Letter

Prescribed-Time Fully Distributed Nash Equilibrium Seeking Strategy in Networked Games

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Dear Editor,

This letter is concerned with prescribed-time fully distributed Nash equilibrium seeking for networked games under directed graphs. An adaptive algorithm is proposed to ensure the convergence of all players to the Nash equilibrium without requiring any knowledge of global parameters. Moreover, it is theoretically proved that the convergence time of the proposed seeking strategy can be predefined based on practical requirements. Finally, a numerical example is presented to validate the effectiveness of the proposed method.

As a key concept in game theory, Nash equilibrium has found wide-ranging applications in practical engineering fields [1], [2]. As a result, numerous research methods have been proposed for the design and analysis of distributed Nash equilibrium seeking strategies depending only on the local neighborhood information [3]. For instance, a distributed Nash equilibrium seeking strategy for general games was proposed by combining a leader-follower consensus protocol and a gradient play [4]. The problem of distributed Nash equilibrium seeking was investigated for networked systems with bounded control inputs [5]. A distributed nonsmooth algorithm with a projected differential inclusion was proposed to solve the generalized Nash equilibrium seeking problem for multi-cluster games [6]. In the presence of external disturbances, a robust distributed algorithm was proposed to drive all agents to reach the Nash equilibrium [7].

Note that, in all aforementioned results, the parameter design of distributed Nash equilibrium seeking strategies is always dependent on the global information such as the eigenvalues of Laplacian matrices, which may be difficult to obtain in practical implementations. As a result, there is a growing demand for the development of fully distributed strategies which can be effective in avoiding global information [8]. In [9], adaptive approaches were utilized to achieve fully distributed Nash equilibrium seeking in networked games with undirected graph. Under directed graph, a fully distributed approach to finding the Nash equilibrium was presented for high-order players subject to actuator limitations [10].

While significant progress has been made in the fully distributed Nash equilibrium seeking, practical scenarios still pose some challenging issues in terms of algorithm convergence speeds. It should be mentioned that the distributed Nash equilibrium seeking strategies [9], [10] can only ensure the asymptotic convergence. However, in certain engineering applications, it may be required to realize the convergence of algorithms within a prescribed time rather than an infinity time [11]. To meet this requirement, various distributed control strategies have been presented. For example, a distributed time-varying seeking strategy that utilizes a prescribed-time observer under undirected graphs was proposed to achieve the convergence within a set time [12]. Utilizing the distributed motion-planning method and the gradient search, a class of prescribed-time distributed Nash equilibrium seeking algorithms have been developed for first and second order multi-agent systems [13]. However, these distributed methods require global information of the game and thus are not fully distributed. Therefore, it is necessary and important to design a prescribed-time fully distributed Nash equilibrium seeking strategy under directed graphs, which is the motivation behind this letter.

Building on the analysis above, this letter makes the following contributions: 1) A new prescribed-time fully distributed Nash equilibrium seeking strategy under directed graphs is proposed by designing an adaptive algorithm to adjust the control parameter according to the consensus error; 2) Theoretical analysis is conducted to prove that the proposed strategy can make all agents convergent to the Nash equilibrium in a prescribed time, which is beneficial for practical engineering applications with specific time constraints.

Notations: The set of real numbers is denoted by $\mathbb{R}$. Let I_n and 1_m denote, respectively, the n-dimensional identity matrix and the m-dimensional column vector with all elements being 1. diag$\{p_{ij}\}$ for $i,j\in\{1,2,\dots,N\}$ stands for a diagonal matrix whose diagonal entries are $p_{11}, p_{12},\dots,p_{1N}, p_{21},\dots,p_{NN}$, successively. Let $[w_i]_{\rm vec}=[w_1^T, w_2^T,\dots,w_N^T]^T$ for $i=1,2,\dots N$. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively, denote the minimum and maximum eigenvalue of a real and symmetric matrix A. Let $\|\cdot\|$ be the Euclidean norm of a vector or the 2-norm of a matrix and the symbol $\otimes$ denote the Kronecker product.

Problem formulation: Consider a game with N players, where $v=\{1,2,\dots,N\}$ denotes the set of players. Let $f_i(x)$ represent player i's cost function, where $x=[x_1^T,x_2^T,\dots,x_N^T]^T$ is a vector of all players' actions, $x_i\in\mathbb{R}^{n_i}$ is the action of player i and n_i is a positive integer. In the considered game, each player is self-interested to minimize its own cost function, i.e.,

$$\min_{x_i} f_i(x) \ \text{ or } \ \min_{x_i} f_i(x_i,x_{-i}) \tag{1}$$

where $x_{-i}=[x_1^T,x_2^T,\dots,x_{i-1}^T,x_{i+1}^T,\dots,x_N^T]^T$.

Then, we present the following Nash equilibrium definition.

Definition 1 ([3]): Nash equilibrium is an action profile on which no player can reduce its cost by unilaterally changing its own action, i.e., an action profile $x^*=(x_i^*,x_{-i}^*)$ is a Nash equilibrium if for all $x_i\in\mathbb{R}^{n_i}$, $i\in v$,

$$f_i(x_i^*,x_{-i}^*)\le f_i(x_i,x_{-i}^*). \tag{2}$$

In this letter, it is assumed that players can communicate with their neighbors via a directed graph denoted as $\mathcal{G}(v,\varepsilon)$, where ε denotes the set of edge among the players. $A=(a_{ij})_{N\times N}$ be the adjacent matrix. For a directed communication topology, edge $e_{ji}\in\varepsilon$ indicates that player j can receive information from player i. If $e_{ij}\in\varepsilon$, then $a_{ij}>0$; otherwise, $a_{ij}=0$. Moreover, $a_{ii}=0$, $i\in v$. A directed communication graph is strongly connected if there exists a directed path between any pair of distinct players.

The following assumptions promote the Nash equilibrium seeking.

Assumption 1: For $i\in v$, the player' objective function $f_i(x)$ is twice-continuously differentiable and $\nabla_i f_i(x)\triangleq\frac{\partial f_i(x)}{\partial x_i}$ is globally Lipschitz, i.e., there exists a positive constant l_i such that

$$\|\nabla_i f_i(x)-\nabla_i f_i(z)\|\le l_i\|x-z\| \tag{3}$$

for all $x,z\in\mathbb{R}^{\bar N}$, where $\bar N=\sum_{i=1}^N n_i$.

Assumption 2: For all $x,z\in R^{\bar N}$, $[\nabla_i f_i(.)]$ vec is strongly monotone

$$(x-z)^T([\nabla_i f_i(x)]\,\text{vec}-[\nabla_{\rm i} {\rm f}_{\rm i}({\rm z})]\,\text{vec})\ge m\|x-z\|^2 \tag{4}$$

where m is a positive constant.

Remark 1: Assumption 1 is a solid base for guaranteeing the existence of gradient vectors with continuous differentiability. Assumption 2 indicates the strong/strict monotonicity of pseudo-gradient vectors, guaranteeing the existence and uniqueness of the Nash equilibrium.

Assumption 3: The communication topology is directed and strongly connected.

Main results: Note that in a networked game, each player's action and objective function are only available for itself but not others. As a result, the traditional centralized algorithm is no longer applicable due to a lack of global information about all players' action and objective functions. In order to solve this problem, it is supposed that each player can generate a local estimate on the players' actions x.

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Citation: C. Qian and L. Ding, "Prescribed-time fully distributed Nash equilibrium seeking strategy in networked games," *IEEE/CAA J. Autom. Sinica*, vol. 11, no. 1, pp. 261–263, Jan. 2024.

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Digital Object Identifier 10.1109/JAS.2023.123933

Let y_{ij} be player i's estimate on player j's action x_j in the game, and $y_i = [y_{i1}^T, y_{i2}^T, \ldots, y_{iN}^T]^T$ denote player i's estimates on the players' actions. Then, we design a fully distributed prescribed-time Nash equilibrium seeking strategy as

$$\dot{x}_i = -\frac{1}{T-t}\nabla_i f_i(y_i) \tag{5}$$

$$\dot{y}_{ij} = -\frac{1}{T-t}\theta_{ij}\eta_{ij}, \quad \theta_{ij} = p_{ij} + \eta_{ij}^T\eta_{ij} \tag{6}$$

$$\dot{p}_{ij} = \frac{1}{T-t}e^{\gamma t}\eta_{ij}^T\eta_{ij} \tag{7}$$

$$\eta_{ij} = \sum_{k=1}^{N} a_{ik}\left(y_{ij} - y_{kj}\right) + a_{ij}\left(y_{ij} - x_j\right) \tag{8}$$

where θ_{ij} is the adaptive gain of player i, $p_{ij}(0) > 0$, $p_{ij}(t) > 0$, $T > 0$ is the set convergence time, and γ is a time-varying parameter shown later.

Let $A_0 = \mathrm{diag}\{a_{ij} \otimes I_{n_i}\}$, $\mathbf{\Theta} = \mathrm{diag}\{\theta_{ij} \otimes I_{n_i}\}$, $M = (m_{ij})_{\bar{N}\times\bar{N}} = L \otimes I_{\bar{N}} + A_0$, $\eta = \left[\eta_1{}^T, \eta_2^T, \ldots, \eta_N^T\right]^T$, $\eta_i = \left[\eta_{i1}^T, \eta_{i2}^T, \ldots, \eta_{iN}^T\right]^T$, $y = [y_1^T, y_2^T, \ldots, y_N^T]^T$. Then, defining $e_{ij} = y_{ij} - x_j$, $r_{ij} = \eta_{ij}^T\eta_{ij}$, $\mathbf{R} = \mathrm{diag}\{I_{n_i} \otimes r_{ij}\}$, and $\mathbf{P} = \mathrm{diag}\{I_{n_i} \otimes p_{ij}\}$, one can obtain that

$$\dot{e} = \dot{y} - 1_N \otimes \dot{x} = -\frac{1}{T-t}\mathbf{\Theta}Me - 1_N \otimes \dot{x} \tag{9}$$

$$\dot{\eta} = M\dot{e} = -\frac{1}{T-t}M\mathbf{\Theta}Me - M(1_N \otimes \dot{x}) \tag{10}$$

where $\mathbf{\Theta} = \mathbf{P} + \mathbf{R}$, $e = [e_1^T, e_2^T, \ldots, e_N^T]^T$ and $e_i = [e_{i1}^T, e_{i2}^T, \ldots, e_{iN}^T]^T$. It is not difficult to know that $-M$ is Hurwitz, which implies that there exist a real positive-definite diagonal matrix $\Xi = \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_N\}$ with $\xi_i < 1$ and a symmetric positive definite matrix $Q \in \mathbb{R}^{(N\bar{N})\times(NN)}$ such that $M^T(\Xi \otimes I_{\bar{N}\times\bar{N}}) + (\Xi \otimes I_{\bar{N}\times\bar{N}})M = Q$.

In the following, we present the theorem of the letter.

Theorem 1: Under Assumptions 1−3 hold, fully distributed Nash equilibrium seeking can be achieved by (5)−(8) in a predefined time T, if the time-varying parameter satisfies $\dot{\gamma} = \frac{1}{(T-t)t}$, where $\gamma(0) > 0$ is the initial condition.

Proof: To facilitate the proof, we define positive constants μ_1, μ_2, ϵ, σ that satisfy $\lambda_{\min}(Q) - \left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) - \epsilon \geq 0$, $\sigma \geq \frac{\bar{l}_2 + \max_{i\in v}\{l_i\}}{m - \vartheta\bar{l}^2\mu_2 N^2}$, where $\max_{i\in v}\{l_i\}$ is the largest value of l_i for $i \in v$ and $\bar{l}_2 = 2N\max_{i\in v}\{l_i\}$.

Define the Lyapunov candidate function as $V_1 = \sum_{i=1}^N \sum_{j=1}^N \xi_j e_{ij}^T e_{ij}$, $V_2 = \frac{1}{2}(x - x^*)^T(x - x^*)$, $V_3 = \sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\xi_j\left(r_{ij}^2 + 2r_{ij}p_{ij}\right)$, $V_4 = \sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\left(p_{ij} - p_{ij}^*\right)^2$, where $V = \sum_{i=1}^4 V_i$ with $p_{ij}^* \geq \frac{\bar{l}_1 + \vartheta\bar{l}^2\mu_1 N + \frac{2\vartheta}{\epsilon} + \sigma(\bar{l}_2 + \max_{i\in v}\{l_i\})}{\lambda_{\min}(M^T M)}$, and $\bar{l}_1 = 2\sqrt{N}\max_{i\in v}\{l_i\}$. Then, we can get that $\dot{V}_1 = -\frac{2}{T-t}\sum_{i=1}^N \sum_{j=1}^N \xi_j e_{ij}^T\theta_{ij}m_{ij}e_{ij} - 2\Xi e^T(1_N \otimes \dot{x}) \leq -\frac{1}{T-t}e^T \mathbf{\Theta}Qe + \frac{\bar{l}_1}{T-t}\|e\|^2 + \frac{\bar{l}_2}{T-t}\|x - x^*\|\|e\| \leq \frac{\bar{l}_1}{T-t}\|e\|^2 + \frac{\bar{l}_2}{T-t}\|x - x^*\|\|e\|$, and $\dot{x}_i = -\frac{1}{T-t}\nabla_i f_i(y_i) = -\frac{1}{T-t}(\nabla_i f_i(y_i) - \nabla_i f_i(x)) - \frac{1}{T-t}(\nabla_i f_i(x) - \nabla_i f_i(x^*))$, so the following inequality is obtained by $\left\|2\Xi e^T(1_N \otimes \dot{x})\right\| \leq \frac{\bar{l}_1}{T-t}\|e\|^2 + \frac{\bar{l}_2}{T-t}\|e\|\|x - x^*\|$.

Under Assumption 2 and $[\nabla_i f_i(x^*)]_{\mathrm{vec}} = \mathbf{0}$, it is easy to obtain that $\dot{V}_2 = -\frac{1}{T-t}(x - x^*)^T([\nabla_i f_i(y_i)]_{\mathrm{vec}} - [\nabla_i f_i(x^*)]_{\mathrm{vec}}) \leq \frac{\max_{i\in v}\{l_i\}}{T-t}\|x - x^*\| \times \|e\| - \frac{m}{T-t}\|x - x^*\|^2$.

Taking the time derivative of V_3 yields

$$\begin{aligned}\dot{V}_3 = &\sum_{i=1}^N \sum_{j=1}^N 2e^{-\gamma t}\xi_j\theta_{ij}\eta_{ij}^T\dot{\eta}_{ij} + \frac{1}{T-t}\sum_{i=1}^N \sum_{j=1}^N \xi_j\left(\left(r_{ij}\eta_{ij}^T\right)\eta_{ij}\right)\\ &-(\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\xi_j\left(r_{ij}^2 + 2r_{ij}p_{ij}\right)\\ \leq &\left|\sum_{i=1}^N \sum_{j=1}^N 2\eta_{ij}^T\theta_{ij}\xi_j m_{ij}\dot{x}_j\right| - \frac{1}{T-t}e^T M^T \mathbf{\Theta}Q\mathbf{\Theta}Me\\ &+\frac{1}{T-t}\sum_{i=1}^N \sum_{j=1}^N \xi_j\left(\epsilon r_{ij}\eta_{ij}^T\eta_{ij}r_{ij} + \frac{1}{\epsilon}\eta_{ij}^T\eta_{ij}\right)\\ &-(\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\xi_j\left(r_{ij}^2 + 2r_{ij}p_{ij}\right).\end{aligned}$$

Taking the time derivative of V_4 gives $\dot{V}_4 \leq \frac{1}{T-t}\sum_{i=1}^N \sum_{j=1}^N(\epsilon p_{ij}\eta_{ij}^T \times \eta_{ij}p_{ij} + \frac{1}{\epsilon}\eta_{ij}^T\eta_{ij} - p_{ij}^*\eta_{ij}^T\eta_{ij}) - (\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}(p_{ij} - p_{ij}^*)^2$.

Hence,

$$\begin{aligned}\dot{V}_3 + \dot{V}_4 \leq &\left|\sum_{i=1}^N \sum_{j=1}^N 2\eta_{ij}^T\theta_{ij}\xi_j m_{ij}\dot{x}_j\right| - \frac{1}{T-t}e^T M^T \mathbf{\Theta}Q\mathbf{\Theta}Me\\ &+\frac{1}{T-t}\epsilon e^T M^T\left(\mathbf{P}^2 + \mathbf{R}^2\right)Me + \frac{1}{T-t}\frac{2}{\epsilon}e^T M^T Me\\ &-\frac{1}{T-t}e^T M^T \mathbf{P}^* Me - (\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\xi_j\left(r_{ij}^2 + 2r_{ij}p_{ij}\right)\\ &-(\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\left(p_{ij} - p_{ij}^*\right)^2\end{aligned}$$

where $\mathbf{P}^* = \mathrm{diag}\{I_{n_j} \otimes p_{ij}^*\}$ and a positive constant ϵ is defined. Then, we can get that $\sum_{i=1}^N \sum_{j=1}^N 2\eta_{ij}^T\theta_{ij}\xi_j m_{ij}\dot{x}_j \leq \frac{1}{T-t}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)e^T M^T \mathbf{\Theta}^2 \left(\Xi \otimes I_{\bar{N}\times\bar{N}}\right)^2 Me + \frac{\lambda_{\max}(M^T M)\bar{l}^2}{T-t}\left(\mu_1 N\|e\|^2 + \mu_2 N^2\|x - x^*\|^2\right)$, where μ_1 and μ_2 are defined. Since $I_{(N\bar{N})\times(N\bar{N})} - \left(\Xi \otimes I_{\bar{N}\times\bar{N}}\right) > 0$, one has $\mathbf{\Theta}^2 > \mathbf{\Theta}^2\left(\Xi \otimes I_{\bar{N}\times\bar{N}}\right)^2$. Hence,

$$\begin{aligned}\dot{V} \leq &\frac{\bar{l}_1}{T-t}\|e\|^2 - \frac{m}{T-t}\left\|x - x^*\right\|^2 + \frac{\bar{l}_2 + \max_{i\in v}\{l_i\}}{T-t}\left\|x - x^*\right\|\|e\|\\ &+\frac{1}{T-t}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)e^T M^T \mathbf{\Theta}^2 Me\\ &+\frac{\lambda_{\max}\left(M^T M\right)\bar{l}^2}{T-t}\left(\mu_1 N\|e\|^2 + \mu_2 N^2\left\|x - x^*\right\|^2\right)\\ &-\frac{\lambda_{\min}(Q)}{T-t}e^T M^T \mathbf{\Theta}^2 Me + \frac{1}{T-t}\epsilon e^T M^T\left(\mathbf{P}^2 + \mathbf{R}^2\right)Me\\ &+\frac{1}{T-t}\frac{2}{\epsilon}e^T M^T Me - \frac{1}{T-t}e^T M^T \mathbf{P}^* Me\\ &-(\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\xi_j\left(r_{ij}^2 + 2r_{ij}p_{ij}\right)\\ &-(\dot{\gamma}t + \gamma)\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\left(p_{ij} - p_{ij}^*\right)^2.\end{aligned}$$

By using $\theta_{ij} = p_{ij} + r_{ij}$, $p_{ij}(0) > 0$, $p_{ij}(t) > 0$, one has

$$\begin{aligned}\dot{V} \leq &\frac{1}{T-t}(\bar{l}_1 + \vartheta\bar{l}^2\mu_1 N + \frac{2\vartheta}{\epsilon} - \lambda_{\min}(\mathbf{P}^*)\lambda_{\min}\left(M^T M\right)\\ &+\sigma\left(\bar{l}_2 + \max_{i\in v}\{l_i\}\right))\|e\|^2\\ &+\frac{1}{T-t}\left(\vartheta\bar{l}^2\mu_2 N^2 - m + \frac{\bar{l}_2 + \max_{i\in v}\{l_i\}}{\sigma}\right)\left\|x - x^*\right\|^2\\ &-\frac{1}{T-t}\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\xi_j\left(r_{ij}^2 + 2r_{ij}p_{ij}\right)\\ &-\frac{1}{T-t}\sum_{i=1}^N \sum_{j=1}^N \frac{e^{-\gamma t}}{2}\left(p_{ij} - p_{ij}^*\right)^2.\end{aligned}$$

where $\vartheta = \lambda_{\max}\left(M^T M\right)$. If p_{ij}^* and σ meet $p_{ij}^* \geq \frac{\bar{l}_1 + \vartheta\bar{l}^2\mu_1 N + \frac{2\vartheta}{\epsilon} + \sigma(\bar{l}_2 + \max_{i\in v}\{l_i\})}{\lambda_{\min}(M^T M)}$, $\sigma \geq \frac{\bar{l}_2 + \max_{i\in v}\{l_i\}}{m - \vartheta\bar{l}^2\mu_2 N^2}$, we conclude that $\dot{V} \leq -\frac{\min\{\lambda, 1\}}{T-t}V$, where λ is a positive constant. Hence, $V(t) \leq \left(\frac{T-t}{T}\right)^{-\min\{\lambda, 1\}} V(0)$, from which it is clear that $\lim_{t\to T} V(t) = 0$. To this end, the conclusions of Theorem 1 can be obtained.

Remark 2: It is noted that, due to the presence of communication networks, the algorithm (5)−(8) will be inevitably subject to a variety of communication constraints such as communication delays and data losses. As a result, it is an interesting topic to further consider the effects of such communication constraints on distributed prescribed-time Nash equilibrium seeking strategy, which will be investigated in our future research work. ■

Remark 3: Different from the existing literature [4], [9], [10], [13], Theorem 1 shows clearly that the proposed distributed Nash equilibrium seeking strategy is fully distributed by utilizing a PI adaptive algorithm to avoid the global information under a directed graph, and also ensures the prescribed convergence time T by introducing $\frac{e^{\gamma t}}{T-t}$ in the adaptive parameter (8).

Numerical example: Consider a 5-player game in which each player's action $x_i \in \mathbb{R}^2$. Let

$$\begin{aligned} f_1(x) &= 3x_{11}^2 + 2x_{12}^2 + 2x_1 x_3 - x_{11} - x_{12} \\ f_2(x) &= 2.5x_{21}^2 + 3x_{22}^2 + 2x_2 x_4 - 2x_{21} - 2x_{22} \\ f_3(x) &= 5x_{31}^2 + 4x_{32}^2 + 6x_3 x_5 - 3x_{31} - 3x_{32} \\ f_4(x) &= 6x_{41}^2 + 5x_{42}^2 + 4x_4 x_2 - 4x_{41} - 4x_{42} \\ f_5(x) &= 7x_{51}^2 + 6x_{52}^2 + 2x_5 x_3 - 5x_{51} - 5x_{52} \end{aligned}$$

in which $x_i = [x_{i1}, x_{i2}]^T$, $x = [x_1^T, x_2^T, x_3^T, x_4^T, x_5^T]^T$. It is calculated that x^* of the game is $x^* = [-0.012, -0.017, 0.308, 0.230, 0.041, 0.043, 0.230, 0.308, 0.259, 0.297]$. The strongly connected communication topology is given in Fig. 1. In addition, $x(0) = [-1, -3, 1.5, -2, -3, 2, 3, 1, -2, 4]^T$, and the initial values of all other variables are zero. Set the prescribed convergence time as $T = 3.5$ s for the proposed seeking strategy. Correspondingly, the simulation result of players' actions in Fig. 2 indicates that they converge to the actual Nash equilibrium. From Fig. 3, it is demonstrated that y_{ij} is convergent to x_j at the prescribed convergence time 3.5 s. Thus, the proposed prescribed-time control strategy has been numerically verified.

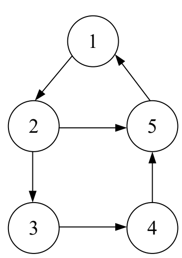

Fig. 1. Strongly connected communication topology.

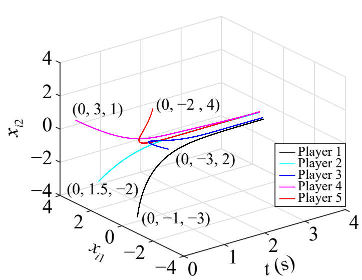

Fig. 2. The trajectories of players' actions.

Conclusion: This letter introduces a new prescribed-time fully distributed Nash equilibrium seeking strategy under directed graphs. The prescribed-time seeking strategy does not require any global information on communication topology and allows to set the convergence time in advance based on specific requirements. Future research will be focused on fully distributed Nash equilibrium seeking in presence of cyberattacks [14] and communication constraints [15].

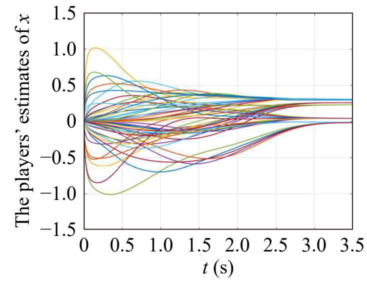

Fig. 3. The estimates y_{ij} on players' actions x for all $i, j \in \{1, 2, , 5\}$.

Acknowledgments: This work was supported by the National Natural Science Foundation of China (NSFC) (62073171) and the Natural Science Foundation of Jiangsu Province (BK20200744).

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