Letter

Data-Driven Fault Compensation Tracking Control for Coupled Wastewater Treatment Process

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Dear Editor,

This letter is concerned with the data-driven fault compensation tracking control for a coupled wastewater treatment process (WWTP) subject to sensor faults. Invariant set theory is introduced to eliminate the completely bounded and differentiable conditions of coupled non-affine dynamics and to explicitly express the control inputs. An adaptive fault compensation mechanism is constructed to accommodate the effects of sensor faults. By employing a cubic absolute-value Lyapunov criteria, it is shown that all the signals are bounded and the tracking error converges to an adjustable neighborhood near the origin. Experiment studies are executed on a standardized platform of WWTP to illustrate the effectiveness of the proposed strategy.

Related work: Effective control of wastewater treatment processes (WWTPs) is especially prominent in whole wastewater industry because inefficiencies in the treatment process can lead to significant production losses and ecosystem issues in the receiving water bodies [1]. At present, the control problem of WWTPs mainly focuses on the information availability on the process to be controlled [2], but the modeling of WWTPs remains a difficult activity due to the highly nonlinear, strong coupling, multivariate and external disturbances of WWTPs. Therefore, the data-driven methods have become an increasingly-used approach to attain control objectives, which does not need the accurate knowledge of the plant and depends merely on the input/output (I/O) data. For example, [3] proposed a data-driven multiobjective predictive control method to address the conflicting control objectives of WWTPs. A data-driven iterative adaptive critic strategy [4] was designed to solve the nonlinear optimal control issue for the complex process dynamics. In fact, high equipment fault rates will bring new uncertainties and challenges to WWTP control system, resulting in inevitable fault trips.

Frequent sensor faults in wastewater treatment operations include dissolved oxygen meter contamination and flowmeter corrosion. The existing WWTP control mechanisms that ignore these sensor faults will directly cause system operating point migration under the action of information feedback. Originally, physical hardware redundancy was the mostly used way for entire WWTP to evade sensor faults and performance degradation [5]. Due to the complex system structure and the excessive consumption of space and cost, tremendous progress has been made in analytic redundancy technologies for sensor faults to replace hardware redundancy. Reference [6] proposed a fault detection (FD) system incorporating the predicting plant and fault diagnosis method, which was smoothly applied to a WWTP. In [7], a fault-tolerant control approach for wastewater treatment was designed by using the data/analytical correlations to achieve fault detection and isolation in the advent of dissolved oxygen sensor faults. However, a FD unit to supervise the system behavior may cause some levels of time delay between fault occurrence and fault accommodation, and cannot provide real-time fault information to the operators. Additionally, the problems on timely and accurate acquisition of sensor fault information to replace hardware redundancy and robust suppression schemes still remain to be resolved in WWTP.

Aiming at the characteristics of strong coupling and frequent failure in WWTP, the research of effective process control scheme, based on a design framework combining a weak conservative decoupling method and an adaptive fault compensation mechanism, has important practical significance and application value for wastewater engineering, which mainly reflects in actively mitigating the fault detriment on the WWTP, effectively ensuring the continuous, safe and stable operation of the control system and sustainably utilizing water resources and protecting environment. Meanwhile, the data-driven fault compensation tracking control strategy studied in this letter also has certain theoretical significance for other complex industrial processes that lack mathematical models but have massive process I/O data. The main contributions of this letter in comparison to some existing results are: 1) Since some of available researches on FD and fault robust suppression [6]–[8] are challenging to guarantee the timely feedback of fault information to operation staff and the security of automatic control system due to the complicated dynamics of the WWTP, an adaptive fault compensation mechanism is derived by sufficiently utilizing the chattering of tracking error to correct the false information transmitted by faulty sensors and accomplish the compensation of fault effects. 2) The invariant set theory is introduced with no any differentiable condition of the non-affine WWTP dynamics, and further promotes explicit expression of the control input. Compared with the existing results [9]–[11] that depend on the bounded conditions of necessarily differentiable process dynamics and known prior knowledge of control coefficients, only one semi-bounded condition is required in this study for weakening the conservatism of such controllable conditions.

Problem statement: A typical WWTP based on activated sludge process, namely, Benchmark Simulation Model No. 1 (BSM1) [12], provides a standardized platform to conduct a comprehensive analysis of the obtained results. The main control goal of BSM1 is to ensure that dissolved oxygen concentration $S_{O,5}$ in the fifth unit and nitrate nitrogen concentration $S_{NO,2}$ in the second unit are regulated at expected setting points, in which the manipulated variables of $S_{O,5}$ and $S_{NO,2}$ are the oxygen transfer coefficient K_La_5 and internal recycle flow rate Q_a, respectively. On the basis of the characteristics of WWTP, a generalized system of WWTP with coupled non-affine dynamics is represented as

$$\begin{aligned}\dot{S}_{O,5}(t) &= \mathcal{D}_1(S_{O,5}(t), S_{NO,2}(t), K_La_5(t)) + \omega_1(t)\\ \dot{S}_{NO,2}(t) &= \mathcal{D}_2(S_{O,5}(t), S_{NO,2}(t), Q_a(t)) + \omega_2(t)\end{aligned} \tag{1}$$

where unknown non-affine functions $\mathcal{D}_1(\cdot,\cdot)$ and $\mathcal{D}_2(\cdot,\cdot)$ represent real-time and coupled WWTP dynamics of physical and biochemical reactions. Noting that the coupled relationship between the components and the manipulated variables indicates that the components interact and affect the dynamics of one another, which does not appear linearly. $\omega_1(t)$ and $\omega_2(t)$ are the external environment disturbances and satisfy $|\omega_i(t)| \le \breve{\omega}_i$ for the given constants $\breve{\omega}_i > 0$, $i = 1,2$. Then, define a function as

$$\mathcal{F}_i(\mathcal{S}, u_i) = \mathcal{D}_i(\mathcal{S}, u_i) - \mathcal{D}_i(\mathcal{S}, 0),\ i = 1,2 \tag{2}$$

where $\mathcal{F}_i(\mathcal{S}, u_i)$ represents the fully coupled process dynamics of the whole WWTP and facilitates the possibility of the feedback linearization, $\mathcal{S} = [S_{O,5}, S_{NO,2}]^T$ and $\mathcal{U} = [K_La_5, Q_a]^T = [u_1, u_2]^T$.

High frequency sensor equipment fault rates in WWTP are caused by harsh operation conditions or self-factors (e.g., weather, wear and

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aging, etc.), and operation under faults will reduce equipment life and introduce instability of process operation. Sudden contamination and blockage of dissolved oxygen/nitrate nitrogen meters are two common sensor faults in WWTP, which generates a certain multiple deviation from the normal transmission value. For developing a fault compensation mechanism to address such sensor issue, the $S_{O,5}$ and $S_{NO,2}$ sensor faults are modeled based on the essential characteristics of sensor working

$$\mathcal{Y}_\iota = \mathcal{P}\mathcal{S},\ \forall t > t_f \tag{3}$$

where $\mathcal{Y}_\iota = [y_{\iota 1}, y_{\iota 2}]^T$ represents the sensor output, $\mathcal{P} = \mathrm{diag}[p_1, p_2]$ is an unknown diagonal matrix and its positive diagonal elements are the effectiveness factors that satisfy $0 < \underline{p}_i \le p_i < 1$, $i = 1,2$, which indicates that the sensors are still functional after partially losing their effectiveness. $\underline{p}_i$ is the lower bound of effectiveness factor p_i, and t_f indicates the uncertain time when a fault occurs.

The uncertainty of fault occurrence time and magnitude can make the tracking error $\mathbf{e} = [\mathbf{e}_1, \mathbf{e}_2]^T$ between the actual process output $\mathcal{S} = [S_{O,5}, S_{NO,2}]^T$ and the desired trajectory $\mathcal{S}_d = [S_{1,d}, S_{2,d}]^T$ difficult to capture for feedback tracking control

$$\mathbf{e} = \mathcal{S} - \mathcal{S}_d \xrightarrow{t_f} \mathbf{e} = \mathcal{P}^{-1}\mathcal{Y}_\iota - \mathcal{S}_d. \tag{4}$$

As the fault severity increases, the control performance and stability of WWTP system are seriously destroyed, namely, the tracking error will quickly deviate from its true value after the fault occurs. Therefore, a sensor fault compensation control strategy is urgently needed to repair the faulty WWTP system performance in time.

Assumption 1: For all $\mathcal{S}$ and u_i, there exist unknown constants $\tau_i > 0$, $\tau_i' > 0$, μ_i and μ_i' such that $\mathcal{F}_i(\mathcal{S}, u_i) \ge \tau_i u_i + \mu_i$ if $u_i \ge 0$ and $\mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i' u_i + \mu_i'$ if $u_i < 0$.

Remark 1: Assumption 1 is widely adopted to address the non-affine function $\mathcal{F}_i(\mathcal{S}, u_i)$ and indicates that the WWTP has the continuous, unnecessarily differentiable and semi-bounded characteristics due to the physical constraints of the control input and the controlled object. Moreover, the upper bound and lower bound of $\mathcal{F}_i(\mathcal{S}, u_i)$ in the scenarios of $u_i \ge 0$ and $u_i \le 0$ are removed, respectively.

Control design: Data-driven control insights are that the designed WWTP controller and adaptive law will not contain mathematical model information of controlled process, and it only uses the online I/O data of the process system to carry out the control design. However, it can be seen from (4) that the process output data becomes spurious and unavailable when the sensor fails. For such issue, an adaptive fault compensation coefficient $\hat{\mathcal{K}}_i$ involving measurable sensor output data $\mathcal{Y}_\iota$ is introduced to achieve the output feedback control by employing the compensated tracking error $\tilde{\mathbf{e}} = \hat{\mathcal{K}}\mathcal{Y}_\iota - \mathcal{S}_d$. Therefore, to achieve accurate output regulation, the following coordinate transformation is utilized:

$$\mathbf{e} = \mathcal{S} - \mathcal{S}_d = \hat{\mathcal{K}}\mathcal{Y}_\iota - \mathcal{S}_d + \tilde{\mathcal{K}}\mathcal{Y}_\iota = \tilde{\mathbf{e}} + \tilde{\mathcal{K}}\mathcal{Y}_\iota \tag{5}$$

where $\tilde{\mathbf{e}} = [\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2]^T$, $\hat{\mathcal{K}} = \mathrm{diag}[\hat{k}_1, \hat{k}_2]$ and $\tilde{\mathcal{K}} = \mathrm{diag}[\tilde{k}_1, \tilde{k}_2]$. $\hat{k}_i$ is named as adaptive fault compensation coefficient and is the estimation of unknown parameter $k_i = 1/p_i$, and its estimation error is $\tilde{k}_i = k_i - \hat{k}_i$. To develop a design procedure, we first define a unknown constant as $\theta_i = \left\{\|W_i\|^2/\mathcal{L}_{i1}, i = 1,2\right\}$, where $\|W_i\|$ is norm of fuzzy weight, and $\mathcal{L}_{i1}$ is unknown parameter and defined later.

Theorem 1: Considering the coupled non-affine WWTP system (1) with sensor faults and satisfying Assumption 1 and initial condition $\mathcal{V}(0) \le \eta$ for a given constant $\eta > 0$, if the process controller u_i and adaptive law of $\hat{\theta}_i$ for WWTP are designed as

$$u_i = -\frac{\tilde{\mathbf{e}}_i}{2a_i^2}\hat{\theta}_i \xi_i^T(\mathcal{Z}_i)\xi_i(\mathcal{Z}_i) - c_i\tilde{\mathbf{e}}_i \tag{6}$$

$$\dot{\hat{\theta}}_i = \frac{\Gamma_i}{2a_i^2}\tilde{\mathbf{e}}_i^2 \xi_i^T(\mathcal{Z}_i)\xi_i(\mathcal{Z}_i) - \lambda_i\hat{\theta}_i \tag{7}$$

and the adaptive fault compensation mechanism is defined as

$$\dot{\hat{k}}_i = \mathrm{Proj}_{[1,1/\underline{p}_i]}\{\mathcal{M}_i\} = \begin{cases} 0, & \text{if } \begin{array}{l} \hat{k}_i = 1 \text{ and } \mathcal{M}_i \le 0 \\ \hat{k}_i = \dfrac{1}{\underline{p}_i} \text{ and } \mathcal{M}_i \ge 0 \end{array} \\ \mathcal{M}_i, & \text{otherwise} \end{cases} \tag{8}$$

with

$$\mathcal{M}_i = \begin{cases} l_i\left[\dfrac{1}{2a_i^2} + \dfrac{1}{2}\left(\dfrac{\tilde{\mathbf{e}}_i\hat{\theta}_i}{2a_i^2}\right)^2 + \dfrac{1}{2}\right]y_{\iota i}^2 + \dfrac{\beta_i}{\beta_i' + y_{\iota i}^2} - r_i\hat{k}_i, & \mathcal{N}_i > 0 \\ 0, & \mathcal{N}_i \le 0 \end{cases}$$

where $i = 1, 2$, $\mathcal{N}_i = l_i[y_{\iota i}^2/2a_i^2 + \frac{1}{2}(y_{\iota i}\tilde{\mathbf{e}}_i\hat{\theta}_i/2a_i^2)^2 + y_{\iota i}^2/2] + \beta_i/(\beta_i' + y_{\iota i}^2) - r_i\hat{k}_i$, $\mathcal{Z}_i = [\mathcal{S}^T, \mathcal{S}_d^T, \dot{S}_{i,d}]^T$, $\hat{\theta}_i$ is an estimation of θ_i with $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\hat{\Theta} = [\hat{\theta}_1, \hat{\theta}_2]^T$, and the a_i, c_i, r_i, l_i, λ_i β_i, β_i' and Γ_i are positive design parameters. Then, $\mathcal{V}(t) \le \eta$ for $\forall t > 0$ and all the signals of the closed-loop WWTP system are uniformly ultimately bounded (UUB) and the tracking error $\mathbf{e}(t)$ can converge to a sufficiently small residual set. Note that $\beta_i/(\beta_i' + y_{\iota i}^2)$ in (8) is an extra modified term to enhance the sensitivity of updating adaptive fault compensation coefficient $\hat{k}_i$.

Proof: Consider a cubic absolute-value Lyapunov function as $V_i = \frac{1}{2}\mathbf{e}_i^2 + \frac{\mathcal{L}_{i1}}{2\Gamma_i}\tilde{\theta}_i^2 + \frac{1}{3l_i}|\tilde{k}_i|^3$, $i = 1,2$. The time derivative of V_i is

$$\begin{aligned}\dot{V}_i &= \mathbf{e}_i[\mathcal{F}_i(\mathcal{S}, u_i) + \mathcal{D}_i(\mathcal{S}, 0) + \omega_i - \dot{S}_{i,d}] \\ &\quad - \frac{\mathcal{L}_{i1}}{\Gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i - \frac{1}{l_i}\tilde{k}_i^2\dot{\hat{k}}_i\mathrm{sgn}(\tilde{k}_i).\end{aligned} \tag{9}$$

Introduce the two functions as $\mathcal{G}_{i1}(\mathcal{S}, u_i) = \frac{2u_i}{u_i^2 + \alpha_{i1}^2}[\mathcal{F}_i(\mathcal{S}, u_i) - \mu_i]$ and $\mathcal{G}_{i2}(\mathcal{S}, u_i) = \frac{2u_i}{u_i^2 + \alpha_{i1}^2}\left[\mathcal{F}_i(\mathcal{S}, u_i) - \mu_i'\right]$, where $\alpha_{i1} > 0$ is unknown parameter. For $\mathcal{G}_{i1}(\mathcal{S}, u_i)$ and $\mathcal{G}_{i2}(\mathcal{S}, u_i)$, along with Assumption 1, one has the relations that $\tau_i u_i + \mu_i \le \mathcal{F}_i(\mathcal{S}, u_i) \le \mathcal{G}_{i1}(\mathcal{S}, u_i)u_i + \mu_i$ if $u_i > \alpha_{i1}$ and $\mathcal{G}_{i2}(\mathcal{S}, u_i)u_i + \mu_i' \le \mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i' u_i + \mu_i'$ if $u_i < -\alpha_{i1}$. Then, there is a continuous function $\kappa_{\mathcal{F}_i}(\mathcal{S})$ such that $|\mathcal{F}_i(\mathcal{S}, u_i)| \le \kappa_{\mathcal{F}_i}(\mathcal{S})$ for $-\alpha_{i1} \le u_i \le \alpha_{i1}$. Consider the compact set $\Omega_\mathbf{e} = \left\{\mathbf{e}\left|\frac{1}{2}\mathbf{e}^T\mathbf{e} \le \eta\right.\right\}$ and $\Omega_0 = \left\{[\mathcal{S}_d, \dot{\mathcal{S}}_d]^T\left|\|\mathcal{S}_d\|^2 + \|\dot{\mathcal{S}}_d\|^2 \le \sigma_d\right.\right\}$, the maximum value of $\kappa_{\mathcal{F}_i}(\mathcal{S})$ can be found on $\Omega_\mathbf{e} \times \Omega_0$, namely, one yields

$$|\mathcal{F}_i(\mathcal{S}, u_i)| \le \kappa_{\mathcal{F}_i}(\mathcal{S}) \le \bar{\kappa}_i, \text{ for } -\alpha_{i1} \le u_i \le \alpha_{i1} \tag{10}$$

where $\bar{\kappa}_i > 0$ is an unknown parameter. Based on (10), it yields $\bar{\kappa}_i \le \bar{\kappa}_i + \tau_i u_1$ if $u_i \ge 0$ and $\tau_i' u_i - \bar{\kappa}_i \le -\bar{\kappa}_i$ if $u_i < 0$. With Assumption 1, one has $\tau_i u_i + \mu_i \le \mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i u_i + \bar{\kappa}_i$ if $0 \le u_i \le \alpha_{i1}$ and $\tau_i' u_i - \bar{\kappa}_i \le \mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i' u_i + \mu_i'$ if $-\alpha_{i1} \le u_i < 0$. Then, we have

$$\begin{cases} \tau_i u_i + \mu_i \le \mathcal{F}_i(\mathcal{S}, u_i) \le \mathcal{G}_{i1}(\mathcal{S}, u_i)u_i + \mu_i, & u_i > \alpha_{i1} \\ \tau_i u_i + \mu_i \le \mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i u_i + \bar{\kappa}_i, & 0 \le u_i \le \alpha_{i1} \\ \tau_i' u_i - \bar{\kappa}_i \le \mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i' u_i + \mu_i', & -\alpha_{i1} \le u_i < 0 \\ \mathcal{G}_{i2}(\mathcal{S}, u_i)u_i + \mu_i' \le \mathcal{F}_i(\mathcal{S}, u_i) \le \tau_i' u_i + \mu_i', & u_i < -\alpha_{i1}. \end{cases} \tag{11}$$

Due to that for $\forall a, b \in \mathbb{R}$, if $a \le \mathbf{x} \le b$, then $\mathbf{x} = \delta a + (1-\delta)b$ with $\delta = (b - \mathbf{x})/(b - a)$, thus there are functions $\zeta_{ij}(t)$ and $\zeta_{ij}'(t) = 1 - \zeta_{ij}(t)$, $j = 1,2,3,4$, in the closed interval $[0,1]$ such that

$$\begin{cases} \mathcal{F}_i(\mathcal{S}, u_i) = \zeta_{i1}'[\mathcal{G}_{i1}(\mathcal{S}, u_i)u_i + \mu_i] + \zeta_{i1}(\tau_i u_i + \mu_i), & u_i > \alpha_{i1} \\ \mathcal{F}_i(\mathcal{S}, u_i) = \zeta_{i2}'(\tau_i u_i + \bar{\kappa}_i) + \zeta_{i2}(\tau_i u_i + \mu_i), & 0 \le u_i \le \alpha_{i1} \\ \mathcal{F}_i(\mathcal{S}, u_i) = \zeta_{i3}'(\tau_i' u_i + \mu_i') + \zeta_{i3}(\tau_i' u_1 - \bar{\kappa}_i), & -\alpha_{i1} \le u_i < 0 \\ \mathcal{F}_i(\mathcal{S}, u_i) = \zeta_{i4}'(\tau_i' u_i + \mu_i') + \zeta_{i4}\left[\mathcal{G}_{i2}(\mathcal{S}, u_i)u_i + \mu_i'\right], & u_i < -\alpha_{i1}. \end{cases} \tag{12}$$

Give the definition of functions $\mathcal{G}_i^*(\mathcal{S}, u_i)$ and $\zeta_i^*(t)$ as follows:

$$\mathcal{G}_i^*(\mathcal{S}, u_i) = \begin{cases} \mathcal{G}_{i1}(\mathcal{S}, u_i)[1 - \zeta_{i1}(t)] + \zeta_{i1}(t)\tau_i, & u_i > \alpha_{i1} \\ \tau_i, & 0 \le u_i \le \alpha_{i1} \\ \tau_i', & -\alpha_{i1} \le u_i < 0 \\ \mathcal{G}_{i2}(\mathcal{S}, u_i)\zeta_{i4}(t) + [1 - \zeta_{i4}(t)]\tau_i', & u_i < -\alpha_{i1}. \end{cases} \tag{13}$$

$$\zeta_i^*(t) = \begin{cases} \mu_i, & u_i > \alpha_{i1} \\ \zeta_{i2}(t)\mu_i + [1 - \zeta_{i2}(t)]\bar{\kappa}_i, & 0 \le u_i \le \alpha_{i1} \\ [1 - \zeta_{i3}(t)]\mu_i' - \bar{\kappa}_i\zeta_{i3}(t), & -\alpha_{i1} \le u_i < 0 \\ \mu_i', & u_i < -\alpha_{i1}. \end{cases} \tag{14}$$

Considering (13) and (14), (12) can be expressed as

$$\mathcal{F}_i(\mathcal{S}, u_i) = \mathcal{G}_i^*(\mathcal{S}, u_i)u_i + \zeta_i^*(t). \tag{15}$$

Next, the characteristics of $\mathcal{G}_{i1}(\mathcal{S}, u_i)$ and $\mathcal{G}_{i2}(\mathcal{S}, u_i)$ are studied. All the variables of the continuous function $\mathcal{G}_{i1}(\mathcal{S}, u_i)$ and $\mathcal{G}_{i2}(\mathcal{S}, u_i)$ are included in the compact set $\Omega_\mathbf{e} \times \Omega_0$. Then, $|\mathcal{G}_{i1}(\mathcal{S}, u_i)|$ and

$|\mathcal{G}_{i2}(\mathcal{S},u_i)|$ have maximum values $\breve{\mathcal{G}}_{i1}$ and $\breve{\mathcal{G}}_{i2}$ in $\Omega_{\mathbf{e}}\times\Omega_0$, namely, $|\mathcal{G}_{i1}(\mathcal{S},u_i)|\le\breve{\mathcal{G}}_{i1}$ and $|\mathcal{G}_{i2}(\mathcal{S},u_i)|\le\breve{\mathcal{G}}_{i2}$ hold. With the aid of (15), one has $0<\mathcal{L}_{i1}\le\mathcal{G}_i^*(\mathcal{S},u_i)\le\mathcal{L}_{i2}$ and $0<\left|\zeta_i^*(t)\right|\le\breve{\zeta}_i$ with unknown parameters $\mathcal{L}_{i1}=\min\{\tau_i,\tau_i'\}$, $\mathcal{L}_{i2}=\max\{\tau_i,\tau_i',\breve{\mathcal{G}}_{i1},\breve{\mathcal{G}}_{i2}\}$ and $\breve{\zeta}_i=\max\{\bar{\kappa}_i,|\mu_i|,|\mu_i'|\}$.

Define an unknown nonlinear function as $\Psi_i(\mathcal{Z}_i)=\mathcal{D}_i(\mathcal{S},0)-\dot{S}_{i,d}$, which cannot be allowed in the manipulated variables K_La_5 and Q_a. Based on the approximative property of fuzzy logic system (FLS), $\Psi_i(\mathcal{Z}_i)$ can be expressed as $\Psi_i(\mathcal{Z}_i)=W_i^T\xi_i(\mathcal{Z}_i)+\varepsilon_i(\mathcal{Z}_i)$ and the approximation error $\varepsilon_i(\mathcal{Z}_i)$ meets $|\varepsilon_i(\mathcal{Z}_i)|\le\breve{\varepsilon}_i$ with $\breve{\varepsilon}_i$ being a unknown parameter. W_i and $\xi_i(\mathcal{Z}_i)$ are the weight and fuzzy basic function vectors, respectively.

Then, the time derivative can be rewritten as

$$\dot{V}_i=\mathbf{e}_i\left[\mathcal{G}_i^*u_i+W_i^T\xi_i(\mathcal{Z}_i)+\varepsilon_i+\zeta_i^*+\omega_i\right]-\frac{\mathcal{L}_{i1}}{\Gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i-\frac{1}{l_i}\tilde{k}_i^2\dot{\hat{k}}_i\mathrm{sgn}(\tilde{k}_i). \tag{16}$$

With the property $\xi_i^T(\mathcal{Z}_i)\xi_i(\mathcal{Z}_i)\le 1$ of FLS, the controller (6) and the facts of $\mathcal{L}_{i2}\ge\mathcal{G}_i^*(\mathcal{S},u_i)\ge\mathcal{L}_{i1}>0$ and $\hat{\theta}_i\ge 0$, we can obtain

$$\begin{aligned}&\mathbf{e}_i\left(\varepsilon_i+\zeta_i^*+\omega_i\right)+(\tilde{\mathbf{e}}_i+\tilde{k}_iy_{ti})W_i^T\xi_i(\mathcal{Z}_i)+\mathbf{e}_i\mathcal{G}_i^*u_i\\&\le\frac{3}{2}\mathbf{e}_i^2+\frac{1}{2}\left(\breve{\varepsilon}_i^2+\breve{\zeta}_i^2+\breve{\omega}_i^2+\mathcal{L}_{i2}^2+a_i^2+a_i^2\mathcal{L}_{i1}\theta_i\right)\\&\quad-c_i\mathcal{L}_{i1}\mathbf{e}_i^2+\frac{c_i^2\mathcal{L}_{i2}^2\mathbf{e}_i^2}{2}+\frac{\mathcal{L}_{i1}}{2a_i^2}\tilde{\mathbf{e}}_i^2\hat{\theta}_i\xi_i^T(\mathcal{Z}_i)\xi_i(\mathcal{Z}_i)\\&\quad+\frac{(\tilde{k}_iy_{ti})^2}{2a_i^2}+\frac{(\tilde{k}_iy_{ti})^2}{2}+\frac{1}{2}\left(\frac{\tilde{k}_iy_{ti}}{2a_i^2}\tilde{\mathbf{e}}_i\hat{\theta}_i\right)^2.\end{aligned} \tag{17}$$

Based on (7) and (8), substituting (17) into (16) gives

$$\dot{V}_i\le-A_i\mathbf{e}_i^2+B_i+\frac{\lambda_i\mathcal{L}_{i1}}{\Gamma_i}\tilde{\theta}_i\hat{\theta}_i+\frac{r_i}{l_i}\tilde{k}_i^2\hat{k}_i\mathrm{sgn}(\tilde{k}_i) \tag{18}$$

where $A_i=c_i\mathcal{L}_{i1}-\frac{1}{2}c_i^2\mathcal{L}_{i2}^2-\frac{3}{2}$ and satisfies relation $A_i>0$. $B_i=(\breve{\varepsilon}_i^2+\breve{\zeta}_i^2+\breve{\omega}_i^2+a_i^2+a_i^2\mathcal{L}_{i1}\theta_i+\mathcal{L}_{i2}^2)/2$.

Stability analysis: Choose the total Lyapunov function as $\mathcal{V}=\sum_{i=1}^2V_i$. From (8), we can get that $\dot{\hat{k}}_i\ge 0$ and $\tilde{k}_i=k_i-\hat{k}_i$, $\lim_{t\to\infty}\hat{k}_i=k_i$, and it means that $k_i>\hat{k}_i$, i.e., $\tilde{k}_i>0$ only if $\hat{k}_i(0)<k_i$. Define constants $\mathcal{A}=\min\{2A_i,\lambda_i,r_i,i=1,2\}$ and $\mathcal{B}=\sum_{i=1}^2B_i+\sum_{i=1}^2(\frac{r_i}{3l_i}k_i^3+\frac{\lambda_i\mathcal{L}_{i1}}{2\Gamma_i}\theta_i^2)$. Then, we can obtain

$$\dot{\mathcal{V}}\le-\mathcal{A}\mathcal{V}+\mathcal{B} \tag{19}$$

where $\frac{\lambda_i\mathcal{L}_{i1}}{\Gamma_i}\tilde{\theta}_i\hat{\theta}_i+\frac{r_i}{l_i}\tilde{k}_i^2\hat{k}_i\mathrm{sgn}(\tilde{k}_i)\le-\frac{\lambda_i\mathcal{L}_{i1}}{2\Gamma_i}\tilde{\theta}_i^2+\frac{\lambda_i\mathcal{L}_{i1}}{2\Gamma_i}\theta_i^2-\frac{r_i}{3l_i}|\tilde{k}_i|^3+\frac{r_i}{3l_i}k_i^3$ is used in (19). Note that reducing a_i and r_i and increasing c_i, λ_i and l_i will make $C=\mathcal{B}/\mathcal{A}$ arbitrarily small. Then, one has $C\le\eta$ through appropriate parameter selection. Since $C\le\eta$, one has $\dot{\mathcal{V}}\le 0$ on $\mathcal{V}=\eta$. Hence $\mathcal{V}(t)\le\eta$ for $\forall t>0$, that is, $\mathcal{V}\le\eta$ is an invariant set. Then, the WWTP system signals are UUB.

Integrating (19) over $[0,t]$, one has $\mathcal{V}(t)\le(\mathcal{V}(0)-C)e^{-\mathcal{A}t}+C$. From the definition of $\mathcal{V}$, it yields $\sum_{i=1}^2\mathbf{e}_i\le\mathcal{V}(t)$ and further obtain that $\lim_{t\to\infty}\|\mathbf{e}\|=\sqrt{2C}$. Similarly, the parameter C can be arbitrarily small by increasing c_i, λ_i and l_i while decreasing a_i and r_i. With proper choice of parameters, the tracking error $\mathbf{e}(t)$ can converge to a sufficiently small residual set. ■

Remark 2: The WWTP controller (6) includes an adaptive control signal to eliminate the impact of unknown nonlinear function $\mathcal{D}_i(\mathcal{S},0)$ and a feedback signal to stabilize the affine linear dynamics in (1). Based on the description of (16) and (17), the adaptive law (7) and adaptive fault compensation mechanism (8) are separately derived through the last term in the third row of (17) and all terms in the fourth row of (17) to achieve online estimation update of fuzzy weight and fault factor. Moreover, since the control inputs in affine systems are explicitly expressed, the WWTP controller (6) can directly stabilize such systems without solving the coupling dynamics problem using the invariant set theory.

Experiments: To validate the effectiveness of the proposed control strategy, the regulations of $S_{O,5}$ and $S_{NO,2}$ are carried out by virtue of BSM1. The environmental disturbances in dry weather and mechanistic model of a WWTP can refer to [12] for details.

Consider the $S_{O,5}$ and $S_{NO,2}$ sensors suddenly lose their 50% and 60% effectiveness at $t_f=4$ day and $t_f=10$ day, respectively, i.e., $k_1=1/p_1=2$ and $k_2=1/p_2=2.5$. The involved parameters on $S_{O,5}$ and $S_{NO,2}$ are chosen as $c_1=4\times10^5$, $c_2=3\times10^7$, $r_1=60$, $r_2=22$, $\beta_1=220$, $\beta_2=22$, $l_i=10^{-8}$ and $\beta_i'=0.01$, $i=1,2$. The remaining parameters are set to one, and the initial values of $\hat{k}_i$ and $\hat{\theta}_i$ are set to one and zero, respectively. The trajectories are set to $S_{1,d}=2$ mg/L for $S_{O,5}$ and $S_{2,d}=1$ mg/L for $S_{NO,2}$.

Performance evaluation: The integral of absolute error (IAE), the integral of square error (ISE) and the maximal absolute deviation of the error ($\mathrm{Dev}_{\max}$) are used as indicators to evaluate control performance. The evaluation start and end times are the first day after the faults and the end of the whole process operation, respectively. For $S_{O,5}$ and $S_{NO,2}$, the IAE (0.0869 and 0.0103), ISE (0.000 84 and 3.56E−05) and $\mathrm{Dev}^{\max}$ (0.009 89 and 0.003 95) under the proposed algorithm are much smaller than the existing algorithms [11]–[14]. Therefore, it is confirmed that the proposed algorithm can ensure transient response and system stability.

Results analysis: With the proposed strategy via data-driven processing, the experimental results are depicted in Figs. 1−4 . It is shown in Fig. 1 that $S_{O,5}$ and $S_{NO,2}$ can achieve the trajectory tracking even if the sensor faults occur during the operation, and the maximum deviations between controlled variables and trajectories are limited to the range of ±0.01 after the faults occur. From Fig. 2, it is illustrated that adaptive fault compensation coefficients $\hat{k}_i$, $i=1,2$, can quickly grow and tend to $k_i=1/p_i$ for alleviating the effects of the sensor faults, although $S_{O,5}$ and $S_{NO,2}$ sensors suffer from varying degrees of faults, respectively. Figs. 3 and 4 indicate that the manipulated variables K_La_5 and Q_a and the adaptive parameter $\hat{\Theta}$ can timely adjust to deal with sensor faults.

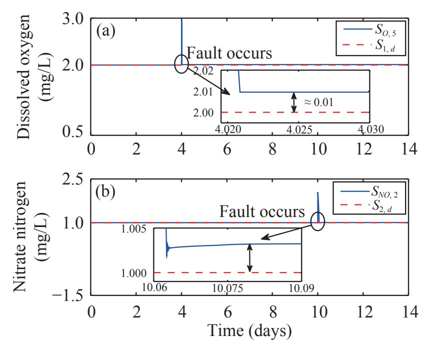

Fig. 1. Tracking performance.

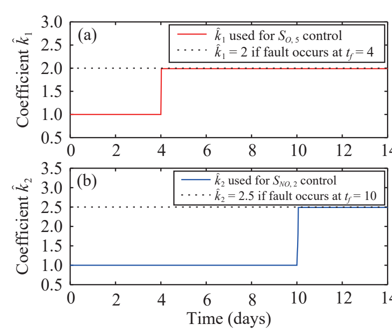

Fig. 2. Fault compensation coefficient.

Results comparison: Fig. 5 plots the curves of $S_{O,5}$ and $S_{NO,2}$, from which the proposed method introducing compensation coefficient $\hat{\mathcal{K}}$ can ensure satisfactory tracking performance and timely fault compensation. If the designed method without such coefficient (green lines), the tracking performance is severely degraded when the faults occur. Thus, the importance of introducing such coefficient is illustrated. Considering a fault-free scenario, Fig. 6 exhibits the compari-

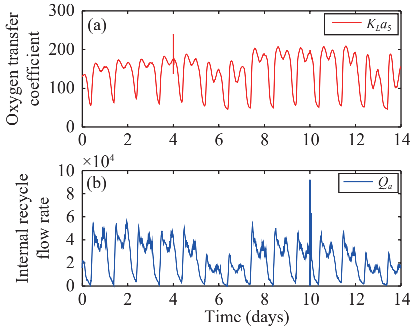

Fig. 3. Manipulated variables.

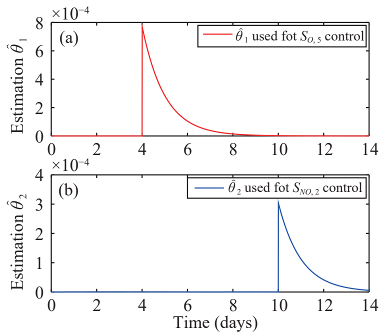

Fig. 4. Adaptive parameter estimation.

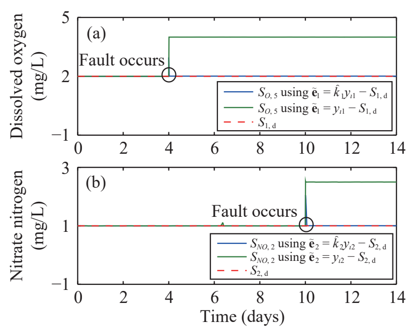

Fig. 5. Performance with/without $\hat{\mathcal{K}}$.

son results with the proposed method (blue lines), the adaptive sliding mode control [10] (ASMC, pink lines) and the default proportional integral control [12] (dPIC, green lines), the performance for $S_{O,5}$ and $S_{NO,2}$ under our strategy is superior to ASMC and dPIC in terms of overshoot, tracking and robustness.

Conclusions: In this letter, the data-driven fault compensation tracking control for coupled WWTP under sensor faults has been investigated. By using invariant set theory, the differential conditions of non-affine dynamics have been removed, and the control inputs have been explicitly expressed. An adaptive fault compensation mechanism has been designed so that the faulty system performance can be repaired in time. The complete theoretical analysis based on a cubic absolute-value Lyapunov criteria has been provided. Experimental results have been carried out on a typical platform of WWTP to validate the effectiveness of the proposed approach. Although this study can achieve performance self-recovery of multivariable tracking control in the presence of sensor faults, it does not explicitly deal with the saturation constraint relationship of the manipulated variables, and ignores the co-existence of sensor and actuator faults, which will be addressed in the future. Additionally, two limitations of applicability or implementation need to be noted: 1) The fuzzy modeling knowledge needs to be obtained through artificial comprehensive judgment of process operation states. 2) The estimation accuracy of the fault factor is adjusted by time-consum-

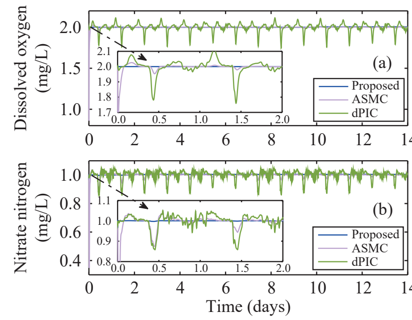

Fig. 6. Performance comparisons.

ing manual intervention to set the design parameters. In view of the above discussion, our future research will thoroughly improve the presented control strategy to realize cooperative control of more control loops in a WWTP.

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