## **Letter**

## **Vision-Based Fixed-Time Uncooperative Aerial Target Tracking for UAV**

Peng Sun, Siqi Li, Bing Zhu, Zongyu Zuo, and Xiaohua Xia

## Dear Editor,

In this letter, a vision-based fixed-time control is proposed for an unmanned aerial vehicle (UAV) with actuator saturation to track an uncooperative target. The fixed-time control is designed in backstepping framework. The relative states between UAV and the target are not directly measured, and are estimated by an onboard monocular camera. It is proved that, the closed loop UAV with actuator saturation is capable of reaching the target before a fixed time. The fixed time can be estimated, and it is independent of initial states.

Uncooperative target tracking is among the challenging problems in UAV [1]. Vision-based measurement is one effective method to estimate target states. It is applicable and economic to detect targets in the absence of RADAR. With position-based visual servo, target states can be estimated by sensor fusion in some certain scenarios, e.g., tracking the ground target [2]. Vision-based measurement and control for UAV tracking can be found in [3].

In the aforementioned works, no restrictions on the tracking time are considered; however, in practice, the interception should be completed within a specific time. Some guidance and control schemes were proposed to solve this problem, e.g., sliding mode technique [4], finite-time scheme [5], and fixed-time scheme [6], etc. Inspired by these methods, a finite-time control was proposed to solve the problem of UAV tracking using only vision-based information, where the converging time depends on the initial states [7].

This letter presents a vision-based fixed-time guidance law for the uncooperative aerial target tracking subject to actuator saturation. The design process is in a backstepping-like framework. Main contributions include: 1) using the proposed fixed-time controller, an uncooperative target is intercepted by the UAV in fixed time, and this fixed convergence time can be estimated independent of the initial state; and 2) the model uncertainty can be treated by the proposed fixed-time controller. The proposed result is proved theoretically and supported by numerical simulation.

**Problem formulation:** The following lemmas are necessary to derive the main result.

Lemma 1 [8]: Consider the system  $\dot{x} = f(x)$ ,  $f(0) = 0$ ,  $x \in \mathbb{R}^n$ , where  $f: U \to \mathbb{R}^n$  is continuous in a neighborhood *U* of the origin. If there exists a continuous radially unbounded function  $V(x)$ , and scalars  $\alpha > 0$ ,  $\beta > 0$ ,  $0 < p < 1$ ,  $g > 1$ , such that  $\dot{V}(x) \le -\alpha \dot{V}(x)^p \beta V(x)^g$ , then the origin is fixed-time stable, and the setting time can be estimated by  $T \leq \frac{1}{\alpha(1-p)} + \frac{1}{\beta(g-1)}$ .

Lemma 2 [9]: If  $0 \le p = \frac{p_1}{p_2} \le 1$ , where  $p_1 > 0$ ,  $p_2 > 0$  are positive

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*x*<sup>*p*</sup> − *y*<sup>*p*</sup> | ≤ 2<sup>1−*p*</sup> |*x* − *y*|<sup>*p*</sup>.

Lemma 3 [10]: For  $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$ , if *c* and *d* are positive real numbers, and  $\mu(x, y) > 0$  is a real-valued function, then,

$$
|x|^c |y|^d \le \frac{c}{c+d} \mu(x, y) |x|^{c+d} + \frac{d}{c+d} \mu^{-\frac{c}{d}}(x, y) |y|^{c+d}.
$$
 (1)

Lemma 4 [4]: If  $x_i \in \mathbb{R}$ ,  $i = 1, 2, ..., n$ , and  $0 < p \le 1$ , then,

$$
\left(\sum_{i=1}^{n}|x_{i}|\right)^{p} \le \sum_{i=1}^{n}|x_{i}|^{p} \le n^{1-p}\left(\sum_{i=1}^{n}|x_{i}|\right)^{p}.
$$
 (2)

The motion of the aerial target is described by

$$
\ddot{P}_{Ta}(t) = a_{Ta}(t), \ \dot{P}_{Ta}(0) = V_{Ta_0}, \ P_T(0) = P_{Ta_0}
$$
 (3)

where  $P_{Ta} = [x_{Ta}(t), y_{Ta}(t), z_{Ta}(t)]^T$  and  $a_{Ta} = [a_{Ta_x}(t), a_{Ta_y}(t),$  $a_{Ta_z}(t)$ <sup>T</sup> are the position and acceleration in north-east-down (NED) frame, respectively, as shown by Fig. 1.



Fig. 1. Relative position of the target and the follower.

The motion of the follower UAV is described by

 $\ddot{P}_F(t) = \text{sat}(a_F(t)), \ \dot{P}_F(0) = V_{F_0}, \ P_F(0) = P_{F_0}$ (4) where  $P_F = [x_F(t), y_F(t), z_F(t)]^T$  denotes the follower position, and  $a_F = [a_{F_x}(t), a_{F_y}(t), a_{F_z}(t)]^T$  is the follower acceleration in NED frame (Fig. 1); and sat( $\cdot$ ) is the saturation function.

The relative position between the target and the follower is

$$
R(t) = P_{Ta}(t) - P_F(t)
$$
\n<sup>(5)</sup>

and its dynamics can be given by

$$
\ddot{R}(t) = a_{Ta}(t) - \text{sat}(a_F(t)), \ \dot{R}(0) = V_0, \ R(0) = R_0.
$$
 (6)

The *objective* of this letter is to design a saturated fixed-time controller to stabilize the relative position based on vision measurement.

optical axis is parallel to the follower  $x_B$  axis, as shown in Fig. 2. According to pinhole camera model, the range  $|R|$  satisfies **Main results:** The vision-based measurement originates from [3]. The vision information is measured by an on-board camera, and its

$$
b_I|R| = b\sqrt{f^2 + y_I^2 + z_I^2} \tag{7}
$$

where  $b_I$  is the width of target in the image plane, and  $b$  is the actual width of the target. It is supposed that  $b$  is uncertain. The relative dynamics can be expressed by

$$
\ddot{r}(t) = \frac{a_T(t) - a_F(t)}{b}, \ \dot{r}(0) = \frac{V_0}{b}, \ r(0) = \frac{R_0}{b}
$$
 (8)

where  $r = [r_1 \ r_2 \ r_3]^T = R(t)/b$  is the scaled relative position.



Fig. 2. Vision-based measurement.

According to (8), the relative motion dynamics are given by  $\dot{x}_1 = x_2, \quad \dot{x}_2 = (1/b)\text{sat}(u) + d$  (9)

where  $x_1 = [r_1, r_2, r_3]^T$  is the relative position;  $x_2 = [r_1, r_2, r_3]^T$  is the relative velocity;  $b_{\text{min}} \le b \le b_{\text{max}}$  is the uncertain target width;  $d = [d_1, d_2, d_3]^T$  is the bounded uncertain target acceleration, and its bound is  $d_{\text{max}} = [d_{1\text{max}}, d_{2\text{max}}, d_{3\text{max}}]^T$ . The saturation function is defined by:  $\text{sat}(u) = \lambda(u)u$ , where  $\lambda(\mu) = \text{diag}\{\lambda(\mu_1), \lambda(\mu_2), \lambda(\mu_3)\},$ and  $\lambda(\mu_i)$  satisfy the relative velocity;  $b_{\text{min}} \le b \le b_{\text{max}}$  is the uncertain target width;

$$
\lambda(u_i) = \begin{cases} 1, & \text{if } |u_i| < |u_{i_{\text{max}}}| \\ \text{sgn}(u_i)u_{i_{\text{max}}}/u_i, & \text{if } |u_i| \ge |u_{i_{\text{max}}}| \end{cases}
$$
(10)

where  $u_{i max}$  denotes the maximum control input of the *i* th component of the control vector, and  $\rho < \lambda(\mu_i) \leq 1$ .

The proposed fixed-time control is designed by

$$
u = -k_1 \xi^{\frac{2}{q}-1} - k_2 \xi^{\frac{1}{q}+\frac{1}{p}-1} - k_3 x_1^{\frac{1}{p}+\frac{1}{q}-1} \tag{11}
$$

*k*<sub>1</sub>, *k*<sub>2</sub>, *k*<sub>3</sub> are feedback gains;  $\xi = x_2^q$  $\frac{q}{2} - x_2^* q$ ,  $x_2^* = -\alpha_1 x_1^{\frac{1}{q}}$  $p = \frac{p_1}{p_2} \in (0, 1), q = \frac{q_1}{q_2} \in (1, 2)$  with odd positive numbers  $p_1, p_2, p_3$ *q*<sub>1</sub>, *q*<sub>2</sub>;  $x_2^q$  $^{q}_{2} = [r_{1}^{q}]$  $^{q}_{1}, \dot{r}^{q}_{2}$  $^{q}_{2}$ ,  $\dot{r}^{q}_{3}$  $\mathcal{L}_{3}^{q}$ ]<sup>T</sup>,  $x_{1}^{\frac{1}{q}} = [r_{1}^{\frac{1}{q}}, r_{2}^{\frac{1}{q}}, r_{3}^{\frac{1}{q}}]^{T}$ ,  $\xi^{\frac{2}{q}-1} = [\xi_{1}^{\frac{2}{q}-1}]$  $\frac{2}{q}-1$ ,  $\xi_2^{\frac{2}{q}-1}$  $\frac{q}{2}$ ,  $\xi_3^{\frac{2}{q}-1}$  $\int_{3}^{\frac{q}{q}-1}$ ]<sup>T</sup> where  $k_1$ ,  $k_2$ ,  $k_3$  are feedback gains;  $\xi = x_2^q - x_2^{q}$ ,  $x_2^* = -\alpha_1 x_1^q$ , and with odd positive numbers  $x_2^q = [r_1^q, r_2^q, r_3^q]^T$ ,  $x_1^q = [r_1^q, r_2^q, r_3^q]^T$ , .

 $V = V_0 + V_1$ , where  $V_0 = \frac{1}{2} x_1^T x_1$  and  $V_1 = \sum_{i=1}^3 W_i$ , To prove the fixed-time stability, define Lyapunov candidate by

$$
W_i = \frac{1}{(2 - (1/q))\alpha_1^{1+q}} \int_{x_2^*}^{x_2} (s^q - x_2^{*q})^{2-\frac{1}{q}}
$$
(12)

and  $x_2^* = [r_1^*, r_2^*, r_3^*]^T$  denote the virtual control that is designed by  $x_2^* = -\alpha_1 x_1^{\bar{q}}$ . The derivative of  $V_0$  can be calculated by  $\dot{V}_0 = x_1^T x_2 =$  $\sum_{i=1}^{3} r_i \dot{r}_i$ , where (according to Lemma 2)

$$
r_i \dot{r}_i = r_1(\dot{r}_i - \dot{r}_i^*) + r_i \dot{r}_i^* \le |r_i| |\dot{r}_i - \dot{r}_i^*| - \alpha_1 |r_i|^{1 + (1/q)}
$$
  
 
$$
\le -\alpha_1 |r_i|^{1 + (1/q)} + 2^{1 - (1/q)} |r_i| |\xi_i|^{1/q}. \tag{13}
$$

It then follows from Lemma 3 that:

$$
\dot{V}_0 \leq - \left(\alpha_1 - \frac{2^{1-\frac{1}{q}}q\gamma_1}{1+q}\right)\sum_{i=1}^3|r_i|^{1+\frac{1}{q}} + \frac{2^{1-\frac{1}{q}}\gamma_1^{-q}}{1+q}\sum_{i=1}^3|\xi_i|^{1+\frac{1}{q}}
$$

where  $\gamma_1$  is a positive constant.

The derivative of  $V_1$  is calculated by  $\dot{V}_1 = \sum_{i=1}^3 \dot{W}_i$ , where

$$
\dot{W}_i = \frac{|\xi_i|^{2-\frac{1}{q}}}{(2 - (1/q))\alpha_1^{1+q}} \ddot{r}_i + \frac{\dot{r}_i}{\alpha_1} \int_{x_2^*}^{x_2} \left(s^q - x_2^{*q}\right)^{1-\frac{1}{q}} ds. \tag{14}
$$

With Lemma 3, the second term satisfies

$$
\frac{\dot{r}_i}{\alpha_1} \int_{x_2^*}^{x_2} \left( s^q - x_2^{*q} \right)^{1 - \frac{1}{q}} ds \le \frac{2^{1 - \frac{1}{q}}}{\alpha_1} |\dot{r}_i| |\xi_i|
$$
\n
$$
\le \frac{2^{1 - \frac{1}{q}}}{\alpha_1 (1 + q)} \gamma_2 |r_i|^{1 + \frac{1}{q}} + \frac{2^{1 - \frac{1}{q}} q}{\alpha_1 (1 + q)} \gamma_2^{-\frac{1}{q}} |\xi_i|^{1 + \frac{1}{q}} \tag{15}
$$

where  $\gamma_2 > 0$ . It follows from (14) and (15) that:

$$
\dot{V} \le -\left(\alpha_1 - \frac{2^{1-\frac{1}{q}}q}{1+q}\gamma_1 - \frac{2^{1-\frac{1}{q}}}{\alpha_1(1+q)}\gamma_2\right) \sum_{i=1}^3 |r_i|^{1+\frac{1}{q}}
$$

$$
+ \left(\frac{2^{1-\frac{1}{q}}}{1+q}\gamma_1^{-q} + \frac{2^{2-\frac{2}{q}}}{\alpha_1} + \frac{2^{1-\frac{1}{q}}}{\alpha_1(1+q)}\gamma_2^{-\frac{1}{q}}\right) \sum_{i=1}^3 |\xi_i|^{1+\frac{1}{q}}
$$

$$
+ \frac{1}{(2-(1/q))\alpha_1^{1+q}} \sum_{i=1}^3 |\xi_i|^{2-(1/q)} \ddot{r_i}.
$$
(16)

Substituting (9), (11) and  $\rho < \lambda(u_i) \le 1$  into (16) yields

$$
\dot{V} \le -h_1 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{q}} - h_2 \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{q}} - h_3 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{p}}
$$

$$
-h_4 \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{p}} + h_d \sum_{i=1}^{3} |\xi_i|^{2 - \frac{1}{q}}
$$
(17)

where

$$
h_1 = \alpha_1 - \frac{2^{1-(1/q)}q}{1+q} \gamma_1 - \frac{2^{1-(1/q)}}{\alpha_1(1+q)} \gamma_2
$$
  
\n
$$
h_2 = \frac{\rho k_1}{b(2-(1/q))\alpha_1^{1+q}} - \frac{2^{1-\frac{1}{q}}\gamma_1^{-q}}{1+q} - \frac{2^{2-\frac{2}{q}}}{\alpha_1} - \frac{2^{1-\frac{1}{q}}\gamma_2^{-\frac{1}{q}}}{\alpha_1(1+q)}
$$
  
\n
$$
h_3 = \frac{\rho k_3((1/p)+(1/q)-1)}{b(2-(1/q))\alpha_1^{1+q}(1+(1/p))}
$$
  
\n
$$
h_4 = \frac{\rho k_2}{b(2-(1/q))\alpha_1^{1+q}} + \frac{\rho k_3(2-(1/q))}{b(2-(1/q))\alpha_1^{1+q}(1+(1/p))}
$$
  
\n
$$
h_d = \frac{1}{(2-(1/q))\alpha_1^{1+q}} ||d_{\text{max}}|| \qquad (18)
$$

1−(1/*q*)

<sup>1</sup>−(1/*q*)*q*

and they can be set positive by control parameters  $\alpha_1$ ,  $k_1$ ,  $k_2$  and  $k_3$ . It then follows from (17) that:

$$
\dot{V} \le -h_1 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{q}} - h_2 (1 - c_1) \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{q}} - h_3 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{p}}
$$

$$
-h_4 (1 - c_2) \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{p}} + h_d \sum_{i=1}^{3} |\xi_i|^{2 - \frac{1}{q}}
$$

$$
-c_1 h_2 \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{q}} - c_2 h_4 \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{p}}
$$
(19)

 $c_1, c_2 \in (0, 1)$  are positive constants. Define  $\Omega_1 = \{\xi_i :$  $c_1 h_2 |\xi_i|^{\frac{2}{q}-1} + c_2 h_4 |\xi_i|^{\frac{1}{p}+\frac{1}{q}-1} \leq h_d$ . If  $\xi_i \notin \Omega_1$ | | where  $c_1, c_2 \in (0, 1)$  are positive constants. Define . If  $\xi_i \notin \Omega_1$ , then

$$
\dot{V} \le -h_1 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{q}} - h_2 (1 - c_1) \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{q}} - h_3 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{p}}
$$

$$
-h_4 (1 - c_2) \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{p}}.
$$
(20)

According to Lemma 2 and (12), it holds that

$$
V_0 \le \frac{1}{2} \sum_{i=1}^{3} |r_i|^2 \quad \text{and} \quad V_1 \le \sum_{i=1}^{3} W_i \tag{21}
$$

 $W_i \leq \frac{1}{\sqrt{2} \cdot (1 + \epsilon)}$ where  $W_i \leq \frac{1}{(2-(1/q))\alpha_1^{1+q}} |x_2 - x_2^*||\xi|^{2-(1/q)}$ . In another aspect, according to Lemma 2,

$$
|x_2 - x_2^*| \le 2^{1 - (1/q)} |\xi|^{1/q}.
$$
 (22)

It then follows form (21) and (22) that  $V_1 \le \sum_{i=1}^3 l_i |\xi|^2$ , where  $l_1 = \frac{2^{1-(1/q)}}{(2-(1/q))^{6}}$  $\frac{2^{1-(1/q)} (2-(1/q))\alpha_1^{1+q}}{(2-(1/q))\alpha_1^{1+q}}$ . Substituting (21) into (20) yields

$$
\dot{V} \le -2^{\frac{1}{2}\left(1+\frac{1}{q}\right)} h_1 V_0^{\frac{1}{2}\left(1+\frac{1}{q}\right)} - l_1^{-\frac{1}{2}\left(1+\frac{1}{q}\right)} h_2 (1-c_1) V_1^{\frac{1}{2}\left(1+\frac{1}{q}\right)} \n-2^{\frac{1}{2}\left(1+\frac{1}{p}\right)} h_3 V_0^{\frac{1}{2}\left(1+\frac{1}{p}\right)} - l_1^{-\frac{1}{2}\left(1+\frac{1}{p}\right)} h_4 (1-c_2) V_1^{\frac{1}{2}\left(1+\frac{1}{p}\right)} \n\le -\tilde{h}_1 V^{\frac{1}{2}\left(1+\frac{1}{q}\right)} - \tilde{h}_2 V^{\frac{1}{2}\left(1+\frac{1}{p}\right)}
$$
\n(23)

 $\tilde{h}_1 = \min \left\{ 2^{(1+(1/q))/2} h_1, l_1^{-(1+(1/q))/2} \right\}$ where  $\tilde{h}_1 = \min \Big\{ 2^{(1+(1/q))/2} h_1, I_1^{-(1+(1/q))/2} h_2 (1-c_1) \Big\}$ , and  $\tilde{h}_2 =$  $\min\left\{2h_3, 2^{1-(1+(1/p))/2} l_1^{-(1+(1/p))/2}\right\}$  $\binom{-(1+(1/p))/2}{1}$ *h*<sub>4</sub>(1−*c*<sub>2</sub>). Based on Lemma 1, if  $\xi \notin \Omega_1$ , then  $\xi$  converges to  $\Omega_1$  in fixed time

$$
T_1 \le \frac{2}{\tilde{h}_1 (1 - (1/q))} + \frac{2}{\tilde{h}_2 ((1/p) - 1)}.
$$
  
Define  $\Omega_2 = \{\xi_i : |\xi_i| \le \Delta_i\}$ , where  $\Delta_i = \min \left\{ \left( \frac{h_d}{c_1 h_2} \right)^{\frac{1}{(2/q)-1}}, \left( \frac{h_d}{c_2 h_4} \right)^{\frac{1}{(1/q)+(1/p)-1}} \right\}$ , such that  $\Omega_1 \subseteq \Omega_2$ , and  $\xi_i$  converges to  $\Omega_2$  within  $T_1$ . If  $|\xi_i| \le \Delta_i$ ,  $\dot{V}$  is calculated by

$$
\dot{V} \le -h_1 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{q}} - h_2 \sum_{i=1}^{3} |\xi_i|^{1 + \frac{1}{q}} - h_3 \sum_{i=1}^{3} |r_i|^{1 + \frac{1}{p}}
$$

$$
-h_4 \sum_{i=1}^3 |\xi_i|^{1+\frac{1}{p}} + h_d \sum_{i=1}^3 \Delta_i^{2-\frac{1}{q}}
$$
 (24)

$$
\leq -h_1(1-c_3) \sum_{i=1}^3 |r_i|^{1+\frac{1}{q}} - h_2 \sum_{i=1}^3 |\xi_i|^{1+\frac{1}{q}}
$$
  

$$
-h_3(1-c_4) \sum_{i=1}^3 |r_i|^{1+\frac{1}{p}} - h_4 \sum_{i=1}^3 |\xi_i|^{1+\frac{1}{p}}
$$
  

$$
+ h_d \sum_{i=1}^3 \Delta_i^{2-\frac{1}{q}} - c_3 \sum_{i=1}^3 |r_i|^{1+\frac{1}{q}} - c_4 \sum_{i=1}^3 |r_i|^{1+\frac{1}{p}}
$$
(25)

where  $c_3$ ,  $c_4 \in (0, 1)$  are positive constants. Define

$$
\Omega_3 = \left\{ c_3 |r_i|^{1 + (1/q)} + c_4 |r_i|^{1 + (1/p)} \le h_d \Delta_i^{2 - (1/q)} \right\}.
$$
 (26)

Based on Lemma 1, if  $r_i \notin \Omega_3$ , then  $r_i$  converge to  $\Omega_3$  in fixed time

$$
T_2 \le \frac{2}{\tilde{h}_3(1 - (1/q))} + \frac{2}{\tilde{h}_4((1/p) - 1)}
$$

 $\tilde{h}_3$  = min $\left\{2^{(1+(1/q))/2}h_1(1-c_3), l_1^{-(1+(1/q))/2}\right\}$ where  $\tilde{h}_3 = \min \Big\{ 2^{(1+(1/q))/2} h_1(1-c_3), l_1^{-(1+(1/q))/2} h_2 \Big\},\$  $\tilde{h}_4$  =  $\min\left\{2h_3(1-c_4), 2^{1-(1+(1/p))/2}l_1^{-(1+(1/p))/2}\right\}$  $\begin{bmatrix}-(1+(1/p))/2 & h_4\end{bmatrix}$ . The relative position converges to  $\Omega_3$  before a fixed time  $T_1 + T_2$ .

Remark 1: In this paper, target-tracking with vision measurement is considered, and it is assumed that initial states can not be infinitely large; otherwise the target cannot be measured by the camera.

Remark 2: Although the final time depends on the upper bound of saturation, it is independent of initial states in the camera vision. The proposed controller is still regarded as "fixed-time controller".

**Numerical example:** In this example, suppose  $f = 12$  mm and  $b = 0.7$  m. The follower acceleration is constrained by  $|u_i| \le 4$  m/s<sup>2</sup>,  $i = 1, 2, 3$ . Suppose that the target acceleration  $a_T(t) = \hat{a}_T(t) + \Delta a_T(t)$ , where  $\hat{a}_T(t)$  is the target maneuver, which is set to  $[1, -0.2, 0.2]$ <sup>T</sup> in  $0 \sim 5$  s,  $[-0.2, 1.4, -0.5]^T$  in  $18 \sim 20$  s,  $[0, -1.4, 0]^T$  in  $28 \sim 30$  s, and  $\Delta a_T(t)$  is . 0.05 [sin( $\pi t/60$ ), sin( $\pi t/60$ ), sin( $\pi t/60$ )]<sup>T</sup> m/s<sup>2</sup>. The target maximum acceleration is  $d_{\text{max}} = [2, 2, 2]^T \text{ m/s}^2$ . Suppose that the follower, e.g., the initial relative positions are given by  $[50, 30, 2]$ , [40, −40, 1] and [30, 20, −4] in three different scenarios. Control parameters are set to  $q = 11/7$ ,  $p = 7/9$ ,  $k_1 = 10$ ,  $k_2 = 5$ ,  $k_3 = 5$ , and  $\alpha_1 = 0.14$ . The parameter  $\rho$  is conservatively set to 0.01 such that the less than  $75.25$  s. camera covers the semi-sphere area with radius 50 m in front of the fixed-time region of attraction contains all states covered by a practical camera. It is calculated that the expected fixed converging time is

It can be seen from simulation results that the follower reaches the target in fixed time. The relative positions are exhibited in Fig. 3, where transient processes are all completed within the expected fixed



Fig. 3. The relative position in case of different initial values: tracking errors of the proposed fixed-time controller converges before the expected 75.25 s.

time 75.25 s in case of different initial value. Besides, the traditional PD control is applied for comparison. It is observed that the proposed fixed-time tracking algorithm provides better disturbance rejection and convergence rate. The control inputs are shown in Fig. 4, and they are bounded within their constraints.



Fig. 4. The control input: they are always within their constraints.

**Conclusion:** A vision-based fixed-time control is proposed to solve the uncooperative UAV target-tracking subject to uncertainties. It is proved that the tracking error converges to a small neighborhood of the origin in a fixed time. The fixed time can be estimated explicitly.

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