## **Letter**

## **A Novel Memory-Based Scheduling Protocol for Networked Control Systems Under Stochastic Attacks and Bandwidth Constraint**

Hongchenyu Yang, Chen Peng, *Senior Member, IEEE*, and Zhiru Cao

## Dear Editor,

In this letter, a novel memory-based scheduling protocol (MBSP) is investigated for networked control systems (NCSs) under bandwidth constraint and stochastic denial-of-service (DoS) attacks. First, considering the bandwidth constraint and that multiple sensors' channels may be attacked, a novel memory-based scheduling protocol is proposed to alleviate the pressure of signal transmission, which can dynamically allocate network sources according to different intensities of DoS attacks. Second, under the proposed MBSP, a stochastic impulse system model is established with consideration of networkinduced delay and packet loss. Then, sufficient conditions for the existence of the desired controller gain and MBSP parameters are given to ensure the mean-square stability of the studied closed-loop system subject to DoS attacks. Finally, a simulation example is provided to verify the effectiveness of the proposed method.

NCSs are spatially distributed systems, where signal transmissions among system components are realized through a shared network. NCSs have been widely used in various fields, including power systems, industrial automation, offshore platform, and so on. However, due to the limited bandwidth of the shared communication network, there may exist some imperfect phenomena such as time delay and data loss, which could degenerate system performances and even make systems unstable. In an open network environment, communication channels are vulnerable to malicious cyber-attacks which could jeopardize NCSs and cause security problems seriously. Generally, deception attacks and DoS attacks are the two most common types of cyber-attacks. Given that DoS attacks occur most frequently and are extremely common, this work focuses mainly on dealing with DoS attacks from a defender's point of view.

To mitigate the pressure of band-limited network communication, lots of studies are devoted to researching proper communication strategies. As one of the most effective transmission strategies which are based on the necessity of communication, the event-triggered mechanism has received much attention during the past decades. However, most of these results take an assumption that all of control signals should be boxed into one data package. But in practical applications, the nodes of NCSs are distributively linked over a large physical extent. That is, only one packet to transmit is unrealistic. Therefore, the research on multiple packet transmission under network bandwidth constraint is indispensable. In view that nodes could preempt network resources from each other, it is unavoidable for the node collision phenomena during multiple packet transmission. To solve this problem, scheduling protocols have been presented to

Corresponding author: Chen Peng.

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The authors are with the School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200444, China (e-mail: yhchenyu@163. com; c.peng@shu.edu.cn; zhiru\_cao@163.com).

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decide which node to gain access to the communication network under multi-channel transmissions. Generally, there are three main types of multi-channel scheduling: Round-Robin (RR) scheduling, try-once-discard (TOD) scheduling, and stochastic scheduling.

**Related work:** Recently, a time-delay approach has been discussed to solve the stabilization problem of NCSs [1] and [2]. In [2], an analysis and comparison between Bernoulli and Markovian stochastic protocols have been made and sufficient exponential stability conditions have been derived for the studied impulsive delayed system with multiple sensor nodes. Besides, the hybrid system method has been employed to analyze NCSs with variable delays

under TOD communication protocol in [3], where a partial exponential stabilization criterion of the hybrid system with time delay has been derived. By utilizing the above three protocols to orchestrate the signal transmission, the stabilization of discrete-time NCSs has been studied in [4] which are provided with uncertain and variable parameters. However, most of these results rarely consider the network security problem, which is one of the most urgent problems to be solved today. Therefore, how to schedule system nodes when NCSs are attacked is one of the topics worth studying.

. missions, which stimulates this work It is well-known that DoS attacks have been widely researched in the past several years [5] and [6]. As discussed in [5], for the case that DoS attacks block the signal transmission of a single communication channel periodically, an event-triggered control method based on state-observer has been presented to realize remote control and observation of the attacked system. In [6], the input-to-state stabilization problem of cyber-physical systems based on multi-channel communication has been addressed under arbitrary DoS attacks of which the feature is characterized by frequency and duration. In [7], each successful transmission interval has been divided into two kinds of time intervals to discuss according to whether attacks exist, and the switching-like event-triggered transmission mechanism has been presented to cope with intermittent DoS attacks so as to accomplish the expected system performances. From the energy-constrained attacker perspective, a stochastic DoS attack energy minimization problem has been solved in [8], where the minimum stochastic attack energy cost could be used to estimate the behavior of the synergetically working sensor network effectively. As mentioned above, most results are based on either a single transmission channel or multiple transmission channels not taking the intensity of DoS attacks into account. Few researchers have studied the scheduling scheme that considers the intensity of DoS attacks under multiple packet trans-

**Problem statement:** In Fig. 1, consider the linear system

$$
\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}
$$
 (1)

*x* (*t*)  $\in \mathbb{R}_x$ ,  $y(t) \in \mathbb{R}_y$ ,  $u(t) \in \mathbb{R}_u$  are state, measurement output and control input.  $A$ ,  $B$  and  $C$  have suitable dimensions. For system (1), consider the cost function

$$
J = \int_0^{+\infty} [x^T(t)\overline{M}x(t) + u^T(t)\overline{N}u(t)]dt
$$
 (2)

where  $\bar{M}$  and  $\bar{N}$  are given symmetric matrices. Denote sampling instant as  $s_q$  satisfying that  $0 = s_0 < s_1 < \cdots < s_q < \cdots$ ,  $s_{q+1} - s_q \leq \overline{s}$ ,



Fig. 1. NCSs subjected to stochastic DoS attacks under MBSP.

 $\lim_{q \to \infty} s_q = +\infty$ ,  $q \in \mathbb{Z}_+$ , where  $\overline{s}$  is the maximum allowable transfer *q*→+∞ *i yi*(*sq*) = *Cix i* (*sq*) interval. The measurement of the th sensor is ,  $i = 1,..., N$ , in which  $x^{i}(s_q)$  is the state corresponding to  $y_i(s_q)$  and *n*(*sq*) = *col*{*y*<sub>1</sub>(*sq*),..., there are *N* sensor nodes in the system. Thus,  $y(s_q) = col(y_1(s_q),...$  $y_N(s_q)$ ,  $C = col\{C_1, \ldots, C_N\}$ .

packet loss by  $\{t_k\} \subseteq \{s_q\}$ , that is, only at sampling instants  $\{t_k\}$ , the induced delay  $\eta_k \in [\eta_1, \bar{\eta}_2]$ . For the convenience of description, let  $\iota_k = t_k + \eta_k$ ,  $\iota_{k+1} = t_{k+1} + \eta_{k+1}$ . In this work, DoS attacks are assumed In an open network environment, packet dropout, cyber-attacks and network-induced delay are inevitable. Denote the sequence after input of the controller can be updated. We assume that the networkto follow a Bernoulli distribution:

$$
\gamma_k^i = \begin{cases} 1, & \text{no DoS attacks} \\ 0, & \text{with DoS attacks} \end{cases}
$$
 (3)

 $\mathbb{E}\{\gamma_k^i\} = Prob\{\gamma_k^i = 1\} = \theta_k^i$ where  $\mathbb{E}\{\gamma_k^l\} = Prob\{\gamma_k^l = 1\} = \theta_k^l$ .

 $\{t_k\}$ . Under i.i.d protocol, let  $\chi_k \in \{1, ..., N\}$  be the active output node at  $t_k$ . The selection of  $\chi_k$  is independent of the previously chosen node and is assumed to be i.i.d with a known invariable  $\beta_k^i$ , In the following, we apply an independent identically distributed (i.i.d) stochastic protocol for the network transmission, which prescribes that only one node gets access to be transmitted at instants

$$
Prob\{\chi_k = i\} = \beta_k^i, \ \ i = 1, ..., N, \ \beta_k^i \ge 0, \ \sum_{i=1}^N \beta_k^i = 1.
$$

The updating law of controller input  $\tilde{y}_i(t_k)$  is described as

$$
\tilde{y}_i(t_k) = \begin{cases}\ny_i(t_k), & i = \chi_k \\
\tilde{y}_i(t_{k-1}), & i \neq \chi_k.\n\end{cases}
$$
\n(4)

*till* new data arriving, that is, for  $t \in [\iota_k, \iota_{k+1})$ , Moreover, the controller keeps its control input signal as constant

$$
\tilde{y}(t) = \tilde{y}(t_k) = col\{\tilde{y}_1(t_{k-1}), \ldots, y_{\chi_k}(t_k), \ldots, \tilde{y}_N(t_{k-1})\}.
$$

has permission to transmit, if  $\gamma_k^i = 1$ ,  $y_i(t_k)$  is transmitted to the con*i* troller side; if  $\gamma_k^i = 0$ ,  $\tilde{y}_i(t_{k-1})$  is used by the controller. In a word, DoS attacks affect only the active sensor node at instant  $t_k$ . And when **Memory-based scheduling protocol:** As Fig. 2 depicts, for the case that the channel has no transmission permission, if NCSs are attacked, the attack will not affect the system, and the historical information stored in the buffer can be still used. When the channel DoS attacks occur, the control input keeps constant with the previous measurement. Therefore, under MBSP, the available control input is

$$
\tilde{y}(t_k) = \begin{cases}\ncol\{\tilde{y}_1(t_{k-1}), \ldots, y_{\chi_k}(t_k), \ldots, \tilde{y}_N(t_{k-1})\}, & \gamma_k^i = 1 \\
col\{\tilde{y}_1(t_{k-1}), \ldots, \tilde{y}_{\chi_k}(t_{k-1}), \ldots, \tilde{y}_N(t_{k-1})\}, & \gamma_k^i = 0.\n\end{cases}
$$

That is, under MBSP, the input of controller can be  $\tilde{y}(t_k) =$  $col{\{\tilde{y}_1(t_{k-1}),..., \gamma_k^i\} \mathcal{Y}_{\chi_k}(t_k) + (1 - \gamma_k^i)\tilde{y}_{\chi_k}(t_{k-1}),..., \tilde{y}_N(t_{k-1})\}}.$ 

Two random variables  $\gamma_k^i$ ,  $\chi_k$  are introduced to build a connection DoS attacks is described with  $\theta_k^i$ . The smaller the value is, the more between scheduling process and DoS attack process. The intensity of frequently the attack occurs, and vice versa. MBSP can dynamically allocate network sources according to different intensities of DoS attacks, so as to be used to relieve the pressure of network communi-



Fig. 2. Memory-based scheduling protocol.

cation under multiple packet transmissions.

**Stochastic impulsive system model:** Define  $\tilde{x}^i(t_{k-1})$  is the state vector corresponding to  $\tilde{y}_i(t_{k-1})$  of the *i*th sensor. Consider state errors  $e_i(t) = \tilde{x}^i(t_{k-1}) - x^i(t_k)$  to describe the control input, so as to build a bridge between attack and scheduling parameters. Thus,

$$
\tilde{y}(t) = \sum_{i=1}^{N} (1 - \gamma_k^i) I_i C_i e_i(t) + C x(t_k) + \sum_{i=1, i \neq \chi_k}^{N} \gamma_k^i I_i C_i e_i(t)
$$

 $I_i = col\{0, ..., 0\}$ | {z } *i*−1 ,*I*,0,...,0 | {z } *N*−*i* where  $I_i = col\{0, ..., 0, I, 0, ..., 0\}$ .

Next, we introduce an output feedback controller

$$
u(t) = K\tilde{y}(t), \ t_k \le t \le t_{k+1}
$$
 (5)

where  $K$  is the controller gain to be designed. The controller is

$$
u(t)=\sum_{i=1}^N(1-\gamma_k^i)KI_iC_ie_i(t)+KCx(t_k)+\sum_{i=1,i\neq \chi_k}^N\gamma_k^iKI_iC_ie_i(t).
$$

Let  $\eta(t) = t - t_k$  for  $t \in [t_k, t_{k+1})$ , then

$$
\eta_1 \leq \eta_k \leq \eta(t) \leq t_{k+1} - t_k + \eta_{k+1} \leq \eta_2
$$

Define the following indicator functions:

$$
\lambda_k^i = \begin{cases} 1, & \text{if } k = i, \\ 0, & \text{if } k \neq i, \end{cases} \quad i = 1, \dots, N
$$

where  $\mathbb{E}\left\{\lambda_k^i\right\} = Prob\{\chi_{t_k} = i\} = \beta_k^i$ . In this work, variables  $\gamma_k^i$  and  $\lambda_k^i$  are mutually independent. The stochastic impulsive system model is written as

$$
\dot{x}(t) = Ax(t) + BKCx(t - \eta(t)) + \sum_{i=1}^{N} (1 - \lambda_k^i \gamma_k^i) BKI_i C_i e_i(t)
$$
 (6)

and state errors update in the form

$$
e_i(t_{k+1}) = (1 - \lambda_k^i \gamma_k^i) e_i(t_k) + x^i(t_k) - x^i(t_{k+1}).
$$
 (7)

The initial of state  $x(t)$  is to be  $\psi(t)$ ,  $t \in [t_0 - \eta_2, t_0]$ ,  $\psi(0) = x_0$ , and  $\psi(t)$  is a continuous function on  $t \in [t_0 - \eta_2, t_0]$ . Define

$$
\pi^i_k = \lambda^i_k \gamma^i_k = \begin{cases} 1, & Prob\left\{\pi^i_k = 1\right\} = \beta^i_k \theta^i_k \\ 0, & Prob\left\{\pi^i_k = 0\right\} = 1 - \beta^i_k \theta^i_k \end{cases}
$$

where  $\mathbb{E} \left\{ \pi_k^i \right\} = \alpha_k^i = \beta_k^i \theta_k^i$ . Therefore, (6) can be rewritten as

$$
\dot{x}(t) = Ax(t) + BKCx(t - \eta(t)) + \sum_{i=1}^{N} (1 - \pi_k^i) BKI_iC_i e_i(t)
$$
 (8)

where  $e_i(t) = (1 - \pi_k^i) e_i(t_k) + x^i(t_k) - x^i(t_{k+1}), t \in [t_k, t_{k+1}).$ 

to defend cyber-attacks and choose the active node. When  $\theta_k^i = 1$ , The scheduling protocol proposed in this work differs from the existing stochastic protocols, such as [2] and [9], which integrates DoS attack parameters. When occurring cyber-attacks, MBSP is used there is no DoS attacks. The situation degenerates into the case in [2]. **Main results:**

Theorem 1: Under the proposed MBSP, for given matrices  $A, B, C$ , *K*,  $\bar{M} > 0$ ,  $\bar{N} > 0$  and scalars  $\eta_1$ ,  $\eta_2$ ,  $\alpha_k^i$ ,  $i = 1,...,N$ , if there exist *M*<sub>1</sub> > 0,  $M_2$  > 0,  $Z$  > 0,  $N_1$  > 0,  $N_2$  > 0, and  $G_i$  > 0,  $S_i$  > 0,  $T_i > 0$ ,  $i = 1, \ldots, N$  with compatible dimensions such that

$$
\Theta_i = \begin{bmatrix} (\eta_2 - \eta_1) T_i - \alpha_k^i S_i & * \\ (1 - \alpha_k^i) S_i & S_i - G_i \end{bmatrix} < 0, \ i = 1, ..., N
$$
 (9)

$$
\left[\Pi_{ef}\right]_{e,f=1,\dots,4} < 0, \ \Pi_{11} = \left[Y_{pq}\right]_{p,q=1,\dots,8} \tag{10}
$$

$$
Y_{11} = A^T Z + ZA + M_1 - 4N_1, \ Y_{42} = -2N_2, \ Y_{43} = -2N_2^T
$$
  

$$
Y_{22} = -M_1 + M_2 - 4N_1 - 4N_2, \ Y_{44} = -8N_2, \ Y_{51} = 6N_1
$$
  

$$
Y_{81}^T = col\{(1 - \alpha_1)ZBKI_1C_1, ..., (1 - \alpha_N)ZBKI_NC_N\}
$$

$$
\Pi_{22} = \Delta_i = \text{diag}\{-N_1, -N_2, -G_1, \dots, -G_N\}, Y_{33} = -M_2 - 4N_2
$$
  
\n
$$
\Pi_{41} = col\{\Phi, \overline{N}\Phi_1', \overline{N}\Phi_{21}', \dots, \overline{N}\Phi_{2N}'\}, \Phi = [M, 0, \dots, 0]
$$
  
\n
$$
\Pi_{21} = col\{\eta_1 N_1 \Phi_1, (\eta_2 - \eta_1) N_2 \Phi_1, \Omega_1, \dots, \Omega_N\}, Y_{21} = -2N_1
$$
  
\n
$$
\Pi_{31} = col\{\Xi_1, \dots, \Xi_N\}, \Pi_{33} = \text{diag}\{\Delta_1, \dots, \Delta_N\}, Y_{41} = C^T K^T B^T Z
$$
  
\n
$$
Y_{52} = 6N_1, Y_{55} = -12N_1, Y_{62} = 6N_2, Y_{64} = 6N_2, Y_{66} = -12N_2
$$
  
\n
$$
Y_{73} = 6N_2, Y_{74} = 6N_2, Y_{77} = -12N_2, Y_{88} = \text{diag}\{-T_1, \dots, -T_N\}
$$
  
\n
$$
\Phi_1 = [A, 0, 0, BK, 0, 0, 0, (1 - \alpha_1)BK I_1 C_1, \dots, (1 - \alpha_N)BK I_N C_N]
$$
  
\n
$$
\Phi_1' = [0, 0, 0, KC, 0, 0, 0, (1 - \alpha_1)KI_1 C_1, \dots, (1 - \alpha_N)KI_N C_N]
$$
  
\n
$$
\Phi_{2i} = [0, \dots, 0, BK I_i C_i, 0, \dots, 0], \Omega_i = \sqrt{\eta_2 - \eta_1} G_i \Phi_1
$$
  
\n
$$
\Phi_{2i}' = [0, \dots, 0, K I_i C_i, 0, \dots, 0], \Lambda_{iv} = \sqrt{\eta_2 - \eta_1} G_v \Phi_{2i}
$$

 $\Xi_i = \sqrt{\alpha_i (1-\alpha_i) \cosh(\eta_1 N_1 \Phi_{2i}, (\eta_2-\eta_1) N_2 \Phi_{2i}, \Lambda_{i1}, \ldots, \Lambda_{iN})}$ then system (8) is asymptotically mean-square stable and  $\mathbb{E}{J} \le$  $J^* = \psi^T(0) Z \psi(0) + \int_{-\eta_2}^{-\eta_1} \int_{\nu}^{0} (\eta_2 - \eta_1) \psi^T(\omega) N_2 \psi(\omega) d\omega dv$ 

$$
+ \int_{-\eta_1}^{0} \psi^T(v) M_1 \psi(v) dv + \int_{-\eta_1}^{0} \int_{v}^{0} \eta_1 \dot{\psi}^T(\omega) N_1 \dot{\psi}(\omega) d\omega dv + \int_{-\eta_2}^{-\eta_1} \psi^T(v) M_2 \psi(v) dv + \sum_{i=1}^{N} \int_{t_k}^{0} (\eta_2 - \eta_1) \sqrt{G_i} \dot{\psi}(v)^2 dv.
$$

Proof: Define the Lyapunov functional candidate as

$$
V(t) = \sum_{i=1}^{N} e_i^T (i_k) S_i e_i (i_k) + V_1(t) + V_2(t), V_1(t) = x^T(t) Zx(t)
$$
  
+ 
$$
\int_{-\eta_2}^{-\eta_1} \int_{t+v}^t (\eta_2 - \eta_1) x^T(\omega) N_2 \dot{x}(\omega) d\omega dv
$$
  
+ 
$$
\int_{-\eta_1}^{0} \int_{t+v}^t \eta_1 x^T(\omega) N_1 \dot{x}(\omega) d\omega dv
$$
  
+ 
$$
\int_{t-\eta_2}^{t-\eta_1} x^T(\omega) M_2 x(\omega) d\omega + \int_{t-\eta_1}^t x^T(\omega) M_1 x(\omega) d\omega
$$
  

$$
V_2(t) = \sum_{i=1}^{N} \int_{t_k}^t (\eta_2 - \eta_1) \sqrt{G_i} \dot{x}(\omega)^2 d\omega.
$$

The detailed derivation process can refer to [9], which is omitted here due to the limited pages.

Theorem 2: Under the proposed MBSP, for given matrices  $A, B, C$ ,  $\overline{M} > 0$ ,  $\overline{N} > 0$  and scalars  $\zeta > 0$ ,  $\eta_1$ ,  $\eta_2$ ,  $\alpha_k^i$ ,  $i = 1,...,N$ , if there exist  $\tilde{M}_1 > 0$ ,  $\tilde{M}_2 > 0$ ,  $X > 0$ ,  $\tilde{N}_1 > 0$ ,  $\tilde{N}_2 > 0$ ,  $\tilde{G}_i > 0$ ,  $S_i > 0$ ,  $T_i > 0$  and  $D, F$  with compatible dimensions such that (11) is solvable, where

$$
\begin{aligned}\n\min \left\{ \sum_{i=1}^{N} G_i o_i \right\} \\
\text{s.t. } (9), (12) \text{ and } (13)\n\end{aligned} \tag{11}
$$

$$
\left[\Pi_{ef}\right]_{e,f=1,\dots,4} < 0,\Pi_{11} = \left[\mathbf{Y}_{pq}\right]_{p,q=1,\dots,8} \tag{12}
$$

$$
\begin{bmatrix} -\zeta I & (DC - CX)^T \\ DC - CX & -I \end{bmatrix} < 0, \begin{bmatrix} o_i & * \\ I & G_i \end{bmatrix} > 0
$$
 (13)

$$
\tilde{Y}_{11} = AX + XA^{T} + \tilde{M}_{1} - 4\tilde{N}_{1}, \ \tilde{Y}_{22} = -\tilde{M}_{1} + \tilde{M}_{2} - 4\tilde{N}_{1} - 4\tilde{N}_{2}
$$
\n
$$
\tilde{Y}_{33} = -\tilde{M}_{2} - 4\tilde{N}_{2}, \ \tilde{Y}_{21} = -2\tilde{N}_{1}, \ \tilde{Y}_{42} = -2\tilde{N}_{2}, \ \tilde{Y}_{44} = -8\tilde{N}_{2}
$$
\n
$$
\tilde{Y}_{51} = 6\tilde{N}_{1}, \ \tilde{Y}_{52} = 6\tilde{N}_{1}, \ \tilde{Y}_{62} = 6\tilde{N}_{2}, \ \tilde{Y}_{64} = 6\tilde{N}_{2}, \ \tilde{Y}_{74} = 6\tilde{N}_{2}
$$
\n
$$
\tilde{Y}_{55} = -12\tilde{N}_{1}, \ \tilde{Y}_{66} = -12\tilde{N}_{2}, \ \tilde{Y}_{73} = 6\tilde{N}_{2}, \ \tilde{Y}_{77} = -12\tilde{N}_{2}
$$
\n
$$
\tilde{Y}_{81}^{T} = col\{(1 - \alpha_{1}) BFC, \dots, (1 - \alpha_{N}) BFC\}, \ \tilde{Y}_{41} = C^{T}F^{T}B^{T}
$$

$$
\tilde{\Pi}_{21} = col\{\eta_1 \tilde{\Phi}_1, (\eta_2 - \eta_1) \tilde{\Phi}_1, \tilde{\Omega}_1, \dots, \tilde{\Omega}_N\}, \ \tilde{Y}_{43} = -2\tilde{N}_2^T
$$
\n
$$
\tilde{\Pi}_{22} = \tilde{\Delta}_i = \text{diag}\{-X\tilde{N}_1^{-1}X, -X\tilde{N}_2^{-1}X, -G_1^{-1}, \dots, -G_N^{-1}\}
$$
\n
$$
\tilde{\Pi}_{33} = \text{diag}\{\tilde{\Delta}_1, \dots, \tilde{\Delta}_N\}, \ \tilde{\Pi}_{31} = col\{\tilde{\Xi}_1, \dots, \tilde{\Xi}_N\}
$$
\n
$$
\tilde{\Pi}_{41} = col\{\tilde{\Phi}, \tilde{N}\tilde{\Phi}_1', \tilde{N}\tilde{\Phi}_{21}', \dots, \tilde{N}\tilde{\Phi}_{2N}'\}, \ \tilde{\Phi} = [MX, 0, \dots, 0]
$$
\n
$$
\tilde{\Pi}_{44} = \text{diag}\{-\bar{M}, -\bar{N}, -\bar{N}, \dots, -\bar{N}\}, \ \tilde{Y}_{88} = \text{diag}\{-T_1, \dots, -T_N\}
$$
\n
$$
\tilde{\Xi}_i = \sqrt{\alpha_i (1 - \alpha_i) col} \{\eta_1 \tilde{\Phi}_{2i}, (\eta_2 - \eta_1) \tilde{\Phi}_{2i}, \tilde{\Lambda}_{i1}, \dots, \tilde{\Lambda}_{iN}\}
$$
\n
$$
\tilde{\Omega}_i = \sqrt{\eta_2 - \eta_1} \tilde{\Phi}_1, \ \tilde{\Lambda}_{i\nu} = \sqrt{\eta_2 - \eta_1} \tilde{\Phi}_{2i}, \ i, \nu = 1, \dots, N
$$

then, system (8) is asymptotically mean-square stable. The controller<br>gain is given by  $K = FD^{-1}$  and  $E\{J\} \le J^* = \psi^T(0)Z\psi(0) + \sum_{i=1}^N \int_{t_k}^0 (\eta_2 - \eta_1) \sqrt{G_i \psi(v)}^2 dv + \int_{-\eta_1}^0 \psi^T(v) (\rho^2 \tilde{M}_1 - 2\rho X) \psi(v) dv + \int_{-\eta_2}^{-\eta_1}$ 

An illustrative example: Suppose that the studied plant is a satellite system in [10]. There are three sensors to be used to measure state signals with  $C_1 = [0, 1, 1, 0]$ ,  $C_2 = [1, 0, 1, 1]$  and  $C_3 = [0, 1, 0, 1]$ and  $x(0) = \text{col}\{0.10, -0.18, -0.07, 0.04\}$  and the matrices of system  $(8)$  are given by

$$
AT = \begin{bmatrix} 0 & 0 & -0.09 & 0.09 \\ 0 & 0 & 0.09 & -0.09 \\ 1 & 0 & -0.04 & 0.04 \\ 0 & 1 & 0.04 & -0.04 \end{bmatrix}, BT = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.
$$

Case 1: Optimal choice of  $\beta_k^l$  for the attacked system under MBSP.

Utilizing the stepwise incremental algorithm similar to [9], given  $\eta_1 = 0.02$ ,  $\rho = 2$ ,  $\theta_k^i = 0.5$ , the optimal solution can be found as  $\eta_2 = 0.2$ ,  $\beta_k^i = [0.05, 0.55, 0.4]$  and the corresponding controller gain can be obtained as  $K = [-0.0275, -1.7848, 0.8237]$ . When  $\rho = 1, 2$ . [9] has no feasible results, while Theorem 2 in this work is solvable and we can reach the upper bound of delay is  $0.25$ ,  $0.22$ . Furthermore, Fig. 3 depicts active sensor nodes, where active node  $= 0$  represents that stochastic DoS attacks occur, from which we can verify the effectiveness of main results.



Fig. 3. Active sensor nodes under MBSP.

Case 2: Resilience of MBSP against different intensities of attacks. Using the same parameter settings of Case 1, Fig. 4 depicts the control input under different intensities of DoS attacks. As Fig. 4 depicts, the proposed MBSP can resist different intensities of DoS attacks and make system stable finally.

Case 3: Comparison between MBSP and stochastic protocol in [2]. Using the same parameter settings of Case 1, state responses with the controller in  $[2]$  are shown in Fig. 5. Clearly, the controller of  $[2]$ cannot defend Dos attacks, which means that the proposed MBSP has significant advantages over the traditional stochastic protocol in [2] when NCSs are subjected to DoS attacks.

Conclusions: In this work, a novel memory-based scheduling pro-



Fig. 4. Control input under  $\theta_k^i = 0.9$ ,  $\theta_k^i = 0.1$ .



Fig. 5. State responses with the controller in [2].

tocol has been proposed for NCSs under multiple transmission channels and stochastic DoS attacks. First, MBSP has been proposed to alleviate the pressure of signal transmissions for NCSs and allocate network sources according to different intensities of DoS attacks dynamically. Second, further considering packet dropout and network-induced delay, a stochastic impulsive delayed system has been established. Then, by using time-delay methods, sufficient stability criteria and the guaranteed cost output feedback controller gain have

been derived. Finally, an illustrative example has been provided to demonstrate the validity of the presented method. This work pays main attention to stochastic protocols. Future research will focus on how to optimally design other scheduling protocols when NCSs are attacked.

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