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Leaky Wave Antennas: Theory and Design

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ABSTRACT

We review old and recent developments on leaky wave antenna theory and design. A leaky wave is treated mathematically as a complex plane wave and the radiation pattern is derived in terms of the complex propagation constant. Several leaky wave antenna structures are introduced and analyzed. An example of a one dimensional leaky wave antenna is a planar surface waveguide with a partially reflecting screen (PRS) made of periodic thin metal strips placed on the dielectric /air interface. This particular configuration has been treated theoretically and experimentally by the present authors. Radiation occurs in this antenna by the spatial harmonic mode with n= -1. The two dimensional (2-D) leaky wave structures depend on radially travelling waves and they produce pencil beams in the broadside direction and conical beams otherwise. Examples of such antennas include a stack of dielectric layers of contrasting alternate high/low permittivities. Results obtained on an antenna of this type are presented as well as some results on new types of planar structures. Future trends and applications for these type of antennas are also featured.

Keywords: *Leaky wave antennas, frequency scanning, high gain antennas.*

I. INTRODUCTION

Leaky wave antennas are one class of travelling wave antennas. They involve a guiding structure on which a travelling wave propagates, but leaks out of a radiating aperture. The illuminated aperture extends over several wavelengths; the longer the aperture, the narrower is the radiation beam. Leaky wave antennas enjoy several advantages that include simplicity of design since no feeding network is needed. It has the capability of frequency scanning of the radiating pattern, which is useful in many applications. Usually leaky wave radiation occurs from a closed waveguide with some means of continuous or periodic power leakage into the exterior region. To have radiation, the mode of propagation should be a fast mode; with $\beta < k_0$, where β is the modal phase constant. The main radiation beam is then directed along $\vartheta = \sin^{-1}(\beta / k_0)$, where ϑ is measured from broadside. It should be noted however that due to radiation loss, β will be associated with an attenuation constant α. The beamwidth becomes narrower as α is reduced. The basic properties of leaky waves was given by the pioneering work of Tamir and Oliner in the sixties [1] and summarized by Tamir in [2]. Recently, the need for high gain antennas has renewed interest in the design and fabrication of leaky wave antennas has risen worldwide [3-4].

In this paper, we review the mathematical treatment of the leaky waves as complex plane waves and derive the radiation pattern in terms of the complex propagation constant in the next section. Several leaky wave antenna structures have evolved and analyzed in the last decade. Broadly speaking, leaky wave antennas are categorized as one dimensional and two dimensional. In the former category, the traveling wave propagates in one direction and therefore the radiation pattern is conical or fan shaped. Examples include a rectangular waveguide with periodic holes or a continuous slit in the side wall. Another example is a planar surface waveguide with periodic strips of metal placed on the dielectric /air interface. This particular configuration has been treated theoretically and experimentally [5] and will be reviewed here in section III. The two dimensional (2-D) leaky wave structures depend on radially travelling waves and they produce pencil beam in the broadside direction and conical beams otherwise. Examples of such antennas include a stack of dielectric layers of contrasting alternate high/low permittivities that are discussed in sec. IV.

II. CHARACTERISTIC PROPERTIES OF LEAKY WAVES

As stated earlier a leaky wave occurs from a closed waveguide with some means of continuous or periodic power leakage into the exterior region. An example of a leaky waveguide is a rectangular waveguide with a slotted side wall as depicted in Fig. 1. The propagation constant of the leaky wave can be considered as a perturbation of that in the closed waveguide. While the longitudinal phase constant in the closed waveguide is real (say β_c) for a lossless guide, the perturbed phase constant is complex since it incorporates attenuation factor. The leaky wave propagation constant may be expressed as $\gamma_z = \alpha_z + j\beta_z$, with both α_z and β_z positive.

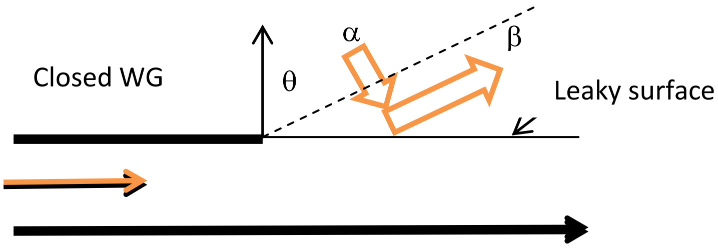

Fig. 1: A rectangular waveguide with slotted side wall acts as a leaky wave antenna

The phase constant β vector and the attenuation vector α are shown in the figure. Note that the attenuation vector is such that the wave is attenuated in the longitudinal direction 'z', but increases exponentially in the transverse direction away from the leaky wave structure. However the leaky wave fields are confined to the wedge defined by the dotted line in Fig.1. The leaky wave radiation pattern can be obtained as that of a travelling wave source with γ_z propagation constant and length L of the leaky surface. With $\alpha_z L >> 1$, the radiation pattern is given by [2]:

$$RP(\vartheta) = \frac{\cos\vartheta}{|\sin\vartheta - \sin w_L|} \tag{1}$$

,where $w_L = \sin^{-1}[(\beta_z - j\alpha_z)/k_0]$

The peak value of the radiation pattern occurs in the direction $\vartheta_p \cong \sin^{-1}[\mathrm{Re}(w_L)]$. It can be shown that the 3-dB beamwidth is given by: $BW \approx 2\,\mathrm{Im}(w_L) \approx 2\alpha_z/\beta_z$. It is noted that the less the attenuation rate, the narrower is the radiation beam. These formulae apply for unidirectional leaky wave. Note that the radiation beam can be scanned in all forward directions except the broadside direction (θ=0) since then $\beta=0$ and the wave is cutoff in the waveguide. To have broadside radiation, the leaky wave should be bi-directional in which case the two oppositely propagating leaky waves produce a forward directed and backward directed beams. When the two beams become close to the broadside, they get mingled in one beam directed in the broadside direction. In this case $\beta_L \sim \alpha_L$ and both are $<< k_0$ [9]. For a bidirectional leaky wave, the radiation pattern is given by [2,6]:

$$RP(\vartheta) = \frac{\cos\vartheta}{|\sin^2\vartheta - \sin^2 w_L|} \tag{2}$$

III. Leaky Wave Antenna Fed by A Surface Wave

An example of one dimensional leaky wave antenna is shown in Fig. 2. Here the leaky wave section is a grounded dielectric layer covered with a partially reflecting screen (PRS), which is made of periodic metallic strips as shown. The period= d and the metal strip thickness is $s<<d$. The leaky wave section is fed by a surface wave which is launched on the grounded dielectric layer on the left hand side. The substrate thickness 'h' and relative permittivity 'ε_r' are chosen so as to support only one surface wave mode; the TM_1 mode, in the frequency range of interest. Defining a normalized frequency as: $F = k_0 h\sqrt{\varepsilon_r - 1}$ with $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, the single mode operation occurs for $F < \pi$ [7]. Denoting the longitudinal phase constant of the surface wave mode by β_{sw}, the corresponding phase constants of the spatial harmonics in the leaky wave section are given by

$$\beta_n = \beta_{sw} + n\pi / d; \qquad n = \pm 1, \pm 2, ... \tag{3}$$

Radiation occurs from a spatial harmonic whenever β_n is less than k_0. Since $\beta_{sw} > k_0$, n must be negative for radiation. The spatial harmonic n= -1 is the main radiating spatial harmonic and the direction of the radiation peak is $\vartheta = \sin^{-1}(\beta_{-1} / k_0)$. As the frequency increases θ goes from backward endfire to broadside to forward endfire. To cover all this range, we need to have $d \le \lambda_0 / 2$, and $\varepsilon_r \ge 9$ [6].

Now we turn attention to the leaky waveguide section. We shall present a perturbation analysis to obtain the radiation properties of the present leaky wave antenna [5]. Since the metal strip width $s<<d$, the metal strips may be considered as small perturbation on the surface wave supporting structure. We may then assume that the surface wave fields remain unperturbed in the region $0<x<h$. Namely the tangential electric field at $x=h^-$ is equal to $E_{z0}(h)$. However, the same field at $x=h^+$ on the metal strip must vanish. To support such discontinuity, we impose periodic magnetic line sources at the positions of the metal strips with magnetic current density m_y given by:

$$\begin{aligned} m_y(z) &= -E_{z0} e^{-j\beta_0 z}; \quad |z| \le s/2 \\ &= 0 \qquad\qquad ; \quad s/2 \le z \le d/2 \end{aligned} \tag{4}$$

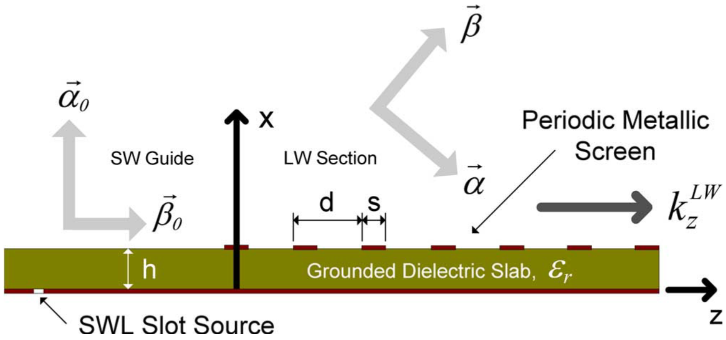

Fig.2. Leaky wave antenna fed by an incident surface wave mode [From Ref. 5]

which is periodic with period d. Note that $\beta_0=\beta_{sw}$. This array of line sources radiate in the air region in the presence of the unperturped fields inside the substrate. As a periodic function $m_y(z)$ can be expressed in a Fourier series form as:

$$m_y(z) = -E_{z0} \frac{s}{d} e^{-j\beta_0 z} + \sum_{r \ne 0} -E_{z0} \frac{s}{d} \operatorname{sinc}\left(\frac{\pi r s}{d}\right) e^{-j\beta_r z} \tag{5}$$

,where $\beta_r = \beta_0 + 2\pi r / d$. The summation in (5) runs over all positive and negative integer values of r except r =0. Now the z-electric field on the air side of the screen ($x=h^+$) is given $E_{z0}+m_y(z)$, so we have:

$$E_z(x = h^+, z) = E_{z0}(1 - \frac{s}{d}) e^{-j\beta_0 z} + \sum_{r \ne 0} -E_{z0} \frac{s}{d} \operatorname{sinc}\left(\frac{\pi r s}{d}\right) e^{-j\beta_r z} \tag{6}$$

It is useful to note at this point that as s tends to zero (the metal strips vanish), the above E_z approaches E_{z0} as expected. Using the wave equation, we get E_z in the air region (for $x>h$) as:

$$E_z(x, z) = E_{z0}(1 - \frac{s}{d}) e^{-u_0(x-h)} e^{-j\beta_0 z} + \sum_{r \ne 0} -E_{z0} \frac{s}{d} \operatorname{sinc}\left(\frac{\pi r s}{d}\right) e^{-u_r(x-h)} e^{-j\beta_r z} \tag{7}$$

, where $u_r = \sqrt{\beta_r^2 - k_0^2}$ and $x \ge h$. The corresponding magnetic field is easily derived from Maxwell's equations. Skipping details, the (complex) radiated power in the upper half space is:

$$P_{rad} = j\omega\varepsilon_0 |E_{z0}|^2 (s/d) \left[(1 - s/d)/u_0^* - (s/d) \sum_{r \ne 0} \operatorname{sinc}(\pi r s / d)/u_r^* \right] \tag{8}$$

The summation is over all positive and negative integers of r. It is useful to note that since $\beta_0 > k_0$, then u_r is real for positive values of r. For negative values of r, u_r can be imaginary whenever $-k_0 < \beta_r < k_0$. The terms that correspond to this condition account for the real part of P_{rad}, or the real power radiation. The other terms account for the reactive component of P_{rad}. The change in the propagation constant of the incident surface wave mode is now obtained as:

$$\Delta\gamma \equiv \alpha + j\Delta\beta = \frac{P_{rad}}{2P_{sw}} \tag{9}$$

, where Psw is the incident surface wave power which depends on $|E_{z0}|^2$, h and ε_r. Assuming the n = -1 spatial harmonic is the only radiating mode, the peak radiation occurs at:

$$\theta_p = \sin^{-1}(\beta_{-1}/k_0) = \sin^{-1}[(\beta_0 + \Delta\beta - 2\pi/d)/k_0] \tag{10}$$

, and the directivity is given by [2,6]:

$$\text{Directivity } D = \sqrt{k_0^2 - \beta_{-1}^2}\,/\,2\alpha \tag{11}$$

To verify the above analysis, we consider the following parameters: ε_r=10.2, d/h=5.9 and L/d=14, where L is the total length of the leaky antenna structure. We shall assume that the screen has s/d=0.2. The residual surface wave power left at $z=L$ after going through the leaky section is assumed to be absorbed with no reflection. This means that the total radiated power is related to the incident surface wave power by: $P_{T,rad} = P_{sw}(1-e^{-2\alpha L})$. The surface wave power is assumed to be launched by a slot excited by a voltage V_s. Reflector and directive slots are assumed to act as parasitic slots that direct all the launched power in the forward direction. The total launched surface wave power P_{sw} normalized to $V_s^2/120\pi$ has been derived in [7] and plotted versus the normalized frequency $F = k_0 h\sqrt{\varepsilon_r - 1}$ in Fig.3. It is seen that the launched P_{sw} has a peak around F=1.6. The radiation starts to occur due to the r = -1 spatial harmonic at F=1.35 with backward endfire beam. As the frequency increases, the radiation beam is scanned towards forward radiation as seen from the plot of θ_p in the Figure.

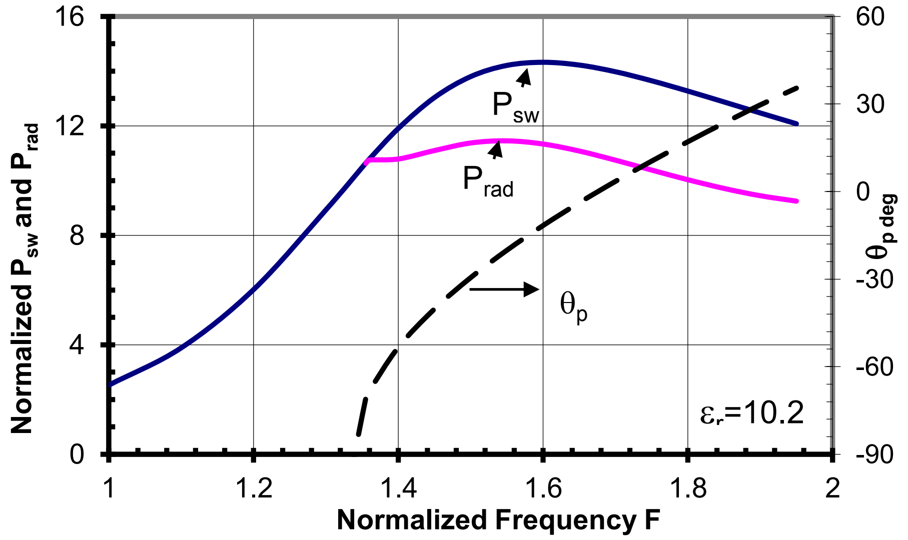

Fig.3. Surface wave power, radiation power and the radiation θ_p versus the normalized frequency.

The directivity (not shown) has a nearly flat value of 18 dB in the range $1.6 \le F \le 2$. The pointing angles θ_p is plotted versus the applied frequency in GHz for different s/d ratios in Fig.4. The substrate has thickness h=1.27

mm and ε_r =10.2 and 12.2. The simulated and calculated results are compared for ε_r =10.2 and show reasonable agreement. The measured results are made on a substrate with ε_r =12.2 and show good comparison with theory.

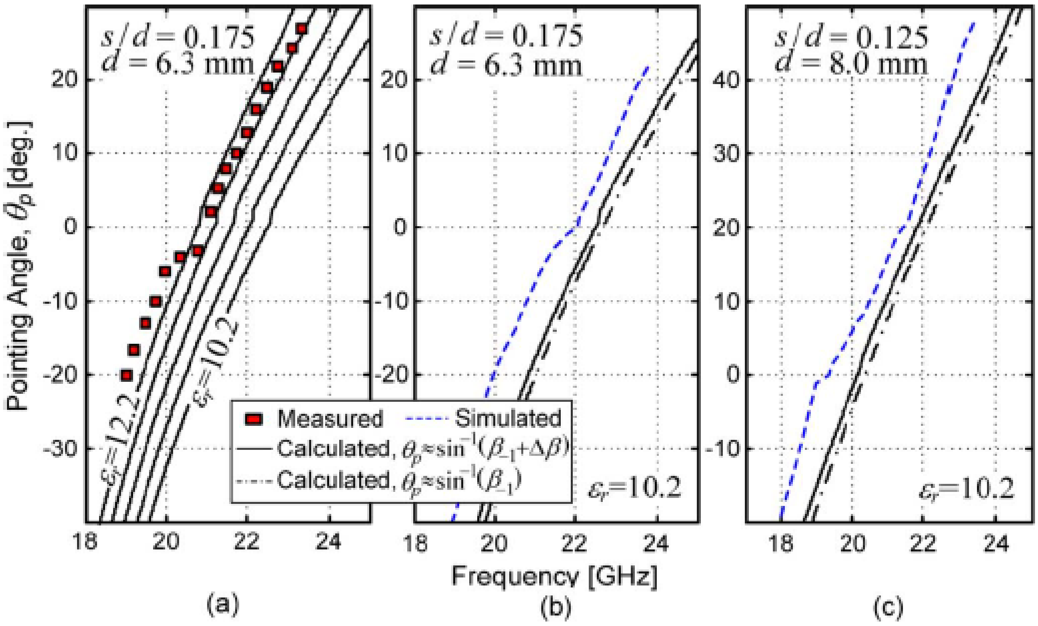

Fig.4. Comparison of the pointing angles θ_p [degrees] for different LWA with different *d, s/d*, and ε_r [from Ref.5]

By changing the metal strips to concentric circles, as shown in Fig.5, the leaky wave turns to be a two-dimensional one [8]. The radiation pattern turns from the fan shaped to conical shape and pencil beam at broadside.

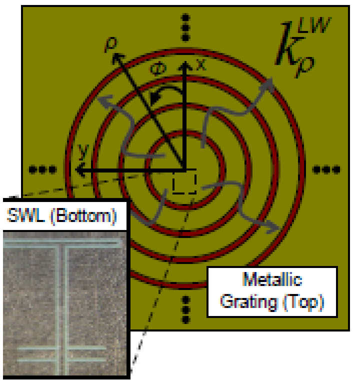

Fig.5. A two dimensional LWA fed by a surface wave. The metal strips form concentric circles [From Ref. 8].

IV. A 3-Layer Leaky Wave Antenna

Another example of a leaky wave antenna structure is a set of horizontal layers of contrasting high/low permittivities. Fig. 6 shows a substrate-air-superstrate structure fed by a horizontal dipole. The three layers are l/4 thick in their media and ε_{r3}>>1 while ε_{r1} can be much or moderately higher than unity. This structure is made vertically resonant to get a high gain antenna . The concept of high gain leaky wave antennas has been introduced by Jackson and Oliner [9], who explain the high gain by the excitation of low attenuated leaky modes. A leaky wave multilayer antenna using periodic or frequency selective screen (FSS) has also been considered and tested recently [10-11].

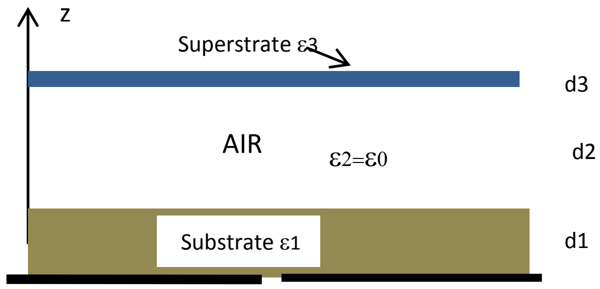

Fig. 6: A 3-dielectric layer leaky wave antenna

With reference to Fig.6, due to the high ε_{r3}, the upper surface of the substrate behaves approximately as a magnetic conductor. So the layers are sandwiched between an electric conductor at z=0 and a magnetic conductor at $z=d_1+d_2+d_3$. The structure is fed by an excited slot in the ground plane as shown. Resonance occurs vertically when the total electrical thickness is 3π, which occurs when each layer is $\lambda/4$ in its own medium. In this case the structure behaves as a leaky wave antenna with maximum gain in the broadside direction. In addition, if the radiation peak is required at an angle θ from broadside, the thickness of each layer should satisfy:

$$k_0 d_i \sqrt{\varepsilon_{ri} - \sin^2 \vartheta} = \pi \qquad (12)$$

The leaky wave mode propagation constant can be obtained using the transverse resonance method. Now we take into account the upper half space air region. The modal equation for TM or TE leaky mode is thus obtained and its solution gives the longitudinal β and α. The radiation pattern is obtained using (2). A plot of α/k_0 and β/k_0 versus the pointing angle θ is shown in Fuig.7. Note that $\alpha\sim\beta$ at broadside as expected in [9]

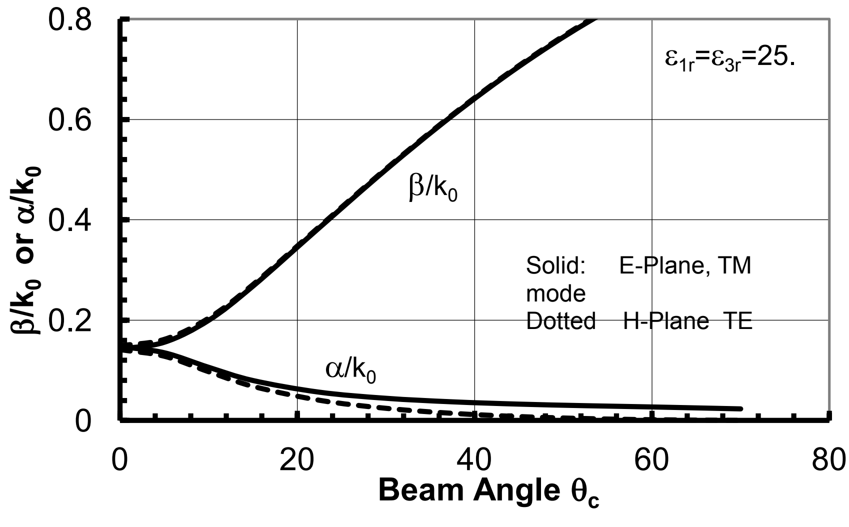

Fig. 7. Phase and attenuation constants of Leaky TM and TE modes.

It is interesting to plot the directivity and radiation efficiency [12] versus ε_{r3} for two values of ε_{r1} in Fig. 8. It is seen that both parameters increase with higher superstrate ε_{r3} and lower substrate ε_{r1}. The radiation efficiency is defined as the percentage power radiated to the sum of this and the surface wavepower.

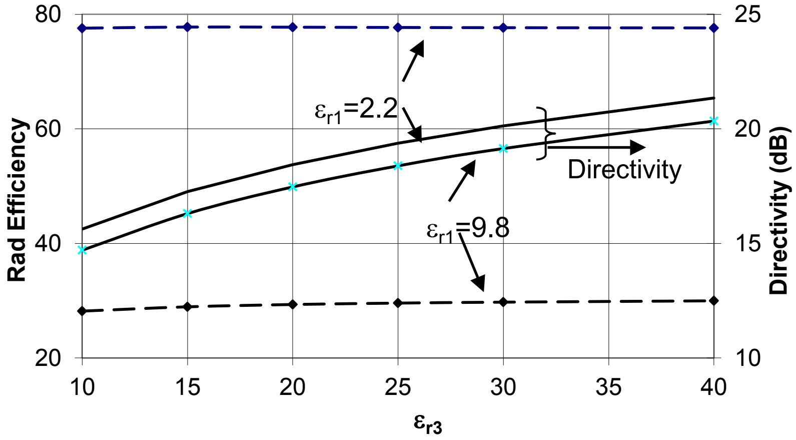

Fig. 8. Radiation Efficiency and directivity versus superstrate ε_{r3} for two values of substrate ε_{r1}.[From 12]

V. Conclusions

A review of the theory and design of leaky wave antennas (LWA) has been given. Examples of both one dimensional and two dimensional LWA's have been studied. Recent trends include the use of metamaterials in the design. Artificial magnetic conductors used as ground plane will reduce the size of the layered microstrip LWA. In addition the use of the class of materials having high permittivity and permeability; named as magneto-dielectrics will reduce the size of superstrates [13]. These new trends will be discussed in the presentation.

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