Letter

Protocol-Based Non-Fragile State Estimation for Delayed Recurrent Neural Networks Subject to Replay Attacks

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Dear Editor,

This letter focuses on the protocol-based non-fragile state estimation problem for a class of recurrent neural networks (RNNs). With the development of communication technology, the networked systems have received particular attentions. The networked system brings advantages such as easy to implement, high flexibility as well as low cost, and also has disadvantages such as limited bandwidth of the communication network which lead to networked-induced phenomena [1], [2]. To alleviate the network-induced phenomena, communication protocols have been introduced in the communication networks of the networked systems [3], [4]. As a widely used communication protocol in real practice, the round-robin (RR) protocol has received research interest and the state estimation problem under the RR protocol is an on-going hotspot in the area of signal processing [5]. Nevertheless, for the RNNs, the corresponding RR protocol-based state estimation problem still needs further research effort which is the first motivation of this letter.

On the other hand, in the networked systems, the signals are prone to be attacked when transmitted through the wireless networks. In general, the cyber attacks can be categorised into three types, namely, denial-of-service attacks, deception attacks, and replay attacks [6]–[8]. Among others, the replay attack has its distinctive characteristic [9]. In the replay attack, the attacker first eavesdrops the historical data transmitted in the network and saves the data in a storage space. Then, the attacker randomly replays the saved data to the system. The replay attacks can destroy the performance of the system even without any prior information of the system. Furthermore, the replay attack is difficult to be detected. Therefore, the state estimation problem under replay attack is of great significance.

Based on the previous analysis, this letter studies the problem of protocol-based non-fragile state estimation for delayed RNNs subject to replay attacks. Two important questions to be addressed are: 1) How to build a suitable mathematical model to describe replay attacks? and 2) How to address the impact of the RR protocol and the replay attacks on the state estimation performance? The main contributions of this letter are that: 1) The replay attacks considered satisfies engineering practice; 2) A new mathematical model is designed to describe the replay attacks; and 3) The effect of the RR protocol and the replay attacks is considered and a non-fragile estimator is designed to ensure the desired performance.

Problem statement: Consider a n-neuron delayed RNNs as follows:

$$\begin{cases} x_i(\zeta+1) = \bar{a}_i x_i(\zeta) + \sum\limits_{j=1}^{n} \bar{b}_{ij} g_j(x_j(\zeta)) + \bar{e}_i v_i(\zeta) \\ \qquad\qquad + \sum\limits_{j=1}^{n} \bar{c}_{ij} g_j(x_j(\zeta - r(\zeta))), \ i = 1,2,\ldots,n \\ y_l(\zeta) = \sum\limits_{j=1}^{n} \bar{d}_{lj} x_j(\zeta), \ l = 1,2,\ldots,n \end{cases} \tag{1}$$

where $x_i(\zeta) \in \mathbb{R}$ represents the state of the ith neuron. $y_l(\zeta) \in \mathbb{R}$ is the lth sensor measurement. $g_j(\cdot)$ is the nonlinear activation function for the jth neuron with the initial condition $g_j(0) = 0$. $\bar{a}_i$ expresses the state feedback coefficient. $\bar{b}_{ij}$ and $\bar{c}_{ij}$ denote connection weight and delayed connection weight, respectively. $v_i(\zeta) \in \mathbb{R}$ is the process noise which belongs to $l_2[0,\infty)$. $\bar{d}_{lj}$ and $\bar{e}_i$ are known scalars. $r(\zeta)$ is the time-varying delay satisfying $0 \le \bar{r}_1 \le r(\zeta) \le \bar{r}_2$ where $\bar{r}_1$ and $\bar{r}_2$ are known constants.

Assumption 1: For any scalars δ_1 and δ_2 ($\delta_1 \ne \delta_2$), the nonlinear function $g_i(\cdot)$ satisfies

$$\phi_i^- \le \frac{g_i(\delta_1) - g_i(\delta_2)}{\delta_1 - \delta_2} \le \phi_i^+, \ i = 1,2,\ldots,n$$

with ϕ_i^- and ϕ_i^+ are known scalars.

In this letter, the RR protocol is used to schedule data communication. Let $\xi(\zeta)$ be the sensor node which can transmit data at time instant ζ with the initial condition $\xi(0) = n$. The value of $\xi(\zeta)$ is obtained by

$$\xi(\zeta) = \text{mod}(\zeta - 1, n) + 1.$$

In the following, the replay attack is considered. We consider the case that each sensor is attacked separately. Under the replay attack, the attacker first eavesdrops and then saves the data in a limited storage space. Since the storage space is limited, we assume that $\tilde{m}$ packets can be stored at most and the set of stored signals is represented by $M_s(\zeta) \triangleq \{y_s(\zeta_1), y_s(\zeta_2), \ldots, y_s(\zeta_{\tilde{m}})\}$ ($s = 1,2,\ldots,n$) where $\zeta_1 \ne \zeta_2 \ne \cdots \ne \zeta_{\tilde{m}}$ and $\zeta_\jmath < \zeta$ ($\jmath = 1,2,\ldots,\tilde{m}$). When a replay attack occurs at time ζ, the attack process is as follows:

1) Delete the measurement data $y_s(\zeta)$ sent at ζth time instant;

2) Replay the historical measurement selected from the storage space $M_s(\zeta)$;

3) Keep the data $y_s(\zeta)$ in the storage space $M_s(\zeta)$ and delete the earliest data in $M_s(\zeta)$. Then, let $M_s(\zeta+1) = M_s(\zeta)$.

To deal with the replay attack, the time stamp approach is used. Each measurement signal $y_s(\zeta)$ is first packaged in a time stamped packet before being sent to the estimator. Therefore, it is easy to detect whether the attack is occurred or not. A variable $\alpha_s(\zeta)$ is introduced to indicate whether an attack occurs or not. When a replay attack occurs, $\alpha_s(\zeta)$ is set to 1. Otherwise, $\alpha_s(\zeta)$ is set to 0. For $\alpha_s(\zeta) = 0$, the measurement signal $y_s(\zeta)$ may also be eavesdropped by the attacker and stored in $M_s(\zeta)$. If the storage space is insufficient, the earliest data in $M_s(\zeta)$ will be deleted and the new data will be stored. When $\zeta = 0$, the storage space is empty and the attacker can only eavesdrop. Therefore, $\alpha_s(0) = 0$.

In this letter, we assume that the attacker has limited energy. Note that the replay attack costs a certain amount of energy. Therefore, the number of consecutive replay attacks is limited. By introducing a variable

$$d_s(\zeta) = \begin{cases} d_s(\zeta - n) + n, & \text{if } \alpha_s(\zeta) = 1 \text{ and } \zeta > n \\ 0, & \text{if } \alpha_s(\zeta) = 0 \end{cases}$$

it is easily known that $d_s(\zeta)$ is related to the number of consecutive replay attacks and therefore $d_s(\zeta) \le \tilde{d}_s$. It is worth noting that $\tilde{d}_s$ is hard to be obtained in practical application. Fortunately, $\tilde{d}_s$ is not required in the proposed algorithm.

Considering the influence of the RR protocol and the replay attacks, the actual signal received by the estimator is

$$\bar{y}_s(\zeta) = \begin{cases} y_s(\zeta - d_s(\zeta)), & \text{if } s = \xi(\zeta) \\ \bar{y}_s(\zeta - 1), & \text{else} \end{cases} \tag{2}$$

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which is rewritten in the following compact form:

$$\bar{y}(\zeta)=\sum_{s=1}^{n}\Big(\Phi_s y(\zeta-d_s(\zeta))+(\tilde{I}_s-\Phi_s)\bar{y}(\zeta-1)\Big) \tag{3}$$

where

$$\mathfrak{R}(\zeta)\triangleq \mathrm{col}\{\mathfrak{R}_1(\zeta),\mathfrak{R}_2(\zeta),\ldots,\mathfrak{R}_n(\zeta)\}\ (\mathfrak{R}=y,\bar{y})$$
$$\Phi_s\triangleq \mathrm{diag}\{\underbrace{0,\ldots,0}_{s-1},\delta(\xi(\zeta)-s),0,\ldots,0\}$$
$$\tilde{I}_s\triangleq \mathrm{diag}\{\delta(\xi(\zeta)-s),\delta(\xi(\zeta)-s),\ldots,\delta(\xi(\zeta)-s)\}.$$

By letting

$$\mathfrak{I}(\zeta)\triangleq \mathrm{col}\{\mathfrak{I}_1(\zeta),\mathfrak{I}_2(\zeta),\ldots,\mathfrak{I}_n(\zeta)\}\ (\mathfrak{I}=x,v)$$
$$\acute{g}(x(\zeta))\triangleq \mathrm{col}\{g_1(x_1(\zeta)),g_2(x_2(\zeta)),\ldots,g_n(x_n(\zeta))\}$$
$$A\triangleq \mathrm{diag}\{\bar{a}_1,\bar{a}_2,\ldots,\bar{a}_n\},\ B\triangleq[\bar{b}_{ij}]_{n\times n},\ C\triangleq[\bar{c}_{ij}]_{n\times n}$$
$$D\triangleq[\bar{d}_{lj}]_{n\times n},\ E\triangleq \mathrm{diag}\{\bar{e}_1,\bar{e}_2,\ldots,\bar{e}_n\}$$

we have

$$\begin{cases}x(\zeta+1)=Ax(\zeta)+B\acute{g}(x(\zeta))+C\acute{g}(x(\zeta-r(\zeta)))+Ev(\zeta)\\ y(\zeta)=Dx(\zeta).\end{cases}$$

Further denoting

$$\bar{x}(\zeta)\triangleq\begin{bmatrix}x^T(\zeta) & \bar{y}^T(\zeta-1)\end{bmatrix}^T$$
$$\bar{g}(\bar{x}(\zeta))\triangleq\begin{bmatrix}\acute{g}^T(x(\zeta)) & \acute{g}^T(\bar{y}(\zeta-1))\end{bmatrix}^T$$

one has

$$\begin{cases}\bar{x}(\zeta+1)=\bar{A}\bar{x}(\zeta)+\bar{B}\bar{g}(\bar{x}(\zeta))+\bar{C}\bar{g}(\bar{x}(\zeta-r(\zeta)))\\ \qquad\qquad+\chi_{s1}\bar{D}\bar{x}(\zeta-d_s(\zeta))+\bar{E}v(\zeta)\\ \bar{y}(\zeta)=\chi_{s2}\bar{D}_1\bar{x}(\zeta-d_s(\zeta))+\bar{D}_2\bar{x}(\zeta)\end{cases} \tag{4}$$

where

$$\bar{A}\triangleq \mathrm{diag}\{A,\chi_{s3}\},\ \bar{B}\triangleq \mathrm{diag}\{B,0\},\ \bar{C}\triangleq \mathrm{diag}\{C,0\}$$
$$\bar{D}\triangleq\begin{bmatrix}0&0\\D&0\end{bmatrix},\ \bar{E}\triangleq\begin{bmatrix}E\\0\end{bmatrix},\ \bar{D}_1\triangleq\begin{bmatrix}D&0\end{bmatrix},\ \bar{D}_2\triangleq\begin{bmatrix}0&\chi_{s3}\end{bmatrix}$$
$$\chi_{s1}\triangleq\sum_{s=1}^{n}(I\otimes\Phi_s),\ \chi_{s2}\triangleq\sum_{s=1}^{n}\Phi_s,\ \chi_{s3}\triangleq\sum_{s=1}^{n}(\tilde{I}_s-\Phi_s).$$

In this letter, the non-fragile state estimator is designed as follows:

$$\begin{aligned}\hat{x}(\zeta+1)=&\bar{A}\hat{x}(\zeta)+\bar{B}\bar{g}(\hat{x}(\zeta))+\bar{C}\bar{g}(\hat{x}(\zeta-r(\zeta)))\\&+\chi_{s1}\bar{D}\hat{x}(\zeta-d_s(\zeta))+(\mathcal{K}+\Delta\mathcal{K})\\&\times(\bar{y}(\zeta)-\chi_{s2}\bar{D}_1\hat{x}(\zeta-d_s(\zeta))-\bar{D}_2\hat{x}(\zeta))\end{aligned} \tag{5}$$

where $\hat{x}(\zeta)$ is the estimate of the neural network state, $\mathcal{K}$ is the gain parameter of the estimator. $\Delta\mathcal{K}$ indicates the variation of the estimator gain and meets the additive norm-bounded condition $\Delta\mathcal{K}=M_1F_1(\zeta)N_1$, where M_1 and N_1 are known matrices with appropriate dimensions. $F_1(\zeta)$ is unknown matrix that satisfies $F_1^T(\zeta)F_1(\zeta)\le I,\forall\zeta\in\mathbb{N}^+$.

Denoting $e(\zeta)\triangleq\bar{x}(\zeta)-\hat{x}(\zeta)$, $\eta(\zeta)\triangleq[\bar{x}^T(\zeta)\ e^T(\zeta)]^T$ and $\breve{g}(\eta(\zeta))\triangleq[\bar{g}^T(\bar{x}(\zeta))\ \bar{g}^T(\bar{x}(\zeta))-\bar{g}^T(\hat{x}(\zeta))]^T$, we obtain the following augmented system:

$$\begin{aligned}\eta(\zeta+1)=&\mathcal{A}\eta(\zeta)+\mathcal{B}\breve{g}(\eta(\zeta))+\mathcal{C}\breve{g}(\eta(\zeta-r(\zeta)))\\&+(\mathcal{D}_1-\mathcal{D}_2)\eta(\zeta-d_s(\zeta))+\mathcal{E}v(\zeta)\end{aligned} \tag{6}$$

where

$$\mathcal{A}\triangleq \mathrm{diag}\{\bar{A},\bar{A}-(\mathcal{K}+\Delta\mathcal{K})\bar{D}_2\},\ \mathcal{B}\triangleq \mathrm{diag}\{\bar{B},\bar{B}\}$$
$$\mathcal{C}\triangleq \mathrm{diag}\{\bar{C},\bar{C}\},\ \mathcal{D}_1\triangleq \mathrm{diag}\{\chi_{s1}\bar{D},\chi_{s1}\bar{D}\}$$
$$\mathcal{D}_2\triangleq \mathrm{diag}\{0,(\mathcal{K}+\Delta\mathcal{K})\chi_{s2}\bar{D}_1\},\ \mathcal{E}\triangleq\begin{bmatrix}\bar{E}^T\bar{E}^T\end{bmatrix}^T.$$

Definition 1: The system (6) is exponentially mean-square stable if there exist scalars $\gamma>0$ and $0<\delta<1$ such that

$$\mathbb{E}\{\|\eta(\zeta)\|^2\}\le\gamma\delta^{\zeta}\max_{j\in[-\bar{r}_2,0]}\|x(j)\|^2,\ \forall\zeta>0.$$

This letter focuses on design a non-fragile state estimator for delayed RNNs (1) such that the following conditions are satisfied:

1) For $v(\zeta)=0$, the augmented system (6) is exponentially mean-square stable.

2) When the initial condition is zero, for a known disturbance attenuation level $\nu>0$ and all nonzero $v(\zeta)$, $\eta(\zeta)$ satisfies

$$\sum_{\zeta=0}^{\infty}\mathbb{E}\{\|\eta(\zeta)\|^2\}\le\nu^2\sum_{\zeta=0}^{\infty}\mathbb{E}\{\|v(\zeta)\|^2\}. \tag{7}$$

Main results:

Theorem 1: Let the scalar $\nu>0$ be given. The augmented system (6) is exponentially mean-square stable and the H_∞ performance index is achieved for nonzero $v(\zeta)$ if there exist positive scalar $\epsilon>0$, positive definite matrices $\mathcal{W}=\mathrm{diag}\{\mathcal{W}_1,\mathcal{W}_2\}>0$, $\mathcal{T}>0$, $\mathcal{S}>0$ and diagonal matrices $\mathcal{O}>0$, $\mathcal{H}>0$, $\mathcal{J}>0$, $\mathcal{Y}>0$, Z satisfying

$$\aleph=\begin{bmatrix}\Xi&\epsilon\mathcal{P}&\mathcal{Q}^T\\*&-\epsilon I&0\\*&*&-\epsilon I\end{bmatrix}<0 \tag{8}$$

where

$$\Xi\triangleq\begin{bmatrix}\Theta_1+I&-\Lambda_{21}&0&0&0&0&\Phi_{17}^T\\*&-\bar{H}&0&0&0&0&\Phi_{27}^T\\*&*&-\bar{Q}&-\Lambda_{22}&0&0&0\\*&*&*&-\bar{J}&0&0&\Phi_{47}^T\\*&*&*&*&-\iota&0&\Phi_{57}^T\\*&*&*&*&*&-\nu^2I&\Phi_{67}^T\\*&*&*&*&*&*&-\mathcal{W}\end{bmatrix}$$

$$\Phi_{17}\triangleq\mathrm{diag}\{\mathcal{W}_1\bar{A},\mathcal{W}_2\bar{A}-Z\bar{D}_2\},\ \Phi_{27}\triangleq\mathrm{diag}\{\mathcal{W}_1\bar{B},\mathcal{W}_2\bar{B}\}$$
$$\Phi_{47}\triangleq\mathrm{diag}\{\mathcal{W}_1\bar{C},\mathcal{W}_2\bar{C}\},\ \Phi_{57}\triangleq\mathrm{diag}\{\Phi_{571},\Phi_{572}\}$$
$$\Phi_{571}\triangleq\mathcal{W}_1\chi_{s1}\bar{D},\ \Theta_1\triangleq(\bar{r}_2-\bar{r}_1+1)\mathcal{T}-\mathcal{W}-\Lambda_{11}+\iota$$
$$\Phi_{572}\triangleq\mathcal{W}_2\chi_{s1}\bar{D}-Z\chi_{s2}\bar{D}_1,\ \Phi_{67}\triangleq\begin{bmatrix}\bar{E}^T\mathcal{W}_1^T&\bar{E}^T\mathcal{W}_2^T\end{bmatrix}^T$$
$$\mathcal{P}\triangleq\begin{bmatrix}\mathcal{R}_1&0&0&0&\mathcal{R}_2&0&0\end{bmatrix},\ \bar{\mathcal{Q}}\triangleq\mathcal{T}+\Lambda_{21}$$
$$\mathcal{Q}\triangleq\begin{bmatrix}0&0&0&0&0&0&\mathcal{Q}_1\end{bmatrix},\ \bar{H}\triangleq\mathrm{diag}\{\mathcal{O},\mathcal{H}\}$$
$$\mathcal{R}_1\triangleq\begin{bmatrix}\mathcal{R}_a^T&0&0&0&0&0&0\end{bmatrix}^T,\ \bar{J}\triangleq\mathrm{diag}\{\mathcal{J},\mathcal{Y}\}$$
$$\mathcal{R}_2\triangleq\begin{bmatrix}0&0&0&0&\mathcal{R}_b^T&0&0\end{bmatrix}^T,\ \iota\triangleq\bar{I}^T\chi_{s2}^T\mathcal{S}\chi_{s2}\bar{I}$$
$$\mathcal{Q}_1\triangleq\begin{bmatrix}\mathcal{Q}_a^T&0&0&0&\mathcal{Q}_a^T&0&0\end{bmatrix}^T,\ \bar{I}\triangleq[0\ I\ 0\ I]$$
$$\mathcal{R}_a\triangleq\mathrm{diag}\{0,\bar{D}_2^TN_1^T\},\ \mathcal{R}_b\triangleq\mathrm{diag}\{0,\bar{D}_1^T\chi_{s2}^TN_1^T\}$$
$$\mathcal{Q}_a\triangleq\mathrm{diag}\{0,-M_1^T\mathcal{W}_2^T\},\ \Lambda_{11}\triangleq\mathrm{diag}\{\mathcal{O}\mathcal{L}_1,\mathcal{H}\mathcal{L}_1\}$$
$$\Lambda_{21}\triangleq\mathrm{diag}\{-\mathcal{O}\mathcal{L}_2,-\mathcal{H}\mathcal{L}_2\},\ \Lambda_{22}\triangleq\mathrm{diag}\{-\mathcal{J}\mathcal{L}_2,-\mathcal{Y}\mathcal{L}_2\}$$
$$\mathcal{L}_1\triangleq\mathrm{diag}\{\phi_1^+\phi_1^-,\phi_2^+\phi_2^-,\ldots,\phi_n^+\phi_n^-\}$$
$$\mathcal{L}_2\triangleq\mathrm{diag}\{(\phi_1^++\phi_1^-)/2,(\phi_2^++\phi_2^-)/2,\ldots,(\phi_n^++\phi_n^-)/2\}.$$

Moreover, the gain matrix is determined by

$$\mathcal{K}=\mathcal{W}_2^{-1}Z. \tag{9}$$

Proof: A Lyapunov functional of the following form is established:

$$V(\zeta)=\sum_{i=1}^{3}V_i(\zeta) \tag{10}$$

where

$$V_1(\zeta)\triangleq\eta^T(\zeta)\mathcal{W}\eta(\zeta)$$
$$V_2(\zeta)\triangleq\sum_{v=\zeta-r(\zeta)}^{\zeta-1}\eta^T(v)\mathcal{T}\eta(v)+\sum_{l=1-\bar{r}_2}^{-\bar{r}_1}\sum_{v=\zeta+l}^{\zeta-1}\eta^T(v)\mathcal{T}\eta(v)$$
$$V_3(\zeta)\triangleq\sum_{w=\zeta-d_s(\zeta)}^{\zeta-1}\eta^T(w)\bar{I}^T\chi_{s2}^T\mathcal{S}\chi_{s2}\bar{I}\eta(w).$$

Calculating the difference of $V(\zeta)$ and noting Assumption 1, we obtain

$$\mathbb{E}\{\Delta V(\zeta)\}+\mathbb{E}\{\eta^T(\zeta)\eta(\zeta)\}-\nu^2\mathbb{E}\{v^T(\zeta)v(\zeta)\}\le\tilde{\Pi}^T(\zeta)\Gamma\tilde{\Pi}(\zeta)$$

where

$$\tilde{\Pi}(\zeta) \triangleq [\eta^T(\zeta) \;\; \breve{g}^T(\eta(\zeta)) \;\; \eta^T(\zeta - r(\zeta)) \;\; \breve{g}^T(\eta(\zeta - r(\zeta))) \;\; \eta^T(\zeta - d_s(\zeta)) \;\; v^T(\zeta)]^T$$

$$\Gamma \triangleq \begin{bmatrix} \tilde{\Theta}_1 + I & \Theta_{12} & 0 & \mathcal{A}^T\mathcal{W}\mathcal{C} & \Theta_{15} & \mathcal{A}^T\mathcal{W}\mathcal{E} \\ * & \Theta_{22} & 0 & \mathcal{B}^T\mathcal{W}\mathcal{C} & \Theta_{25} & \mathcal{B}^T\mathcal{W}\mathcal{E} \\ * & * & -\bar{Q} & -\Lambda_{22} & 0 & 0 \\ * & * & * & \Theta_{44} & \Theta_{45} & C^T\mathcal{W}\mathcal{E} \\ * & * & * & * & \Theta_{55} & \Theta_{56} \\ * & * & * & * & * & \Theta_{66} \end{bmatrix}$$

$$\tilde{\Theta}_1 \triangleq \Theta_1^* - \Lambda_{11},\;\; \Theta_{12} \triangleq \mathcal{A}^T\mathcal{W}\mathcal{B} - \Lambda_{21},\;\; \Theta_{22} \triangleq \mathcal{B}^T\mathcal{W}\mathcal{B} - \bar{H}$$

$$\Theta_{44} \triangleq C^T\mathcal{W}C - \bar{J},\;\; \Theta_{15} \triangleq \mathcal{A}^T\mathcal{W}(\mathcal{D}_1 - \mathcal{D}_2)$$

$$\Theta_{25} \triangleq \mathcal{B}^T\mathcal{W}(\mathcal{D}_1 - \mathcal{D}_2),\;\; \Theta_{45} \triangleq C^T\mathcal{W}(\mathcal{D}_1 - \mathcal{D}_2)$$

$$\Theta_{55} \triangleq (\mathcal{D}_1 - \mathcal{D}_2)^T\mathcal{W}(\mathcal{D}_1 - \mathcal{D}_2) - \iota,\;\; \Theta_{56} \triangleq (\mathcal{D}_1 - \mathcal{D}_2)^T\mathcal{W}\mathcal{E}$$

$$\Theta_{66} \triangleq \mathcal{E}^T\mathcal{W}\mathcal{E} - \nu^2 I,\;\; \Theta_1^* \triangleq \mathcal{A}^T\mathcal{W}\mathcal{A} + (\bar{r}_2 - \bar{r}_1 + 1)\mathcal{T} - \mathcal{W} + \iota.$$

By applying Schur complement lemma, we derive that

$$\sum_{\zeta=0}^{\infty}\mathbb{E}\{\|\eta(\zeta)\|^2\} - \nu^2\sum_{\zeta=0}^{\infty}\mathbb{E}\{\|v(\zeta)\|^2\} < V(0) - V(\infty).$$

It is easy to have $V(\infty) \geq 0$ and subsequently (7) is obtained. Using Schur complement lemma, S-procedure lemma and matrix analysis technique, it is obtained that if there exists a positive scalar ϵ satisfying $\Xi + \epsilon^{-1}\mathcal{P}\mathcal{P}^T + \epsilon^{-1}(\epsilon\mathcal{Q})^T(\epsilon\mathcal{Q}) < 0$, then the augmented system is exponentially mean-square stable and the H_∞ performance index is achieved for $v(\zeta) \neq 0$. The estimator gain can be obtained by $Z = \mathcal{W}_2\mathcal{K}$.

Numerical example: Consider a 2-neuron delayed RNNs (1) with

$$A = \text{diag}\{-0.8, -0.085\},\; E = \text{diag}\{0.01, 0.01\}$$

$$B = \begin{bmatrix} -0.013 & 0.015 \\ 0.012 & -0.011 \end{bmatrix},\; C = \begin{bmatrix} 0.002 & -0.001 \\ 0.003 & -0.002 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.3 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}.$$

The activation function $g(x(\zeta)) = \tanh(4x(\zeta))$. $\bar{r}_1 = 1$, $\bar{r}_2 = 3$. Fig. 1 is the error between the true states and the estimated states. Fig. 2 shows the instants when replay attacks occur. Fig. 3 expresses the norm of the estimation error $e(\zeta)$.

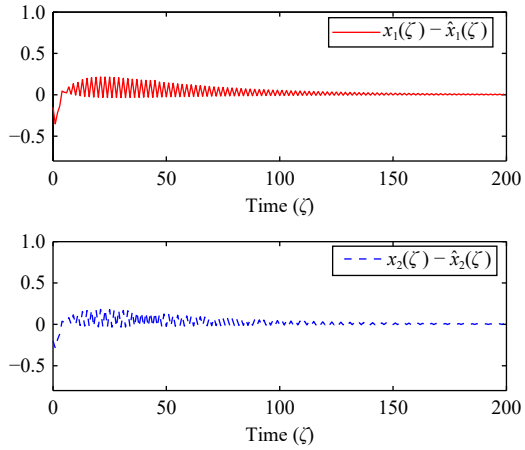

Fig. 1. The error between true states and estimated states.

Conclusion: In this letter, we have studied the protocol-based non-fragile state estimation problem of delayed RNNs under replay attacks. While ensuring exponentially mean square stability of the system, the H_∞ performance index has been satisfied. Then, by using matrix analysis technique, the estimator gain has been solved. Finally, the effectiveness of the proposed estimation method is verified by numerical simulation.

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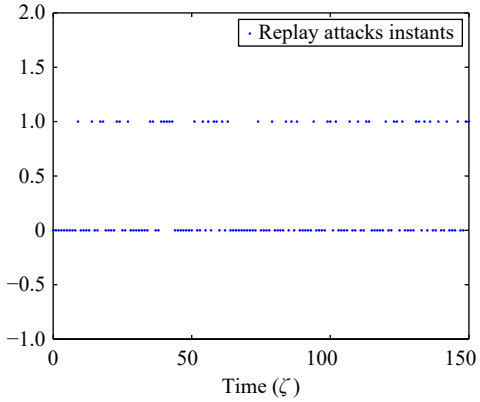

Fig. 2. The instants of replay attacks occurred.

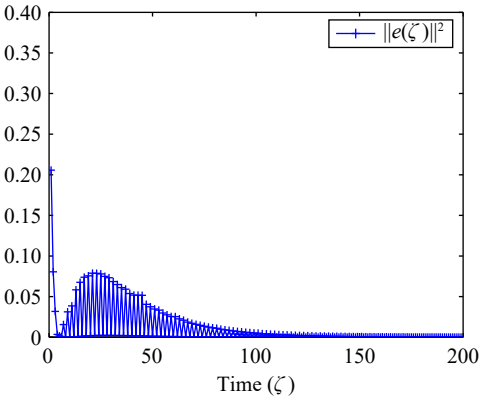

Fig. 3. The norm of the estimation error $e(\zeta)$.

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