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Consensus-based Distributed Intentional Controlled Islanding of Power Grids

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***Abstract*—The problem of partitioning a power grid into a set of islands can be a solution to restore power dispatchment in sections of a grid affected by an extreme failure. Current solutions to this problem usually involve finding the partition of the grid into islands that minimizes the sum of their absolute power imbalances. This combinatorial problem is often solved through heuristic centralized methods. In this paper, we propose instead a distributed online algorithm through which nodes can migrate among islands, self-organizing the network into a suitable partition. We prove that, under a set of appropriate assumptions, the proposed solution yields a partition whose absolute power imbalance falls within a given bound of the optimal solution. We validate our analytical results by testing our partitioning strategy on the IEEE 118 and 300 benchmark problems.**

I. Introduction

The penetration of renewable and distributed generation, e.g., [1]–[3], and the possible occurrence of cascading failures [4] have made the problem of devising control strategies to govern the operation of power grids of crucial concern. Examples of problems of interest include those reported in [5]–[7], [8], [9] and [10]–[12].

When the control architecture fails to guarantee reliable operation of the transmission grid, last resort strategies have been devised so as to ensure power dispatchment across at least some of its sections. Strategies for *Intentional Controlled Islanding* (ICI) address this issue [13]–[17] by identifying sections of the grid (or *islands*) that can isolate and operate independently from the rest of the network. Recently, intentional islanding has also been proposed in the framework of distribution networks—see [18] and references therein—as the presence of storage devices and renewable energy generation allow these grids to be partitioned into *networks of microgrids*, e.g., [19]–[21]. Finally, sectioning a grid into islands can be instrumental for Parallel Power System Restoration where, to accelerate recovery after a blackout, the islands are restored in parallel and then reconnected [22].

The problem of partitioning a grid into a set of islands (that do not exchange power among each other) is usually posed as a combinatorial problem—see for example [19], [23]–[25]—and sometimes is recast as a graph optimization problem [15]–[17]. Solving this problem numerically can be computationally expensive, so that heuristic strategies are frequently used to seek a suboptimal solution, while meeting the required computational time that allows the network to stabilize after a contingency [13], [26]–[28]. The centralized techniques used to solve the islanding problem include spectral clustering over simple [22] or multi-layer graphs [29], ant colony optimization algorithms [30], or particle swarm optimization [31]. Recently, different scientists and institutions have encouraged a shift towards more distributed operation of the power infrastructure, to facilitate the inclusion of distributed energy resources (DERs). Examples include self-organizing capacity for open, distributed, and clean energy communities [32], changes in the regulatory structure and the role of network operators [33], [34], and the distributed control of microgrids [35]–[37]. To the best of our knowledge, no distributed Intentional Controlled Islanding techniques have been proposed in the current literature.

In this paper, we bridge this gap and propose a distributed approach to solve the ICI problem, where network nodes can migrate from an island to another so as to *self-organise* into a partition minimizing the power imbalance between different islands and avoiding large amounts of load shedding. Specifically, starting from some initial partition of the grid, we endow the nodes with the ability of locally estimating the power imbalance of their island and of those neighboring it, and decide whether to migrate or not to a different island from their own. The estimation strategy is completely distributed and decentralized and relies on nodes running a virtual consensus dynamics parameterized so that the consensus equilibrium the nodes reach can be used to estimate the power imbalance of the island of interest. Under suitable assumptions, we analytically show that our migration strategy generates a sequence of partitions that converge in finite time to a configuration whose average absolute power imbalance falls within a certain bound of the minimal one. We validate our strategy by partitioning the IEEE 118 and IEEE 300 test systems, comparing the viable partitions we obtain to others suggested in previous papers in the Literature.

A concise list of our contributions is as follows:

i. we present a novel decentralized consensus-based estimation rule for the nodes of a grid to accurately estimate the power imbalance of the island to which they belong. It is worth noting that our strategy can be applied to estimate any property of a subgraph that is obtained as the sum of the contributions of the nodes belonging to that subgraph. This makes our approach versatile and applicable to a

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range of scenarios beyond power imbalance estimation;

ii. we provide a migration rule allowing the nodes of a grid to self-organize into a partition whose average power imbalance falls within a precomputable distance from the minimal one.

The remainder of the manuscript is organized as follows: in Section II we introduce some notation and give our problem statement, in Section III we give our decentralized ICI strategy whose convergence is then proved in Section IV, while its effectiveness is validated numerically in Section V. We then apply our strategy in a realisic case study presenting the results in Section VI, and discuss the perspectives and limitations of our work in Section VII. Finally, conclusions are drawn in Section VIII.

II. Preliminaries and problem statement

Notation: Given a set $\mathcal{Q}$, we denote by $|\mathcal{Q}|$ its cardinality; $\mathbf{1}$ is the column vector of ones, with appropriate dimension.

Power grid: We model a power grid as an *undirected connected* graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of $n \in \mathbb{N}_{>0}$ grid nodes (loads or generators) and $\mathcal{E}$ is the set of edges representing transmission lines interconnecting them. Without loss of generality, the $n_{\mathrm{g}} \in \mathbb{N}_{>0}$ generators are labeled as nodes $1, \dots, n_{\mathrm{g}}$, while the $n_{\mathrm{l}} \in \mathbb{N}_{>0}$ loads as nodes $n_{\mathrm{g}} + 1, \dots, n$. We let $p_i \in \mathbb{R}$ be the active power generated or consumed at node i; $p_i > 0$ if i is a generator, while $p_i \leq 0$ if i is a load. We let A be the (symmetric) adjacency matrix associated to the graph $\mathcal{G}$; its (i, j)-th element a_{ij} being 1 if $\{i, j\} \in \mathcal{E}$ or 0 otherwise.

Islands and neighbors: We define an *island* as a connected subgraph $\mathcal{M}_l = (\mathcal{V}_l, \mathcal{E}_l)$ of $\mathcal{G}$, where $\mathcal{V}_l \subseteq \mathcal{V}$ and $\mathcal{E}_l = (\mathcal{V}_l \times \mathcal{V}_l) \cap \mathcal{E}$. Given a set of nodes $\mathcal{V}_l$, we denote by $\mathcal{N}(\mathcal{V}_l)$ the set of *neighbors* of the nodes in $\mathcal{V}_l$, i.e., $\mathcal{N}(\mathcal{V}_l) := \{i \in \mathcal{V} \setminus \mathcal{V}_l \mid \exists j \in \mathcal{V}_l : \{i, j\} \in \mathcal{E}\}$. We say that island $\mathcal{M}_m$ is a *neighbor* of island $\mathcal{M}_l$ if and only if $\mathcal{N}(\mathcal{V}_m) \cap \mathcal{V}_l \neq \emptyset$. Moreover, we denote by $\mathcal{N}_i$ the set of neighbors of node i.

Grid partitions: We say that the grid is *partitioned* into $n_\mu \in \mathbb{N}_{>0}$ islands, described by the subgraphs $\mathcal{M}_l, \dots, \mathcal{M}_{n_\mu}$, with corresponding node sets $\mathcal{V}_1, \dots, \mathcal{V}_{n_\mu}$, if $\Pi = \{\mathcal{V}_1, \dots, \mathcal{V}_{n_\mu}\}$ is a *partition* of $\mathcal{V}$. Additionally, a node, say i, in an island, say $\mathcal{M}_l$, is a *boundary node* if $\mathcal{N}_i \cap (\mathcal{V} \setminus \mathcal{V}_l) \neq \emptyset$. Furthermore, we define the condensed graph $\mathcal{G}^\Pi = (\mathcal{V}^\Pi, \mathcal{E}^\Pi)$ induced by the partition Π, where node l in $\mathcal{V}^\Pi$ is associated to $\mathcal{V}_l$ in Π, and an edge $\{l, m\}$ exists in $\mathcal{E}^\Pi$ if and only if $\mathcal{V}_l \cap \mathcal{N}(\mathcal{V}_m) \neq \emptyset$.

Power imbalance: The *power imbalance* of an island $\mathcal{M}_l$ is defined as

$$P_l := \sum_{i \in \mathcal{V}_l} p_i, \tag{1}$$

while the overall *grid's power imbalance* is given by

$$P_{\mathrm{tot}} := \sum_{i=1}^{n} p_i = \sum_{l=1}^{n_\mu} P_l. \tag{2}$$

The power imbalance in (1) is associated to the deviation of the synchronous frequency of the island from its nominal value, which in turn is related to its stability [9], [38]. Indeed, if the generated power exceeds loads' demand, the frequency increases, while it decreases otherwise. Excessively large variations in the operating frequency with respect to the nominal one can cause faults.

Control problem: The problem we study is to find a partition of the power grid $\mathcal{G}$ into $n_\mu \geq 2$ islands (that do not exchange power among each other) so as to minimize the *average absolute power imbalance*, defined as

$$J := \frac{1}{n_\mu} \sum_{l=1}^{n_\mu} |P_l|. \tag{3}$$

Note that, as $\sum_{l=1}^{n_\mu} |P_l| \geq \left|\sum_{l=1}^{n_\mu} P_l\right| = |P_{\mathrm{tot}}|$, then

$$J \geq J^* := \left|\frac{P_{\mathrm{tot}}}{n_\mu}\right|. \tag{4}$$

The cost function in (3) has been used in previous work in the literature on grid partitioning, e.g. [13], [19], [26], as an indicator of the ability of a power system to satisfy the loads' demand, which is also known as *adequacy* [39].

III. A consensus based partitioning strategy

We propose a strategy that, given an initial partition $\Pi(0)$ of the power grid into n_μ islands, uses a consensus algorithm to let the nodes self-organise into a new partition that minimizes J, as defined in (3). In particular, at each step k of the algorithm, one node can migrate between islands. We denote by $\Pi(k)$ the partition after k migrations have occurred; $\mathcal{M}_l(k) = (\mathcal{V}_l(k), \mathcal{E}_l(k))$, $l \in \{1, \dots, n_\mu\}$ being the corresponding islands, $P_l(k)$, $l \in \{1, \dots, n_\mu\}$ their power imbalances, and $J(k)$ the corresponding value of the cost function.

Our strategy is based on two fundamental ingredients:

1) a *distributed dynamic estimator* based on average consensus dynamics that nodes can use to estimate the power imbalance in their island and in those of their neighbors;
2) a *migration condition* according to which a boundary node can decide whether to migrate from its island to a neighboring one.

Next, we describe the two elements above.

A. Distributed power imbalance estimation

At any step k, each node, say i, can obtain an estimate of the power imbalance, say $P_l(k)$, of the island it belongs to or of an island neighboring it, say $\mathcal{M}_l(k) = (\mathcal{V}_l(k), \mathcal{E}_l(k))$, by running a consensus based estimation strategy.

Specifically, let us define the auxiliary graph $\widehat{\mathcal{M}}_l(k) := (\widehat{\mathcal{V}}_l(k), \widehat{\mathcal{E}}_l(k))$ with

$$\widehat{\mathcal{V}}_l(k) := \begin{cases} \mathcal{V}_l(k) \setminus i, & \text{if } i \in \mathcal{V}_l(k), \\ \mathcal{V}_l(k) \cup i, & \text{if } i \notin \mathcal{V}_l(k), \end{cases} \tag{5}$$

and $\widehat{\mathcal{E}}_l(k) := (\widehat{\mathcal{V}}_l(k) \times \widehat{\mathcal{V}}_l(k)) \cap \mathcal{E}$. To estimate $P_l(k)$, node i must trigger the distributed solution of the two *virtual* continuous-time consensus dynamics given by

$$\dot{x}_h(t) = p_h + \sum_{\{j,h\} \in \mathcal{E}_l(k)} (x_j(t) - x_h(t)), \quad \forall h \in \mathcal{V}_l(k), \tag{6a}$$

$$\dot{\widehat{x}}_h(t) = p_h + \sum_{\{j,h\}\in\widehat{\mathcal{E}}_l(k)} (\widehat{x}_j(t) - \widehat{x}_h(t)), \quad \forall h \in \widehat{\mathcal{V}}_l(k), \quad (6b)$$

starting from null initial conditions. Here, $x_h(t)$ and $\widehat{x}_h(t)$ are the virtual states associated to each node $h \in \mathcal{V}_l(k)$ and $h \in \widehat{\mathcal{V}}_l(k)$, respectively.

Remark 1. *To run the consensus dynamics* (6) *in a distributed manner, we assume the virtual states x_h and $\widehat{x}_h$ are broadcast to all neighboring nodes $\mathcal{N}_h \cap \mathcal{V}_l(k)$.*

Now, dynamics (6a) can be recast in matrix form as

$$\dot{\boldsymbol{x}}(t) = \mathbf{p} - L\boldsymbol{x}(t), \quad (7)$$

where $\boldsymbol{x}$ is the stack vector of the virtual states x_h, $\mathbf{p}$ is the stack vector of the power values p_h, and L is the (symmetric) Laplacian matrix associated to $\mathcal{M}_l(k)$. Let us recall that $\mathbf{1}^\mathsf{T}$ is an eigenvector of the symmetric Laplacian L, with 0 as an associated eigenvalue. To obtain the asymptotic behaviour of (7), we premultiply (7) by $\mathbf{1}^\mathsf{T}$, obtaining, for all time t,

$$\mathbf{1}^\mathsf{T}\dot{\boldsymbol{x}}(t) = \mathbf{1}^\mathsf{T}\mathbf{p} = P_l(k). \quad (8)$$

Moreover, differentiating (7), we obtain the dynamical system $\ddot{\boldsymbol{x}}(t) = -L\dot{\boldsymbol{x}}(t)$, whose dynamics, determined by the spectral properties of L, are such that

$$\lim_{t\to\infty} \dot{\boldsymbol{x}}(t) \in \operatorname{span}(\mathbf{1}). \quad (9)$$

Altogether, (8) and (9) imply that $\lim_{t\to\infty}\dot{\boldsymbol{x}}(t) = \mathbf{1}\omega_l$, where

$$\omega_l := \frac{P_l(k)}{|\mathcal{V}_l(k)|}, \quad (10)$$

Similarly, from (6b), we obtain that $\lim_{t\to\infty}\dot{\widehat{\boldsymbol{x}}}(t) = \mathbf{1}\widehat{\omega}_l$, with

$$\widehat{\omega}_l := \frac{1}{|\widehat{\mathcal{V}}_l(k)|} \sum_{j\in\widehat{\mathcal{V}}_l(k)} p_j. \quad (11)$$

Exploiting (5), (11) can be recast as

$$\widehat{\omega}_l = \begin{cases} \frac{1}{|\mathcal{V}_l(k)|-1}\left(P_l(k) - p_i\right), & \text{if } i \in \mathcal{V}_l(k), \\ \frac{1}{|\mathcal{V}_l(k)|+1}\left(P_l(k) + p_i\right), & \text{if } i \notin \mathcal{V}_l(k). \end{cases} \quad (12)$$

Then, (10) and (12) can be solved together for the unknowns $P_l(k)$ and $|\mathcal{V}_l(k)|$, obtaining

$$P_l(k) = a_l\omega_l \frac{p_i - \widehat{\omega}_l}{\widehat{\omega}_l - \omega_l}, \quad (13a)$$

$$|\mathcal{V}_l(k)| = a_l \frac{p_i - \widehat{\omega}_l}{\widehat{\omega}_l - \omega_l}, \quad (13b)$$

with $a_l = -1$ if $i \in \mathcal{V}_l(k)$ and $a_l = 1$ if if $i \notin \mathcal{V}_l(k)$.

From (13a), to estimate $P_l(k)$, node i needs to compute ω_l and $\widehat{\omega}_l$. To do so in a distributed manner, node i starts the distributed computation of the consensus dynamics (6a) and (6b) by broadcasting its virtual states x_i and $\widehat{x}_i$ to the nodes in $\mathcal{N}_i \cap \mathcal{V}_l(k)$. In turn, each of these starts sharing its virtual state with its neighbors (within $\mathcal{V}_l(k)$), until all nodes in $\mathcal{V}_l(k)$ join the distributed simulation. Note that the aforementioned procedure can be conducted through one-hop communication if each node h has knowledge of the index $l \in \{1, \dots, n_\mu\}$ of the island it belongs to, and of its consumed or generated power p_h. Obviously, in a practical implementation, the grid nodes should be equipped with sufficient computational and communication capabilities to run the virtual consensus dynamics on a timescale that is compatible with the grid requirements.

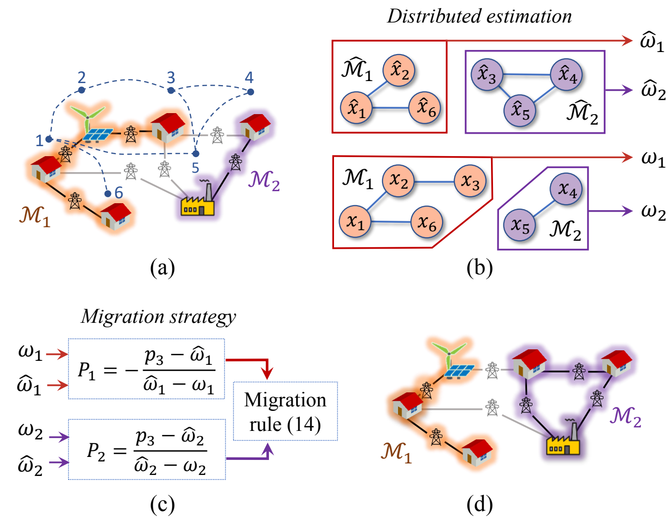


Fig. 1: (a) Initial partition of the power network, with dashed lines representing the communication links among nodes; the topology being equal to that of the power network itself. (b) Boundary node 3 triggers the distributed simulation of the virtual consensus dynamics in (6) for both islands $\mathcal{M}_1$ and $\mathcal{M}_2$. (c) The migration rule (14) is used to decide whether to migrate from island $\mathcal{M}_1$ to $\mathcal{M}_2$ and (d) a new partition is eventually generated.

In what follows, we will show how the network nodes can exploit this estimation process to self-organise into a partition of the power network whose power imbalance (3) is rendered minimal.

B. Migration condition

A boundary node (see § II), say i, in island $\mathcal{M}_m(k)$, can decide whether to migrate to a neighboring island $\mathcal{M}_l(k)$ (see Figure 1) by assessing the power imbalances $P_l(k)$ and $P_m(k)$ (computed through our estimation strategy in § III-A).

Specifically, at step k, node i will migrate from $\mathcal{M}_m(k)$ to $\mathcal{M}_l(k)$ if and only if

$$\begin{cases} \min(P_l(k), P_m(k)) < \min(P_l(k+1), P_m(k+1)), & (14a) \\ \mathcal{M}_m(k+1) \text{ is connected}, & (14b) \end{cases}$$

with

$$P_l(k+1) = P_l(k) + p_i, \quad (15a)$$

$$P_m(k+1) = P_m(k) - p_i, \quad (15b)$$

$$\mathcal{V}_l(k+1) = \mathcal{V}_l(k) \cup i, \quad (15c)$$

$$\mathcal{V}_m(k+1) = \mathcal{V}_m(k) \setminus \{i\}. \quad (15d)$$

Remark 2. *Condition* (14b) *concerning connectivity can be ensured using the estimation strategy in § III-A. Indeed, given an island $\mathcal{M}_l(k)$, if there exists a node $i \in \mathcal{V}_l(k)$ such that*

$\mathcal{M}_l(k) \setminus \{i\}$ is not connected, the virtual derivatives $\dot{\hat{x}}_h$ of its neighbors in $\widehat{\mathcal{M}}_l$ (see (6b)) will in general converge to different values, thus providing a warning signal.

C. Migration algorithm

According to our decentralized partitioning strategy, starting from some initial partition at step $k = 0$, each boundary node must trigger the distributed estimation of the power imbalance of the island it belongs to and of its neighboring islands by running the virtual consensus dynamics (6). Then, depending on these power imbalances, exploiting the migration condition (14), the boundary nodes will decide whether to migrate or not to neighboring islands.

For the sake of clarity, we illustrate the process by referring to the exemplary situation depicted in Figure 1, where a grid with $n = 6$ nodes is initially partitioned in $n_\mu = 2$ islands, $\mathcal{M}_1(0)$ and $\mathcal{M}_2(0)$ (Figure 1(a)). Then, each boundary node, as for instance node $3 \in \mathcal{V}_1(0)$, must decide whether to migrate to the other island ($\mathcal{M}_2$) or not. To this aim, node 3 triggers the distributed estimation of the power imbalances $P_1(0)$ and $P_2(0)$ in both the islands $\mathcal{M}_1(0)$ and $\mathcal{M}_2(0)$ (see Figure 1(b)), by running two virtual consensus processes of the form (6) involving all nodes belonging to each of the islands. Once a steady state in the distributed simulation of (6) has been reached, node 3 uses the pairs $(\omega_1, \hat{\omega}_1)$ and $(\omega_2, \hat{\omega}_2)$ to estimate $P_1(0)$ and $P_2(0)$, which it then uses to evaluate the migration condition (14) (Figure 1(c)) to assess whether to migrate from $\mathcal{M}_1$ to $\mathcal{M}_2$. Once this decision is taken, a new partition is generated (Figure 1(d)).

In general, our strategy prescribes that the grid nodes get involved in all the (possibly multiple) distributed consensus processes invoked according to (6) by the boundary nodes of their island or of neighboring ones so as to allow the estimation of the power imbalances of interest. Hence, at any time, each node will have a number of virtual states corresponding to the number of estimation processes it is asked to contribute to. These steps are summarized in Algorithm 1. Additionally, as soon as a node becomes a boundary node (see § II), it must trigger additional virtual dynamics to decide whether to migrate or not from its island to a neighboring one. This additional procedure is summarized in Algorithm 2.

IV. Proof of convergence

The following Lemma and Theorem state that the migration process governed by rule (14) generates a finite sequence $\{\Pi(k)\}_{k\in\{0,\dots,K\}}$ of $K \in \mathbb{N}$ migration steps, and give a bound on the difference between the cost $J(K)$ of the final partition and the optimal cost J^* computed in (4). To give their proof, we must first define the stack vector $\mathbf{P}(k) := [P_1(k) \ \cdots \ P_{n_\mu}(k)]^\top$ and $\mathbf{P}^* := p^*\mathbf{1}$.

Lemma 1. *If*

$$|P_l(k) - P_m(k)| \le \bar{p} \quad \forall l, m : \mathcal{N}(\mathcal{V}_m(k)) \cap \mathcal{V}_l(k) \neq \varnothing, \tag{16}$$

where $\bar{p} := \max_{i\in\mathcal{V}} |p_i|$, then

$$J(k) - J^* \le \frac{2}{n_\mu}\left(\sum_{l=l^*+1}^{n_\mu} p^* + \bar{p}\left(l - \frac{n_\mu+1}{2}\right)\right) - (p^* + |p^*|), \tag{17}$$

with

$$l^* = \left\lceil -\frac{p^*}{\bar{p}} + \frac{n_\mu+1}{2}\right\rceil, \tag{18}$$

and $p^* := P_{\mathrm{tot}}/n_\mu$.

Proof. From (3), we have that

$$J(k) = \frac{1}{n_\mu}\left(\sum_{l:P_l(k)>0} P_l(k) - \sum_{l:P_l(k)\le 0} P_l(k)\right). \tag{19}$$

Moreover, as

$$\sum_{l:P_l(k)>0} P_l(k) + \sum_{l:P_l(k)\le 0} P_l(k) = P_{\mathrm{tot}} = n_\mu p^*,$$

we can recast (19) as

$$J(k) = \frac{1}{n_\mu}\left(2\sum_{l:P_l(k)>0} P_l(k) - n_\mu p^*\right).$$

Hence, as $J^* = |p^*|$ [from (4)], we obtain

$$J(k) - J^* = \frac{2}{n_\mu}\sum_{l:P_l(k)>0} P_l(k) - (p^* + |p^*|). \tag{20}$$

Without loss of generality, let us relabel the islands so that $P_1(k) \le P_2(k) \le \cdots \le P_{n_\mu}(k)$. Then, as the graph $\mathcal{G}$ (defined in § II) and all the islands remain connected for all k, at each step also the graph $\mathcal{G}^{\Pi(k)}$ (defined in § II) will be connected and thus (16) implies that

$$P_{l+1}(k) \le P_l(k) + \max_{i\in\mathcal{V}} |p_i|, \quad \forall l \in \{1,\dots,n_\mu - 1\}. \tag{21}$$

Note that, from (2), $\sum_{l=1}^{n_\mu} P_l(k) = P_{\mathrm{tot}} = n_\mu p^*$, and hence from (21) we obtain

$$P_l(k) \le p^* + \bar{p}\left(l - \frac{n_\mu+1}{2}\right), \quad \forall l \in \{1,\dots,n_\mu\}, \tag{22}$$

with $\bar{p} := \max_{i\in\mathcal{V}} |p_i|$. From (20), $J(k) - J^*$ is maximized (worst case) when (22) is an equality. In such a case, to compute $J(k) - J^*$ by leveraging (20), we must first find

$$l^*: \quad P_l(k) \ge 0, \ \forall l \in \{l^*,\dots,n_\mu\}. \tag{23}$$

Hence, to find l^* we must find the smallest integer l such that

$$p^* + \bar{p}\left(l - \frac{n_\mu+1}{2}\right) \ge 0, \tag{24}$$

yielding (18). Then, from (23), (22), and (20), we obtain (17) and the Lemma is proved. □

Theorem 1. *Assume that at each step k there exist a node i and islands $\mathcal{M}_l(k)$ and $\mathcal{M}_m(k)$ (that is a triplet (l, m, i)) such that*

$$\begin{cases} i \in \{\mathcal{V}_m(k) \cap \mathcal{N}(\mathcal{V}_l(k))\} \\ \qquad\wedge \\ \mathcal{M}_m(k) \setminus i \text{ is connected} \end{cases} \tag{25a}$$

and

$$\begin{cases} P_l(k) > P_m(k) \ \wedge \ p_i < 0 \\ \qquad\vee \\ P_l(k) < P_m(k) \ \wedge \ p_i > 0. \end{cases} \tag{25b}$$

Then, the sequence $\Pi(k)$ *obtained under the migration rule* (14) *is finite and converges in* $K < +\infty$ *steps to a partition* $\Pi(K)$ *such that* $J(k)$ *fulfills* (17) *at* $k = K$.

Proof. Consider a triplet (l, m, i) fulfilling (25), and

$$|P_m(k) - P_l(k)| > |p_i|; \tag{26}$$

we start by showing that, when assuming (25), (26) is equivalent to (14), i.e., a migration of node i from island $\mathcal{M}_m$ to $\mathcal{M}_l$ will occur.

Firstly, we show that (14) implies (26). When $P_m(k) < P_l(k)$, we have $p_i < 0$ from (25b), and from (14) we have that

$$P_m(k) < P_l(k+1). \tag{27}$$

Differently, when $P_l(k) < P_m(k)$, we have $p_i > 0$ from (25b), and from (14) we have that

$$P_l(k) < P_m(k+1). \tag{28}$$

From (27) and (28), recalling (15a) and (15b), we have

$$\begin{cases} P_m(k) - P_l(k) < p_i, & \text{if } p_i < 0, \\ P_m(k) - P_l(k) > p_i, & \text{if } p_i > 0. \end{cases} \tag{29}$$

As (29) implies (26), we have proved that (14) implies (26).

Now, let us prove that (26) implies (14). To do so, note that (26) is equivalent to

$$\begin{cases} P_l(k) > P_m(k) + |p_i|, & \text{if } P_l(k) > P_m(k), \\ P_m(k) > P_l(k) + |p_i|, & \text{if } P_l(k) < P_m(k). \end{cases} \tag{30}$$

Moreover, exploiting (25b) and recalling (15a) and (15b), (30) can be recast as

$$\begin{cases} P_m(k) < P_l(k) + p_i = P_l(k+1), & \text{if } P_l(k) > P_m(k), \\ P_l(k) > P_m(k) - p_i = P_m(k+1), & \text{if } P_l(k) < P_m(k). \end{cases} \tag{31}$$

It is straightforward to see that (31) immediately leads to (14). Therefore, we have proved that (when (25) holds) (26) $\Leftrightarrow$ (14).As (14) is equivalent to (26) and (25), then if at some step, say K, no triplet (l, m, i) existed fulfilling (26), the migration process would stop and, as the network $\mathcal{G}$ is connected and so is the graph $\mathcal{G}^{\Pi(K)}$ at that step, we would have

$$|P_l(K) - P_m(K)| \leq \max_{i \in \mathcal{V}} |p_i| \quad \forall l, m : \mathcal{V}_m(K) \cap \mathcal{N}(\mathcal{V}_l(K)) \neq \varnothing. \tag{32}$$

As from Lemma 1, (32) implies that the bound (17) holds, to prove our thesis we are left with showing that a stopping time instant K exists. Firstly, note that such a step K exists if (14) fulfills

$$\|\mathbf{P}(k+1) - \mathbf{P}^*\|_2 \leq \alpha \, \|\mathbf{P}(k) - \mathbf{P}^*\|_2 \;\; \forall k \in \{0, \dots, K-1\} \tag{33}$$

for some positive scalar $\alpha < 1$ as if (33) were satisfied, then our migration rule would be a contraction mapping. In such case, from the Banach-Caccioppoli theorem [40], there would be no limit cycles in the sequence $\{\mathbf{P}(k)\}$ and thus also in $\{\Pi(k)\}$. Hence, as the number of possible partitions is finite, so would be the sequence $\{\mathbf{P}(k)\}$ and thus, to complete our proof, we need to show that (14) implies (33). As we have enforced that only one migration occurs at each step k, then $\mathbf{P}(k+1)$ only differs from $\mathbf{P}(k)$ for the l-th and m-th entries. Hence, proving (33) only requires showing that

$$\begin{aligned} (P_l(k+1) - p^*)^2 &+ (P_m(k+1) - p^*)^2 \\ &< (P_l(k) - p^*)^2 + (P_m(k) - p^*)^2 \end{aligned} \tag{34}$$

for all $k \in \{0, \dots, K-1\}$. After a few algebraic simplifications, (34) can be rewritten as

$$p_i(P_l(k) - P_m(k) + p_i) < 0 \quad k \in \{0, \dots, K-1\}, \tag{35}$$

which is trivially fulfilled by any triplet (l, m, i) fulfilling (25) and (26), yielding that (25) and (26) imply (33). In turn, as (25) and (26) imply (14), the existence of K and thus our thesis remains proved. □

Algorithm 1: Default routine for any node h.

```
Broadcast all virtual states to neighboring nodes
Obtain virtual states from neighboring nodes
Integrate (6) for all simulations where h is involved
```

Algorithm 2: Additional steps for a *boundary node* $h \in \mathcal{V}_m$.

```
Communicate with the nodes in N_h ∩ V_m to trigger a
  distributed simulation of (6)
for l : N_h ∩ V_l ≠ ∅ do
    Communicate with the nodes in N_h ∩ V_l to trigger a
      distributed simulation of (6)
    Wait for steady state in such simulations
    Estimate P_m and P_l, ∀l : N_h ∩ V_l ≠ ∅ using (13)
    Decide whether to migrate from M_m to M_l via (14)
```

Remark 3. *A sufficient (but not necessary) condition to fulfill the assumption of Theorem 1 is that neighboring islands have at least a load on the boundary between them.*

In the following section, we validate the strategy numerically.

V. Numerical validation

We demonstrate the effectiveness of our algorithm by deploying it to partition the IEEE 118 and 300 testbed cases [41]. The nodal power values p_i are computed by solving an Optimal Power Flow (OPF) problem on the whole non-partitioned grid, leveraging Matpower 6.0 [42]. As the test cases include nodes with null nodal power $p_i = 0$, we allow for these nodes to migrate from their island, say $\mathcal{M}_m(k)$, to a neighboring island, say $\mathcal{M}_l(k)$, as long as (i) their migration does not render $\mathcal{M}_m(k)$ disconnected and (ii) $P_l(k) \neq P_l(k'), \forall k' < k : i \in \mathcal{V}_l(k')$.

To apply our partitioning strategy (Algorithms 1 and 2), we need some initial partitions $\Pi(0)$. To test our algorithm under different conditions, we considered different choices $\Pi(0)$. In some cases, we took as $\Pi(0)$ some selected partitions from [13], [27], [43]. In other cases, to generate $\Pi(0)$, we first employ the Search Space Reduction Procedure (SSRP) [27], which generates a spanning tree connecting groups of coherent generators (these are taken from [27]). Then, the remaining nodes are aggregated to the tree using the Breadth-First Search (BFS) algorithm [44].

Remark 4. *Throughout our numerical analysis, whenever a node, say $i \in \mathcal{V}_m(k)$, can choose to migrate to more than one island, it will select the one maximizing the difference*

$$\Delta P_l = \min\{P_l(k) + P_i, P_m(k) - P_i\} - \min\{P_l(k), P_m(k)\}.$$

This choice ensures a maximal improvement of the average absolute power imbalance after the migration.

A. IEEE 118 bus system

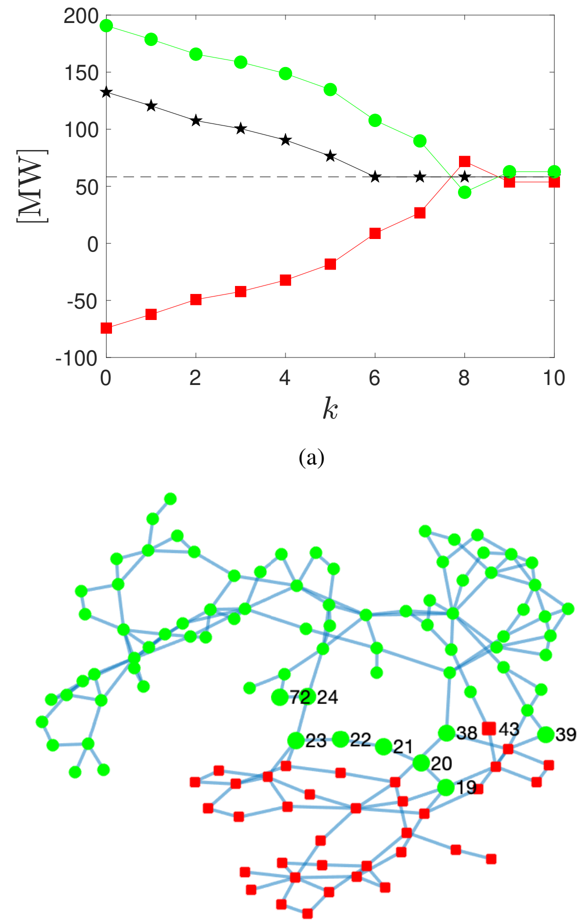
We used our algorithm to partition the IEEE 118 test system in $n_\mu = 2$ and $n_\mu = 3$ islands, considering only $n_g = 19$ generators (excluding the reactive compensators). We assume that the migration process is triggered by a three phase solid ground fault at bus 15 forcing line 14-15 to disconnect. With $n_\mu = 2$, we ran the algorithm first by considering as initial partition $\Pi(0)$ the one generated by SSRP+BFS and then using as $\Pi(0)$ the final partition reported in [13] to evaluate how it performed when started from different initial partitions. For the case $n_\mu = 3$, we considered as $\Pi(0)$ the partition generated via SSRP+BFS and then the final one obtained in [27]. All relevant information and the results of our distributed partitioning strategy are reported in Table I.

We observe that the proposed algorithm is indeed capable of converging in all cases towards partitions that minimize J, as $J(K) = J^*$. As a representative example, we depict in Figure 2 the case that $n_\mu = 2$ and $\Pi(0)$ is generated by SSPR+BFS; namely, Figure 2a portrays the power imbalances $P_1(k)$ and $P_2(k)$ at the various steps, while the final partition $\Pi(K)$ is reported in Figure 2b. Note that from the OPF results we have $\max_i |p_i| = 542.78$ MW and $J^* = 58.25$ MW and thus the bound given in Theorem 1 is satisfied as $|J(K) - J^*| = 0$ (see Table I).

B. IEEE 300 bus system

We used Algorithms 1 and 2 to partition the IEEE 300 test system in $n_\mu = 3$ and in $n_\mu = 4$ islands, assuming a failure affects line 194-195. With $n_\mu = 3$, as $\Pi(0)$ we considered the partition obtained via SSRP+BFS. We also repeat the partitioning starting from the arbitrary initial partition reported in Table II. When $n_\mu = 4$, as initial partition $\Pi(0)$ we consider one obtained via SSRP+BFS and one from [27]. In both cases, the groups of coherent generators were selected as in Table II of [27]. All relevant information and the results are reported in Table II.

Again, in all cases, our algorithm is capable of finding an optimal partition, as $J(K) = J^*$. Additionally, we note that the initial partitions obtained via SSRP+BFS and that from [27] are already optimal with respect to minimizing J; however, by performing a few more migration steps, our algorithm is able to further decrease the standard deviation between the power imbalances of the three islands (compare $P_l(0)$ and $P_l(K)$ in Table II). This happens routinely, as the migration law (14) aims at equalizing the power imbalances in all the islands (with the result of minimizing J).



(a)

(b)

Fig. 2: Partitioning of the IEEE 118 test system into $n_\mu = 2$ islands, through Algorithms 1 and 2. (a) $P_1(k)$ (red squares), $P_2(k)$ (green circles), $J(k)$ (black stars), and J^* (dashed line); all are in MW. (b) Final network partition $\Pi(K)$; red square denote $\mathcal{V}_1(K)$, while green circles denote $\mathcal{V}_2(K)$. Nodes $72, 24, 23, 22, 21, 39, 20, 19, 38$ migrated from $\mathcal{M}_1$ to $\mathcal{M}_2$ in the given order, while node 43 migrated from $\mathcal{M}_2$ to $\mathcal{M}_1$ at $k = 9$. Note that the last migration does not change the power imbalances as it involves node 38 whose nodal power is zero.

In Figure 3, we report the representative case where $n_\mu = 3$ and $\Pi(0)$ is the arbitrary partition. The power imbalances $P_1(k)$, $P_2(k)$, $P_3(k)$ are depicted in Figure 3a, while the final partition $\Pi(K)$ is portrayed in Figure 3b. Interestingly, across all our numerical experiments, not only does our algorithm ensure fulfillment of the bound given in Theorem 1, but it also always ensures $J(K) = J^*$, and in all cases it succeeds in reducing the standard deviation among the power imbalances of the islands with respect to that of the initial partition (see Table II).

VI. Case study

As shown in our numerical analysis, our decentralized strategy is not only capable of minimizing (3), but also of

Case	n_μ	K	Cut-set at $\Pi(0)$	$\Pi(0)$	Cut-set at $\Pi(K)$	$J(0)$	$J(K)$	J^*	$P_l(0)$	$P_l(K)$	Bound (17)
IEEE 118	2	10	{24-70, 34-43, 37-40, 38-65, 39-40, 71-72}	SSRP+BFS	{15-19, 18-19, 19-34, 23-25, 23-32, 30-38, 37-38, 37-39, 37-40, 43-44}	120.5	58.25	58.25	{−74.26, 190.75}	{53.74, 62.75}	213.14
IEEE 118	2	9	{1-2, 3-12, 5-8, 6-7, 11-12, 15-17, 15-19, 24-70, 30-38, 34-36, 44-45, 70-71}	[13]	{4-5, 5-11, 11-12, 15-17, 15-19, 30-38, 34-37, 35-37, 43-44, 69-70, 70-75, 74-75}	265.5	58.25	58.25	{−258.25, 374.74}	{65.75, 50.74}	213.14
IEEE 118	3	7	{24-70, 34-43, 37-40, 38-65, 39-40, 68-81, 69-77, 71-72, 75-77, 76-118}	SSRP+BFS	{19-34, 21-22, 23-25, 23-32, 30-38, 34-36, 34-37, 37-38, 37-39, 37-40, 68-81, 69-77 75-77, 76-118}	80.34	38.83	38.83	{−74.26, 1.98, 188.77}	{53.74, 1.98, 60.77}	335.97
IEEE 118	3	8	{24-70, 24-72, 38-65, 40-42, 41-42, 44-45, 69-77, 75-77, 81-80, 118-76}	[27]	{24-70, 42-49, 44-45, 61-64, 63-64, 65-66, 65-68, 69-77, 71-72, 75-77, 76-118, 80-81}	147	38.83	38.83	{−199.26 313.77 1.98}	{83.66, 30.86, 1.98}	335.97

TABLE I: Results after applying Algorithms 1 and 2 to the IEEE 118 test case, considering different initial partitions $\Pi(0)$. Power values are reported in MW. Note that bound (17) is computed for $k = K$.

diminishing the standard deviation between power imbalances; thus making the islands equally robust to unforeseen power fluctuations. Unsurprisingly, this comes at the price of cutting several lines, as minimization of the power imbalance and of the *power flow disruption* are known to be conflicting goals [13]. While both could benefit from some load shedding, ultimately, a partition with high power imbalance and/or power flow disruption leads to the existence of islands where the the power flow is unfeasible and thus loads will not be served. Our migration strategy does not take explicitly into account the number of lines cut. We present here a case study to show that, nevertheless, it can still lead to nodes self-organising into a partition where power is not only available but can be dispatched, i.e., the power flow is feasible for all the islands.

To this aim, we consider the problem of refining the partition of the IEEE 118 test case into $n_\mu = 3$ islands proposed in [27], to react to a failure in bus 26[1]. Solving the power flow for the test case in the absence of a fault shows that this generator is responsible for 7.2% of the total active power generated and, consistently, post-fault, an island of the partition will be endowed with negative power imbalance. To overcome this problem, we apply our strategy using the partition proposed in [27] as initial partition $\Pi(0)$ for our distributed algorithm. We take the groups of coherent generators as in [27], this time assuming these nodes cannot change island throughout the migration process. Furthermore, following [22], we assume that post-fault load can be shed by 15%[2] in non-critical loads belonging to islands affected by the refinement process, and select the set of critical loads as in [22]. Finally, as our aim is that of refining $\Pi(0)$ so as to make sure all islands are such that $P_l(K) \geq 0$, we enforce the additional rule that node i can migrate from $\mathcal{M}_m(k)$ to $\mathcal{M}_l(k)$ only if $\text{sign}(P_l(k)) = -\text{sign}(P_m(k))$ which will cause migrations to cease if all islands have positive power imbalance. To evaluate the quality of the partition $\Pi(K)$ resulting from applying our strategy, we will compare the properties of the power flow solutions of the islands in $\Pi(K)$ to those in $\Pi(0)$ in terms of

(i) minimal and maximal voltage magnitude $V_{\min}$ and $V_{\max}$;
(ii) minimal and maximal voltage angles $\delta_{\min}$ and $\delta_{\max}$;
(iii) active and reactive power losses λ_P and λ_Q.

We find that in this scenario $K = 17$ migrations are required to make sure all islands have positive power imbalance, resulting in the nodes self-organizing into a partition defined by the cutset {15-19, 17-113, 18-19, 23-25, 27-32, 31-32, 34-37, 35-37, 37-39, 37-40, 38-65, 69-77, 75-77, 76-118, 80-81, 114-115}. Let us start describing the outcome of our case-study underlining that $\mathcal{V}_3(K) = \mathcal{V}_3(0)$, that is, no migrations occurred that changed island $\mathcal{M}_3$. For this reason, we shed no load in this island post-fault, resulting in no changes in its power flow. Conversely, all 17 migrations involve nodes in islands $\mathcal{M}_1$ and $\mathcal{M}_2$. Consistently, as shown in Table III, for these two islands we observe changes in all the variables we chose to describe the power flow solution. However, the only variable whose value degrades substantially is the worst minimal voltage magnitude $V_{\min}$ that, for island $\mathcal{M}_2(k)$ takes the value of 0.87, quite far from the desired value of 1 and lower than the initial value of 0.946, but still consistent with other islanding results available in the literature (see for

[1]For the purpose of this case study, it is irrelevant whether islanding is performed to avoid a blackout or for Parallel Power Grid Restoration purposes.

[2]This value is half that considered in [22] when solving the islanding problem for Parallel Power System Restoration.

Case	n_μ	K	Cut-set at $\Pi(0)$	$\Pi(0)$	Cut-set at $\Pi(K)$	$J(0)$	$J(K)$	J^*	$P_I(0)$	$P_I(K)$	Bound (17)
IEEE 300	3	3	{3-129, 7-110, 40-68, 54-123, 57-66, 66-190, 67-190, 68-73, 185-186}	SSRP+BFS	{3-129, 40-68, 54-123, 57-66, 64-67, 66-190, 68-73, 109-110, 184-185, 185-187}	102.92	102.92	102.92	{6.11, 129.98, 172.65}	{6.11, 145.98, 156.65}	1254.95
IEEE 300	3	12	{40-68, 57-66, 66-190, 67-190, 68-73, 106-113, 112-116, 122-123, 185-186}	Arbitrary	{36-40, 39-40, 61-66, 64-67, 65-66, 68-73, 105-106, 106-107, 106-147, 112-116, 119-121, 121-154, 122-124, 122-128, 127-157, 154-158, 157-158, 168-189, 172-187, 177-188, 184-185}	529.49	102.92	102.92	{−639.87, 775.96, 172.65}	{129.21, 18.89, 160.65}	1254.95
IEEE 300	4	5	{3-129, 7-110, 40-68, 54-123, 61-66, 64-67 65-66, 68-73, 68-173, 174-198, 185-186}	SSRP+BFS	{3-129, 7-110, 40-68, 54-123, 57-180, 57-190, 66-190, 67-190, 68-73, 68-173, 168-187, 172-187, 174-198, 184-185}	77.187	77.187	77.187	{19.76, 6.11, 205.98, 76.9}	{114.76, 6.11, 110.98, 76.9}	1908.2
IEEE 300	4	3	{57-66, 64-67, 66-190, 68-173, 109-110, 109-129, 122-123, 174-191, 174-198, 184-185, 185-187}	[27]	{7-110, 57-66, 66-190, 67-190, 68-173, 109-129 122-123, 168-187, 172-187, 174-191, 174-198, 184-185}	77.187	77.187	77.187	{145.98, 79.76, 6.11 76.9}	{110.98, 114.76, 6.11, 76.9}	1908.2

TABLE II: Results after applying Algorithms 1 and 2 to the IEEE 300 test cases, considering different initial partitions $\Pi(0)$. Power values are reported in MW. Note that bound (17) is computed for $k = K$.

instance Figure 4 in [45]). Conversely, the largest voltage angle (in magnitude) is comparable before and after the migration process, and while we observe a general increase in the active and reactive losses, the highest reactive power loss in the final partition is just 20% larger than that pre-fault.

Overall, this case study shows that our strategy can also be deployed for post-fault refinement of a reasonable partition. Next, we discuss the steps required towards a real-world implementation of such a decentralized islanding strategy.

VII. Towards a practical implementation

In this section, we discuss some issues that are important to achieve a practical implementation of the strategy.

A. Power flow feasibility

In this work, we chose to seek a partition of a grid that minimizes the islands' power imbalance. As is known in the literature [13], this can come at the price of obtaining a large power flow disruption which can cause the power flow to be unfeasible for some of the islands. However, this choice allowed us to prove rigorously that nodes of a power system can self-organize into islands that fulfill some electrical property of interest.

The test case described in Section VI shows that our results can also be exploited in a realistic scenario. Nevertheless, for practical implementation, power flow disruption should also be explicitly taken into account to systematically ensure the available power to be dispatchable. We are currently adapting the tools developed in this work to optimize the trade-off between power imbalance and the power flow disruption. Our preliminary numerical investigations are encouraging, and suggest that a multi-objective distributed strategy based on the same estimation tools presented in this work can consistently lead to islands where power is both available and can be dispatched.

B. Implementation issues

A real world implementation of the tools presented in this paper relies on three main assumptions: the availability of (i) computational power at each node, (ii) a communication infrastructure between the nodes having the same topology as the physical grid, and (iii) measurements of the generated/consumed power at each node. Which one of these assumptions proves more restrictive depends on the reason for which islanding is needed. In case islanding is needed to avoid a cascading outage, the largest drawback with respect to a centralized approach is that substantial effort must be devoted in designing efficient distributed simulation protocols. Indeed, to avoid a blackout, the migration process must be executed on a timescale compatible with that of the development of post-fault instability. In our numerical experiments, we found that the numerical simulations required to perform a migration step lasted around 8 ms to run (on a personal computer equipped with an Intel Core i5 processor with six cores at 3 GHz, and 16GB of RAM memory) yielding, for the test case in section VI, a total time of 0.12 s for the nodes to obtain the final partition. Note that this is in line with the computational time

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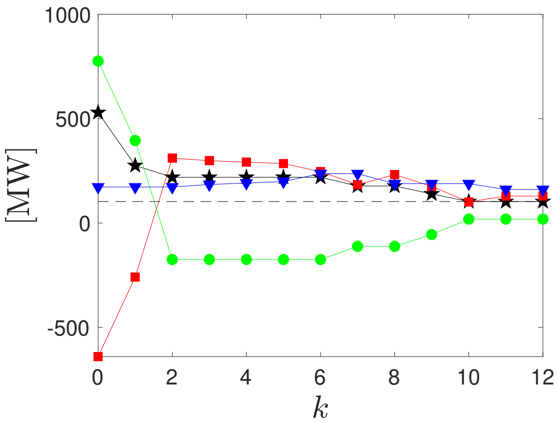
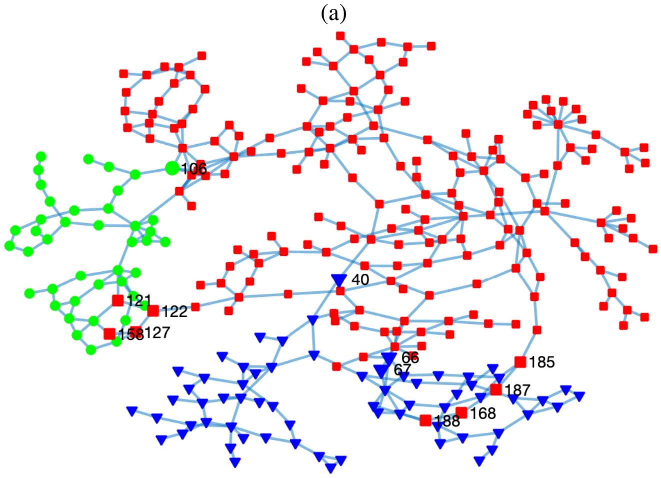
Island	P_I [MW]	$V_{\min}$ [p.u.]	$V_{\max}$ [p.u.]	$\delta_{\min}$ [deg]	$\delta_{\max}$ [deg]	λ_P [MW]	λ_Q [MVar]
$\mathcal{M}_1(0)$	−410	0.936	1.050	−20.66	21.25	6.27	59.22
$\mathcal{M}_2(0)$	474	0.946	1.050	17.47	32.61	6.32	19.03
$\mathcal{M}_3(0)$	216	0.943	1.040	17.13	41.77	3.96	20.18
$\mathcal{M}_1(K)$	14	0.955	1.050	8.51	33.68	4.94	59.22
$\mathcal{M}_2(K)$	58	0.874	1.050	−44.47	30.27	16.74	70.87
$\mathcal{M}_3(K)$	216	0.943	1.040	17.13	41.77	3.96	20.18

TABLE III: Comparison between the properties of the power flow solution pre-fault for the islands defined by the initial partition $\Pi(0)$ defined in [27] and post-fault after the initial partition is refined through our distributed strategy.



(a)



(b)

Fig. 3: Partitioning of the IEEE 300 test system into $n_\mu = 3$ islands, through Algorithms 1 and 2. (a) $P_1(k)$ (red squares), $P_2(k)$ (green circles), $P_3(k)$ (blue triangles), $J(k)$ (black stars), and J^* (dashed line). (b) Final network partition $\Pi(K)$; red squares denote $\mathcal{V}_1(K)$, green circles denote $\mathcal{V}_2(K)$, and blue triangles denote $\mathcal{V}_3(K)$. The nodes' migration order is 106, 122, 185, 187, 168, 188, 127, 66, 121, 158, 67 and 40.

required to solve the islanding problem on the same test case (the IEEE 118 bus system) in [46].

If the strategy is applied for Parallel Power System Restoration, i.e., post black-out, ensuring reliable communication among the nodes might be the most pressing issue.

Finally, in energetic communities, privacy issues may arise and one might want to make sure nodes cannot infer the power generated or absorbed by their peers from the communicated signals. Furthermore, in this latter case, choosing the number of energetic communities n_μ could also be exploited for optimization purposes, whereas in post-fault scenarios it is usually fixed and determined by the groups of coherent generators and, for the case of Parallel Power System Restoration, by the availability of blackstart units. Addressing these issues is the subject of ongoing work and will be reported elsewhere.

C. Limitations

Although the framework we have developed offers several advantages, such as effective power management and fault tolerance, it does have a significant limitation. Specifically, it requires an external initial partition to be provided, which could be a potential bottleneck in the decentralization process. While it is realistic to assume the availability of such a partition, fully decentralizing the islanding process would necessitate the development of a distributed strategy enabling generators to self-organize into coherent groups and recursively add loads to form the initial islands. Addressing this limitation is an important direction for future research, as it could lead to greater efficiency and scalability of the overall system.

VIII. Conclusions

We introduced a power network islanding algorithm that solves the Intentional Controlled Islanding problem in a distributed manner. Our strategy allows the network nodes to self-organise so as to minimize the average absolute power imbalance among islands. To allow the nodes to make informed decisions, we devised a consensus-based estimator which is instrumental to the migration process, as it allows nodes to estimate the power imbalances of neighboring islands in a distributed manner. We demonstrated analytically that our algorithm converges in finite time to a partition whose average absolute power imbalance is in a given neighborhood of the optimal one. We tested the strategy on two benchmark power networks, the IEEE 118 and 300 bus systems under different fault conditions showing the effectiveness of the proposed approach.

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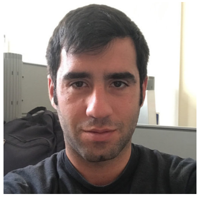

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