p^*_{ei}(\tau|t_k)$ is in Ω_ε at t_k, since Ω_ε is invariant for the nominal system, $p^*_{ei}(\tau|t_k)$ stays in the Ω_ε, and the actual tracking error $p_{ei}(t_{k+1})$ stays in $\Omega=\{p_e|\|p_e\|\le\eta\delta e^{a\delta}+\varepsilon r\}$. ■

Numerical example: An example of three followers with fixed formation is given to verify the effectiveness of the MPC algorithm.

The trajectory of the virtual leader is a circle with $v_r=2$ m/s and $\omega_r=0.5$ rad/s. The initial leader state $z_r(0)=[0\text{ m},-4\text{ m},0\text{ rad}]^T$. The control constraint is $\{|v_i(t)|\le 3\text{ m/s},|\omega_i(t)|\le 1.5\text{ rad/s}\}$ for $i=1,2,3$. The disturbance is assumed to satisfy $\{n_{xi}(t)\le 0.001;n_{yi}(t)\le 0.001\}$. The desired positions fixed on the virtual leader are $p_{d_1}=[0.5;0.1]$ m, $p_{d_2}=[-0.5;0.5]$ m and $p_{d_3}=[-0.5;-0.5]$ m. $l=0.5$ m. $T=0.5$ s, and $\delta=0.05$ s. Set $r=10$, $\varepsilon=0.001$, $Q=H=\text{diag}(1,1)$, $P=\text{diag}(0.1,0.1)$.

It is shown in Fig. 2 that three followers reach the desired positions and keep a fixed configuration. Norms of tracking errors are shown in Fig. 2, where tracking errors converge to the neighborhood of the origin. Control inputs are displayed in Fig. 3. It can be judged that, with the proposed robust distributed MPC, formation tracking is achieved with satisfactory performance, and no constraints are violated in presence of bounded disturbance. Admittedly, the theoretical bound of tolerated disturbance is small due to conservative calculation.

Conclusion: A distributed robust MPC is designed for formation tracking of nonholonomic systems with input constraints and bounded disturbances. A robust constraint is presented to resist the external disturbances. Coupling costs are constructed to keep agents in formation. Theoretical proof and simulation example show that the recursive feasibility is ensured, and tracking errors are stabilized within a robust invariant region.

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