# Letter

## Control of 2-D Semi-Markov Jump Systems: A View from Mode Generation Mechanism

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Dear Editor,

Two-dimensional (2-D) systems have wide applications in image data processing, gas absorption and fluid dynamics analysis [1]–[3]. When there exist abrupt changes in 2-D systems, they are usually modeled by 2-D Markov jump systems (MJSs) or 2-D semi-Markov jump systems (SMJSs). This letter investigates the control of 2-D SMJSs based on a novel mode generation mechanism, which could avoid mode ambiguousness phenomenon caused by the evolution of system mode in two different directions. The criterion that guarantees the almost surely exponential stability of the system is obtained. A thermal process is studied to demonstrate the availability of the proposed method.

**Engineering background:** A wide range of real-world systems have inherent 2-D dynamical structures, and the information broadcasts in two directions in such systems. For example, the multipass metal rolling process is a typical type of linear multipass process in industrial manufacturing [4]. Its system dynamics depend not only on the time direction, but also on the batch direction. In the heat exchanger and long transmission line, system dynamics are determined by both time and position [5]. The Roesser state-space model was established to deal with linear image processing, and two-state sets were used to describe system dynamics in two directions [6]. In this way, many methods for temporal systems can be extended to such a spatial model. In addition, Roesser model has drawn much attention attribute to its simple structure and intuitive form compared with other 2-D models such as Attasi model and Fornasini-Marchesini (FM) model [7]. In engineering, the machine may switch to different working modes or the parameter of components may be changed due to many factors, and it is important to ensure that the control strategy matches with working mode. In this regard, it is practical to consider 2-D systems with jumping parameters.

**Related work:** In recent years, 2-D MJSs have become a hot topic which focuses on the modeling of abrupt changes occurring to 2-D systems. In [8], the authors designed a kind of asynchronous controller for 2-D MJSs based on the hidden Markov model. In [9], the $H_\infty$ control issue was addressed in the sense of finite-region. The sliding mode control was studied in [10], where a novel 2-D sliding surface was presented. Considering the existences of time delay and uncertainties in transition information, the stability analysis was investigated in [11]. The filtering problem was studied for 2-D MJSs with deficient mode information in [12]. To the best of our knowledge, the above works are based on an assumption that the transition probabilities in two directions are equal [13]. However, only one variable was used in these works to determine a system mode. When current modes in the horizontal and vertical direction are different, how to determine the next system mode becomes unclear. For instance, the mode in the horizontal direction switches from mode 2 to mode 3 ($2 \rightarrow 3$), and the mode in the vertical direction switches from mode 1 to mode 3 ($1 \uparrow 3$). In this case, the transition probabilities in two directions are not equal, and we cannot determine the next system mode based solely on one direction. Such a phenomenon is named as mode ambiguousness. This problem can be eliminated by assuming that mode evolutions in two directions are identical all the time. Unfortunately, this assumption is too strict or may be unreasonable for a stochastic system. Furthermore, it may be meaningless to consider the case that two mode evolution directions follow different transition probabilities, if the problem of mode ambiguousness is not removed.

As is well known, the transition probability of discrete-time MJSs is constant, which is independent of the sojourn time [14], [15]. In many fault-tolerant systems, the transition rate function follows a bathtub curve, and it is time-varying. SMJSs could model such systems more accurately since their transition processes are governed by the sojourn time as well as transition probabilities. Based on the theories developed for SMJSs, many applications for 2-D SMJSs have also been launched, e.g., broadcast wireless network, edge detection techniques and inverse heat conduction [16]–[18]. System dynamics and the mode evolution process of 2-D SMJSs are more complex due to the effect of sojourn time on two directions. In this regard, how to avoid mode ambiguousness phenomenon in 2-D SMJSs is a crucial subject. Although 2-D SMJSs have a vast application prospect, they have not yet aroused attention in the control community, which motivates this letter.

**Our contributions:** Motivated by the discussions above, this letter makes attempt to investigate the control problem of 2-D SMJSs. The contributions are: 1) The designed controller can obtain mode information more accurately by removing mode ambiguousness problem, which improves its adaptability to abrupt changes in the system. 2) By introducing adjustment parameters and sojourn time, the ability of the controller to stabilize the system can be enhanced. 3) The almost surely exponential stability condition is established by introducing some concepts in the switched systems.

**Notation:** Throughout the paper, $\mathbb{Z}^+$ is the positive integer and $\mathbb{Z}_{\geq n}$ is the integer no less than $n$. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation. Symbol "$*$" denotes the symmetric term, and $()'$ is the matrix transpose. $\lambda_{\min}(\cdot)$ is the smallest eigenvalue of a matrix.

**Problem statement:** Consider the following 2-D SMJSs in Roesser model:

$$x^+(i,j) = A_{\theta(i,j)}x(i,j) + B_{\theta(i,j)}u(i,j) \tag{1}$$

where

$$x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, x^+(i,j) = \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}$$

$$A_{\theta(i,j)} = \begin{bmatrix} A_{1\theta(i,j)} & A_{2\theta(i,j)} \\ A_{3\theta(i,j)} & A_{4\theta(i,j)} \end{bmatrix}, B_{\theta(i,j)} = \begin{bmatrix} B_{1\theta(i,j)} \\ B_{2\theta(i,j)} \end{bmatrix}$$

and $x^h(i,j)$, $x^v(i,j)$, $u(i,j)$ are the horizontal state, vertical state and control input, respectively. System matrices are governed by a semi-Markov chain $\theta(i,j)$, and it is noted that $\theta(i,j)$ only depends on the value of $i+j$. Define $k_l$ as the time index of the $l$th jump, and $\varpi_l$ is the index of system mode of the $l$th jump. When $i+j=k\in[k_l,k_{l+1})$, we denote $\theta(i,j)=\theta_k=p\in\mathbb{S}=\{1,2,...s\}$. The sojourn time of mode $p$ is defined by $\tau_p=k_{l+1}-k_l$, which satisfies $\mathbb{E}\{\tau_p\}=\sigma_p\in\mathbb{Z}^+$. The transition probability matrix of the embedded chain $\{\varpi_l, l\in\mathbb{Z}^+\}$ is given by

$$\Pi = [\pi_{pq}]_{s\times s}, \Pr\{\varpi_{l+1}=q|\varpi_l=p\}=\pi_{pq} \tag{2}$$

where $0\leq\pi_{pq}\leq 1$, $p\neq q$, $\pi_{pp}=0$. As stated in [19], the embedded Markov chain has a unique stationary distribution

$$w = [\ w_1 \quad w_2 \quad \cdots \quad w_s\ ], w\Pi = w, w_p > 0, \sum_{p=1}^{s} w_p = 1, \forall p\in\mathbb{S}.$$

The boundary conditions $(X_0, \mathcal{G}_0)$ of system (1) are

$$X_0 = \{x^h(0,j), x^v(i,0)|i,j\in\mathbb{Z}\}, \mathcal{G}_0 = \{\theta(0,j),\theta(i,0)|i,j\in\mathbb{Z}\}. \tag{3}$$

Assumption 1 [8]: Assume that $X_0$ satisfies the following condition:

$$\lim_{N\to\infty}\mathbb{E}\{\sum_{n=0}^{N}(||x^h(0,n)||^2+||x^v(n,0)||^2)\}<\infty. \tag{4}$$

**Control objectives:** The main objectives are to employ the proposed mode generation mechanism to avoid mode ambiguousness problem, and to design a state feedback controller $u(i,j) = K_{\theta(i,j)}x(i,j) = [\ K_{1\theta(i,j)} \quad K_{2\theta(i,j)}\ ]x(i,j)$ to stabilize system (1).

**Mode ambiguousness problem:** In the previous works about 2-D MJSs, the transition probability satisfies

$$\Pr\{\theta(i+1,j)=q|\theta(i,j)=p\}=\Pr\{\theta(i,j+1)=q|\theta(i,j)=p\}=\pi_{pq} \tag{5}$$

besides, transition probabilities of the horizontal and vertical directions are allowed to be different. The drawback is that these works need an implicit assumption, that is, the mode evolution in two directions are identical. Otherwise, the index of system mode cannot be simply defined by $\theta(i,j)$. We use Fig. 1(a) to explain the phenomenon of mode ambiguousness, which shows the mode evolution process in two directions obeying condition (5), and we regard $(i,j)$ as a coordinate

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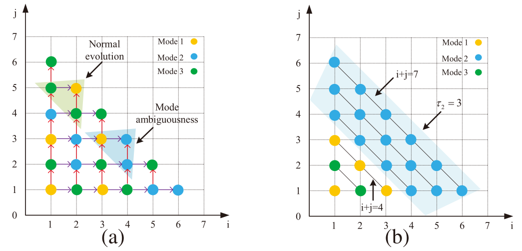



Fig. 1. Two different mode generation mechanisms.

that contains mode information. Denote symbols $\rightarrow$ and $\uparrow$ as the mode evolution in the horizontal direction and vertical direction, respectively. It can be indicated that the mode of a coordinate can evolve along two directions, e.g., $(1,4)\uparrow(1,5)$ and $(1,4)\rightarrow(2,4)$. They follow the same transition probability $\pi_{23}$, and the condition (5) can be guaranteed. In the green triangle area of Fig. 1(a), there are two mode evolution processes: $(1,5)\rightarrow(2,5)$ and $(2,4)\uparrow(2,5)$. It is a normal case of mode evolution, and both the horizontal and vertical modes could switch to the same mode with the transition probability $\pi_{31}$. However, in the blue triangle area of Fig. 1(a), evolution processes $(3,3)\rightarrow(4,3)$ and $(4,2)\uparrow(4,3)$ follow different transition probabilities ($\pi_{12}$ and $\pi_{22}$). In this situation, how to determine the mode of $(4,3)$ if coordinates $(3,3)$ and $(4,2)$ switch to different modes? To avoid mode ambiguousness phenomenon, it is required that $\theta(i+1,j)=\theta(i,j+1)$, whereas no literature has explicitly stated this hypothesis. Furthermore, assuming that transition probabilities in both directions are different cannot avoid this problem effectively.

Fig. 1(b) shows a mode generation mechanism subject to the value of $i+j$. It is intuitive that coordinates in the line $i+j=k$ have the same mode, which is regarded as the global mode ($\theta_k$). Actually, system mode evolves along each line with different $k$ under this generation mechanism. Such a mechanism could ensure that the left-adjacent and lower-adjacent coordinates to a certain coordinate share the same global mode, thus avoiding mode ambiguousness phenomenon. For SMJSs, a system mode for different $k$ may be the same due to the effect of sojourn time, which may create a mode region as shown in the blue area of Fig. 1(b).

The following lemma and definition are necessary for the stability analysis.

Lemma 1 [19], [20]: Let $T_p(k)$ and $N_p(k)$ be the total activation time and the occurrence number of mode $p$ in time interval $[0,k]$, then the following equations hold:

$$\lim_{k\to\infty}\frac{T_p(k)}{N_p(k)}=\sigma_p,\ \text{a.s.},\ \lim_{k\to\infty}\frac{T_p(k)}{k}=\bar{w}_p,\ \bar{w}_p=\frac{w_p\sigma_p}{\sum_{l=1}^{s}w_l\sigma_l},\ \text{a.s.}$$

Definition 1 [20], [21]: System (1) is almost surely exponentially stable (ASES), if for any boundary conditions $X_0$ and $\mathcal{G}_0$, the following condition holds:

$$\limsup_{k\to\infty}\frac{\ln\sum_{i+j=k}||x(i,j)||^2}{k}<0,\ \text{a.s.} \tag{6}$$

**Main results:** The stability criterion will be given in the form of linear matrix inequalities (LMIs), and gains of the controller will be determined.

Theorem 1: Given scalars $\lambda_p>0$, $\mu_p>0$ and $\sigma_p\in\mathbb{Z}_{\geq 1}$, if there exist matrices $\bar{Q}_p>0$, $\bar{U}_p>0$, $\bar{K}_{1p}$ and $\bar{K}_{2p}$, such that the following LMIs hold $\forall p,q\in\mathbb{S}$:

$$\begin{bmatrix}\Theta_1 & \Theta_2\\ * & \Theta_3\end{bmatrix}<0 \tag{7}$$

$$\bar{Q}_q<\mu_p\bar{Q}_p,\bar{U}_q<\mu_p\bar{U}_p,p\neq q \tag{8}$$

$$\sum_{p=1}^{s}\bar{w}_p(\ln\lambda_p+\frac{\ln\mu_p}{\sigma_p})<0 \tag{9}$$

where

$$\Theta_1\triangleq\text{diag}\{-\lambda_p\bar{Q}_p,-\lambda_p(\bar{U}_p-\bar{Q}_p-\bar{Q}_p')\},\Theta_3\triangleq\text{diag}\{-\bar{Q}_p,-\bar{U}_p\}$$

$$\Theta_2\triangleq\begin{bmatrix}\bar{Q}_p'A_{1p}'+\bar{K}_{1p}'B_{1p}' & \bar{Q}_p'A_{3p}'+\bar{K}_{1p}'B_{2p}'\\ \bar{Q}_p'A_{2p}'+\bar{K}_{2p}'B_{1p}' & \bar{Q}_p'A_{4p}'+\bar{K}_{2p}'B_{2p}'\end{bmatrix}$$

and the definition of $\bar{w}_p$ is the same as that in the Lemma 1, then system (1) under Assumption 1 is ASES. The controller gain can be determined by

$$\bar{K}_p=\begin{bmatrix}\bar{K}_{1p} & \bar{K}_{2p}\end{bmatrix},K_p=\begin{bmatrix}\bar{K}_{1p}\bar{Q}_p^{-1} & \bar{K}_{2p}\bar{Q}_p^{-1}\end{bmatrix}.$$

Proof: For any $i+j=k\in[k_l,k_{l+1})$ and $\theta_k=p$, constructing a Lyapunov function as follows:

$$\begin{aligned}V(x(i,j),\theta_k)&=x'(i,j)\text{diag}\{Q_{\theta(i,j)},U_{\theta(i,j)}\}x(i,j)\\&=x'(i,j)\text{diag}\{\bar{Q}_{\theta(i,j)}^{-1},\bar{U}_{\theta(i,j)}^{-1}\}x(i,j)\end{aligned} \tag{10}$$

and in order to simplify the notation, we denote $x(i,j)\triangleq x(k)$ for any $i$ and $j$ satisfying $i+j=k$.

Since $(\bar{U}_p-\bar{Q}_p)'\bar{U}_p^{-1}(\bar{U}_p-\bar{Q}_p)\geq 0$, one has $-\bar{Q}_p'\bar{U}_p^{-1}\bar{Q}_p\leq\bar{U}_p-\bar{Q}_p-\bar{Q}_p'$. Then, it can be inferred from (7) that

$$\begin{bmatrix}\bar{\Theta}_1 & \Theta_2\\ * & \Theta_3\end{bmatrix}<0 \tag{11}$$

where $\bar{\Theta}_1=\text{diag}\{-\lambda_p\bar{Q}_p,-\lambda_p\bar{Q}_p'\bar{U}_p^{-1}\bar{Q}_p\}$. Pre- and post-multiplying (11) with $\text{diag}\{\bar{Q}_p^{-1},\bar{Q}_p^{-1},I,I\}$, and defining $\bar{K}_p=\begin{bmatrix}K_{1p}\bar{Q}_p & K_{2p}\bar{Q}_p\end{bmatrix}$, yields

$$\begin{bmatrix}-\lambda_p\text{diag}\{\bar{Q}_p^{-1},\bar{U}_p^{-1}\} & (A_p+B_pK_p)'\\ * & -\text{diag}\{\bar{Q}_p,\bar{U}_p\}\end{bmatrix}<0 \tag{12}$$

which implies that

$$V(x(k+1),\theta_{k+1})<\lambda_{\theta_k}V(x(k),\theta_k). \tag{13}$$

Hence, for any $i$, $j$ satisfying $i+j=k+1$ and $i+j=k$, we have

$$\sum_{i+j=k+1}V(x(k+1),\theta_{k+1})<\lambda_{\theta_k}\sum_{i+j=k}V(x(k),\theta_k). \tag{14}$$

In addition, inequalities in (8) result in

$$V(x(k_l),\theta_{k_l})<\mu_{\theta_{k_l}}V(x(k_l),\theta_{k_{l-1}}),\forall\theta_{k_{l-1}}=q. \tag{15}$$

For $k\in[k_l,k_{l+1})$, iterating inequalities (14) and (15) leads to

$$\begin{aligned}&\sum_{i+j=k}V(x(k),\theta_k)<\lambda_{\theta_{k-1}}\sum_{i+j=k-1}V(x(k-1),\theta_{k-1})\\&<\lambda_{\theta_{k-1}}^{k-k_l}\sum_{i+j=k_l}V(x(k_l),\theta_{k_l})<\mu_{\theta_{k_l}}\lambda_{\theta_{k-1}}^{k-k_l}\sum_{i+j=k_l}V(x(k_l),\theta_{k_{l-1}})\\&<\prod_{p=1}^{s}\mu_p^{N_p(k)}\lambda_p^{T_p(k)}\sum_{i+j=0}V(x(0),\theta_0).\end{aligned} \tag{16}$$

According to Lemma 1, it can be established from conditions (9) and (16) that

$$\begin{aligned}&\limsup_{k\to\infty}\frac{\ln\sum_{i+j=k}V(x(k),\theta_k)}{k}\\&<\limsup_{k\to\infty}\frac{\ln\prod_{p=1}^{s}\mu_p^{N_p(k)}\lambda_p^{T_p(k)}\sum_{i+j=0}V(x(0),\theta_0)}{k}\\&=\limsup_{k\to\infty}\frac{\sum_{i+j=0}V(x(0),\theta_0)}{k}\\&+\limsup_{k\to\infty}\sum_{p=1}^{s}[\frac{N_p(k)\ln\mu_p}{k}+\frac{T_p(k)\ln\lambda_p}{k}]=\sum_{p=1}^{s}\bar{w}_p(\ln\lambda_p+\frac{\ln\mu_p}{\sigma_p})<0\end{aligned}$$

which further implies inequality (6) since

$$\begin{aligned}&\limsup_{k\to\infty}\frac{\ln\sum_{i+j=k}||x(i,j)||^2}{k}\\&\leq\limsup_{k\to\infty}\frac{\ln\lambda_{\min}^{-1}(\text{diag}\{Q_p,U_p\})}{k}+\sum_{p=1}^{s}\bar{w}_p(\ln\lambda_p+\frac{\ln\mu_p}{\sigma_p})<0.\end{aligned}$$

Therefore, system (1) is ASES recalling to Definition 1. ■

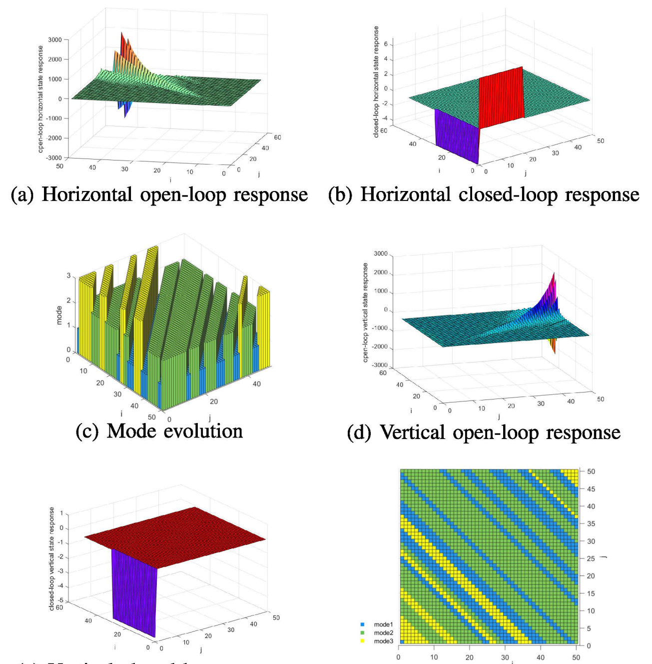
Remark 1: The existing literature about the control of discrete-time SMJSs is mainly based on the concept of semi-Markov kernel proposed in [22], and analyzes the $\sigma$-error mean-square stability. However, the above method is hard to be extended to 2-D SMJSs due to the existence of matrix power and the truncation on sojourn time. By introducing the concepts of activation time $T_p(k)$ and jump number $N_p(k)$ which are commonly employed in switched systems, the matrix power could be avoided in this letter. And the almost surely exponential stability criterion can be established by using stationary distribution $w$ and Lemma 1. Condition (8) is a typical stability constraint in switched systems.

**Simulations:** This section provides an example to verify the effectiveness of the proposed approach. Consider a thermal process described by

$$\frac{\partial T(x,t)}{\partial x}=-\frac{\partial T(x,t)}{\partial x}-h_{\theta(x,t)}T(x,t)+f_{\theta(x,t)}u(x,t)$$

where $T(x,t)$ denotes the temperature at space $x\in[0,x_s]$ and time $t\in[0,\infty)$. Constant coefficients $h_{\theta(x,t)}$ and $f_{\theta(x,t)}$ are subject to a semi-Markov signal $\theta(x,t)$. According to [23], the state-space model is given by

$$x^+(i,j)=A_px(i,j)+B_pu(i,j),p=\theta(i,j)=1,2,3$$




(a) Horizontal open-loop response (b) Horizontal closed-loop response

(c) Mode evolution (d) Vertical open-loop response

(e) Vertical closed-loop response (f) Platform of mode

Fig. 2. Simulation results.

where

$$A_p = \begin{bmatrix} 0 & 1 \\ \frac{\Delta t}{\Delta x} & 1 - \frac{\Delta t}{\Delta x} - h_p \Delta t \end{bmatrix}, B_p = \begin{bmatrix} 0 \\ f_p \Delta t \end{bmatrix}$$
$$x^h(i,j) = T(i-1,j), x^v(i,j) = T(i,j).$$

For different subsystems, relevant parameters are taken as $h_1 = 1.25$, $h_2 = 1.58$, $h_3 = 2.86$, $f_1 = 1$, $f_2 = 0.5$, $f_3 = 1$, $\Delta t = 0.1$ and $\Delta x = 0.1$. Other simulation parameters are $\lambda_1 = 0.7$, $\lambda_2 = 0.8$, $\lambda_3 = 1.02$, $\mu_1 = 1.1$, $\mu_2 = 1.05$ and $\mu_3 = 1.12$. Assume that the sojourn time of each mode follows Poisson distribution, that is, $\tau_1 \sim Pois(3)+1$, $\tau_2 \sim Pois(4)+1$ and $\tau_1 \sim Pois(5)+1$. Clearly, $\sigma_1 = 4$, $\sigma_2 = 5$, $\sigma_3 = 6$. The transition probability matrix and the corresponding stationary probability matrix are

$$\Pi = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.6 & 0 & 0.4 \\ 0.58 & 0.42 & 0 \end{bmatrix}, w = \begin{bmatrix} 0.3708 & 0.2906 & 0.3386 \end{bmatrix}$$

and it can be easily verified that condition (9) is satisfied.

By solving Theorem 1, we get a feasible controller with the following gains:

$$K_1 = \begin{bmatrix} -10 & 1.25 \end{bmatrix}, K_2 = \begin{bmatrix} -20 & 3.16 \end{bmatrix}, K_3 = \begin{bmatrix} -10 & 2.86 \end{bmatrix}.$$

The boundary conditions are $x^h(0,j) = 7$ $(0 \le j \le 20)$ and $x^v(i,0) = -4.7$ $(0 \le j \le 30)$. Applying the designed controller to stabilize system (1), simulation results are presented in Figs. 2 and 3. It is obvious that the controller could stabilize the system effectively. Besides, Figs. 2(c) and 2(f) show the mode evolution process that follows rule (2). Seeing from the platform of Fig. 2(c), the generated modes create multiple mode regions, which are consistent with Fig. 1(b).

Multiplying the system matrix $A_p$ by a scalar $\beta$, and $\beta_{\max}$ can be regarded as a performance evaluation indicator which reflects the the difficulty of stabilizing a system. We obtain a set of data of $(\lambda_1, \beta_{\max})$ by changing the value of $\lambda_1$, that is, $(0.2, 0.6324)$, $(0.3, 0.7745)$, $(0.4, 0.8944)$, $(0.5, 0.9999)$ and $(0.6, 1.0954)$. It is obvious that a larger $\lambda_1$ helps to stabilize the system. The value of $\lambda_1$ is allowed to be larger if we increase $\sigma_p$, meanwhile, the condition (9) can be guaranteed.

**Conclusions:** The control issue of 2-D SMJSs has been studied in this letter. Considering that a system mode can be generated from two directions in the Roesser model, which gives rise to mode ambiguousness phenomenon, a novel mode generation mechanism has been presented to avoid such a problem. The proposed mechanism has taken the effect of sojourn time into account. The almost surely exponential stability criterion has been obtained for the system based on the proposed mechanism. The developed method has been applied to the control of a thermal process. Future research topics may include 2-D SMJSs with time-delay and actuator saturation [24]–[26].

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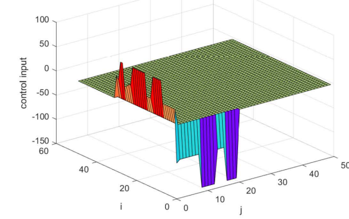

Fig. 3. Control input.

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