Letter

Vibration Control of an Experimental Flexible Manipulator Against Input Saturation

Zhijia Zhao, *Member, IEEE*, Sentao Cai, Ge Ma, and F. Richard Yu, *Fellow, IEEE*

Dear Editor,

This letter considers the control problem of an experimental flexible manipulator in position tracking, vibration suppression, and saturation compensation. Based on the backstepping technology and a Nussbaum function, we develop an anti-windup control to restrain the manipulator's vibration, realize the desire trajectory tracking, and eliminate the saturation. Applying the Lyapunov's method, the control system's stability with the proposed control is proved. Finally, the practicability and effectiveness of the control methodology are verified on a Quanser experiment platform.

Recently, the modeling and control of the flexible manipulator system have aroused the interest of the scholars and made some progresses [1]. Due to the advantages of lightweight, low energy consumption, and fast response, the flexible manipulator plays a significant role in diverse applications, such as aerospace, healthcare, industry, etc., [2]. Facing the complicate working circumstance, the manipulator system generally produces unexpected vibration during operation. Hence, developing relevant control schemes to suppress vibration of the flexible manipulator is urgent and essential.

Considering that the flexible manipulator is a distributed parameter system, the boundary control is regarded as an effective strategy to avoid the spillover by arranging a small number of actuators at the boundary. In [3], an active boundary controller was applied in flexible Timoshenko manipulator to handle the input backlash and the output states constraints. In [4], an adaptive iterative learning algorithm was designed with the boundary control to address external disturbances and the system parametric uncertainty existing in a flexible manipulator. In [5], the authors proposed a boundary controller based on a robust state observer, which could regulate the joint position and suppress elastic vibration of the flexible manipulator.

For the practical flexible manipulator system, the saturation characteristic exists inevitably in the implement of the actuators, which is caused by the inherent physical constraints [6]. Without considering this characteristic, it may reduce the system's control precision apparently and affect the performance seriously. To resolve the problem, the researchers have presented several effective strategies in the vibration suppression of the flexible manipulator system. In [7], a boundary output feedback control based on high-gain observers was designed to compensate for the input saturation, regulate the position, and restrain the oscillation of a flexible two-link manipulator system. In [8], the authors presented an adaptive neural-network boundary controller to achieve stability of a flexible manipulator, using neural-network scheme to address the model uncertainties and the input saturation. In [9], an adaptive stochastic tracking control strategy based on an auxiliary system and the backstepping technique was designed to eliminate the effect of the input saturation. However, the saturation was handled by the piecewise sign function in the above literatures, which may result in the chattering effect of the control input and even damage the actuator severely. To resolve the chattering, the researchers adopted a hyperbolic tangent function (HTF) to handle the saturation. In [10], the HTF was introduced to regulate angular position and inhibit elastic oscillation of a flexible-link manipulator under input constraint. In [11], the authors fulfilled the modeling of a two-link flexible manipulator and presented a control strategy with the HTF to address the input saturation. Although the significant progress for handling the saturation has been recorded, few attempts about employing the HTF and the Nussbaum function to handle the input saturation have been applied in the vibration suppression and position regulation of a flexible manipulator with a control torque, which inspires our study.

Corresponding author: Ge Ma.

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Z. J. Zhao, S. T. Cai, and G. Ma are with the School of Mechanical and Electrical Engineering, Guangzhou University, Guangzhou 510006, and also with the Guangdong Laboratory of Artificial Intelligence and Digital Economy (SZ), Shenzhen 518060, China (e-mail: zhjzhaoscut@163.com; 2112007142@e.gzhu.edu.cn; m_ge@gzhu.edu.cn).

F. R. Yu is with the Guangdong Laboratory of Artificial Intelligence and Digital Economy (SZ), College of Computer Science & Software Engineering, Shenzhen University, Shenzhen 518060, China (e-mail: yufei@gml.ac.cn).

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Motivated by the above literatures, this letter aims to develop a vibration control for a flexible manipulator under the presence of the input saturation. The main contributions of this letter can be summarized as: 1) Compared with the methods in [12], [13], the proposed control strategy is more targeted and appropriate to apply in the flexible manipulator system than the conventional ordinary differential equation systems. With the backstepping technology and a Nussbaum function, the system's trajectory tracking, oscillation suppression, and compensation of the input saturation can be achieved, which can effectively solve the problem of control signal chattering. 2) Using the Lyapunov's analysis, the system output responses are proven to be stable. The control performance of this scheme is also verified on a Quanser's experiment platform.

Problem formulation: The flexible manipulator system's simplified model is described in [14]. For the system, $o(z,t)$ and $\theta(t)$ are output variables that denote the manipulator's elastic deflection and rotation angle, respectively. The objective of this letter lies in exploiting a control strategy to make $\theta(t)$ track the desired angle θ_d and reduce $o(z,t)$ as small as possible. The displacement of the manipulator is deduced as $d(z,t)=o(z,t)+z\theta(t)$ and the control torque $\tau(t)$ is installed in the starting point of the manipulator with the length of L.

Remark 1: For the concise expression, some derivative terms $(\star)(z,t)$, $\partial(\star)/\partial z$, $\partial^2(\star)/\partial z^2$, $\partial^3(\star)/\partial z^3$, $\partial^4(\star)/\partial z^4$, $\partial(\star)/\partial t$, $\partial^2(\star)/\partial t^2$, $(\star)(0,t)$, and $(\star)(L,t)$ can be replaced by these notations $(\star)$, $(\star)'$, $(\star)''$, $(\star)'''$, $(\star)''''$, $(\dot\star)$, $(\ddot\star)$, $(\star)_0$, and $(\star)_L$.

The flexible manipulator's governing equation is derived as [14]

$$\rho\ddot{d}=-EIo''''+Td''-p\dot{d} \tag{1}$$

$\forall(z,t)\in(0,L)\times[0,+\infty)$, ρ, p, T, and EI denote the density, damping coefficient, tension, and bending stiffness of flexible manipulators, respectively.

The boundary conditions are described as

$$o_0=o_0'=o_L''=0 \tag{2}$$

$$To_L'=EIo_L'''. \tag{3}$$

The dynamics of the rotation angle can be deduced as

$$I\ddot{\theta}-EIo_0''-To_L=\tau(t) \tag{4}$$

$\forall t\in[0,+\infty)$, I expresses the rotation inertia of flexible manipulators, and $\tau(t)$ is the system input.

The input saturation characteristic's model is proposed as

$$\tau(t)=\begin{cases}\operatorname{sign}(c_i(t))u_{\max}, & |c_i(t)|\geq u_{\max}\\ c_i(t), & |c_i(t)|<u_{\max}\end{cases} \tag{5}$$

where $u_{\max}$ is a known positive constant and defined as the upper bound of $\tau(t)$. With the effect of sign function, $\tau(t)$ has a sharp corner when the designed control signal $c_i(t)$ satisfies $|c_i(t)| = u_{\max}$, which may cause a non-differentiable problem in the backstepping design. Hence, we introduce a smooth function to approximate the saturation as

$$\tau(t) = h(c_i(t)) + D(c_i(t)) = u_{\max} \times \tanh\left(\frac{c_i(t)}{u_{\max}}\right) + D(c_i(t)) \tag{6}$$

where $D(c_i(t))$ can be regarded as the error between $\tau(t)$ and $h(c_i(t))$ with the bound deduced as

$$|D(c_i(t))| \le u_{\max}(1 - \tanh(1)) = D_1. \tag{7}$$

To solve the unknown term $D(c_i(t))$, we design a adaptive variable $\hat{D}_1(t)$ to estimate $D(c_i(t))$ with the error $\tilde{D}_1(t) = \hat{D}_1(t) - D(c_i(t))$.

Control design: In this section, we adopt the backstepping technology and a Nussbaum function to conduct the vibration control for a flexible manipulator to track the desired trajectory, restrain the oscillation, and remove the effect of the input saturation. Subsequently, we prove the closed-loop manipulator system's stability with the Lyapunov's theory.

First, we define some transformations of state variables as

$$b_1 = a_1 = \theta(t) - \theta_d \tag{8}$$

$$b_2 = a_2 - v_1 = \dot{\theta}(t) - v_1 \tag{9}$$

$$b_3 = h(c_i(t)) - v_2 \tag{10}$$

where $\theta_d > 0$ is a tracked angle, and v_1 and v_2 are the virtual control signals.

A Lyapunov candidate function is selected as

$$\Upsilon_1(t) = \frac{1}{2}b_1^2 + \frac{1}{2}Ib_2^2. \tag{11}$$

The time derivative of (11) is derived as

$$\dot{\Upsilon}_1(t) = b_1(b_2 + v_1) + b_2(EIo_0'' + To_L - I\dot{v}_1 + b_3 + v_2 + D(c_i(t))). \tag{12}$$

Then, we can design the virtual control according to (12)

$$v_1 = -k_1 b_1 \tag{13}$$

$$v_2 = -k_2 b_2 - b_1 + I\dot{v}_1 - \hat{D}_1(t). \tag{14}$$

Substituting (13) and (14), (12) can be rewritten as

$$\dot{\Upsilon}_1(t) \le -k_1 b_1^2 - k_2 b_2^2 + b_2 b_3 + b_2 EIo_0'' + b_2 To_L - b_2\tilde{D}_1(t). \tag{15}$$

Next, we establish an auxiliary system as

$$\dot{c}_i(t) = -\alpha c_i(t) + \beta. \tag{16}$$

where $\alpha > 0$ and β is the control law to be designed.

Considering (9), (10), and (16), we deduce

$$I\dot{b}_3 = Ig(-\alpha c_i(t) + \beta) - I\dot{v}_2 \tag{17}$$

where $g = \frac{\partial c_o}{\partial c_i(t)} = \frac{4}{(e^{c_i(t)/u_{\max}} + e^{-c_i(t)/u_{\max}})^2} > 0$.

It is obvious that the varying g enhances the difficulties and complexities to the control design and analysis. In order to solve the problem, we introduce a Nussbaum function $N(\phi)$ to conduct the control law β of (16). The control law is then designed as

$$\beta = N(\phi)\bar{\beta}, \ \bar{\beta} = g\alpha c_i(t) + \dot{v}_2 - \frac{1}{I}b_2 - \frac{1}{I}k_3 b_3 \tag{18}$$

where k_i, $i = 1, \ldots, 3$ are control gains.

The Nussbaum function $N(\phi)$ is defined as

$$N(\phi) = \phi^2 cos(\phi), \ \dot{\phi} = \mu I b_3 \bar{\beta} \tag{19}$$

where $\mu > 0$.

The Nussbaum function satisfies the following properties:

$$\lim_{m\to\pm\infty} \sup \frac{1}{m}\int_0^m N(\phi)d\phi = \infty \tag{20}$$

$$\lim_{m\to\pm\infty} \inf \frac{1}{m}\int_0^m N(\phi)d\phi = -\infty. \tag{21}$$

Then, we designed the adaptive law of $\hat{D}_1(t)$ as

$$\dot{\hat{D}}_1(t) = b_2 - k_4\hat{D}_1(t) \tag{22}$$

where $k_4 > 0$ is a control gain.

Now, we select a Lyapunov function as

$$\Upsilon_u(t) = \Upsilon_1(t) + \frac{1}{2}Ib_3^2 + \frac{1}{2}\tilde{D}_1^2(t). \tag{23}$$

deriving (23), we derive

$$\dot{\Upsilon}_u(t) \le -k_1 b_1^2 - k_2 b_2^2 - k_3 b_3^2 + \frac{1}{\mu}[gN(\phi) - 1]\dot{\phi} + b_2 EIo_0'' + b_2 To_L + \frac{1}{2}k_4\tilde{D}_1^2(t) + \frac{1}{2}k_4 D_1^2. \tag{24}$$

Thus, the Lyapunov function candidate is established as

$$\Upsilon(t) = \Upsilon_u(t) + \Upsilon_v(t) + \Upsilon_w(t) \tag{25}$$

where

$$\Upsilon_v(t) = \frac{1}{2}\rho\int_0^L \dot{d}^2 dz + \frac{1}{2}EI\int_0^L o''^2 dz + \frac{1}{2}T\int_0^L o'^2 dz \tag{26}$$

$$\Upsilon_w(t) = k_1\rho\int_0^L \dot{d}d_e dz \tag{27}$$

with $d_e = o + z[\theta(t) - \theta_d]$.

Lemma 1: The time derivative of (25) is upper bounded as

$$\dot{\Upsilon}(t) \le -w\Upsilon(t) + \sigma \tag{28}$$

where $w, \sigma > 0$.

Theorem 1: For the flexible manipulator system given by (1)−(4), choosing appropriate control gains, the proposed control strategy can achieve the expected performance for the single-link flexible manipulator system. It is concluded that the closed-loop system signal $o(z,t)$ and the error $[\theta(t) - \theta_d]$ are uniformly bounded.

Experiment verification: In this section, we verify the proposed control scheme on the rotary flexible link system of Quanser Inc as shown in [14]. The experimental platform is composed of a lightweight stainless steel link, the foil gauge measuring the deformation data, a DC motor unit, a power amplifier, and a data acquisition board, which is convenient to achieve the communication between the PC and the platform. To examine the practicality and effectiveness of the proposed method, we conduct two comparison groups which are the boundary control [14] without considering the input saturation and the anti-saturation control [7] using the auxiliary system, respectively. The desired angle is set as 30° and the signal delays 0.2 s after the system's operation.

With the proposed control: We conduct the simulink control model according to (11), (12), and (16)−(19). The control gains are selected as $\alpha = 0.01$, $k_1 = 33$, $k_2 = 0.08$, $k_3 = 5$, $k_4 = 15$, and $\mu = 0.5$. The determination of the above parameters should first satisfy the inequalities in (28), and then continuously debug in the simulation and experiment until a better control effect achieved. The input saturation is restrained from −2 N·m to 2 N·m on the platform. As shown in Figs. 1 and 2, the red solid line denotes the output responses under the proposed control. Although the system's vibration amplitude reaches the maximum about 0.054 m, it also converges to 0 in the rapidest speed and the vibration is suppressed form the maximum to 0.012 m within 0.4 s. In the angle tracking, the angle response can track the desired position in a short time about 0.28 s, and the flexible link is stabilized at 30° with 0.85 s.

With the previous boundary control: For comparison, applying the control scheme without considering the saturation in [14], the control results are depicted as the green solid line. From the Figs. 1 and 2, we can conclude that the vibration is also suppressed well, but the inhibitory process is slower relatively than the former. Comparing with the proposed method, the boundary control requires more time to realize system stability and the rotation angle can not trace 30° entirely.

With the anti-saturation control: Referring to [7], the control strategy using the auxiliary system is designed for the flexible manipulator with the control law as $u_2(t) = -a_2 m(t) + a_3 u_a(t) - I\beta\dot{\theta}(t) - k_\theta[\theta(t) - \theta_d] - (a_4 + T)o_L$, which $m(t)$ and $u_a(t)$ are the auxiliary systems. In Figs. 1 and 2, the performance of this control scheme is depicted by a blue solid line. It can seen that the trajectory tracking executes better than the boundary control within 0.8 s, but the ability of suppressing vibration is weaker than the proposed method.

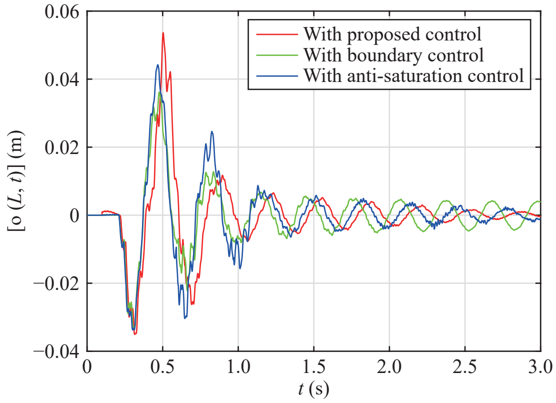

Fig. 1. Tip deflection o (L, t) under control.

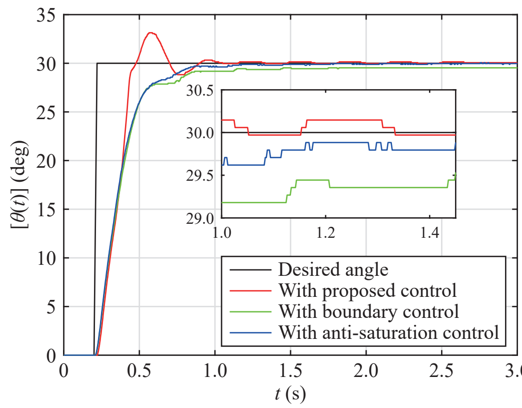

Fig. 2. Angular position $\theta(t)$ with under control.

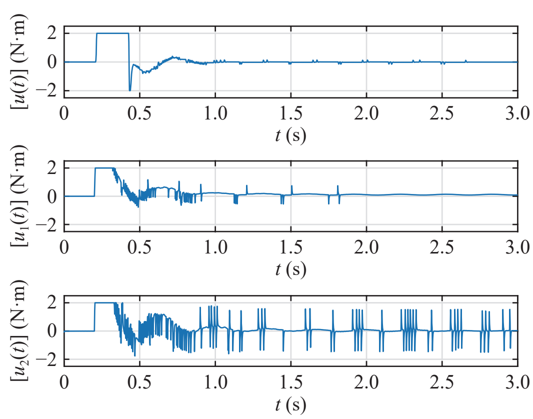

Fig. 3. Control input of the system.

The physical control input of three control methods is shown as Fig. 3 and we can conclude that the proposed control avoids the chattering apparently. From the comparison, the proposed strategy exhibits a ascendant performance than the comparison groups. In a word, it validates that the proposed control is effective and possesses the capacity against the input saturation.

Conclusion: In this letter, we have investigated the anti-windup control problem of a flexible manipulator system existing the input saturation. Employing the backstepping technology and a Nussbaum function, a vibration control strategy has been presented to achieve the vibration suppression, position tracking, and remove the influence of the input saturation. With the stability analysis and experimental verification, the proposed control has been proven to be practical and effective.

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