

## Letter

## Finite-Horizon $l_2$ - $l_\infty$ State Estimation for Networked Systems Under Mixed Protocols

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Dear Editor,

This letter is concerned with the finite-horizon  $l_2$ - $l_\infty$  state estimation (SE) problem for a class of networked time-delay systems where the measurements are transmitted through two individual communication channels. With the aim of preventing the transmitted data from conflicts, the signal transmissions over such the communication channels are scheduled by different communication protocols, namely the stochastic communication protocol (SCP) and Round-Robin protocol (RRP). A mixed scheduling model is proposed to describe the transmission behaviors subject to such two protocols. The objective of this letter is to design an estimator such that the estimation error achieves the desired finite-horizon  $l_2$ - $l_\infty$  performance in the mean square. By using the Lyapunov stability theory, a sufficient condition is proposed to guarantee the existence of the desired estimator. An illustrative simulation example is given to verify the effectiveness and correctness of the proposed design scheme.

In the past decades, the SE problem has become a hot research topic in the field of signal processing and control leading to a great number of excellent results published in the literature, see e.g., [1] and [2]. At the same time, a lot of SE techniques have been reported such as the set-membership SE [3],  $l_2$ - $l_\infty$  SE [4] and variance-constrained SE [5]. Among others, the  $l_2$ - $l_\infty$  SE method has been found suitable to deal with square-summable noises. Note that due to various reasons such as temperature fluctuations and component ageing [6], almost all practical systems contain certain time-varying parameters. In such a case, the traditional  $l_2$ - $l_\infty$  SE technique that is implemented over an infinite-horizon is no longer suitable. Accordingly, it makes practical sense to study the  $l_2$ - $l_\infty$  SE problem over a finite-horizon.

For two decades, due to the rapid development of network techniques, networked systems (NSs) have received tremendous research attention leading to a great many references reported [2], [7] and [8]. In NSs, due to the limited bandwidth, it is unrealistic to allow all the sensors to connect to the shared communication channel simultaneously [9]. As such, many communication protocols have been employed to schedule the order of the sensors such as the RRP [3] and [10], the SCP [9] and the try-once-discard protocol [11]. It is worth mentioning that in most existing results concerning the SE/control problems for NSs with protocols, it is assumed that only one-type communication protocol is utilized to schedule network resources. Such an assumption, however, is not always true since it is necessary sometimes to adopt two or more different protocols (namely the mixed protocols) simultaneously in some complex wireless networks such as heterogeneous wireless networks applied in the context of 5G networks which contain multiple infrastructure points

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with various wireless access technologies [12] and [13].

Summarizing the aforementioned discussion, it seems interesting to study the finite-horizon  $l_2$ - $l_\infty$  SE problem for NSs under mixed protocols. It is worth mentioning that this is a non-trivial problem since there are two essential problems to be solved: 1) How to analyze the  $l_2$ - $l_\infty$  performance under the effects of mixed protocols? and 2) How to establish a criterion for the existence of the desired estimator? Therefore, the main purpose of this letter is to answer the above questions. The primary contributions of this letter are highlighted as follows. 1) The finite-horizon  $l_2$ - $l_\infty$  SE problem is, for the first time, studied for NSs under mixed protocols. 2) By utilizing the stochastic analysis technique, a theoretical framework is established to design the estimator parameters ensuring the finite-horizon  $l_2$ - $l_\infty$  performance for the estimation error under mixed protocols.

### Problem formulation:

**Plant:** The following NS on the finite horizon  $[0, N]$  is considered:

$$\begin{cases} x(s+1) = A(s)x(s) + B(s)x(s-d) + E(s)\omega(s) \\ y_l(s) = C_l(s)x(s) + D_l(s)v_l(s), \quad l = 1, 2 \\ z(s) = H(s)x(s) \\ x(s) = \phi(s), \quad -d \leq s \leq 0 \end{cases} \quad (1)$$

where  $x(s) \in \mathbb{R}^{n_x}$  denotes the system state;  $y_l(s) \in \mathbb{R}^{n_{y_l}}$  represents the measurement signal of the  $l$ th sensor node;  $z(s) \in \mathbb{R}^{n_z}$  is the signal to be estimated;  $\omega(s) \in l_2([0, N], \mathbb{R}^\omega)$  is the process noise;  $v_l(s) \in l_2([0, N], \mathbb{R}^{n_{v_l}})$  denotes the measurement noise;  $d$  is the known constant;  $\phi(s)$  is a given initial value; and  $A(s)$ ,  $E(s)$ ,  $B(s)$ ,  $C_l(s)$ ,  $D_l(s)$  and  $H(s)$  are known matrices with appropriate dimensions.

**Transmission model:** Consider the case that the sensors of system (1) are grouped into two sensor nodes according to their physical locations. In this situation, two communication channels, namely Channels 1 and 2, are deployed to transmit the signals from such two sensor nodes. As discussed in Introduction, the so-called mixed protocols are utilized to schedule the signal transmissions over Channels 1 and 2. To be specific, RRP and SCP are employed to schedule the signal transmissions in Channels 1 and 2, respectively. In the following, let us introduce the transmission behaviors over such mixed protocols in Channels 1 and 2 in detail.

For the channel  $l \in \{1, 2\}$  with the protocol scheduling, only one measurement output of the sensor node is allowed to utilize the corresponding channel to send its measurement to the remote estimator at each time instant for the purpose of preventing transmissions from data collisions. Particularly, letting

$$y_l(s) \triangleq \begin{bmatrix} y_{l,1}(s) & y_{l,2}(s) & \cdots & y_{l,n_{y_l}}(s) \end{bmatrix}^T \quad (2)$$

with  $y_{l,\epsilon}(s) \in \mathbb{R}$  being the  $\epsilon$ th measurement output of the  $l$ th sensor node for  $\epsilon = 1, 2, \dots, n_{y_l}$  at a certain time instant  $s$ . Only one measurement signal  $y_{l,\epsilon}(s)$  ( $\epsilon = 1, 2, \dots, n_{y_l}$ ) is permitted to access to the channel  $l$  at each transmission instant. Letting the label of the signal having right to use channel  $l$  at time  $s$  be  $\xi_l(s)$ , we have from the scheduling of the RRP [3] and SCP [9] that

1)  $\xi_1(s)$  can be described by

$$\xi_1(s) = \text{mod}(s-1, n_{y_1}) + 1. \quad (3)$$

2)  $\xi_2(s)$  is governed by a homogeneous Markov chain with the transition probability matrices  $\Pi = [p_{ij}]_{n_{y_2} \times n_{y_2}}$  being

$$\text{Prob}\{\xi_2(s+1) = j | \xi_2(s) = i\} = p_{ij}, \quad i, j \in \{1, \dots, n_{y_2}\} \quad (4)$$

where  $p_{ij} \geq 0$  is the transition probability from  $i$  to  $j$ .

Denote the measurement output after being transmitted at time instant  $s$  via the channel  $l$  as  $\bar{y}_l(s)$ . Then, by employing the zero-order holder, the update principle of  $\bar{y}_l(s)$  can be modeled by

$$\bar{y}_l(s) = \Phi_{\xi_l(s)} y_l(s) + (I - \Phi_{\xi_l(s)}) \bar{y}_l(s-1) \quad (5)$$

where  $\Phi_{\xi_l(s)} \triangleq \text{diag}\{\sigma(\xi_l(s)-1), \dots, \sigma(\xi_l(s)-n_{y_l})\}$  and  $\sigma(a) \in \{0, 1\}$  is the Kronecker delta function which is a binary function that equals 0 if  $a = 1$  and equals 1 otherwise.

**Structure of estimator:**

Denote  $v(s) \triangleq [v_1^T(s) \ v_2^T(s)]^T$ ,  $\bar{y}(s) \triangleq [\bar{y}_1^T(s) \ \bar{y}_2^T(s)]^T$  and  $y(s) \triangleq [y_1^T(s) \ y_2^T(s)]^T$ . From (1) and (5), we have

$$\begin{cases} \bar{y}(s) = \Phi_{\xi_1(s), \xi_2(s)} v(s) + (I - \Phi_{\xi_1(s), \xi_2(s)}) \bar{y}(s-1) \\ y(s) = C(s)x(s) + D(s)v(s) \end{cases} \quad (6)$$

where  $C(s)$ ,  $D(s)$  and  $\Phi_{\xi_1(s), \xi_2(s)}$  can be straight-forwardly derived from (1) and (5).

Proposition 1:  $\xi_1(s) \in \{1, 2, \dots, n_{y_1}\}$  and  $\xi_2(s) \in \{1, 2, \dots, n_{y_2}\}$  can be mapped to the sequence  $\delta(s) \in \{1, 2, \dots, n_{y_1} n_{y_2}\}$  by the following mapping  $\Psi(\cdot, \cdot)$ :

$$\delta(s) = \Psi(\xi_1(s), \xi_2(s)) \triangleq \xi_2(s) + (\xi_1(s) - 1)n_{y_2}. \quad (7)$$

Moreover, for a given  $\delta(s)$ , the values of  $\xi_1(s)$  and  $\xi_2(s)$  can be derived by

$$\begin{cases} \xi_2(s) = \Psi_2(\delta(s)) \triangleq \text{mod}(\delta(s) - 1, n_{y_2}) + 1 \\ \xi_1(s) = \Psi_1(\delta(s)) \triangleq \left\lfloor \frac{\delta(s) - 1}{n_{y_2}} \right\rfloor + 1. \end{cases} \quad (8)$$

Proof: The proof is quite straightforward and is therefore omitted for conciseness. ■

It is obvious from Proposition 1 that there is a one-to-one correspondence between the pair  $(\xi_1(s), \xi_2(s))$  and the variable  $\delta(s)$ . Hence, we can easily derive from (8) that for  $\forall m, n \in \{1, 2, \dots, n_{y_1} n_{y_2}\}$ ,

$$\begin{aligned} q_{mn} &\triangleq \text{Prob}\{\delta(s+1) = n | \delta(s) = m\} \\ &= \text{Prob}\{\xi_1(s+1) = \Psi_1(n) | \xi_1(s) = \Psi_1(m)\} \\ &\quad \times \text{Prob}\{\xi_2(s+1) = \Psi_2(n) | \xi_2(s) = \Psi_2(m)\} \\ &= \begin{cases} p_{\bar{i}\bar{j}}, & \text{if } \Psi_1(n) = \text{mod}(s, n_{y_1}) + 1 \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (9)$$

where  $\bar{i} \triangleq \Psi_2(m)$  and  $\bar{j} \triangleq \Psi_2(n)$ .

According to Proposition 1, we can rewrite  $\bar{y}(s)$  as follow:

$$\bar{y}(s) = \Phi_{\delta(s)} v(s) + (I - \Phi_{\delta(s)}) \bar{y}(s-1). \quad (10)$$

Denoting  $\bar{\omega}(s) \triangleq \begin{bmatrix} \omega(s) \\ v(s) \end{bmatrix}$  and  $\eta(s) \triangleq \begin{bmatrix} x(s) \\ \bar{y}(s-1) \end{bmatrix}$ , we have

$$\begin{cases} \eta(s+1) = \bar{A}_{\delta(s)} \eta(s) + \bar{B}(s) \eta(s-d) + \bar{C}_{\delta(s)} \bar{\omega}(s) \\ \bar{y}(s) = \bar{A}_{\delta(s)} \eta(s) + \bar{C}_{\delta(s)} \bar{\omega}(s) \\ z(s) = \bar{H}(s) \eta(s) \end{cases} \quad (11)$$

where  $\bar{A}_{\delta(s)}(s)$ ,  $\bar{H}(s)$ ,  $\bar{B}(s)$ ,  $\bar{A}_{\delta(s)}(s)$ ,  $\bar{C}_{\delta(s)}(s)$  and  $\bar{C}_{\delta(s)}(s)$  can be straight-forwardly derived from (1), (6) and (10).

Let  $\hat{\eta}(s)$  and  $\hat{z}(s)$  be the estimates of  $\eta(s)$  and  $z(s)$ , respectively. Then, construct the estimator for system (11) as follows:

$$\begin{cases} \hat{\eta}(s+1) = \bar{B}(s) \hat{\eta}(s-d) + \bar{A}_{\delta(s)} \hat{\eta}(s) \\ \quad + K_{\delta(s)} (\bar{y}(s) - \hat{y}(s)) \\ \hat{z}(s) = \bar{H}(s) \hat{\eta}(s) \end{cases} \quad (12)$$

where  $K_{\delta(s)}(s)$  ( $\delta(s) \in \{1, 2, \dots, n_{y_1} n_{y_2}\}$ ) is the estimator gain matrix to be designed.

Denote  $\bar{z}(s) \triangleq z(s) - \hat{z}(s)$  and  $e(s) \triangleq \eta(s) - \hat{\eta}(s)$ . We have

$$\begin{cases} e(s+1) = \check{B}(s)e(s-d) + \check{A}_{\delta(s)} e(s) + \check{C}_{\delta(s)} \bar{\omega}(s) \\ \bar{z}(s) = \check{H}(s)e(s) \end{cases} \quad (13)$$

where  $\check{A}_{\delta(s)}(s)$ ,  $\check{H}(s)$ ,  $\check{C}_{\delta(s)}(s)$  and  $\check{B}(s)$  can be derived straight-forwardly from (11) and (12).

Definition 1 [6]: Let the positive integer  $N$  and the disturbance attenuation level  $\gamma > 0$  be given. The SE error system (13) is said to achieve the finite-horizon  $l_2$ - $l_\infty$  performance index if

$$\sup_{0 \leq s \leq N} \mathbb{E}\{\bar{z}^T(s)\bar{z}(s)\} < \gamma^2 \sum_{\varsigma=0}^N \bar{\omega}^T(\varsigma)\bar{\omega}(\varsigma) + \gamma^2 \sum_{\iota=-d}^0 e^T(\iota)S_1 e(\iota) \quad (14)$$

where  $S_1$  is a given positive definite matrix (PDM).

The purpose of this letter is to design the estimator (12) such that the SE error system (13) achieves the finite-horizon  $l_2$ - $l_\infty$  performance index.

**Main results:** In this section, We will give a sufficient condition to

guarantee that the system (13) achieves the prescribed finite-horizon  $l_2$ - $l_\infty$  performance index.

Theorem 1: Let the disturbance attenuation level  $\gamma$  and the PDM  $S_1$  be given. Assume that, with the initial conditions  $\max\{P(0), Q(\iota)_{\iota \in \{-d, -d+1, \dots, 0\}}\} < \gamma^2 S_1$ , there exists sequences of PDMs  $\{P_m(s)\}_{s \in [0, N]}$  and  $\{Q(s)\}_{s \in \{-d, -d+1, \dots, 0\} \cup [0, N-1]}$  and  $\{Z_m(s)\}_{s \in [0, N-1]}$  such that the following linear matrix inequalities (LMIs) hold for  $\forall m, n \in \{1, 2, \dots, n_{y_1} n_{y_2}\}$ :

$$\Lambda_m(s) \triangleq \begin{bmatrix} -P_m(s) + Q(s) & 0 & 0 & \Lambda_m^{(14)}(s) \\ * & -Q(s-d) & 0 & \check{B}^T(s)P(s+1) \\ * & * & -I & \Lambda_m^{(34)}(s) \\ * & * & * & -\check{P}(s+1) \end{bmatrix} < 0 \quad (15)$$

$$\check{H}^T(s)\check{H}(s) < \gamma^2 P_m(s) \quad (16)$$

where

$$\Lambda_m^{(14)}(s) \triangleq \bar{A}_m^T \bar{P}(s+1) - \bar{A}_m^T Z_m^T(s)$$

$$\Lambda_m^{(34)}(s) \triangleq \bar{C}_m^T \bar{P}(s+1) - \bar{C}_m^T Z_m^T(s)$$

$$\bar{P}(s+1) \triangleq \sum_{n=1}^{n_{y_1} n_{y_2}} q_{mn} P_n(s+1).$$

Then, the SE error system (13) achieves the finite-horizon  $l_2$ - $l_\infty$  performance index, and the state estimator gains  $K_m(s)$  can be calculated as  $K_m(s) = (\bar{P}(s+1))^{-1} Z_m^T(s)$ .

Proof: We use the following Lyapunov-like functional:

$$V(s) = e^T(s)P_{\delta(s)} e(s) + \sum_{\iota=s-d}^{s-1} e^T(\iota)Q(\iota)e(\iota). \quad (17)$$

Denote  $\delta(s) = m$  and  $\delta(s+1) = n$ . From (13) and (17), we have

$$\begin{aligned} \mathbb{E}\{\Delta V(s) - \bar{\omega}^T(s)\bar{\omega}(s) | \delta(s) = m\} \\ = \mathbb{E}\{V(s+1) - V(s) - \bar{\omega}^T(s)\bar{\omega}(s) | \delta(s) = m\} \\ = \mathbb{E}\{\zeta^T(s)\mathcal{H}_m(s)\zeta(s) | \delta(s) = m\} \end{aligned} \quad (18)$$

where

$$\begin{aligned} \zeta^T(s) &\triangleq [e^T(s) \quad e^T(s-d) \quad \bar{\omega}^T(s)] \\ \mathcal{H}_m(s) &\triangleq \begin{bmatrix} \mathcal{H}_m^{(11)}(s) & \bar{A}_m^T \bar{P}(s+1) \check{B}(s) & \bar{A}_m^T \bar{P}(s+1) \check{C}_m(s) \\ * & \mathcal{H}_m^{(22)}(s) & \check{B}^T \bar{P}(s+1) \check{C}_m(s) \\ * & * & \check{C}_m^T \bar{P}(s+1) \check{C}_m(s) - I \end{bmatrix} \\ \mathcal{H}_m^{(11)}(s) &\triangleq -P_m(s) + Q(s) + \bar{A}_m^T \bar{P}(s+1) \bar{A}_m(s) \\ \mathcal{H}_m^{(22)}(s) &\triangleq \check{B}^T \bar{P}(s+1) \check{B}(s) - Q(s-d). \end{aligned}$$

Then, according to the well-known Schur Complement lemma [14],  $\mathcal{H}_m(s) < 0$  holds if and only if  $\Lambda_m(s) < 0$ . Therefore, based on (15) and (18), we can see that

$$\mathbb{E}\{\Delta V(s)\} < \bar{\omega}^T(s)\bar{\omega}(s). \quad (19)$$

Summing up (19) on both sides from 0 to  $t-1$  ( $1 \leq t \leq N$ ) with respect to  $s$  gives

$$\mathbb{E}\{V(t)\} < \mathbb{E}\{V(0)\} + \sum_{\varsigma=0}^{t-1} \bar{\omega}^T(\varsigma)\bar{\omega}(\varsigma). \quad (20)$$

According to (13), (16), (17) and (20) and taking the supremum of  $\mathbb{E}\{\bar{z}^T(s)\bar{z}(s)\}$  over the finite horizon  $[0, N]$ , we have

$$\sup_{0 \leq s \leq N} \mathbb{E}\{\bar{z}^T(s)\bar{z}(s)\} < \gamma^2 \sum_{\varsigma=0}^N \bar{\omega}^T(\varsigma)\bar{\omega}(\varsigma) + \gamma^2 \sum_{\iota=-d}^0 e^T(\iota)S_1 e(\iota). \quad (21)$$

As such, there exist the estimator gains  $K_m(s)$  which ensure the SE error system (13) satisfies the finite-horizon  $l_2$ - $l_\infty$  performance index. Moreover,  $P_n(s+1)$  and  $Z_m(s)$  are given as part of the recursive LMIs solution, and the estimator gains  $K_m(s)$  are given by  $K_m(s) = (\bar{P}(s+1))^{-1} Z_m^T(s)$ . ■

Remark 1: So far, we have designed the desired finite-horizon  $l_2$ - $l_\infty$  estimator for NSSs under mixed protocols. By means of the Lyapunov stability theory, a sufficient condition has been derived to guarantee that the SE error system (13) achieves the finite-horizon

$l_2-l_\infty$  performance index. In addition, the explicit expression of the filter gain matrices  $K_m(s)$  ( $m \in \{1, 2, \dots, n_{y_1}, n_{y_2}\}$ ) have been provided in Theorem 1 according to the solution of certain recursive LMIs.

**Numerical example:** Consider a NS in the form (1) whose parameters are given as follows:

$$A(s) = \begin{bmatrix} 1.05 - 0.01 \sin(s) & 0.01 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.21 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.39 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.22 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.11 \end{bmatrix}$$

$$B(s) = 0, \quad C_1(s) = \begin{bmatrix} 0.28 & 0.3 & 0.1 & 0.3 & 0.2 \\ 0.15 & 0.19 & 0 & 0.23 & 0.41 \end{bmatrix}$$

$$C_2(s) = \begin{bmatrix} 0.38 & 0.1 & 0.2 & 0.2 & 0.4 \\ 0.25 & 0.9 & 0.11 & 0.3 & 0.1 \end{bmatrix}, \quad d = 2$$

$$D_1(s) = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix}, \quad D_2(s) = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}, \quad \Pi = \begin{bmatrix} 0.3 & 0.7 \\ 0.2 & 0.8 \end{bmatrix}$$

$$E^T(s) = \begin{bmatrix} 0.02 & 0.02 & 0.02 & 0.01 & 0.03 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 0.9 & 0.01 & 0.02 & 0.03 & 0.04 \end{bmatrix}, \quad n_{y_2} = 2$$

$$\phi(s) = \begin{bmatrix} 0.1 & 9 & 0 & 0 & 0 \end{bmatrix}, \quad n_{y_1} = 2$$

$$S_1 = \text{diag}\{50, 50, 50, 50, 50, 50, 50, 50, 50\}$$

$$v_2(s) = v_1(s) = 6e^{-0.01s}, \quad \omega(s) = 5 \cos(-0.1s).$$

The simulation run length is selected as 80. Set the disturbance attenuation level  $\gamma = 0.3$ . Let the initial state be

$$\hat{x}(0) = \begin{bmatrix} 1 & 10 & 0 & 0 & 0 \end{bmatrix}^T, \quad x(0) = \begin{bmatrix} 0.9 & 9.9 & 0 & 0 & 0 \end{bmatrix}^T.$$

Based on the derived estimator parameters, numerical simulation results are shown in Figs. 1 and 2. Fig. 1 plots the state trajectories of  $\hat{z}(s)$  and  $z(s)$ , respectively. Fig. 2 gives information about  $\gamma^* \triangleq \sqrt{\frac{\sup_{0 \leq s \leq 80} \bar{z}^T(s)\bar{z}(s)}{\sum_{s=0}^{80} \bar{\omega}^T(s)\bar{\omega}(s) + \sum_{t=-2}^0 e^T(t)S_1 e(t)}}$  with 50 Monte-Carlo trials. All these figures imply that the proposed estimation method performs indeed well, therefore verifying the effectiveness and correctness of the proposed finite-horizon  $l_2-l_\infty$  estimation scheme.

**Conclusion:** In this letter, the finite-horizon  $l_2-l_\infty$  SE problem has been studied for NSs under mixed protocols. Two communication channels have been considered where the network accessing orders of the sensor nodes are scheduled by the SCP and RRP, respectively. A novel finite-horizon  $l_2-l_\infty$  estimator has been designed through the solution of certain recursive LMIs. The effectiveness of the proposed scheme has been illustrated through a simulation example. Future research topics would include the moving horizon estimation problem under mixed protocols [2] and set-membership filtering subject to mixed protocols [3] and [15].

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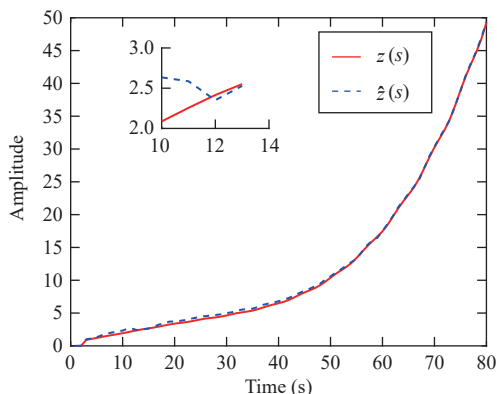


Fig. 1. State trajectories of  $z(s)$  and  $\hat{z}(s)$  under mixed protocols.

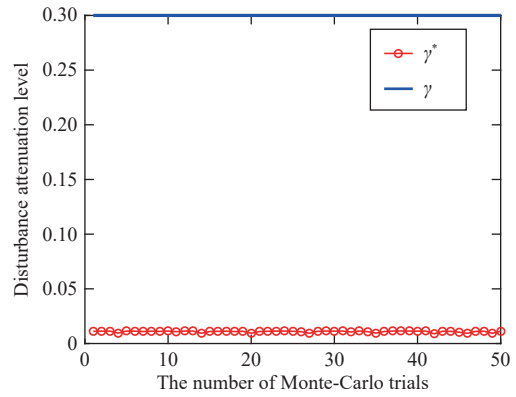


Fig. 2.  $\gamma^*$  and the disturbance attenuation level.

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