Letter

Dynamic Target Enclosing Control Scheme for Multi-Agent Systems via a Signed Graph-Based Approach

Weihao Li, Kaiyu Qin, Mengji Shi, Jingliang Shao, and Boxian Lin

Dear Editor,

This letter investigates the target enclosing control problem of multi-agent systems. A signed graph-based control strategy is presented, where the agents are steered to enclose the dynamic target from both sides as they move. This is inspired by the phenomenon that signed networks exhibit bipartite clustering if the underlying graph is structurally balanced, so that the agents may naturally enclose the zero point from opposite sides (+ and −) if proper controllers are applied. By adopting a distributed observer to estimate the information of dynamic target, a consensus-based enclosing control scheme is designed. Furthermore, a complete Laplacian matrix (CLM)-based Lyapunov analysis method is introduced to prove the control stability, which provides simpler theoretical validation for the stability analysis of average consensus than the conventional method based on state transition matrix. Finally, some numerical simulations are shown to verify the effectiveness of the proposed control scheme.

Networked agents use to require convergence of their states as a part of their cooperation, namely, a consensus reached under properly configured interaction topology and control coefficients [1]–[3]. Numerous significant publications have created various collective behavior from extending the consensus algorithm by specifying different placement rules (formation [4]) or information sources (tracking [5], containment [6]), which applies to diverse vehicle group tasks [7]–[9]. The analyses of consensus-based system enjoy plenty mathematical tools since the behavior can be simply expressed by convergence of states, while some increasingly demanded multi-agent behavior with divergence involved, including area coverage and target enclosing, have not yet been formulated in uniform definitions. Fortunately, target enclosing can be easily exemplified by the scenario of enclosed target(s) being closed off on opposite sides by the agents for diverse purpose like protection, seizure or attack [10]. In that sense, several analogous behavior such as surrounding [11], fencing [12] introduced in many past works fall within the scope of "enclosing problems" but expressed in different terms. This also evidences the great value of target enclosing in military surveillance, rescue operation, protection and exploration.

Various ways are found to define the target enclosing problem in literature. Kobayashi *et al*. [10] describe it as a sub-task of target capturing. Chen *et al*. [11] formulate the problem with leaders (targets) contained in the convex hull of followers, and specially introduce the concept of symmetric enclosing ("balanced surrounding") for final configuration of symmetry (in the form of regular-polytope-shaped deployment of the followers whose center overlaps with the leaders'). Target enclosing problems are mainly solved by achieving dynamic formations (with tunable surrounding density depending on the knowledge of targets' collective motion [11]) or specified deployment in regular shapes (such as the circular/rectangular arrangement [13]) around the target(s). In the final analysis, the agents should be finally deployed on the opposite sides of the target(s) for enclosing.

Different from existing research, this letter comes up with a novel dynamic target enclosing control scheme where the agents are connected by signed networks. Agents may understand neighbors' information from signed edges in opposite ways, so that their states could eventually diverge in sign (+ and −) to perform enclosing of the zero point (the origin). Specifically, if the signed network is structurally balanced, symmetric enclosing would take place when applying bipartite consensus laws [14]–[17]. Inspired by this idea, we develop enclosing control rules in a "consensus" fashion. In our rules, the stationary is replaced by traveling target whose data is processed in the signed network, so that the agents will symmetrically move between the two sides of the enclosed target at last. The contributions of this letter include: 1) A novel signed graph-based dynamic target enclosing control scheme is proposed. The consensus-based control algorithm is designed utilizing the natural bipartite clustering phenomena in structurally balanced networks to create divergence of agents around a target. This strategy extends the methodology for dynamic target enclosing problems, and can be generalized to numerous applications by bringing formation control or coordinate transformation into the algorithm. 2) This article has also designed an efficient approach to stability proof of consensus-based algorithms by constructing a CLM − the Laplacian matrix of a normalized complete graph that shares the same node partition as that of the original signed networks. With the help of several properties of CLM, target enclosing among networked agents can be conveniently validated in theory, which is simpler than the conventional method based on state transition matrix.

Notations: The diagonal matrix is defined as $\mathbf{diag}_i^n[\alpha_i] \triangleq \mathbf{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ and $\mathbf{col}_i^n[\alpha_i] \triangleq [\alpha_1^T, \alpha_2^T, \ldots, \alpha_n^T]^T$ or $\mathbf{col}^n[\alpha] \triangleq [\alpha^T, \alpha^T, \ldots, \alpha^T]^T$ denotes a column vector.

Preliminary and problem formulation: In this part, we will first introduce some relevant basics of graph for network connection and useful lemmas. Then, the target enclosing problem is formulated.

In this letter, an undirected signed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is adopted to denote the communication topology among the multi-agent system, where the set of agents is $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$, and the set of links is $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The value a_{ij} of matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ represents the information exchange between the agent i and agent j. Then, the Laplacian matrix L is defined as $L = H - A$, where $H \in \mathbb{R}^{n\times n}$ is the weighted degree matrix and $H = \mathrm{diag}\{h_1, h_2, \ldots, h_n\}$, $h_i = \sum_{j=1}^n |a_{ij}|$. Without loss of generality, a signed graph $\mathcal{G}$ is structurally balanced if there exists a bipartition $\mathcal{V}_1, \mathcal{V}_2$ of the nodes with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \geq 0$ when v_i, v_j are in the same subgroup, and $a_{ij} \leq 0$ when v_i and v_j are in different subgroups. Thus, for a structurally balanced graph, denote $\mathcal{W} = (w_1, w_2, \ldots, w_n)^T \in \mathbb{R}^{n\times 1}$ as a signature matrix, where $w_i = 1$ if $v_i \in \mathcal{V}_1$ and $w_i = -1$ if $v_i \in \mathcal{V}_2$.

Lemma 1: Consider a connected undirected graph with n nodes whose Laplacian matrix L has a single zero eigenvalue and the corresponding eigenvector is $\mathcal{W}$. In other words, L is semi-positive definite, and $L\mathcal{W} = (\mathcal{W}^T L)^T = \mathbf{0}_{n\times 1}$.

Proof: We can obtain that $L\mathcal{W} = \mathrm{col}_i^n\left[\sum_{j=1}^n |a_{ij}| w_i - \sum_{j=1}^n a_{ij} w_j\right]$, one gets $|a_{ij}| = w_i w_j a_{ij}$, then $\mathrm{col}_i^n\left[\sum_{j=1}^n |a_{ij}| w_i - \sum_{j=1}^n a_{ij} w_j\right] = \mathrm{col}_i^n\left[\sum_{j=1}^n a_{ij} w_i^2 w_j - \sum_{j=1}^n a_{ij} w_j\right] = \mathbf{0}_{n\times 1}$. ■

To provide simpler theoretical validation for the stability analysis, a special Laplacian matrix $C \in \mathbb{R}^{n\times n}$ (called CLM) which shares the same node partition as that of the original signed networks is defined, where $C \triangleq nI_n - \mathcal{W}\mathcal{W}^T$, and I_n is a identity matrix. Obviously,

Corresponding author: Mengji Shi.

Citation: W. H. Li, K. Y. Qin, M. J. Shi, J. L. Shao, and B. X. Lin, "Dynamic target enclosing control scheme for multi-agent systems via a signed graph-based approach," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 2, pp. 560–562, Feb. 2023.

W. H. Li, K. Y. Qin, M. J. Shi, and B. X. Lin are with the School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu 611731, and also with the Aircraft Swarm Intelligent Sensing and Cooperative Control Key Laboratory of Sichuan Province, Chengdu 611731, China (e-mail: liweihao@std.uestc.edu.cn; kyqin@uestc.edu.cn; maangat@126.com; linbx@uestc.edu.cn).

J. L. Shao is with the School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: jinliangshao@uestc.edu.cn).

Digital Object Identifier 10.1109/JAS.2023.123234

$C\mathcal{W} = n\mathcal{W} - \mathcal{W}\mathcal{W}^T\mathcal{W} = n\mathcal{W} - \mathbf{col}_i^n[nw_i] = \mathbf{0}_{n\times 1}$.

Lemma 2: If C, L are the CLM and Laplacian matrix of a normalized complete graph $\bar{\mathcal{G}}$ and connected graph $\mathcal{G}$, respectively. The following propositions hold.

1) $C^lL = LC^l = n^lL$, for $\forall l \in \mathbb{N}$;

2) $(L+\mathcal{W}\mathcal{W}^T)$ is positive definite, and $(L+\mathcal{W}\mathcal{W}^T)C = LC$.

Proof: According to the Lemma 1, noting that $\mathcal{W}^TL = \mathbf{0}_{1\times n}$, one gets $CL = (nI - \mathcal{W}\mathcal{W}^T)L = nL = LC$. Therefore, $C^lL = C^{l-1}CL = n^lL = LC^l$, and the proof of the first proposition is achieved. Based on the Lemma 1, there has an orthogonal matrix Q so that $QLQ^T = \mathbf{diag}\{0, J_2, \ldots, J_m\} \triangleq \Lambda$, where J_k ($k = 2,3,\ldots,m$) is a Jordan block with respect to a positive eigenvalue of L. Then we can write $Q = \mathbf{col}_k^m[q_k^T]$, $(k = 1,2,\ldots,m)$, where $q_1 = \frac{1}{\sqrt{n}}\mathcal{W}$, and $q_k \in \mathbb{R}^{\dim(J_k)\times n}$ for $k > 1$. Since $QL = QL(Q^TQ) = \Lambda Q$, then $q_k^TL = J_kq_k^T$, and $q_k^T = J_k^{-1}q_k^TL$. It follows that $q_1^T\mathcal{W} = \sqrt{n}$, and for $k > 1$, $q_k^T\mathcal{W} = J_k^{-1}q_k^T(L\mathcal{W}) = \mathbf{col}^{\dim(J_k)}[0]$, which conduces to $Q\mathcal{W} = \mathrm{col}_k^n[q_k^T\mathcal{W}] = [\sqrt{n}, 0, \ldots, 0]_n^T \triangleq Q_n^{\mathbf{1}}$. Thus,

$$Q(L+\mathcal{W}\mathcal{W}^T)Q^T = \Lambda + Q_n^{\mathbf{1}}(Q_n^{\mathbf{1}})^T = \mathbf{diag}\{n, J_2, \ldots, J_m\}$$

which indicates that all the eigenvalues of $(L+\mathcal{W}\mathcal{W}^T)$ are positive. That means $(L+\mathcal{W}\mathcal{W}^T)$ is positive definite. Furthermore, $(L+\mathcal{W}\mathcal{W}^T)C = LC + \mathcal{W}(C\mathcal{W})^T = LC$, then the proof of the second proposition is achieved. ■

Considering a first-order multi-agent system, whose model dynamics can be described as $\dot{x}_i(t) = u_i(t)$, $i \in \mathcal{I}$, where x_i and u_i denote the position and control input of agent i, respectively. The index set is $\mathcal{I} = (1,2,\ldots,n)$. To describe the target enclosing problem of the multi-agent system, we first define the position of moving target is x_0, which is generated by an external reference system with provided system information: $\dot{\varsigma} = A_\xi\varsigma$, $x_0 = C_\xi\varsigma$, where ς is the state of external reference system, $A_\xi \in \mathbb{R}^{m\times m}$ and $C_\xi \in \mathbb{R}^{p\times m}$, $p \le m$ are the system matrix and output matrix, respectively. In this letter, only the position of target is accessible to informed agents.

Definition 1 (Target enclosing problem): The control law u_i is applied to achieve target enclosing, if the states of multi-agent systems finally satisfy $\lim_{t\to\infty}|x_i - \mathrm{sgn}(a_{ij})x_j + (\mathrm{sgn}(a_{ij})-1)x_0| = 0$.

For i and j-th agents which in the same sub-graph, i.e., $a_{ij} > 0$, one sees their convergence $(x_i(t) \to x_j(t))$ as $t \to \infty$, while for two agent in different sub-graph, i.e., $a_{ij} < 0$, then they will finally be evenly located on the two sides of the target x_0, that is, $(x_i(t) - x_0(t)) \to -(x_j(t) - x_0(t))$ as $t \to \infty$. In other words, the agents will be deployed to enclose the moving target and keep it as their geometry center. According to the Definition 1, the enclosing error is defined as

$$e_i = \sum_{j=1}^{n}|a_{ij}|(x_i - \mathrm{sgn}(a_{ij})x_j + (\mathrm{sgn}(a_{ij})-1)x_0). \tag{1}$$

The main objective is to design the target enclosing control protocol for agents.

Main results: This part provides solutions to the target enclosing problems. The controller is produced by constructing a distributed observer to estimate the state of target for each agent. The target information is generated by the external reference system, and the following assumptions are made throughout this part.

Assumption 1: The system matrix A_ξ and output matrix C_ξ are available for each agent. The target can be regarded as a virtual leader for the multi-agent system, and we define the communication matrix between agents and target is $\Gamma = \mathbf{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_n\} \in \mathbb{R}^{n\times n}$, if agent i can measure the target's information, $\Gamma_i > 0$, otherwise $\Gamma_i = 0$. At least, one agent can measure the information of target, that is $\exists i$, $\Gamma_i > 0$.

In order to realize the distributed observation of moving target among the topology network, the estimation information of the agent to external reference system is defined as $\hat{\varsigma}_i$, and the observed value of the target is $\hat{x}_{i0}$. Furthermore, the distributed observation errors can be defined as $e_{\varsigma i} = -\sum_{j=1}^n |a_{ij}|(\hat{\varsigma}_i - \hat{\varsigma}_j) - \Gamma_i(\hat{\varsigma}_i - \varsigma)$. The distributed observer is further constructed as

$$\dot{\hat{\varsigma}}_i = A_\xi\hat{\varsigma}_i + k_\varsigma e_{\varsigma i}, \quad \hat{x}_{i0} = C_\xi\hat{\varsigma}_i$$

where $k_\varsigma > 0$ is the observation gain. Define the distributed observation error of agent i is $\tilde{\varsigma}_i = \hat{\varsigma}_i - \varsigma$, and it's compact form is $\tilde{\varsigma} = \mathrm{col}_i^n[\varsigma_i] \in \mathbb{R}^{n\times 1}$, then the derivative form of $\tilde{\varsigma}$ is

$$\dot{\tilde{\varsigma}} = (I_n \otimes A_\xi - k_\varsigma(\bar{L}\otimes I_m))\tilde{\varsigma}, \quad \bar{L} = L + \Gamma. \tag{2}$$

Lemma 3: If $k_\varsigma > 0$, the observation error $\tilde{\varsigma}(t) \to 0$ as $t \to \infty$. That is, $(\hat{x}_{i0}(t) - x_0(t)) \to 0$ as $t \to \infty$.

Although that coefficient matrix $(I_n \otimes A_\xi - k_\varsigma(\bar{L}\otimes I_m))$ is negative definite, the proof of Lemma 3 can be obtained by analyzing the stability of observation error system (2). Therefore, the observed value of target's position and velocity are $\hat{x}_{i0} = C_\xi\hat{\varsigma}_i$ and $\dot{\hat{x}}_{i0} = C_\xi(A_\xi\hat{\varsigma}_i + k_\varsigma e_\varsigma)$, respectively. Then, the enclosing error of (1) is modified to

$$\hat{e}_i = \sum_{j=1}^{n}|a_{ij}|(x_i - \mathrm{sgn}(a_{ij})x_j + (\mathrm{sgn}(a_{ij})-1)\hat{x}_{i0}). \tag{3}$$

Then, the control input is designed as

$$u_i = -k\hat{e}_i + \dot{\hat{x}}_{i0} = -k\hat{e}_i + C_\xi(A_\xi\hat{\varsigma}_i + k_\varsigma e_{\varsigma i}) \tag{4}$$

where $k > 0$ is the controller gain.

Theorem 1: By adopting the control input (4), if the Assumption 1 and Lemma 3 are hold, the target enclosing of multi-agent systems is achieved.

Proof: Define $\rho_{ij} = (x_i - \mathrm{sgn}(a_{ij})x_j + (\mathrm{sgn}(a_{ij})-1)\hat{x}_{i0})$ as the enclosing error for agent i, then the control goal is to provide protocol u_i to achieve $\rho_{ij}(t) \to 0$ as $t \to \infty$. Thus, let $z_{1i} = \sum_{j=1}^n \rho_{ij}$, and define $Z_1 = \mathbf{col}_i^n[z_{1i}]$, $X = \mathbf{col}_i^n[x_i]$, $X_0 = \mathbf{col}^n[\hat{x}_{i0}]$ and $E = \mathbf{col}_i^n[\hat{e}_i]$. Then,

$$Z_1 = \mathbf{col}_i^n\left[\sum_{j=1}^{n}((x_i - \hat{x}_{i0}) - \mathrm{sgn}(a_{ij})(x_j - \hat{x}_{i0})\right] = C(X - X_0). \tag{5}$$

And the compact form of $\hat{e}_i$ and u_i are $E = L(X - X_0)$ and $U = -kE + \dot{X}_0$. The derivative from of Z_1 is

$$\dot{Z}_1 = C(U - \dot{X}_0) = -kCE. \tag{6}$$

Consider a candidate Lyapunov function $V_1 = \frac{1}{2}Z_1^T(L+\mathcal{W}\mathcal{W}^T)Z_1$ where $V_1 \ge 0$. According to Lemma 2 and (6), the derivative form of Lyapunov function along t leads to

$$\begin{aligned}\dot{V}_1 &= Z_1^T(L+\mathcal{W}\mathcal{W}^T)\dot{Z}_1 = -kZ_1^T(L+\mathcal{W}\mathcal{W}^T)CE \\ &= -kZ_1^TL^TCL(X - X_0) = -k(LZ_1)^T(LZ_1) \le 0.\end{aligned} \tag{7}$$

This implies $V_1(\infty) \le V_1(0) < \infty$, which guarantees the boundedness of V_1. As long as $Z_1 \neq 0$, the Lyapunov function V_1 keeps decreasing and $\lim_{t\to\infty}Z_1 \to 0$. That is, $Z_1 = C(X - X_0) = n(X - X_0) - \mathcal{W}\mathcal{W}^T(X - X_0) \to 0$ and $(X - X_0) = \frac{1}{n}\mathcal{W}\mathcal{W}^T(X - X_0)$. Thus, for i-th agent, $x_i - \hat{x}_{i0} = \frac{1}{n}\sum_{j=1}^n w_iw_j(x_j - \hat{x}_{i0})$. And based on $w_i = w_j\mathrm{sgn}(a_{ij})$, then $x_i - \mathrm{sgn}(a_{ij})x_j + (\mathrm{sgn}(a_{ij})-1)\hat{x}_{i0} = 0$ holds. When Lemma 3 holds, $(\hat{x}_{i0}(t) - x_0(t)) \to 0$, as $t \to \infty$. Therefore, $(x_i(t) - \mathrm{sgn}(a_{ij})x_j(t) + (\mathrm{sgn}(a_{ij})-1)x_0(t)) \to 0$. According to the Definition 1, the proof of Theorem 1 is complete. ■

Numerical example: In this part, some simulation results are given to verify the effectiveness of the proposed control scheme. Fig. 1(a) shows the topology among the agents, and the symbol matrix $\mathcal{W} = \mathbf{col}_i^n[w_i] = [1,1,-1,-1]^T$. Moreover, consider that only 1st agent can measure/receive the state information x_0 of target, thus the weight matrix is $\Gamma = \mathbf{diag}\{1,0,0,0\}$. The system and output matrices of external reference system are

$$A_\xi = \begin{bmatrix} 0 & 0.5\pi \\ -0.5\pi & 0 \end{bmatrix}, \; C_\xi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where the initial values $\varsigma(0) = [5,5]^T$ and $\hat{\varsigma}_i(0) = [0,0]^T$, $\forall i \in \mathcal{I}$. The parameters of distributed observer and controller are $k_\varsigma = 10$, $k = 0.5$. In this simulation, the target enclosing problem is considered in the two-dimensional space (x-y). The initial value of agents' state are: $x_1(0) = [-1,1]^T$, $x_2(0) = [3,3]^T$, $x_3(0) = [-4,-4]^T$, $x_4(0) = [2,2]^T$. At first, by adding the formation offset p_{ij} of agent i and j, whose in the same sub-graph, the target enclosing error is $e_i = \sum_{j=1}^n |a_{ij}|(x_i - \mathrm{sgn}(a_{ij})x_j + (\mathrm{sgn}(a_{ij})-1)\hat{x}_{i0} - p_{ij}\eta_{ij})$, where $x_i =$

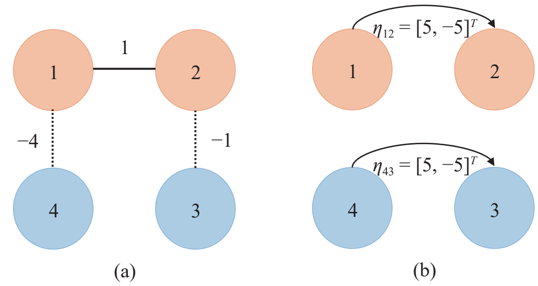

Fig. 1. The interactions among the agents. (a) The topology among the agents; (b) The formation of agents.

$[x_{ix}, x_{iy}]^T$, $x_0 = [x_{0x}, x_{0y}]^T$ and $\eta_{ij} = [\eta_{xij}, \eta_{yij}]^T$. p_{ij} is the formation offset weight which satisfies

$$p_{ij} = 0.5(\mathrm{sgn}(a_{ij}) + 1) = \begin{cases} 1, & \text{if } a_{ij} > 0 \\ 0, & \text{if } a_{ij} \le 0. \end{cases}$$

It follows that, for i and j-th agents which in the same sub-graph, their x-axis states deviation finally approach to $(x_{ix} - x_{jx}) = \eta_{xij}$, while for the two agents in different sub-graph, that is, $(x_{ix} + x_{jx}) = 2x_{0x}$. The formation interval η_{ij} between agent i and j is shown in Fig. 1(b), where $\eta_{xij} = -\eta_{xji}$. The simulation result is shown in Fig. 2. Eventually, each agent moves in different circle, and their formation surrounds the target.

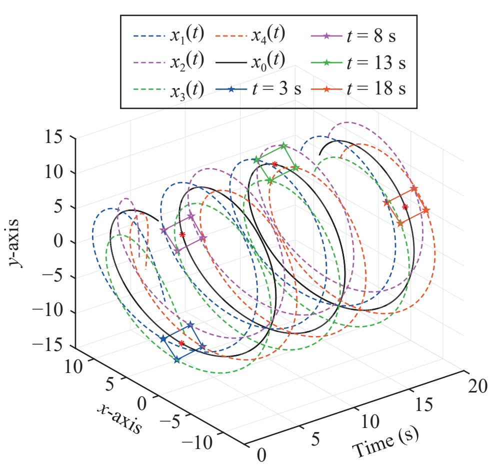

Fig. 2. The positions of agents and target in three-dimensions (circular motion).

Conclusion: This letter investigates the target enclosing control problem of multi-agent systems, different from the existing methods in literature, a novel distributed signed graph-based control scheme is proposed. The dynamic target enclosing control is completed by designing an distributed observer for the moving center of the signed network to achieve enclosing tracking around the target. Moreover, a CLM, the Laplacian matrix of a normalized complete graph which shares the same node partition as that of the original signed networks, is constructed to prove the closed-loop stability of system. In future work, we may focus on the extension of the proposed signed graph-based to the target(s) enclosing problem in real-world applications, such as protection, seizure or attack for military operations.

Acknowledgments: This work was supported by the Science & Technology Department of Sichuan Province (2022NSFSC0037, 2022JDR0107, 2020YJ0044, 2021YFG0131), the Fundamental Research Funds for the Central Universities (ZYGX2020J020) and the National Numerical Wind Tunnel Project, China (NNW2021 ZT6-A26).

References

[1] Y. Zheng, Q. Zhao, J. Ma, and L. Wang, "Second-order consensus of hybrid multi-agent systems," *Systems & Control Letters*, vol. 125, pp. 51–58, 2019.

[2] L. Pan, H. Shao, M. Mesbahi, Y. Xi, and D. Li, "Consensus on matrix-weighted switching networks," *IEEE Trans. Automatic Control*, vol. 66, no. 12, pp. 5990–5996, 2021.

[3] M. Ye, D. Li, Q.-L. Han, and L. Ding, "Distributed nash equilibrium seeking for general networked games with bounded disturbances," *IEEE/CAA J. Autom. Sinica*. DOI: 10.1109/JAS.2022.105428.

[4] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[5] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.

[6] Z. Meng, W. Ren, and Z. You, "Distributed finite-time attitude containment control for multiple rigid bodies," *Automatica*, vol. 46, no. 12, pp. 2092–2099, 2010.

[7] Z. Gao and G. Guo, "Fixed-time sliding mode formation control of auvs based on a disturbance observer," *IEEE/CAA J. Autom. Sinica*, vol. 7, no. 2, pp. 539–545, 2020.

[8] W. Li, H. Zhang, Y. Zhou, and Y. Wang, "Bipartite formation tracking for multi-agent systems using fully distributed dynamic edge-event-triggered protocol," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 5, pp. 847–853, 2022.

[9] J. Huang, S. Zhou, H. Tu, Y. Yao, and Q. Liu, "Distributed optimization algorithm for multi-robot formation with virtual reference center," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 4, pp. 732–734, 2022.

[10] Y. Kobayashi, K. Otsubo, and S. Hosoe, "Design of decentralized capturing behavior by multiple mobile robots," in *Proc. IEEE Workshop on Distributed Intelligent Systems: Collective Intelligence and Its Applications*, 2006, pp. 13–18.

[11] F. Chen, W. Ren, and Y. Cao, "Surrounding control in cooperative agent networks," *Systems & Control Letters*, vol. 59, no. 11, pp. 704–712, 2010.

[12] Z. Chen, "A cooperative target-fencing protocol of multiple vehicles," *Automatica*, vol. 107, pp. 591–594, 2019.

[13] B. Hu, H. Zhang, B. Liu, H. Meng, and G. Chen, "Distributed surrounding control of multiple unmanned surface vessels with varying interconnection topologies," *IEEE Trans. Control Systems Technology*, vol. 30, no. 1, pp. 400–407, Jan. 2022.

[14] C. Altafini, "Consensus problems on networks with antagonistic interactions," *IEEE Trans. Automatic Control*, vol. 58, no. 4, pp. 935–946, 2012.

[15] J. Wu, Y. Zhu, Y. Zheng, and H. Wang, "Resilient bipartite consensus of second-order multiagent systems with event-triggered communication," *IEEE Systems J.*, 2021, DOI: 10.1109/JSYST.2021.3132623.

[16] L. Shi, W. X. Zheng, J. Shao, and Y. Cheng, "Sub-super-stochastic matrix with applications to bipartite tracking control over signed networks," *SIAM J. Control and Optimization*, vol. 59, no. 6, pp. 4563–4589, 2021.

[17] J. Shao, W. X. Zheng, L. Shi, and Y. Cheng, "Bipartite tracking consensus of generic linear agents with discrete-time dynamics over cooperation-competition networks," *IEEE Trans. Cybernetics*, vol. 51, no. 11, pp. 5225–5235, 2020.