Letter

A Looped Functional Method to Design State Feedback Controllers for Lurie Networked Control Systems

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Dear Editor,

This letter deals with the stabilization of Lurie networked control systems with network-induced delays (NID). By constructing a two-sided looped Lyapunov functional, a sufficient condition is derived to ensure the absolute stability of the resultant closed-loop system under a state feedback controller. Then, based on this condition, a cone complementary linearisation (CCL) iterative algorithm is presented to design state feedback controller. It is shown via a numerical example that the proposed method can deliver less conservative results as well as fewer iterations if compared with existing ones.

With the rapid development of computer science and communication technology, networked control systems (NCSs) have gained wide attention due to advantages such as low cost, simple diagnosis and maintenance, flexibility of operation. In an NCS, data exchange among the devices is implemented through a shared network medium. Consequently, NID like transmission delays in the S-C and C-A channels cannot be avoided. These delays may result in the degradation of system performance and even destabilize the NCS [1]–[3]. Therefore, research interests on this topic are usually focused on designing a certain controller to ensure that the NCS is stable if network-induced delays vary within a proper range [4].

For example, in [5], a stability condition is presented with an assumption that the upper bound of NID is no more than the sampling period. When the upper bound of NID is larger than the sampling period, a novel model is proposed in [6] based on an input delay approach. In [7], by introducing some free matrices to reflect the relationship between NID and its upper bound, less conservative stability conditions for NCSs are obtained. However, the literature aforementioned above focuses on the linear NCSs rather than nonlinear ones that are more practical.

In [8], a kind of nonlinear NCS, namely Lurie NCS, is investigated based on the input delay approach and its stability is analyzed, in which some useful terms are ignored in the derivative of the chosen Lyapunov functional. By retaining those ignored terms and employing an improved free-weighting matrix method, less conservative conditions than [8] are proposed in [9]. Further improvement can be found in [10]. However, those results aforementioned above do not take NID into account, leading to limited application scopes of them.

Usually, a stability criterion for a closed-loop NCS is a set of nonlinear matrix inequalities since the control gain is unknown. To solve the control gain, there are three methods available. The first one is called a parameter-tuning method [6]. By setting two matrix variables to be linear with a tuning parameter, the nonlinear matrix inequalities are turned into linear matrix inequalities (LMIs) with tuning parameters. Then suitable control gains can be calculated if the LMIs are feasible by tuning those parameters. The second one is based on some skills to enlarge a nonlinear term such that the nonlinear matrix inequalities are linearized [8]. Nevertheless, it is well known that these two methods just produce conservative results. The third method is the CCL iterative algorithm [11], by which control gains can be designed with less conservativeness after finite iterations. Whereas, the stopping conditions involved in the iterative algorithm [11] are somewhat strict. By revising the stopping condition, an improved iterative algorithm is developed in [7]. This method is employed to design suitable controllers for Lurie NCSs in [9] and [10].

In this letter, a looped functional method is used to deal with the stabilization of a Lurie NCS with NID. By constructing a two-sided looped functional, a novel stability criterion is presented for the closed-loop Lurie NCS. Then, by introducing a CCL algorithm, a stabilizing state-feedback controller can be designed. It is shown through a numerical example that the proposed method can provide less conservative results and the number of iterations is reduced.

Notation: Throughout the letter, $\mathbb{S}^n(\mathbb{S}^n_+)$ is the set of $n\times n$ real (positive-definite) symmetric matrices; Symmetric terms in a symmetric matrix are represented by the symbol "$*$" and $\text{He}\{U\} = U + U^T$.

Problem statement: Consider the following system:

$$\begin{cases}\dot{x}(t) = \mathcal{A}x(t)+\mathcal{B}u(t)+\mathcal{D}\sigma(t)\\ z(t) = Cx(t)\\ \sigma(t) = -\varpi(t,z(t))\end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ and $u(t)\in\mathbb{R}^m$ are state vector and input vector, respectively. $z(t)\in\mathbb{R}^p$ is the measured output. $\varpi(t,z(t))$ is a piecewise continuous nonlinear function that is global Lipschitz in $z(t)$, $\varpi(t,0)=0$, and satisfies

$$\varpi^T(t,z(t))[\varpi(t,z(t))-\Gamma z(t)]\le 0 \tag{2}$$

for $\forall t\ge 0$ and $\forall z(t)\in\mathbb{R}^p$, where Γ is a real diagonal matrix. The set of all functions that satisfy the sector condition above is denoted by $F[0,\Gamma]$.

Under Assumption 1 presented in [6], the digital control law for networked control systems may be represented as

$$u(t)=\mathcal{K}x(t_k),\quad t\in[t_k,t_{k+1}). \tag{3}$$

Then, the closed-loop system can be represented as

$$\begin{cases}\dot{x}(t)=\mathcal{A}x(t)+\mathcal{B}\mathcal{K}x(t_k)+\mathcal{D}\sigma(t), t\in[t_k,t_{k+1})\\ z(t)=Cx(t)\\ \sigma(t)=-\varpi(t,z(t))\end{cases} \tag{4}$$

where $\{t_0,t_1,t_2,\dots,t_k,\dots\}$ is a time sequence satisfying $t_{k+1}-t_k = h_k\in[\eta,\overline{\eta}]$.

Remark 1: By defining $\tau(t)=t-t_k$, system (4) can be represented as the following system with time-delay:

$$\dot{x}(t)=\mathcal{A}x(t)+\mathcal{B}\mathcal{K}x(t-\tau(t))+\mathcal{D}\sigma(t),\quad t\in[t_k,t_{k+1}) \tag{5}$$

where $\tau(t)$ is the network-induced delay with $\tau(t)\le\overline{\eta}$. It is observed that $\dot{\tau}(t)=1$ for $t\ne t_k$, yet this condition is ignored in [8]–[10], leading to conservative stability conditions.

In the sequel, we introduce the following lemma, which is indispensable in deriving the main results.

Lemma 1 [12]: Let x be a differentiable signal in $[\alpha,\beta]\to\mathbb{R}^n$. For any matrices $\mathcal{R}\in\mathbb{S}^n_+$, and $\mathcal{N}_1,\mathcal{N}_2\in\mathbb{R}^{m\times n}$, the following inequality holds:

$$-\int_\alpha^\beta \dot{x}^T(s)\mathcal{R}\dot{x}(s)ds \le \bar{\xi}^T\{\text{He}\{\mathcal{N}_1(\bar{e}_1-\bar{e}_2)+\mathcal{N}_2(\bar{e}_1+\bar{e}_2-2\bar{e}_3)\}+(\beta-\alpha)\times(\mathcal{N}_1\mathcal{R}^{-1}\mathcal{N}_1^T+\frac{1}{3}\mathcal{N}_2\mathcal{R}^{-1}\mathcal{N}_2^T)\}\bar{\xi} \tag{6}$$

where $\bar{\xi}\in\mathbb{R}^m$ and $\bar{e}_i (i=1,2,3)$ are entry matrices such that $x(\beta)=\bar{e}_1\bar{\xi}, x(\alpha)=\bar{e}_2\bar{\xi}$, $\frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds=\bar{e}_3\bar{\xi}$.

Main results: For simplifying the description of vectors and matrices, we denote

$$\xi(t)=col\Big\{x(t),x(t_k),x(t_{k+1}),\int_{t_k}^t x(s)ds,\int_t^{t_{k+1}}x(s)ds,\int_{t_k}^t\frac{x(s)}{t-t_k}ds,\int_t^{t_{k+1}}\frac{x(s)}{t_{k+1}-t}ds,\sigma(t)\Big\}$$

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and e_i, $i \in \{1,2,\ldots,8\}$ are row-block vectors such that $x(t) = e_1\xi(t)$, $x(t_k) = e_2\xi(t), \ldots, \sigma(t) = e_8\xi(t)$.

First, consider Lurie NCS (4) with a given $\mathcal{K}$. By using looped-functional method, the following stability condition is obtained.

Theorem 1: Given scalars $\overline{\eta}$ and $\underline{\eta}$ with $\overline{\eta} \ge \underline{\eta} \ge 0$, and the controller gain $\mathcal{K}$, the closed-loop system (4) with nonlinear function $\varpi(\cdot) \in F[0,\Gamma]$ is absolutely stable if there exist $\mathcal{P} \in \mathbb{S}_+^n$, $Q_1, \in \mathbb{R}^{n\times n}$, $Q_2, \in \mathbb{S}^{3n}$, $\mathcal{R}_1,\mathcal{R}_2 \in \mathbb{S}^n$, $Y_1,Y_2 \in \mathbb{R}^{(7n+p)\times n}$, $M,N \in \mathbb{R}^{3n\times 2n}$, such that, for $h_k \in [\underline{\eta},\overline{\eta}]$, (7) and (8) hold

$$\bar{\Phi}_1(h_k) = \begin{bmatrix} \Phi_0 + h_k\Phi_1 & \sqrt{h_k}\Pi_4^T M & \sqrt{h_k}\Pi_1^T\mathcal{R}_1 \\ * & -\bar{R}_2 & 0 \\ * & * & -\mathcal{R}_1 \end{bmatrix} < 0 \tag{7}$$

$$\bar{\Phi}_2(h_k) = \begin{bmatrix} \Phi_0 + h_k\Phi_2 & \sqrt{h_k}\Pi_3^T N & \sqrt{h_k}\Pi_1^T\mathcal{R}_2 \\ * & -\bar{R}_1 & 0 \\ * & * & -\mathcal{R}_2 \end{bmatrix} < 0 \tag{8}$$

where

$$\begin{aligned}
\Phi_0 &= \mathrm{He}\{e_1^T\mathcal{P}\Pi_1 - e_1^T Q_1 e_4 + e_5^T Q_1 e_1\} - \mathrm{He}\{Y_1 e_4 + Y_2 e_5 \\
&\quad + e_8^T e_8 + e_8^T\Gamma C e_1\} + \mathrm{He}\{\Pi_3^T N\Pi_5 + \Pi_4^T M\Pi_6\} \\
\Phi_1 &= \mathrm{He}\{Y_1 e_6\} - \Pi_2^T Q_2\Pi_2, \quad \Phi_2 = \mathrm{He}\{Y_2 e_7\} + \Pi_2^T Q_2\Pi_2 \\
\Pi_1 &= \mathcal{A}e_1 + \mathcal{B}\mathcal{K}e_2 + \mathcal{D}e_8, \quad \Pi_2 = \begin{bmatrix} e_2^T & e_3^T & e_4^T + e_5^T \end{bmatrix}^T \\
\Pi_3 &= \begin{bmatrix} e_3^T & e_1^T & e_7^T \end{bmatrix}^T, \quad \Pi_4 = \begin{bmatrix} e_1^T & e_2^T & e_6^T \end{bmatrix}^T \\
\Pi_5 &= \begin{bmatrix} e_3 - e_1 \\ e_3 + e_1 - 2e_7 \end{bmatrix}, \quad \Pi_6 = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \end{bmatrix} \\
\bar{R}_1 &= \mathrm{diag}\{\mathcal{R}_1, 3\mathcal{R}_1\}, \quad \bar{R}_2 = \mathrm{diag}\{\mathcal{R}_2, 3\mathcal{R}_2\}.
\end{aligned}$$

Proof: Choose a Lyapunov functional candidate as

$$V(x_t) = V_0(t) + \mathcal{W}(t), \ \ t \in [t_k, t_{k+1}) \tag{9}$$

where $V_0(t) = x^T(t)\mathcal{P}x(t)$ and

$$\begin{aligned}
\mathcal{W}(t) = \ & 2\int_t^{t_{k+1}} x^T(s)ds Q_1 \int_{t_k}^t x(s)ds \\
& + (t_{k+1} - t)(t - t_k)\zeta^T Q_2\zeta \\
& - (t - t_k)\int_t^{t_{k+1}} \dot{x}^T(s)\mathcal{R}_1\dot{x}(s)ds \\
& + (t_{k+1} - t)\int_{t_k}^t \dot{x}^T(s)\mathcal{R}_2\dot{x}(s)ds
\end{aligned}$$

with $\mathcal{P} \in \mathbb{S}_+^n$, $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{S}^{3n}$, $\mathcal{R}_1,\mathcal{R}_2 \in \mathbb{S}^n$ to be determined and $\zeta = col\left\{x(t_k), x(t_{k+1}), \int_{t_k}^{t_{k+1}} x(s)ds\right\}$.

Calculating the derivative of $V(x_t)$ yields

$$\begin{aligned}
\dot{V}(t) = \ & 2x^T(t)\mathcal{P}\dot{x}(t) - 2x^T(t)Q_1\int_{t_k}^t x(s)ds \\
& + 2\int_t^{t_{k+1}} x^T(s)dsQ_1 x(t) + (t_{k+1} - t)\zeta^T Q_2\zeta \\
& - (t - t_k)\zeta^T Q_2\zeta + (t - t_k)\dot{x}^T(t)\mathcal{R}_1\dot{x}(t) \\
& + (t_{k+1} - t)\dot{x}^T(t)\mathcal{R}_2\dot{x}(t) - \int_t^{t_{k+1}} \dot{x}^T(s)\mathcal{R}_1\dot{x}(s)ds \\
& - \int_{t_k}^t \dot{x}^T(s)\mathcal{R}_2\dot{x}(s)ds.
\end{aligned} \tag{10}$$

Denote $\vartheta_1 = -\int_t^{t_{k+1}} \dot{x}^T(s)\mathcal{R}_1\dot{x}(s)ds$ and $\vartheta_2 = -\int_{t_k}^t \dot{x}^T(s)\mathcal{R}_2\dot{x}(s)ds$. It follows from Lemma 1 that:

$$\vartheta_1 \le \xi^T(t)[(t_{k+1} - t)\Pi_3^T N\bar{R}_1^{-1}N^T\Pi_3 + \mathrm{He}\{\Pi_3^T N\Pi_5\}]\xi(t) \tag{11}$$

$$\vartheta_2 \le \xi^T(t)[(t - t_k)\Pi_4^T M\bar{R}_2^{-1}M^T\Pi_4 + \mathrm{He}\{\Pi_4^T M\Pi_6\}]\xi(t) \tag{12}$$

for any matrices $M,N \in \mathbb{R}^{3n\times 2n}$.

For any matrices $Y_1,Y_2 \in \mathbb{R}^{(7n+p)\times n}$, the following zero equations are true:

$$0 = 2\xi^T(t)Y_1((t - t_k)e_6 - e_4)\xi(t) \tag{13}$$

$$0 = 2\xi^T(t)Y_2((t_{k+1} - t)e_7 - e_5)\xi(t). \tag{14}$$

It follows from (2) that:

$$0 \le -2\xi^T(t)[e_8^T e_8 + e_8^T\Gamma C e_1]\xi(t). \tag{15}$$

Adding the right sides of (13)–(15) to (10) and applying (11) and (12) yield

$$\dot{V}(x_t) \le \xi^T(t)\left[\frac{t - t_k}{h_k}\bar{\Phi}_1(h_k) + \frac{t_{k+1} - t}{h_k}\bar{\Phi}_2(h_k)\right]\xi(t) \tag{16}$$

where

$$\begin{aligned}
\bar{\Phi}_1(h_k) &= \Phi_0 + h_k\Phi_1 + h_k\Pi_1^T\mathcal{R}_1\Pi_1 + h_k\Pi_4^T M\bar{R}_2^{-1}M^T\Pi_4 \\
\bar{\Phi}_2(h_k) &= \Phi_0 + h_k\Phi_2 + h_k\Pi_1^T\mathcal{R}_2\Pi_1 + h_k\Pi_3^T N\bar{R}_1^{-1}N^T\Pi_3.
\end{aligned}$$

Thus, if $\bar{\Phi}_1(h_k) < 0$ and $\bar{\Phi}_2(h_k) < 0$, which are, respectively, equivalent to (7) and (8) in the sense of the Schur complement, $\dot{V}(x_t) < -\varepsilon\|x(t)\|^2$ for a sufficiently small $\varepsilon > 0$. Hence, system (4) is absolutely stable. ■

Remark 2: A two-sided looped functional approach was proposed in [13], which shows great potential in the reduction of conservativeness. However, the derived condition in [13] is difficult to be applied to controller design due to the fact that there are a lot of free matrices coupled with system matrices. Inspired by [13], an improved looped functional, $\mathcal{W}(t)$, is constructed and introduced in the Lyapunov functional (9). As the characteristics of the networked-induced delay was considered in the looped functional, the derived condition is less conservative than [8]–[10].

Next, Theorem 1 is extended to design a stabilizing controller $\mathcal{K}$ for system (4).

Theorem 2: Given scalars $\overline{\eta}$ and $\underline{\eta}$ with $\overline{\eta} \ge \underline{\eta} \ge 0$, the closed-loop system (4) with nonlinear function $\varpi(\cdot) \in F[0,\overline{\Gamma}]$ is absolutely stable if there exist matrices $L \in \mathbb{S}_+^n$, $\hat{Q}_1 \in \mathbb{R}^{n\times n}$, $\hat{Q}_2 \in \mathbb{S}^{3n}$, $\mathcal{Z}_1,\mathcal{Z}_2 \in \mathbb{S}^n$, $\hat{Y}_1,\hat{Y}_2 \in \mathbb{R}^{(7n+p)\times n}$, $\hat{M},\hat{N} \in \mathbb{R}^{3n\times 2n}$, such that, for $h_k \in [\underline{\eta},\overline{\eta}]$, the following matrix inequalities hold:

$$\hat{\Phi}_1(h_k) = \begin{bmatrix} \Psi_0 + h_k\hat{\Psi}_1 & \sqrt{h_k}\Pi_4^T\hat{M} & \sqrt{h_k}\hat{\Pi}_1^T \\ * & -\hat{Z}_2 & 0 \\ * & * & -\mathcal{Z}_1 \end{bmatrix} < 0 \tag{17}$$

$$\hat{\Phi}_2(h_k) = \begin{bmatrix} \Psi_0 + h_k\hat{\Psi}_2 & \sqrt{h_k}\Pi_3^T\hat{N} & \sqrt{h_k}\hat{\Pi}_1{}^T \\ * & -\hat{Z}_1 & 0 \\ * & * & -\mathcal{Z}_2 \end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned}
\hat{\Psi}_0 &= \mathrm{He}\{e_1^T\hat{\Pi}_1 - e_1^T\hat{Q}_1 e_4 + e_5^T\hat{Q}_1 e_1\} - \mathrm{He}\{\hat{Y}_1 e_4 + \hat{Y}_2 e_5 \\
&\quad + e_8^T e_8 + e_8^T\Gamma C L e_1\} + \mathrm{He}\{\Pi_3^T\hat{N}\Pi_5 + \Pi_4^T\hat{M}\Pi_6\} \\
\hat{\Psi}_1 &= \mathrm{He}\{\hat{Y}_1 e_6\} - \Pi_2^T\hat{Q}_2\Pi_2, \quad \hat{\Psi}_2 = \mathrm{He}\{\hat{Y}_2 e_7\} + \Pi_2^T\hat{Q}_2\Pi_2 \\
\hat{\Pi}_1 &= \mathcal{A}Le_1 + \mathcal{B}Ve_2 + \mathcal{D}e_8, \quad \hat{Z}_1 = \mathrm{diag}\{L\mathcal{Z}_1^{-1}L, 3L\mathcal{Z}_1^{-1}L\} \\
\hat{Z}_2 &= \mathrm{diag}\{L\mathcal{Z}_2^{-1}L, 3L\mathcal{Z}_2^{-1}L\}
\end{aligned}$$

with Π_i, $i \in \{2,3,\ldots,6\}$ being defined in Theorem 1. Moreover, the controller gain is obtained by $\mathcal{K} = VL^{-1}$.

Proof: Donate

$$\begin{aligned}
\Lambda_1 &= \mathrm{diag}\{\mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{P}^{-1}, I\} \\
\Lambda_2 &= \mathrm{diag}\{\mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{R}_1^{-1}\}, \quad \Lambda_3 = \mathrm{diag}\{\mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{R}_2^{-1}\} \\
\Lambda_4 &= \mathrm{diag}\{\mathcal{P}^{-1}, \mathcal{P}^{-1}, \mathcal{P}^{-1}\}, \quad \Lambda_5 = \mathrm{diag}\{\mathcal{P}^{-1}, \mathcal{P}^{-1}\}.
\end{aligned}$$

Pre-multiply and post-multiply $\bar{\Phi}_1(h_k)$ by $\mathrm{diag}\{\Lambda_1, \Lambda_2\}$, and $\bar{\Phi}_2(h_k)$ by $\mathrm{diag}\{\Lambda_1, \Lambda_3\}$, respectively. Make the following changes on the variables:

$$\begin{aligned}
& L = \mathcal{P}^{-1}, \quad \hat{Q}_1 = \mathcal{P}^{-1}Q_1\mathcal{P}^{-1}, \quad \hat{Q}_2 = \Lambda_4 Q_2\Lambda_4 \\
& \mathcal{Z}_i = \mathcal{R}_i{}^{-1}, \quad \hat{Y}_i = \Lambda_1 Y_i\mathcal{P}^{-1}, \quad i \in \{1,2\} \\
& \hat{M} = \Lambda_4 M\Lambda_5, \quad \hat{N} = \Lambda_4 N\Lambda_5, \quad V = \mathcal{K}\mathcal{P}^{-1}
\end{aligned}$$

then (17) and (18) are derived. ■

Note that there are nonlinear terms $L\mathcal{Z}_2^{-1}L$ and $L\mathcal{Z}_1^{-1}L$ in (17) and (18). Thus, the conditions given in Theorem 2 cannot be directly implemented by using existing numerical software. The following CCL algorithm is presented to deal with this non-convex problem.

Define two new variables $\mathcal{U}_1$ and $\mathcal{U}_2$ such that $\mathcal{U}_1 \le L\mathcal{Z}_1^{-1}L$ and $\mathcal{U}_2 \le L\mathcal{Z}_2^{-1}L$. Replace the conditions (17) and (18) with

$$\hat{\Phi}_1(h_k)=\begin{bmatrix}\hat{\Psi}_0+h_k\hat{\Psi}_1 & \sqrt{h_k}\Pi_4^T\hat{M} & \sqrt{h_k}\hat{\Pi}_1^T\\ * & -\mathrm{diag}\{\mathcal{U}_2,3\mathcal{U}_2\} & 0\\ * & * & -\mathcal{Z}_1\end{bmatrix}<0 \quad (19)$$

$$\hat{\Phi}_2(h_k)=\begin{bmatrix}\Psi_0+h_k\hat{\Psi}_2 & \sqrt{h_k}\Pi_3^T\hat{N} & \sqrt{h_k}\hat{\Pi}_1^T\\ * & -\mathrm{diag}\{\mathcal{U}_1,3\mathcal{U}_1\} & 0\\ * & * & -\mathcal{Z}_2\end{bmatrix}<0 \quad (20)$$

and

$$\mathcal{U}_i\le L\mathcal{Z}_i^{-1}L,\ \ i=1,2. \quad (21)$$

Notice that (21) is equal to $\mathcal{U}_i^{-1}-L^{-1}\mathcal{Z}_iL^{-1}\ge 0$. By the Schur complement, it is equivalent to

$$\begin{bmatrix}\mathcal{U}_i^{-1} & L^{-1}\\ L^{-1} & \mathcal{Z}_i^{-1}\end{bmatrix}\ge 0,\quad i=1,2. \quad (22)$$

Thus, by introducing new variables $\mathcal{P}$, H_i, $\mathcal{R}_i$, $i=1,2$, the original conditions (17) and (18) are represented as (19), (20) and

$$\begin{bmatrix}H_i & \mathcal{P}\\ \mathcal{P} & \mathcal{R}_i\end{bmatrix}\ge 0,\ \mathcal{P}=L^{-1},\ H_i=\mathcal{U}_i^{-1},\ \mathcal{R}_i=\mathcal{Z}_i^{-1},\ i=1,2.$$

Then, this non-convex problem can be transformed to become the following LMI-based nonlinear minimization problem:

$$\begin{aligned}&\text{Minimize Tr}\Big\{L\mathcal{P}+\sum_{i=1}^{2}(\mathcal{U}_iH_i+\mathcal{Z}_i\mathcal{R}_i)\Big\}\\ &\text{s.t. (19), (20) and}\\ &\begin{bmatrix}H_i & \mathcal{P}\\ \mathcal{P} & \mathcal{R}_i\end{bmatrix}\ge 0,\quad \begin{bmatrix}L & I\\ I & \mathcal{P}\end{bmatrix}\ge 0\\ &\begin{bmatrix}\mathcal{U}_i & I\\ I & H_i\end{bmatrix}\ge 0,\quad \begin{bmatrix}\mathcal{Z}_i & I\\ I & \mathcal{R}_i\end{bmatrix}\ge 0,\quad i=1,2.\end{aligned} \quad (23)$$

Algorithm 1: Design the gain $\mathcal{K}$ with the maximum $\overline{\eta}_{\max}$.

Step 1: Choose sufficiently small initial $\overline{\eta}$ and $\underline{\eta}$ with $\overline{\eta}\ge\underline{\eta}\ge 0$ such that there exists a set of feasible solution to (19)–(23). Set $\overline{\eta}_{\max}=\overline{\eta}$.

Step 2: Find a feasible set $(\mathcal{P}_0,L_0,\mathcal{U}_{10},\mathcal{U}_{20},H_{10},H_{20},\mathcal{Z}_{10},\mathcal{Z}_{20},\mathcal{R}_{10},\mathcal{R}_{20},V)$ satisfying (19)–(23).

Step 3: Solve the following LMI problem:

$$\begin{aligned}&\text{Minimize Tr}\Big\{\sum_{i=1}^{2}(\mathcal{U}_{ik}H_i+\mathcal{U}_iH_{ik}+\mathcal{Z}_{ik}\mathcal{R}_i+\mathcal{Z}_i\mathcal{R}_{ik})\\ &\qquad +L\mathcal{P}_k+L_k\mathcal{P}\Big\},\quad \text{s.t. (19), (20) and (23).}\end{aligned}$$

Set $L_{k+1}=L$, $\mathcal{P}_{k+1}=L^{-1}$, $\mathcal{U}_{i(k+1)}=\mathcal{U}_i$, $H_{i(k+1)}=\mathcal{U}_i^{-1}$, $\mathcal{Z}_{i(k+1)}=\mathcal{Z}_i$, $\mathcal{R}_{i(k+1)}=\mathcal{Z}_i^{-1}, i=1,2$.

Step 4: If LMIs (7) and (8) hold with the controller gain $\mathcal{K}$ obtained in Step 3, then set $\overline{\eta}_{\max}=\overline{\eta}$, increase $\overline{\eta}$ to some extent and return to Step 2. If (7) and (8) hold within a given times of iteration, then exit. Otherwise, set $k=k+1$ and go to Step 3.

Numerical example: This section provides a numerical example to verify the efficiency of the proposed approach.

Example 1: Consider system (1) with

$$\mathcal{A}=\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix},\ \mathcal{B}=\begin{bmatrix}1\\0\end{bmatrix},\ C=[1\quad -0.5],\ \mathcal{D}=\begin{bmatrix}0\\1\end{bmatrix}$$
$$\varpi(\cdot)\in F[0,1].$$

It is reported in [8]–[10] that system (4) is stable for $\overline{\eta}=1.2841$ with the controller gain $\mathcal{K}=[-0.5324\ -0.2419]$ in [8], and $\overline{\eta}=1.5250$ with $\mathcal{K}=[-0.5347\ -0.2469]$ after 84 times of iteration in [9], and $\overline{\eta}=1.5279$ with the controller gain $\mathcal{K}=[-0.5296\ -0.2532]$ after 39 times of iteration in [10]. For the purpose of comparison, set $\underline{\eta}=0$. By applying Algorithm 1, it is obtained that $\overline{\eta}=2.4558$ with $\mathcal{K}=[-0.5858\ -0.2074]$ after 29 times of iteration. It is obvious that the approach presented in this letter can yield less conservative results with fewer iterations in comparison with [8]–[10].

Conclusion: This letter has investigated the problem of stabilizing Lurie NCSs. Based on a looped function method, an improved stability condition for the closed-loop Lurie NCSs has been formulated. Then, a CCL algorithm has been presented to design suitable state feedback controllers. Finally, a numerical example has been carried out to demonstrate the effectiveness of the proposed method.

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